Math 155: Instructor: Wen Li. Teaching Assistant: Siting Liu.

Homework # 2 Due on Friday, January 24

[**1**] Give a single intensity transformation function T for spreading the intensities of an image so the lowest intensity is 0 and the highest is $L-1$. f denotes the original image intensities, f_{min} denotes the minimum value of f and f_{max} denotes the maximum value of f.

[**2**] (Histogram equalization in continuous variables) An image has the gray-level PDF

$$p_r(r) = \begin{cases} \frac{6r+2}{3(L-1)^2+2(L-1)} \text{ if } 0 \le r \le L-1 \\ 0, \text{ otherwise} \end{cases}$$

with $L-1>0$.

(a) Verify some of the properties that a PDF has to satisfy: $p_r(r) \ge 0$ for all $r \in (-\infty, \infty)$ and $\int_{-\infty}^{\infty} p_r(r)dr = 1$.

(b) Find the transformation function $s = T(r)$ obtained through "histogram equalization" in continuous variables.

(c) Verify that $p_s(s)$ is a uniform "flat" distribution for $s \in [0, L-1]$ (recall the formula $p_s(s) = p_r(r)\left|\frac{dr}{ds}\right|$).

[**3**] (Histogram matching in continuous variables) An image has the gray-level PDF $p_r(r) = -2r+2$, with $0 \le r \le 1$. It is desired to transform the gray levels of this image so that they will have the specified $p_z(z) = 2z$, $0 \le z \le 1$. Find the transformation from r to z (express z in terms of r, here $L-1=1$, r and z are continuous).

[**4**] (Spatial linear filtering) Download Fig. 3.33(a), filter this image by applying a linear average filter with $m \times m$ mask (use the average mask with entries $w_{s,t} = \frac{1}{m^2}$, for all $s,t \in [-a,a]$, $a = (m-1)/2$), where $m = 3, 9, 15$. You can keep the border pixels unchanged. (Please print out your computer program, original image and smoothing images with $m = 3, 9, 15$.)