Mass and Lifetime Ladders in Quantum Geometry

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today

Abstract

We develop a formalism for extracting geometric depth, conduit counts, and lifetime scaling from observed particle masses. Using the operator-algebraic budgets $(\kappa, d, m, \Delta_{\text{int}})$, we invert the mass formula, derive convexity bounds (K-rigidity), and propose falsifiable constraints on ladder fits across sectors.

1 Introduction

We investigate the ladder-like structure of particle masses, interpreting each step as a geometric process in a spin-network background. Parameters are obtained from operator-algebraic entropy budgets and inverted to yield geometric depths and lifetimes.

2 Parameter Definitions and Provenance

We work with four fundamental parameters derived from the operator-algebraic framework:

- $\kappa = -2 \ln s_2$: geometric suppression from bridge transfer
- d: quantum dimension of neutralizer representations
- m: number of independent intertwiners
- Δ_{int}: modular gap of boundary algebra

3 Budgets and Provenance

We recall the definitions of κ (geometric suppression constant), d (channel multiplicity), m (conduit count), and Δ_{int} (interaction depth). These arise from the operator-algebraic entropy framework of [**?**, **?**, **?**].

4 Inversion Formula

Given a particle mass proxy μ_i and sector budgets $(\kappa, d, \Delta_{\text{int}})$, the geometric depth is

$$\Delta L_i = \frac{m_i \ln d - \ln(\mu_i \, \Delta_{\text{int}})}{\kappa}. \tag{1}$$

Here m_i is obtained from the spin resource $X_i = \sum_a \ln(2j_a + 1)$ via a convex response $m_i = f(X_i)$.

5 K-Rigidity: Convexity Constraints

We formalize "K-rigidity" as a set of convexity bounds on distributing index-cost along a mass ladder.

Definition 5.1 (Spin resource and response)**.** Let $X := \sum_a \ln(2j_a + 1)$ be the additive spin resource for a step. Assume $m = f(X)$ with $f''(X) \geq 0$.

Theorem 5.2 (Jensen bound)**.** *For fixed* X_{tot}, $\sum m_i \geq n f(X_{\mathrm{tot}}/n)$.

Corollary 5.3 (Mean depth bound)**.** *Averaging the inversion formula and applying Jensen yields a lower bound on the mean depth* $\frac{1}{n} \sum \Delta L_i$.

Theorem 5.4 (Adjacent spacing bound)**.** *If* f *is* L*-Lipschitz,* $|\Delta L_{i+1} - \Delta L_i|$ *is bounded by a function of* $|X_{i+1} - X_i|$ *and adjacent mass ratios.*

Proposition 5.5 (Minimax depth)**.** *Equal* X_i *minimizes* $\max \Delta L_i$.

These bounds restrict allowable oscillations in geometric depth, providing a falsifier if experimental data violates them without invoking nonconvex f or exotic resources.

6 Sector Fits and Cross-Anchor Protocol

We outline a procedure for fitting $(\kappa, d, \Delta_{\mathrm{int}})$ across sectors, using known particles as anchors and verifying cross-sector consistency.

7 Robustness and Falsifiers

We list testable predictions: (i) Violations of K-rigidity imply missing states or hidden resources. (ii) Ladder fits with large oscillations in ΔL_i are disfavored.

8 Worked Examples

Example fit for a lepton ladder: table of μ_i, m_i, ΔL_i, plots of depths vs step index, and K-rigidity score.

References

References

[1] M. Sandoz, "Bridge-Monotonicity in Spin Networks," (2025), preprint.

[2] M. Sandoz, "Entropy Monotonicity via Local Graph Rewrites," (2025), preprint.

[3] M. Sandoz et al., "Operator-Algebraic Perspective on Entropy Flow," (2025), preprint.