# **APMA 3100**

# **Probability**

**Fall 2024** 

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Office: Small Building Room 107

Office hours: Mon & Wed 2-3 pm

Tues 9:30-11:30 am

**TEXT:** *Probability and Stochastic Processes*, 3<sup>rd</sup> edition by R. D. Yates and D. J. Goodman. Access the free ebook through this <u>link</u>.

**PRE-REQUISITE:** APMA 2120 [Multivariable Calculus] or equivalent. A working knowledge of double integrals, infinite series, and functions of one and several variables is required.

**COURSE DESCRIPTION:** A calculus-based introduction to probability theory and its applications in engineering and applied science. Topics covered include counting techniques, conditional probability, independence, discrete and continuous random variables, expected value and variance, joint distributions, covariance, correlation, central limit theorem, and an introduction to statistical inference.

**OBJECTIVES:** We hope to provide you with

- 1. A working knowledge of concepts and tools of probability theory.
- 2. Experience with a variety of typical applications of probability.
- 3. A foundation for further studies in probability, statistics, and stochastic processes.
- 4. An enhancement of overall mathematical skills.

### HOW WILL I HAVE A GOOD EXPERIENCE?

**I want you to be successful in this class.** This is my #1 priority as a professor. Everything in this class is built to lead you toward having a deep learning experience that is also enjoyable. My goal is to both challenge and support you so you'll learn and have a great experience in the process.

- 1. **Active engagement during class time**. The best way to learn anything is to be an active participant in the process. Students who approach the class with a passive mindset typically struggle. Those who approach it with an active mindset, on the other hand, often surprise themselves with how much and how well they learn. Make it a priority not to just attend and take notes but get involved.
- 2. **Be ready to work hard**. Many concepts and problems will be new to you. They will stretch your thinking. You will experience frustration and failure before you experience understanding. This is part of the normal learning process. You will have support on all sides from your classmates, TAs and me.
- 3. **Good management of time, tasks, and information**. Understanding the material won't help you if you procrastinate, skip announcements, or don't use a calendar. All course information will be clearly laid out for you, but it's up to you to import that information into your own lives and act on it.

If you can commit to these three things, then **I have every expectation that you'll succeed in the course**, no matter what your math background or perceived math skill is.

### WHAT WILL CLASS MEETINGS BE LIKE?

Your work in the class will follow a pattern that will involve you before, during, and after our meetings:

**BEFORE each class**: You will read parts of the text or lecture notess that will get you up to speed on the basic ideas of new material.

**DURING each class**: Class time is reserved for doing math, together and individually. We will work through some class notes together, then you will work in groups on a worksheet covering the topic at hand.

**AFTER each class**: You will practice and apply the ideas we discuss during class through homework assignments, as well as getting started on reading for next day's material.

#### WHAT ASSIGNMENTS WILL THERE BE?

**Engagement Points:** Your participation in our learning community is essential for your success and the success of others. To earn Engagement Points, you can complete various activities and assignments that demonstrate your commitment to engaging with the class.

- **Group work:** Group work activities will be given in class throughout the semester. Our group work will contribute to a supportive learning environment. Working in groups allows you to ask and answer questions of your peers which helps to deepen your understanding of the concepts and applications. **All the group work will be due by the end of the next class meeting day.** For example, Friday's group work will be due on Monday at 11:59 pm. Late submissions will be accepted until noon the following day (a *12-hour grace period*). However, please note that no submissions will be accepted after the grace period.
- **Homework:** You will have weekly online assignments, administered via WeBWorK (usually due Monday night). All WeBWorK assignments are open-book and open-note, but you should work through them without aid whenever possible. You'll learn the most by carefully working through the solution process from start to finish without any type of assistance. In addition, several *textbook exercises* will be assigned every week. (See Appendix D) It is strongly recommended that you use them as a supplement to the WeBWorK assignments.

**Projects:** There will be two projects assigned during the semester that will build on and extend the knowledge you will gain in class. Course projects will be completed in groups and will involve a written summary report. You are encouraged to discuss your project with your instructor or TAs. There will be an opportunity to revise each project to improve your grade.

### **HOW DO I EARN A FINAL GRADE?**

Learning happens over time, as we revisit ideas and reflect on them. Your final grade will reflect how well you *eventually understand* each topic. You can make mistakes without penalty, as long as you *eventually* demonstrate mastery of the topic.

**Learning Targets**: There are 25 Learning Targets in the course, listed in Appendix A to this syllabus. These are the main tasks that you should be able to do if you are successful in APMA3100. Your main goal in the course is to provide evidence of skill on as many targets as possible.

Checkpoints (every other Friday): Learning Targets are assessed through checkpoints, which are

offered once every two weeks. Every checkpoint lists several targets, one problem per Learning Target. Each learning target will appear on *two consecutive* checkpoints.

You will earn a mark for each target. Marks are:

M	Mastered	Demonstrates understanding of the relevant target. The work is correct, complete, and clear. May include some trivial errors, but no additional study or review is needed.
P	Progressing	Demonstrates partial understanding, but with a fundamental error, misunderstanding, or is incomplete. Needs review and revision.

**Mastering Targets**: There is no penalty for earning a P. Once you've successfully earned an M **once** on a learning target, the target is *completed* and no further assessment on that target is necessary.

**Reassessment:** In addition to biweekly in-class checkpoints, you can request reassessment on a specific learning target during a reassessment session. You will receive and attempt a new instance of a problem for that learning target. During each session, you may reattempt any particular learning target no more than once. Overall, in-class checkpoints and reassessments will provide you with *three opportunities* to demonstrate your skill on these targets. The schedule for Checkpoints and Reassessment sessions are provided in Appendices B and C. *Note: The third attempt for Learning targets 18-25 will be included as part of the final exam.* 

**Final Exam**: There is also a *cumulative points-based* final exam (scheduled on <u>Wednesday</u>, <u>Dec 11</u>, <u>7-10 pm</u>) that will provide you with an opportunity to demonstrate everything you have learned! Please note that the final exam is a mandatory part of all APMA classes. It can't be taken early; please make your travel plans accordingly.

Your base grade, BEFORE the final: Your base grade is determined by the following table. Each grade has a requirement specified in its row in the table. To earn a grade, you will need to meet all the requirements in the row for that grade. For example, to earn a B+, you will need to master 22 learning targets and earn at least 85% of engagement points and 85% of project points.

Letter Grade	Learning Targets	Engagement Pts	Project Pts with at			
	Completed (out	with at least (out	least (out of total)			
	of 25)	of total)				
A	24	90%	90%			
A-	23	90%	90%			
B+	22	85%	85%			
В	21	85%	85%			
B-	20	85%	85%			
C+	19	75%	75%			
С	18	75%	75%			
C-	17	75%	75%			
D	15	55%	55%			
F	Have n	ot <b>fully</b> completed a	ny row			

**Your course grade, AFTER the final:** The final exam only contributes to your course grade by potentially adding a modifier to your base grade. For example, if you receive a B before the final and score 88% on the final, then you will move up one level and your final course grade will be a B+.

	Scores on	Scores on	Scores on	Scores	Scores
	the final	the final	the final	below this	below this
	needed to	needed to	needed to	threshold	threshold
	move up	move up	keep your	will result in	will result in
	two levels	one level	base grade	moving	moving
				down one	down two
				level	levels
Before final: A		≥ 95%	≥ 80%	80%	60%
Before final: A-	≥ 95%	≥ 90%	≥ 75%	75%	55%
Before final: B+, B, B-	≥ 95%	≥ 85%	≥ 70%	70%	50%
Before final: C+, C, C-	≥ 85%	≥ 75%	≥ 60%	60%	45%
Before final: D	≥ 80%	≥ 70%	≥ 45%	45%	
Before final: F	≥ 75%	≥ 65%			

#### WHAT ELSE SHOULD I KNOW?

**CLASSROOM ETIQUETTE:** Please (1) arrive on time and (2) turn off your cell phone. Laptops and tablets can only be used for the purpose of reading or taking notes. You are expected to attend every class and fully participate in a professional manner. You should prepare for class by reading ahead in the textbook or the lecture notes on our Canvas site. To get the most out of our precious class time, your peers and I will rely on you to come prepared and ready for discussion.

# **CALCULATORS:** You must use a **scientific calculator** similar to:

Casio: fx-260 or fx-260Solar Staples scientific calculator

Texas Instruments: TI-30Xa or TI-30X IIS (solar)

Sharp: EL-501XBGR (has factorial key but no permutations or combination keys) or EL-531XBWH (has permutation and combination keys, **better** choice)

You may not use a graphing calculator or any more advanced calculator. For example, your calculator cannot possess any calculus capabilities (such as computing integrals). You may use an online scientific calculator such as <a href="https://www.desmos.com/scientific">https://www.desmos.com/scientific</a> for your homework.

**LEARNING NEEDS**: The University of Virginia strives to provide accessibility to all students. If you require an accommodation to fully access this course, please contact Student Disability Access Center (SDAC) at (434)243-5180 or <a href="sdac@virginia.edu">sdac@virginia.edu</a>. If you are unsure if you require an accommodation, or to learn more about their services, you may contact the SDAC at the number above or by visiting their website at <a href="http://studenthealth.virginia.edu/student-disability-access-center/faculty-staff">http://studenthealth.virginia.edu/student-disability-access-center/faculty-staff</a>.

**HONOR CODE:** Tests and the final exam must be performed **independently** and must be pledged. Aids must be **limited** to those permitted in the instructions. The Honor Code will be strictly observed: any instance of cheating will be reported to the Honor Committee; it will also incur an automatic failure in the course with a grade of F.

# **Appendix A: Learning Targets**

### Basics of Probability (Chapters 1-2)

- 1. I can compute and interpret probabilities of events, including compound events involving operations. (1.1-1.3)
- 2. I can compute and interpret probabilities of events involving conditional probabilities and determine whether two events are independent. (1.4, 1.6)
- 3. I can apply the Law of Total Probability and Bayes Theorem in the computation and interpretation of probabilities. (1.5)
- 4. I can apply basic counting techniques to determine number of outcomes in a sample space or event. (2.2)
- 5. I can determine the reliability of systems. (2.4)

## Discrete RVs (Chapter 3)

- 6. I can construct PMFs and CDFs of discrete RVs and functions of one discrete RV. (3.1, 3.2, 3.4, 3.6)
- 7. I can use PMFs and CDFs of discrete RVs to compute probabilities, expected values, variances, and standard deviations. (3.1, 3.2, 3.4, 3.5, 3.7, 3.8)
- 8. I can identify and compute probabilities for Binomial, Geometric and Pascal RVs. (3.3)

## Continuous RVs (Chapter 4)

- 9. I can use PDFs and CDFs of continuous RVs to compute probabilities, expected values, variances, and standard deviations. (4.1-4.4)
- 10. I can distinguish between PDFs and CDFs and use one to compute the other. (4.1-4.3)
- 11. I can explain the connection between Poisson and Exponential RVs and compute probabilities for each. (3.3, 4.5)
- 12. I can compute probabilities for Gaussian (Normal) RVs. (4.6)
- 13. I can find the PDF and expected value of a function of one continuous RV. (6.2)

## Joint Distributions (Chapter 5)

- 14. I can construct and use joint PMFs and marginal PMFs to compute probabilities and expected values. (5.1-5.3, 5.7)
- 15. I can construct and use joint PDFs and marginal PDFs to compute probabilities and expected value. (5.4-5.5)
- 16. I can compute covariance and correlation of two random variables and use the relationship between covariance and independence. (5.6, 5.8)
- 17. I can construct and interpret the PDF or CDF of a function of two continuous RVs. (6.4, 6.5)

#### Conditioned RVs (Chapter 7)

- 18. I can compute conditional PMFs or PDFs when conditioning a RV by an event and use it to compute probabilities and expected values. (7.1, 7.2)
- 19. I can compute conditional PMFs or PDFs when conditioning a RV by another RV and use it to compute probabilities and expected values. (7.4, 7.5)

#### Sums, Means, and CLT (Chapter 9 & 10)

- 20. I can apply the CLT for sums and averages of RVs and know when it is appropriate to do so. (9.1, 9.5)
- 21. I can apply Markov's and Chebyshev's Inequalities and the Weak Law of Large Numbers and correctly interpret and use my results. (10.1-10.3)

# Hypothesis Testing (Chapter 11)

- 22. I can design a significance test for a claimed probability model. (11.1)
- 23. I can design a binary hypothesis test to decide between two probability models. (11.2)

#### Estimation (Chapter 12)

- 24. I can estimate RVs using minimum mean square error estimates. (12.1)
- 25. I can estimate RVs using linear estimates. (12.2)

**Appendix B: Course Calendar** 

APN	APMA 3100 Course Calendar (Subject to Change)								
Week	M	W	F						
26-Aug		1.1-1.3	1.1-1.3 Day2						
2-Sep	1.4, 1.5	1.6	2.1, 2.2						
9-Sep	2.2	2.4	Checkpoint 1						
3 304	2.2	2. 1	CHECKPOINT I						
16-Sep	3.1, 3.2, 3.4	3.5, 3.8	Discussion						
	Project 1 Due 9/16								
23-Sep	3.3	3.3	Checkpoint 2						
30-Sep	3.6, 3.7	4.1-4.4	4.5						
7.0-+	4.6	Chadraint 2	6.2						
7-Oct	4.6	Checkpoint 3	6.2						
14-Oct	NO CLASS	5.1-5.3	5.4-5.5						
21-Oct	5.6, 6.1	6.4	Checkpoint 4						
28-Oct	6.5	5.7, 5.8	7.1, 7.2						
4-Nov	7.4, 7.5	9.1, 9.5	Checkpoint 5						
	Project 2 Due 11/4	•	'						
11-Nov	9.5	10.1-10.3	11.1						
18-Nov	11.2	11.2	Checkpoint 6						
25-Nov	12.1	NO CLASS	NO CLASS						
2-Dec	12.2	Checkpoint 7	Checkpoint 8						
Final Exam: Wednesday, Dec 11, 7-10pm									

# **Appendix C: Checkpoint Schedule**

	Checkpoint	1	2	3	4	5	6	7	8	8 Reassessment Session		essions	Total #
Target	Sections	13-Sep	27-Sep	9-Oct	25-Oct	8-Nov	22-Nov	4-Dec	6-Dec	30-Oct	2-Dec	Final	Opportunities
LT1	1.1-1.3												3
LT2	1.4-1.6												3
LT3	1.5												3
LT4	2.1-2.2												3
LT5	2.4												3
LT6	3.1-3.2, 3.4, 3.6												3
LT7	3.1, 3.2, 3.4, 3.5, 3.7, 3.8												3
LT8	3.3												3
LT9	4.1-4.4												3
LT10	4.1-4.3												3
LT11	3.3, 4.5												3
LT12	4.6												3
LT13	6.2												3
LT14	5.1-5.3, 5.7												3
LT15	5.4-5.5												3
LT16	5.6, 5.8												3
LT17	6.4, 6.5												3
LT18	7.1-7.2												3
LT19	7.4-7.5												3
LT20	9.1, 9.5												3
LT21	10.1-10.3												3
LT22	11.1												3
LT23	11.2												3
LT24	12.1												3
LT25	12.2									_			3
Total #	LTs Covered	4	6	6	7	7	6	6	6	10	7	8	

# **Appendix D: Practice Problems for LTs**

LT	Textbook Sections	Textbook Problems
1	1.1-1.3	1.1.2, 1.2.2, 1.3.1, 1.3.4, 1.3.6
2	1.4, 1.6	1.4.1, 1.4.3, 1.4.5, 1.4.6, 1.4.8, 1.6.6, 1.6.8, 1.6.10
3	1.5	1.5.1, 1.5.2
4	2.2	2.2.5, 2.2.6, 2.2.7, 2.2.12
5	2.3, 2.4	2.3.1, 2.3.3, 2.4.1, 2.4.2, 2.4.3
6	3.1, 3.2, 3.4, 3.6	3.2.1(a), 3.2.2(a), 3.2.3(a), 3.2.4, 3.2.5(a), 3.2.6, 3.2.7, 3.2.10, 3.4.1(d), 3.4.2, 3.4.3, 3.4.7, 3.6.2, 3.6.4, 3.6.7
7	3.1, 3.2, 3.4, 3.5, 3.7, 3.8	3.2.1(b), 3.2.2(bcd), 3.2.3(bcd), 3.2.5(bc), 3.4.1(abc), 3.5.2, 3.5.3, 3.5.5, 3.5.7, 3.5.12, 3.5.14, 3.7.5, 3.7.6, 3.8.1, 3.8.4, 3.8.6, 3.8.8
8	2.1, 3.3	2.1.4, 2.1.6, 2.1.9, 3.3.3, 3.3.5, 3.3.6, 3.3.14, 3.3.16
9	4.1-4.4	4.2.1, 4.2.2, 4.2.4, 4.3.1(abc), 4.3.2, 4.4.2(a), 4.4.4, 4.4.6(a), 4.4.7(a). 4.5.10, 4.5.12
10	4.1-4.3	4.3.1(d), 4.3.2, 4.3.3, 4.3.4, 4.3.6
11	3.3, 4.5	3.3.10, 4.5.4, 4.5.5
12	4.6	4.6.3, 4.6.4, 4.6.6, 4.6.10
13	4.4, 6.2	4.4.6(b), 4.4.7(b), 6.2.2, 6.2.3, 6.2.5
14	5.1, 5.2, 5.3, 5.7	5.1.1, 5.2.2, 5.2.3, 5.2.4, 5.3.2, 5.3.3, 5.3.4, 5.7.2, 5.7.4, 5.7.5, 5.7.8
15	5.4, 5.5	5.4.2, 5.4.3, 5.5.2, 5.5.3, 5.5.5, 5.5.8
16	5.6, 5.8	5.6.2, 5.6.6, 5.6.7, 5.8.1, 5.8.3, 5.8.6, 5.8.7
17	6.4, 6.5	6.4.1, 6.4.2, 6.4.4, 6.4.5, 6.5.2, 6.5.3, 6.5.4
18	7.1, 7.2	7.1.1, 7.1.3, 7.1.7*, 7.2.2, 7.2.3, 7.2.4, 7.2.6, 7.2.7 *use Gaussian (160,40) for patient with diabetes
19	7.4, 7.5	7.4.4, 7.4.5, 7.4.6, 7.5.1, 7.5.3
20	9.1, 9.5, 10.1	9.1.1, 9.1.2, 9.1.3, 9.1.5, 9.5.1, 9.5.2, 9.5.4, 9.5.7, 10.1.1, 10.1.3, 10.1.4
21	10.2, 10.3	10.2.1, 10.2.3(omit part (c)) 10.3.1, 10.3.2
22	11.1	11.1.1, 11.1.4, 11.1.5, 11.1.6
23	11.2	11.2.1, 11.2.2*, 11.2.4 *use: exp(1/3); mu = 3, 6, 10 in lieu of 60, 120, 200; omit part (f)
24	12.1	12.1.3, 12.1.4, 12.1.5
25	12.2	12.2.1, 12.2.4, 12.2.5