Math 155. Instructor: Wen Li. Teaching Assistant: Siting Liu.

Homework # 4   Due on Friday, February 7

Section 3.6 Sharpening Spatial Filters

[**1**] (Composite Sharpening Mask by Laplacian). Write a computer program that implements the operation $g(x, y) = f(x, y) - \nabla^2 f(x, y)$ in the form of a spatial linear filter with a 3x3 mask. Apply the program to the image of the North Pole of the moon (Fig3.40(a).jpg). (You need to rescale g by using the formula you obtained in HW2 Problem 1, or use imagesc() to plot the output image.) You should turn in the form of the composite sharpening mask, details of the method, the computer program, the input and output images, explanation of your result. Perform your calculations only for interior pixels, not for boundary pixels.

[**2**] (Detect edges by gradient) Download Fig5.26a and plot a gradient image g by using following formula

$$g(x, y) = |\nabla f(x, y)| \approx \left| \frac{\partial f(x, y)}{\partial x} \right| + \left| \frac{\partial f(x, y)}{\partial y} \right|$$

and the Sobel operators to approximate the gradient magnitude. (You need to rescale g by using the formula you obtained in HW2 Problem 1, or use imagesc() to plot the output image). You should turn in the details of the method, the computer program, the input and output images, explanation of your result. Ignore the pixels on the boundary of the image for simplicity when computing the discrete gradient.

Chapter 4

[**3**] Compute in continuous variables the Fourier transform of the function

$$f(t) = \begin{cases} A, & \text{if } 0 \le t \le K, \\ 0, & \text{otherwise,} \end{cases}$$

where A and K are positive constants. Evaluate $F(0)$.

[**4**] Recall that the 1D DFT is

$$F(u) = \sum_{x=0}^{M-1} f(x) e^{-i2\pi ux/M}.$$

(a) Assuming the formula above, prove

$$f(x) = \frac{1}{M} \sum_{u=0}^{M-1} F(u) e^{i2\pi ux/M},$$

using the following orthogonality of exponentials

$$\sum_{u=0}^{M-1} e^{-i2\pi ur/M} e^{i2\pi ux/M} = \begin{cases} M & \text{if } r = x \\ 0 & \text{otherwise.} \end{cases}$$

(b) Show now the converse of (a): assume given $f(x)$ function of $F(u)$ in the discrete case, and show $F(u) = \sum_{x=0}^{M-1} f(x) e^{-i2\pi ux/M}$. (use the same orthogonality of exponentials).

[**5**] Assume that $f(x)$ is given by the IDFT (inverse discrete Fourier transform) formula in one dimension. Show the periodicity property $f(x) = f(x + kM)$, where k is an integer.