Geometric Genesis III: Mass
Lifetime Ladders in Quantum Geometry

Matthew Sandoz

today

**Abstract**

We develop a formalism for extracting geometric depth, conduit counts, and lifetime scaling from observed particle masses. Using the operator-algebraic budgets $(\kappa, d, m, \Delta_{\text{int}})$, we invert the mass formula, derive convexity bounds (K-rigidity), and propose falsifiable constraints on ladder fits across sectors.

# 1 Introduction

We investigate the ladder-like structure of particle masses, interpreting each step as a geometric process in a spin-network background. Parameters are obtained from operator-algebraic entropy budgets and inverted to yield geometric depths and lifetimes.

# 2 Parameter Definitions and Provenance

We work with four fundamental parameters derived from the operator-algebraic framework:

- $\kappa = -2 \ln s_2$: geometric suppression from bridge transfer
- $d$: quantum dimension of neutralizer representations
- $m$: number of independent intertwiners
- $\Delta_{int}$: modular gap of boundary algebra

# 3 Budgets and Provenance

We recall the definitions of $\kappa$ (geometric suppression constant), $d$ (channel multiplicity), $m$ (conduit count), and $\Delta_{\text{int}}$ (interaction depth). These arise from the operator-algebraic entropy framework of [?, ?, ?].

# 4 Inversion Formula

Given a particle mass proxy $\mu_i$ and sector budgets $(\kappa, d, \Delta_{\text{int}})$, the geometric depth is

$$\Delta L_i = \frac{m_i \ln d - \ln(\mu_i \, \Delta_{\text{int}})}{\kappa}. \tag{1}$$

Here $m_i$ is obtained from the spin resource $X_i = \sum_a \ln(2j_a + 1)$ via a convex response $m_i = f(X_i)$.

## 5 K-Rigidity: Convexity Constraints

We formalize "K-rigidity" as a set of convexity bounds on distributing index-cost along a mass ladder.

**Definition 5.1** (Spin resource and response)**.** Let $X := \sum_a \ln(2j_a + 1)$ be the additive spin resource for a step. Assume $m = f(X)$ with $f''(X) \geq 0$.

**Theorem 5.2** (Jensen bound)**.** *For fixed* $X_{\text{tot}}$*,* $\sum m_i \geq n f(X_{\text{tot}}/n)$*.*

**Corollary 5.3** (Mean depth bound)**.** *Averaging the inversion formula and applying Jensen yields a lower bound on the mean depth* $\frac{1}{n} \sum \Delta L_i$*.*

**Theorem 5.4** (Adjacent spacing bound)**.** *If* $f$ *is L-Lipschitz,* $|\Delta L_{i+1} - \Delta L_i|$ *is bounded by a function of* $|X_{i+1} - X_i|$ *and adjacent mass ratios.*

**Proposition 5.5** (Minimax depth)**.** *Equal* $X_i$ *minimizes* $\max \Delta L_i$*.*

These bounds restrict allowable oscillations in geometric depth, providing a falsifier if experimental data violates them without invoking nonconvex $f$ or exotic resources.

## 6 Sector Fits and Cross-Anchor Protocol

We outline a procedure for fitting $(\kappa, d, \Delta_{\text{int}})$ across sectors, using known particles as anchors and verifying cross-sector consistency.

## 7 Robustness and Falsifiers

We list testable predictions: (i) Violations of K-rigidity imply missing states or hidden resources. (ii) Ladder fits with large oscillations in $\Delta L_i$ are disfavored.

## 8 Worked Examples

Example fit for a lepton ladder: table of $\mu_i$, $m_i$, $\Delta L_i$, plots of depths vs step index, and K-rigidity score.

## References

## References

[1] M. Sandoz, "Bridge-Monotonicity in Spin Networks," (2025), preprint.

[2] M. Sandoz, "Entropy Monotonicity via Local Graph Rewrites," (2025), preprint.

[3] M. Sandoz et al., "Operator-Algebraic Perspective on Entropy Flow," (2025), preprint.