Epistemology

Five Paradoxes of Inductive Reasoning

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Some Questions

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- What does it mean to be rational?
- What is the nature of good reasoning?
- What is the nature of good **inductive** reasoning?

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- Principle of the Uniformity of Nature (PUN): the future resembles the past.
- Why accept the PUN? Because up until now, the future always has resembled the past.
- But that is an inductive argument.

Induction $\rightleftarrows$ PUN

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(ii) Give some other kind of justification of PUN that's neither deductive or inductive.

(iii) Accept that induction can only be justified in a circular fashion, but argue that this is okay.

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- More general problem: how do we distinguish good inductive inferences from bad ones?
- When does a piece of evidence lend inductive support to a hypothesis?

Two Axioms of Confirmation

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2. If we've never encountered an F that wasn't G, then every F we encounter that is G supports the hypothesis `all F's are G'.

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- So the hypothesis `all ravens are black' is confirmed by every non-black object that is not a raven.
- But this is absurd. Encountering a white shoe or a blue sky or a green car does not lend support to the hypothesis that every raven is black.

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- Quine tries to solve the paradox by rejecting axiom 1.
- Hempel and Goodman argue that this isn’t really a paradox at all — it’s actually true that white shoes and red cars confirm that all ravens are black.

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- If we accept axiom 2, then Walter's evidence supports the hypothesis that all emeralds are green.

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- But now Walter defines the property `grue’ as follows: an object is grue if and only if it either (i) was observed by time t and is green, or (ii) was not observed by time t and is blue.

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- And Walter notices that, under this definition, every emerald he’s observed has been grue, as well as being green.
- So axiom 2 says that Walter’s evidence supports the hypothesis that all emeralds are grue.
- But this is inconsistent with the hypothesis that all emeralds are green, since it entails that all unobserved emeralds are blue.

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- So this looks like another reason to give up axiom 2.
- But just giving up axiom 2 isn’t enough. We need to understand why observing green emeralds does confirm the generalization `all emeralds are green’ but observing green emeralds does not confirm the generalization `all emeralds are grue’.

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- Response: it's true that we defined grue in terms of green/blue and time, but we could equivalently start with grue/bleen, and define green as `either observed before t and grue, or observed after t and bleen' (where `bleen' means `either observed before t and blue, or observed after t and green').

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- Relative to one way of choosing our initial vocabulary, grue and been need to be defined in terms of time, and relative to another, green and blue need to be defined in terms of time.
- Since what counts as a good inductive inference shouldn't depend on which language we start with, this suggests that the problem with grue/bleen can't be that they are defined in terms of time.

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- According to Bayesianism, there are two basic rules that a rational agent's credences should always satisfy.

Bayesianism

- Firstly, rational credences should behave like probabilities, i.e.

(i) cr(X or Y) = cr(X) + cr(Y) if X and Y are mutually exclusive,

(ii) cr(T) = 1, where T is a logical truth.

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- Secondly, when a rational agent learns a piece of evidence E, they should update their credences by Bayesian conditionalisation:

$$Cr_E(X) = cr(X|E) = Cr(E\&X)/Cr(E)$$

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- Don't be more confident in one hypothesis than you are in another unless you have some evidence which justifies that preference.

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- But note: the hypothesis `the area is between 0 and 1' is logically equivalent to the hypothesis `the length is between 0 and 1', and we assigned the former a credence of 1/4 and the latter a credence of 1/2.

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- But note: the hypothesis `the area is between 0 and 1' is logically equivalent to the hypothesis `the length is between 0 and 1', and we assigned the former a credence of 1/4 and the latter a credence of 1/2.
- POI is inconsistent.

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- So what credences should we adopt at the beginning of our investigations, before we have any evidence?
- We can't just use POI, but abandoning POI seems to force us into arbitrarily preferring some hypotheses to others in the absence of any justifying evidence.
- Subjectivists think that we can rationally choose any credences we want to begin with, but that seems wrong — surely it's irrational to be almost certain that a hypothesis is true in the absence of any evidence.

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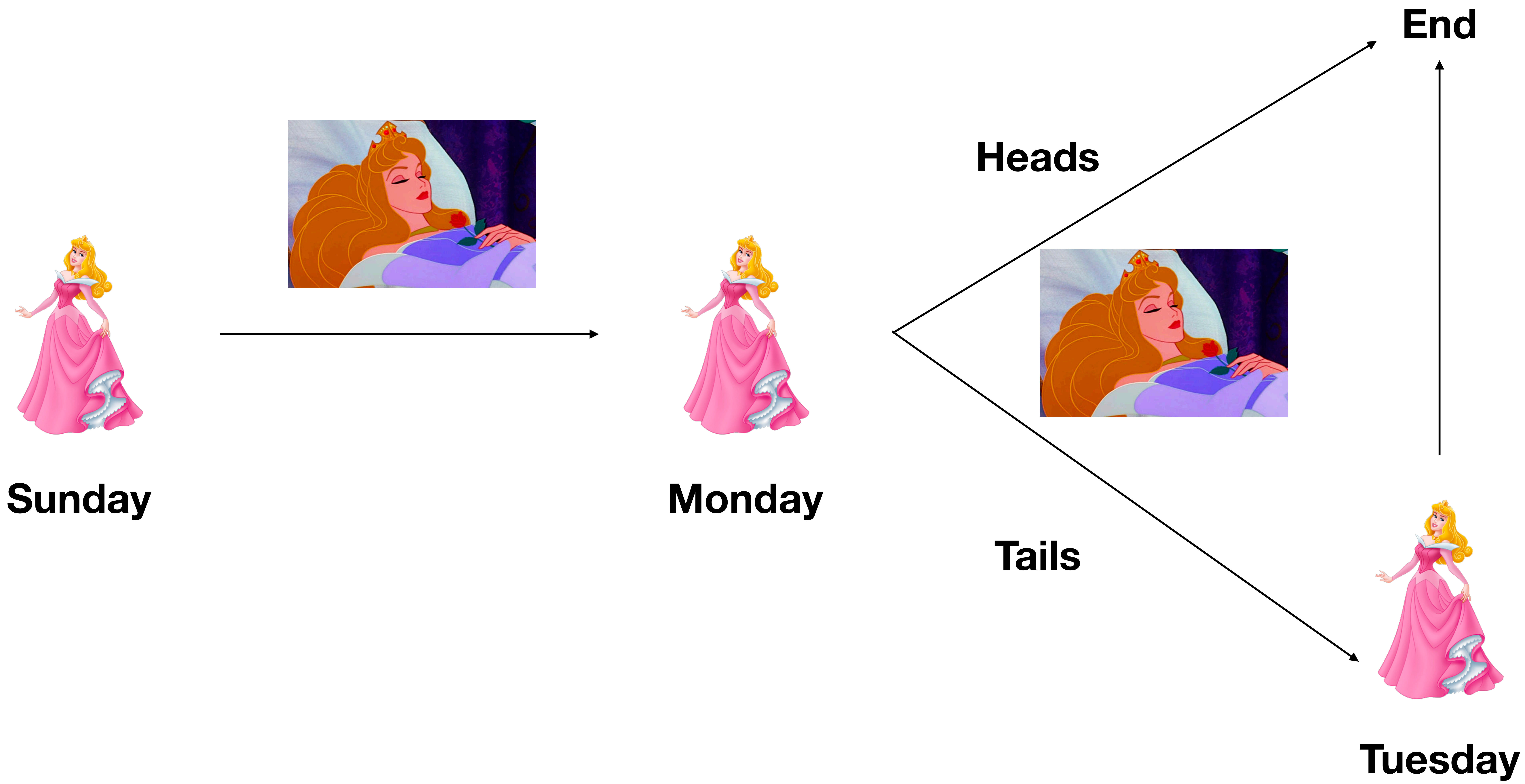
- On Sunday, Beauty will be put to sleep. On Monday, she will be awakened, interviewed, and then put back to sleep with a memory erasing drug that makes her forget her Monday awakening.
- The experimenters will then flip a fair coin. If it lands heads, they will leave Beauty asleep until the end of the experiment. If it lands tails, they will wake her again on Tuesday, interview her and put her back to sleep once more before they end the experiment.


End
Heads
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- Answer 1: she should have credence 1/2 in the coin landing heads, since we specified that it's a fair coin.
- Answer 2: she should have credence 1/3 in the coin landing heads, since there are three possibilities — it's Monday and the coin will land heads, it's Monday and the coin will land tails, it's Tuesday and the coin landed tails, and only one of them involves the coin landing heads.

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- But that seems strange: Beauty knows that the coin hasn't been tossed yet, she knows it's fair and she's more confident in heads than she is in tails. Answer 1 was originally motivated by the idea that she should be equally confident in heads and tails because the coin is fair.

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- But that means that, after waking up, Beauty's credence in heads has changed from 1/3 to 1/2 even though she doesn't seem to have learned anything relevant to the outcome of the coin toss.
- It seems like you shouldn't change your credences unless you gain new evidence that actually justifies that change.

The Upshot

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- Inductive reasoning is a mess!

Thanks!