Math 155: Instructor Wen Li. Teaching Assistant: Siting LIU.

- Midterm on Friday, February 14, 2020, 9.00-9.50am (usual lecture room).
- Please review notes, homework, and textbook to prepare for the midterm. Sections covered for the midterm (referring to the 3rd edition of the textbook): Chapter 2: Section 2.4. Chapter 3: Sections 3.1, 3.2 (except bit-plane slicing), 3.3, 3.4, 3.5, 3.6. Chapter 4: Sections 4.2, 4.4, 4.5 (except 4.5.3, 4.5.4), 4.6, 4.7.

Homework # 5, due on Friday, February 14

[1] Show that the continuous 2D Fourier transform is a linear process.

[2] Denote $H(\mu, \nu)$ as the 2D continuous Fourier transform of $h(t, z)$. Show that if the transform $H(\mu, \nu)$ is real and symmetric, i.e. if

$$H(\mu, \nu) = \overline{H(\mu, \nu)} = \overline{H(-\mu, -\nu)} = H(-\mu, -\nu),$$

then the corresponding IFT $h(t, z)$ is also real and symmetric.

[3] (a) Show, in discrete variables, the translation property

$$\mathcal{F}\Big(f(x, y)e^{2\pi i(u_0 \frac{x}{M} + v_0 \frac{y}{N})}\Big) = F(u - u_0, v - v_0),$$

where $F = \mathcal{F}(f)$.

(b) Using (a), deduce the formula

$$\mathcal{F}\Big(f(x, y)(-1)^{x+y}\Big) = F(u - M/2, v - N/2),$$

where M and N even positive integers.

[4]

(a) Show, in discrete variables, the translation property

$$\mathcal{F}\Big(f(x - x_0, y - y_0)\Big) = F(u, v)e^{-2\pi i(x_0 u/M + y_0 v/N)},$$

where $F(u, v) = \mathcal{F}(f(x, y))$.

(b) Consider the linear difference operator $g(x, y) = f(x+1, y) - f(x, y)$. Obtain the filter transfer function, $H(u, v)$, for performing the equivalent process in the frequency domain.

[5] Prove the validity of the discrete convolution theorem in one variable (you may need to use the translation properties).

[6] (Computational Project) **Fourier Spectrum and Average Value**

(a) Use in Matlab "help fft" and "help fft2" to learn the commands for computing discrete Fourier transforms. Sample codes using the Fourier transform in 1D and 2D are posted on CCLE Section Week 5.

(b) Download Fig5.26a and compute its (centered) Fourier spectrum.

(c) Display the spectrum image.

(d) Using your algorithm, obtain the average intensity value of the input image.

(Please print out the code, the pectrum image, and the average intensity value, explain how did you get the the average intensity value of input image.)