Math 155: Hw # 7, due on Friday, Feb. 28

[**1**] We have seen in continuous variables that $\mathcal{F}(\delta) \equiv 1$, thus δ and 1 form a Fourier pair; we also have that $\mathcal{F}(1) = \delta$. Using the second property and the translation property, show that the Fourier transform of $f(t) = \sin(2\pi\mu_0 t)$, where μ_0 is a real number, is $F(\mu) = (i/2)\Big[\delta(\mu + \mu_0) - \delta(\mu - \mu_0)\Big]$.
(hint: you could express f function of exponentials).

[**2**] We have seen in continuous variables that $\mathcal{F}(\delta) \equiv 1$, thus δ and 1 form a Fourier pair; we also have that $\mathcal{F}(1) = \delta$. Using the second property and the translation property, show that the Fourier transform of the continuous function $f(t, z) = A\sin(2\pi\mu_0 t + 2\pi\nu_0 z)$ is

$$F(\mu, \nu) = A\frac{i}{2}\Big[\delta(\mu + \mu_0, \nu + \nu_0) - \delta(\mu - \mu_0, \nu - \nu_0)\Big].$$

(hint: you could express f function of exponentials as done in the 1D case).

[**3**] Assume that $\mathcal{F}(1) = \delta$ also holds in the discrete case (this can be shown). Using this property and the translation property, show that the Fourier transform of the discrete function $f(x, y) = \sin(2\pi u_0 x + 2\pi v_0 y)$ is

$$F(u, v) = \frac{i}{2}\Big[\delta(u + Mu_0, v + Nv_0) - \delta(u - Mu_0, v - Nv_0)\Big].$$

[4] **Periodic Noise Reduction Using a Notch Filter**

(a) Write a program that implements sinusoidal noise of the form:
$\eta(x, y) = A\sin(2\pi u_0 x + 2\pi v_0 y)$.

The input to the program must be the amplitude, A, and the two frequency components u_0 and v_0.

(b) Download image 5.26(a) of size $M \times N$ and add sinusoidal noise to it, with $v_0 = 0$. The value of A must be high enough for the noise to be quite visible in the image (for example, you can take $A = 200$, $u_0 = 0.25$, $v_0 = 0$).

(c) Compute and display the degraded image and its spectrum (you may need to apply a log transform to visualize the spectrum).

(d) Notch-filter the image using a Gaussian notch reject filter of the form shown in Fig. 5.18(c) to remove the periodic noise.

(Please print out your code, the degraded image and its spectrum, the image after denoising.)