Robotics Notes

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Contents

1 Introduction

This is my personal learning notes for robotics.

[**TODO:**]

Abbreviations

RL	Reinforcement Learning
prob.	probability
a.k.a.	also known as
func.	function
vs.	versus
KF	Kalman Filter
EKF	Extended Kalman Filter
IF	Information Filter
EIF	Extended Information Filter
MHEKF	Multi-Hypothesis Extended Kalman Filter
MDP	Markov Decision Process
POMDP	Partially Observable Markov Decision Process

2 Probabilistic Robotics

Reference from the great book: [TBF06].

2.1 State Estimation

2.1.1 Bayes Filters

$$\begin{aligned} bel(x_t) &= p(x_t|z_{1:t}, u_{1:t}) && \text{belief over a state} \\ \overline{bel}(x_t) &= p(x_t|z_{1:t-1}, u_{1:t}) && \text{a posterior (before adapt to } z_t\text{)} \\ \Rightarrow \text{Calculating } bel(x_t) &\text{ from } \overline{bel}(x_t) && \text{correction/measurement update} \end{aligned}$$

Bayes Filter Algorithm:

2	for all x_t do:	
3	$\overline{bel}(x_t) = \int p(x_t\|u_t, x_{t-1}) bel(x_{t-1}) dx$	prediction step
4	$bel(x_t) = \eta\, p(z_t\|x_t)\, \overline{bel}(x_t)$	update step

$\Rightarrow$ Can only be implemented for very simple estimation problems, finite state space

Important assumption: Markov property (each state s a complete summary of the past)

Problem: can not be implemented on digital computers

The next subsections describe two Gaussian filters (Kalman Filter (KF) and Information Filter (IF)) and two extensions of them (Extended Kalman Filter (EKF) and Extended Information Filter (EIF)). The Gaussian filters have major advantage in computational cost, with the disadvantage of having assumption on uni-model distribution.

2.1.2 The Kalman Filter (KF)

- ***Learning Resources:*** www.bzarg.com
- For continuous space, not discrete or hybrid

- **Assumption:** posterior are Gaussians and Markov property

$p(x_t\|u_t, x_{t-1})$	must be linear func. (linear system dynamics)
$p(z_t, x_t)$	also linear
$bel(x_0)$	initial belief must be Gaussian

$$
\begin{aligned}
x_t &= A_t x_{t-1} + B_t u_t + \varepsilon_t \\
p(x_t|u_t, x_{t-1}) &= \det\left(2\pi\Sigma_t\right)^{-\frac{1}{2}} \exp\left[-\frac{1}{2}\left(x_t - A_t x_{t-1} - B_t u_t\right)^T R_t^{-1}\left(x_t - A_t x_{t-1} - B_t u_t\right)\right] \\
z_t &= C_t x_t + \delta_t \quad (= y) \\
p(z_t|x_t) &= \det\left(2\pi Q_t\right)^{-\frac{1}{2}} \exp\left[-\frac{1}{2}\left(z_t - C_t x_t\right)^T Q_t^{-1}\left(z_t - C_t x_t\right)\right] \\
bel(x_0) = p(x_0) &= \det\left(2\pi\Sigma_0\right)^{-\frac{1}{2}} \exp\left[-\frac{1}{2}\left(x_t - \mu_0\right)^T \Sigma_0^{-1}\left(x_t - \mu_0\right)\right]
\end{aligned}
$$

Kalman Filter Algorithm: $(\mu_{t-1}, \Sigma_{t-1}, u_t, z_t)$

$$
\left.\begin{array}{ll}
2 & \overline{\mu}_t = A_t \mu_{t-1} + B_t u_t \\
3 & \overline{\Sigma}_t = A_t \Sigma_{t-1} A_t^T + R_t
\end{array}\right\} \text{incorporate } u_t \text{ - prediction step - } \mathcal{O}(n^2)
$$

$$
\left.\begin{array}{lll}
4 & K_t = \overline{\Sigma}_t C_t^T \left(C_t \overline{\Sigma}_t C_t^T + Q_t\right)^{-1} & \text{(Kalman gain)} \\
5 & \mu_t = \overline{\mu}_t + K_t(z_t - C_t \overline{\mu}_t) & \\
6 & \Sigma_t = (I - K_t C_t)\overline{\Sigma}_t &
\end{array}\right\} \text{incorporate } z_t \text{ - correction step - } \mathcal{O}(n^{2,8})
$$

$$
\begin{array}{ll}
7 & \text{return } \mu_t, \Sigma_t \qquad \Rightarrow \text{ belief at time } t
\end{array}
$$

$\Rightarrow$ quite computationally expensive, **everything are Gaussians**

2.1.3 Extended Kalman Filter (EKF)

Overcome the assumption on linearity by only approximate by Gaussians

$$
\begin{aligned}
x_t &= g(u_t, x_{t-1}) + \varepsilon_t \\
z_t &= h(x_t) + \delta_t
\end{aligned}
$$

$$
\begin{aligned}
g(u_t, x_{t-1}) &\approx g(u_t, \mu_{t-1}) + g'(u_t, \mu_{t-1})(x_t - \mu_{t-1}) \\
&= g(u_t, \mu_{t-1}) + G_t(x_t - \mu_{t-1}) \\
&\quad g' \text{ is the Jacobian of state } (n \times n \text{ matrix}) \\
p(x_t|u_t, x_{t-1}) &\approx \det(2\pi R_t)^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}[x_t - g(u_t, x_{t-1})]^T R_t^{-1}[x_t - g(u_t, x_{t-1})]\right\} \\
h(t) &\approx h(\overline{\mu}_t) + h'(\mu_t)(x_t - \mu_{t-1}) \\
&= h(\overline{\mu}_t) + H_t(x_t - \mu_{t-1}) \\
p(z_t|x_t) &= \det(2\pi Q_t)^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}[z_t - h(x_t)]^T Q_t^{-1}[z_t - h(x_t)]\right\}
\end{aligned}
$$

Extended Kalman Filter Algorithm: $(\mu_{t-1}, \Sigma_{t-1}, u_t, z_t)$

2 $\overline{\mu}_t = g(u_t, \mu_{t-1})$

3 $\overline{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + R_t$

4 $K_t = \overline{\Sigma}_t H_t^T \left(H_t \overline{\Sigma}_t H_t^T + Q_t\right)^{-1}$

5 $\mu_t = \overline{\mu}_t + K_t[z_t - h(\overline{\mu}_t)]$

6 $\Sigma_t = (I - K_t H_t)\overline{\Sigma}_t$

7 return μ_t, Σ_t

- Can extend EKF $\Rightarrow$ Multi-Hypothesis Extended Kalman Filter (MHEKF)
- EKF's performance depends on degree of nonlinearities and uncertainty
- Unscented KF and moments matching KF are better

2.1.4 Information Filter (IF)

- Moment representation: μ & Σ
- Canonical representation: ξ & Ω

  Information precision matrix: $\Omega = \Sigma^{-1}$; $\Sigma = \Omega^{-1}$

  Information vector: $\xi = \Sigma^{-1}\mu$; $\mu = \Omega^{-1}\xi$

$$
\begin{aligned}
p(x) &= \det(2\pi\Sigma)^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}(x - \mu)^T \Sigma^{-1}(x - \mu)\right\} \\
&= \eta \exp\left\{-\frac{1}{2}x^T \Omega x + x^T \xi\right\}
\end{aligned}
$$

Information Filter Algorithm: $(\xi_{t-1}, \Omega_{t-1}, u_t, z_t)$

$$\left.\begin{aligned} &\overline{\Omega}_t = (A_t \Omega_{t-1}^{-1} A_t^T + R_t)^{-1} \\ &\overline{\xi}_t = \overline{\Omega}_t (A_t \Omega_{t-1}^{-1} \xi_{t-1} + B_t u_t) \end{aligned}\right\} \mathcal{O}(n^{2,8})$$

$$\left.\begin{aligned} &\Omega_t = C_t^T Q_t^{-1} C_t + \overline{\Omega}_t \\ &\xi_t = C_t^T Q_t^{-1} z_t + \overline{\xi}_t \end{aligned}\right\} \mathcal{O}(n^2)$$

return ξ_t, Ω_t

2.1.5 Extended Information Filter (IF)

$$\begin{aligned} x_t &= g(u_t, x_{t-1}) + \varepsilon_t \\ z_t &= h(x_t) + \delta_t \\ G_t &= g'(u_t, \mu_{t-1}) \\ H_t &= h'(\mu_t) \end{aligned}$$

Extended Information Filter Algorithm: $(\xi_{t-1}, \Omega_{t-1}, u_t, z_t)$

$$\begin{aligned} &\mu_{t-1} = \Omega_{t-1}^{-1} \xi_{t-1} \\ &\overline{\Omega}_t = (G_t \Omega_{t-1}^{-1} G_t^T + R_t)^{-1} \\ &\overline{\xi}_t = \overline{\Omega}_t \, g(u_t, \mu_{t-1}) \\ &\overline{\mu}_t = g(u_t, \mu_{t-1}) \\ &\Omega_t = \overline{\Omega}_t + H_t^T Q_t^{-1} H_t \\ &\xi_t = \overline{\xi}_t + H_t^T Q_t^{-1} \left[z_t - h(\overline{\mu}_t) - H_t \overline{\mu}_t \right] \end{aligned}$$

- ***Global uncertainty:*** set $\Omega = 0$ is better than set $|\Sigma| = \infty$
- IF tends to be numerically more stable than KF
- IF is better for multi-robot problems
- For high dimensional state, EKF is computational better than EIF

2.2 Measurements

2.2.1 Map Representation

Maps: $m = \{m_1, m_2, \ldots, m_N\}$
There are 2 ways to represent a map:

[TODO: Add images]

feature-based	location-based
m_n: properties of a feature and location of feature	a specific location
only the shape of the environment **at the specific locations**	volumetric: **label for any location** in the world
easy to adjust positions of objects $\Rightarrow$ popular in the robotic mapping field	occupancy grid map

2.2.2 Measurement Noise

The 4 types of measurement noise:

- Correct range with local measurement noise
  With z_t^{k*} as the correct distance
  $$p_{hit}(z_t^k|x_t, m) = \begin{cases} \eta\, \mathcal{N}(z_t^k|z_t^{k*}, \sigma_{hit}^2) & \text{if } 0 \leq z_t^k \leq z_{\max} \\ 0 & \text{otherwise} \end{cases} \tag{2.1}$$
- Unexpected object
  $$p_{short}(z_t^k|x_t, m) = \begin{cases} \eta \lambda_{short} e^{-\lambda_{short} z_t^k} & \text{if } 0 \leq z_t^k \leq z_t^{k*} \\ 0 & \text{otherwise} \end{cases} \tag{2.2}$$
- Failures
  $$p_{\max}(z_t^k|x_t, m) = I(z = z_{\max}) \tag{2.3}$$
- Random measurements
  $$p_{rand}(z_t^k|x_t, m) = \begin{cases} \frac{1}{z_{\max}} & \text{if } 0 \leq z_t^k < z_{\max} \\ 0 & \text{otherwise} \end{cases} \tag{2.4}$$

[**TODO: Add image, plot**]

$$p(z_t^k|x_t, m) = \begin{bmatrix} z_{hit} \\ z_{short} \\ z_{max} \\ z_{rand} \end{bmatrix}^T \cdot \begin{bmatrix} p_{hit}(z_t^k|x_t, m) \\ p_{short}(z_t^k|x_t, m) \\ p_{max}(z_t^k|x_t, m) \\ p_{rand}(z_t^k|x_t, m) \end{bmatrix} \tag{2.5}$$

2.3 Robot Motion

Pose: $[x, y, \theta]^T$ at location $[x, y]^T$ and orientation θ

2.3.1 Motion Model

Motion Model, also known as (a.k.a.) Probabilistic Kinematic Model: $p(x_t, u_t, x_{t-1})$

Velocity commands	Odometry (distance traveled, angle turned, etc.)
Use for Probabilistic motion planning	**more accurate** **but post-the-fact** **(not for motion planning)** Use for estimation

Each has closed form calculation and sampling algorithm.

2.3.2 Velocity Motion Model

Assuming we can control a robot through velocities:

$$u_t = \begin{bmatrix} v_t \\ \omega_t \end{bmatrix}; \qquad x_{t-1} = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}; \qquad x_t = \begin{bmatrix} x' \\ y' \\ \theta' \end{bmatrix}$$

Motion Model Velocity Algorithm: (x_t, u_t, x_{t-1})

$$\left.\begin{aligned}
&\mu = \frac{1}{2}\frac{(x-x')\cos\theta + (y-y')\sin\theta}{(y-y')\cos\theta - (x-x')\sin\theta} \\
&x^* = \tfrac{x+x'}{2} + \mu(y-y') \\
&y^* = \tfrac{y+y'}{2} + \mu(x'-x) \\
&r^* = \sqrt{(x-x^*)^2 + (y-y^*)^2}
\end{aligned}\right\}\text{Invert the motion model}$$

$$\Delta\theta = \operatorname{atan2}(y'-y^*, x'-x^*) - \operatorname{atan2}(y-y^*, x-x^*)$$

$$\left.\begin{aligned}
&\hat{v} = \tfrac{\Delta\theta}{\Delta t} r^* \\
&\hat{\omega} = \tfrac{\Delta\theta}{\Delta t} \\
&\hat{\gamma} = \tfrac{\theta'-\theta}{\Delta t} - \hat{\omega}
\end{aligned}\right\}\text{compared actual velocities with the desired}$$

return $p(v-\hat{v}, \alpha_1|v| + \alpha_2|\omega|) \cdot p(\omega-\hat{\omega}, \alpha_3|v| + \alpha_4|\omega|) \cdot p(\hat{\gamma}, \alpha_5|v| + \alpha_6|\omega|)$

Sample Motion Model Velocity Algorithm: (u_t, x_{t-1})

$$\hat{v} = v + sample(\alpha_1|v| + \alpha_2|\omega|)$$

$$\hat{\omega} = \omega + sample(\alpha_3|v| + \alpha_4|\omega|)$$

$$\hat{\gamma} = sample(\alpha_5|v| + \alpha_6|\omega|)$$

$$x' = x - \frac{\hat{v}}{\hat{\omega}} \sin\theta + \frac{\hat{v}}{\hat{\omega}} \sin(\theta + \hat{\omega}\Delta t)$$

$$y' = y + \frac{\hat{v}}{\hat{\omega}} \cos\theta - \frac{\hat{v}}{\hat{\omega}} \cos(\theta + \hat{\omega}\Delta t)$$

$$\theta' = \theta + \hat{\omega}\Delta t + \hat{\gamma}\Delta t$$

return $x_t = [x', y', \theta']^T$

- Probability normal distribution(a, b): return $\frac{1}{\sqrt{2\pi b}} e^{-\frac{a^2}{2b}}$
- Probability triangular distribution(a, b): return $\begin{cases} 0 & \text{if } |a| > \sqrt{6b} \\ \frac{\sqrt{6b}-|a|}{6b} & \text{else} \end{cases}$
- Sample normal distribution(b): return $\frac{b}{6} \sum_{i=1}^{12} rand(-1, 1)$
- Sample triangle distribution(b): return $b.rand(-1, 1).rand(-1, 1)$

2.3.3 Odometry Motion Model

Only available after the robot has moved
$\Rightarrow$ only use for filter algorithm
not for accurate motion planning and control

Motion Model Odometry Algorithm: (x_t, u_t, x_{t-1})

$$\delta_{rot1} = \text{atan2}(\bar{y}' - \bar{y}, \bar{x}' - \bar{x}) - \bar{\theta}$$

$$\delta_{trans} = \sqrt{(\bar{x} - \bar{x}')^2 + (\bar{y} - \bar{y}')^2}$$

$$\delta_{rot2} = \bar{\theta}' - \bar{\theta} - \delta_{rot1}$$

$$\left.\begin{array}{l} \hat{\delta}_{rot1} = \text{atan2}(y' - y, x' - x) - \theta \\ \hat{\delta}_{trans} = \sqrt{(x - x')^2 + (y - y')^2} \\ \hat{\delta}_{rot2} = \theta' - \theta - \hat{\delta}_{rot1} \end{array}\right\} \Rightarrow \text{inverse motion model}$$

$$p_1 = prob(\delta_{rot1} - \hat{\delta}_{rot1}, \alpha_1\hat{\delta}_{rot1} + \alpha_2\hat{\delta}_{trans})$$

$$p_2 = prob(\delta_{trans} - \hat{\delta}_{trans}, \alpha_3\hat{\delta}_{trans} + \alpha_4(\hat{\delta}_{rot1} + \hat{\delta}_{rot2}))$$

$$p_3 = prob(\delta_{rot2} - \hat{\delta}_{rot2}, \alpha_1\hat{\delta}_{rot2} + \alpha_2\hat{\delta}_{trans})$$

return $p_1.p_2.p_3 \qquad (= p(x_t|u_t, x_{t-1}))$

NOTE:

- Bar $\Leftrightarrow$ measurements

$$\bar{x}_{t-1} = [\bar{x} \quad \bar{y} \quad \bar{\theta}]^T$$
$$\bar{x}_t = [\bar{x}' \quad \bar{y}' \quad \bar{\theta}']^T$$

- Hat $\Leftrightarrow$ estimations
- No bar and hat $\Leftrightarrow$ hypothesized final pose x, y

Sample Motion Model Odometry Algorithm: (u_t, x_{t-1})

$$\delta_{rot1} = \text{atan2}(\bar{y}' - \bar{y}, \bar{x}' - \bar{x}) - \bar{\theta}$$
$$\delta_{trans} = \sqrt{(\bar{x} - \bar{x}')^2 + (\bar{y} - \bar{y}')^2}$$
$$\delta_{rot2} = \bar{\theta}' - \bar{\theta} - \delta_{rot1}$$
$$\hat{\delta}_{rot1} = \delta_{rot1} - sample(\alpha_1 \delta_{rot1} + \alpha_2 \delta_{trans})$$
$$\hat{\delta}_{trans} = \delta_{trans} - sample(\alpha_3 \delta_{trans} + \alpha_4 (\delta_{rot1} + \delta_{rot2}))$$
$$\hat{\delta}_{rot2} = \delta_{rot2} - sample(\alpha_1 \delta_{rot2} + \alpha_2 \delta_{trans})$$
$$x' = x + \hat{\delta}_{trans} \cos(\theta + \hat{\delta}_{rot1})$$
$$y' = y + \hat{\delta}_{trans} \sin(\theta + \hat{\delta}_{rot1})$$
$$\theta' = \theta + \hat{\delta}_{rot1} + \hat{\delta}_{rot2}$$
$$\text{return} \quad x_t = [x' \quad y' \quad \theta']^T$$

2.3.4 Map-based Motion Model

Map-based Motion Model: $p(x_t | u_t, x_{t-1}, m)$

- Occupancy maps: $p(x_t | m) = 0 \Leftrightarrow$ the robot collides
- If the distance from $x_{t-1} \rightarrow x_t$ is small enough ($<$ half the robot's diameter), we can estimate the probability (prob.) $p(x_t | u_t, x_{t-1}, m) \approx \eta p(x_t | u_t, x_{t-1}) p(x_t | m)$, which discards the info relating the robot's path to x_t

Motion Model with Map Algorithm: (x_t, u_t, x_{t-1}, m)
return $p(x_t | u_t, x_{t-1}).p(x_t | m)$

Sample Motion Model with Map Algorithm: (u_t, x_{t-1}, m)

$$
\begin{aligned}
& do: \\
& \qquad x_t = sample\_motion\_model(u_t, x_{t-1}) \\
& \qquad \pi = p(x_t|m) \\
& until \quad \pi > 0 \\
& return < x_t, \pi >
\end{aligned}
$$

3 Markov Decision Process

3.1 Definitions

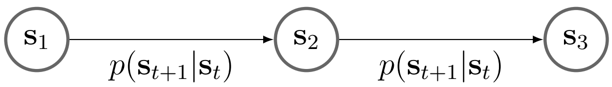
3.1.1 Markov Chain

A Markov Chain $\mathcal{M}$ is a tuple consisting of:

$\mathcal{M} = \{\mathcal{S}, \mathcal{T}\}$

$\mathcal{S}$ – state space	$s \in \mathcal{S}$ (discrete or continuous)
$\mathcal{T}$ – transition operator	$p(s_{t+1}\|s_t)$ or $\vec{\mu}_{t+1} = \mathcal{T}\vec{\mu}_t$ ($\mathcal{T}$ is a matrix)

Markov chain is a process without memories. In other words, the next state s_{t+1} depends only on the current state s_t, not the previous state s_{t-1}.

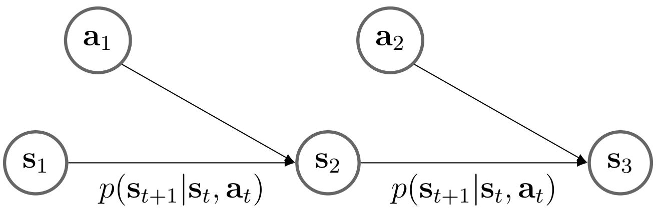


3.1.2 Markov Decision Process (MDP)

A Markov Decision Process (MDP) $\mathcal{M}$ is a tuple consisting of:

$\mathcal{M} = \{\mathcal{S}, \mathcal{A}, \mathcal{T}, r, \gamma\}$

$\mathcal{S}$ – state space	$s \in \mathcal{S}$ (discrete or continuous)
$\mathcal{A}$ – action space	$a \in \mathcal{A}$ (discrete or continuous)
$\mathcal{T}$ – transition operator	$p(s_{t+1}\|s_t)$ (now a tensor)
r – reward function	$r : \mathcal{S} \times \mathcal{A} \to \mathbb{R}$, $r(s_t, a_t)$
γ – discount factor	$\gamma \in [0, 1]$ (optional)



There are definitions relating to MDP:

- Policy: choice of action (at each state): $\pi_\theta(\mathbf{a}_t|\mathbf{s}_t)$
- Utility: sum of (discounted) rewards

A MDP can also be considered as a Markov chain on the augmented state $(\mathbf{s}, \mathbf{a})$, a.k.a. Q-state. Knowing the state transition $p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t)$ and policy $\pi_\theta(\mathbf{a}_t|\mathbf{s}_t)$, the transition of these Q-states can be derived as follows:

$$p((\mathbf{s}_{t+1}, \mathbf{a}_{t+1})|(\mathbf{s}_t, \mathbf{a}_t)) = p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t)\pi_\theta(\mathbf{a}_{t+1}|\mathbf{s}_{t+1}) \tag{3.1}$$

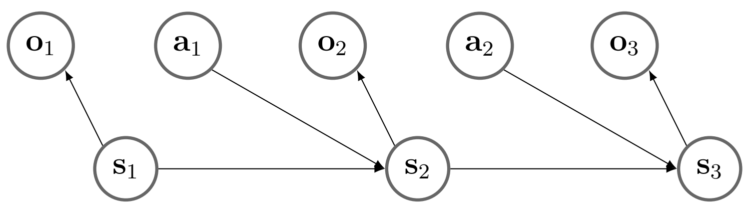
MDP state projects an expectimax-like search tree [**TODO: add image**]

3.1.3 Partially Observable Markov Decision Process (POMDP)

A Partially Observable Markov Decision Process (POMDP) $\mathcal{M}$ is a tuple consisting of:

$$\mathcal{M} = \{\mathcal{S}, \mathcal{A}, \mathcal{O}, \mathcal{T}, \mathcal{E}, r, \gamma\}$$

$\mathcal{S}$ – state space	$s \in \mathcal{S}$	(discrete or continuous)
$\mathcal{A}$ – action space	$a \in \mathcal{A}$	(discrete or continuous)
$\mathcal{O}$ – observation space	$o \in \mathcal{O}$	(discrete or continuous)
$\mathcal{T}$ – transition operator	$p(s_{t+1}|s_t)$	
$\mathcal{E}$ – emission prob.	$p(o_t|s_t)$	
r – reward function	$r : \mathcal{S} \times \mathcal{A} \to \mathbb{R}$	
γ – discount factor	$\gamma \in [0, 1]$	(optional)



Finite versus (vs.) infinite horizon: [**TODO:**]

To solve infinite utilities:

- Finite horizon
- Discounting
- Absorbing state (like the fire hole, overheating)

3.2 Bellman equations

- $T(s, a, s') = P(s'|s, a)$ the prob. of reaching state s' from taking action a at state s
- $R(s, a, s')$ the reward of making the transition
- $Q^*(s, a)$: expected utility starting in state s and having taken action a, then act optimally

$$Q^*(s, a) = \sum_{s'} T(s, a, s') \left[R(s, a, s') + \gamma V^*(s')\right] \tag{3.2}$$

It is the sum over possible next state s', because there is uncertainty of which state s' will be reached, even with the same starting state s and action a.

- $V^*(s)$: value of a state - expected utility starting in state s and acting optimally

$$V^*(s) = \max_a Q^*(s, a) \tag{3.3}$$

$$\Rightarrow \quad V^*(s) = \max_a \sum_{s'} T(s, a, s') \left[R(s, a, s') + \gamma V^*(s')\right] \tag{3.4}$$

- $\pi^*(s)$: optimal action / policy from state s

[**TODO: Explain this??**]

$$\mathbf{V(s)} = \mathbb{E}[\mathbf{G_t}|\mathbf{S_t} = \mathbf{s}] = \mathbb{E}\left[\mathbf{R_{t+1}} + \gamma \mathbf{V(S_{t+1})}|\mathbf{S_t} = \mathbf{s}\right] = \mathbf{R_s} + \gamma \sum_{\mathbf{s'}} \mathbf{P_{ss'}} \mathbf{V(s')} \tag{3.5}$$

3.3 Partially Observable Markov Decision Process

POMDP is defined with a tuple: $< S, A, O, P, R, Z, \gamma >$:

- S: state
- A: action
- O: **observation**
- P: transition matrix
- R: reward
- Z: **observation func.**
- γ: discount factor (optional)

$$Z^a_{s'o} = P\left[O_{t+1} = o|S_{t+1} = s', A_t = a\right] \tag{3.6}$$

$$\Rightarrow \quad \begin{cases} V^*(b) \leftarrow \max_a \left[R^a_b + \gamma \sum_{s'} P^a_{bb'} V(b')\right] \\ \pi^*(b) = \arg\max_a \left[R^a_b + \gamma \sum_{s'} P^a_{bb'} V(b')\right] \end{cases} \tag{3.7}$$

4 Research Proposal

4.1 Introduction and Background of Interest

[**TODO: ignore for now**]

Robots are complex machines designed to support our lives.

Since mid 19th century, its emergence has received research attention in both the academic and industry. The first major application was in the automotive industry. Its presence is entering our lives in different fields is spreading even more and more.

Its appearance has improved

4.2 Literature Review

To my best current knowledge, robot arms have made significant improvement, but are still far from delivering general-purpose tasks. Initially, robot arms are programmed explicitly and only capable of working in the known and constrained environment of factories. Progress in different fields of technology has provided the robots more inputs (e.g., vision, audio, haptic) to operate in dynamics and unstructured environment. State-of-the-art robot manipulators can deliver complex human tasks, e.g., cooking [Mol], making coffee [Cof], and cleaning around the house [Bot]. However, the behaviors of these models are still awkwardly disruptive and far from the mastery level of human hand's dexterity. In addition, instead of having the adaptability for a wide range of conditions, most models still operate in designed environments with known tools and settings.

4.2.1 Robot Grasping

Grasping is one of the critical and unavoidable problems for robot arm manipulators, especially for logistics and service robots. Among approaches that used tactile input, most use as the sole input to improve grasp stability, adaptability to object's weight [BLJ+11; LBK+14], a few combine with other modality for improvement [COU+17]. Overall, most successful grasping approaches are empirical, using deep learning on large dataset to find the best grasping candidates. Majority of models are supervised learning using single-modal input, i.e. RGB images. A few has considered multi-modal inputs: RGB with depth images or tactile input. [BMA+13; CRC18; LLC+19; KBK+20]

4.2.2 Robot Learning

Works on reaching target, handling objects

the data gap between different approaches: some with million of samples (Sanctuary AI) in simulation, some with collective robots learning (google) → a data-efficient approach for grasping

efficient and smoothing object grasping and manipulation would concerns with both both visual and force sensor. Thus input from computer vision and tactile sensor have to be integrated [HJL18]

The gap between what robots can utilize and what human can utilize

faster learning from demonstration

different type of demonstration (visual, instead of guiding physically)

A meta learning approach for grasping by human demonstration. Prior works approach: Given 3D models, using advanced physical simulation, ... sensing for grasping examining object shape, geometries, center of gravity, etc.

a meta learning approach may also be applicable to non-rigid, deformable object (e.g., clothes, towels)

I argue that it's irrelevant to learn and optimize where to grasp an object or the object characteristics. The optimal grasp depends on the usage of the tools, objects. Someday the robot will have more context about what all tools/objects are for, but, a few demonstration would be straight forward. Thus, I want to develop a model that able to input only a few of human demonstration and be able to learn how to grasp the object appropriately. plan to approach this problem with meta learning. Meta learning is also helpful for transfer learning experience to other task, deal with the current lack of large available dataset for different arms and different tasks.

With the current attention on 3D CV, I believe that soon there will be a approach to extract more information regarding object geometries, materials and usage context. Thus, a multi-modal approach would be nice to pave the way later.

4.3 Approaches and Choice of Methods

4.3.1 Key Points for Improvement

[**FORMAT:** Prior works suffer from, limited, make assumptions on ...]

Prior works on robotic manipulators were limited on their utilization of tactile sensors. Complex hand and arm behaviors require some level of force and pressure feedback. Thus, these sensors can potentially leverage the manipulators to the dexterity level as of human's arms. However, tactile sensors are currently been used mostly for the development of soft robotics [HJL18]. For grasping task, despite certain successes from using visual and tactile input separately, there hasn't been a multi-modal approach capable of combining the strengths of both side, i.e., accurate localization, adaptability to object's weight and deformability. A model, which has knowledge from both visual and tactile sources, promises to deliver complex manipulation tasks with diverse objects.

On the other hand, generalization is still a major challenge for Reinforcement Learning (RL).

Based on previous progresses on teleoperation [HVWY+20], visual imitation learning [FYZ+17; SPG19], visual imitation learning and meta-RL can be extended further to learn more generalized and complex tasks. [**TODO: say something about previous work limitations**]. In addition, incorporating tactile input is of course an obvious direction for improvement.

Prior RL works on robotic grasping are value-based approaches, policy gradient, and make certain assumption about object's characteristics. [LLC+19]

4.3.2 Approaches and Research Contributions

[**FORMAT:** I want to extend, clarify these questions, extend these directions, improve ...]

The progress I wish to work towards concerns robotic collective learning for collaborative tasks. For example, a system of multiple robot arms playing 2v2 table tennis, packaging an item or cooking. Building up towards this progress, I intend to work and gain more experiences on the following problems:

- Using visual and tactile input in multi-modal deep learning.
- Transfer learning for RL
- Data efficiency for RL with meta-RL
- Few shot learning with meta RL
- Fast RL
- Visual imitation learning: learning by visual demonstration [HVWY+20]

- Exploration from demonstration

Directions / smaller problems I want to pursue:

- A data collection pipeline for robot manipulators: preferably as a mapping from human's arm movements. The data contains both visual and tactile information. It would then be used for the learning of other tasks, e.g. grasping, object sensing, manipulation.
- Integrate tactile information to robotic object grasping. Combining with visual data, process the tactile data to extract possible object's characteristics, e.g., weight, material, deformability. This will then possibly extend to some learning on object manipulation.
- An approach for faster, purposeful exploration and generalization: not necessarily exploration from scratch, but based on some prior expert inputs.
- A meta-RL approach for robotic grasping with few-shot learning. A meta learning approach could be
- Learning from visual demonstration [**TODO:**]

Integration with generative models. Could generative models be incorporated effectively into RL algorithms? They can perhaps generate expert-like samples for imitation learning. Should and how can we evaluate the importance of different samples?

sth can be broken down into different approaches, sub-problem, aspects, . . .

Prior work mostly concern with rigid objects. tactile inputs would bring adaptability, optimal grip force, and perhaps more underlying information regarding material, etc. for non-rigid, soft, deformable object. dynamics load

in addition, it would enable more complex object manipulation tasks that computer vision alone would be insufficient

- A approach to map human behaviors to single-arm manipulator. Use for visual and tactile data collection. Possibly generalized for different arm manipulator, gripper, sensor input, etc. As demonstration for other approach, imitation learning, object grasping, reaching, object manipulation, etc.
- Robotic grasping as few-shot meta-RL

I also curious with the usage of some deep learning techniques: generative model, score matching, etc.

4.4 Proposed Research Plan

[**TODO:**]

[**TODO: How the IAS lab is related to this research plan?**]

- There is alignment in our directions
- t thay vi tri m tuyen phu hop vs t => nen t muon join
- It's amazing that there is a great diversity at IAS, not just in terms of backgrounds, but also academic focuses. Each person works on different topics, MCTS, robot dynamics, imitation learning, etc. Regardless, different works support others and the whole team is pushing the frontier of robotics.

[**TODO: Give compliments to their works!**]

- mention more explicitly regarding the topics, what research they have done
- I read your papers about ... let alone the works on ...

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