MWoKS

Machine Dynamics of Rigid System

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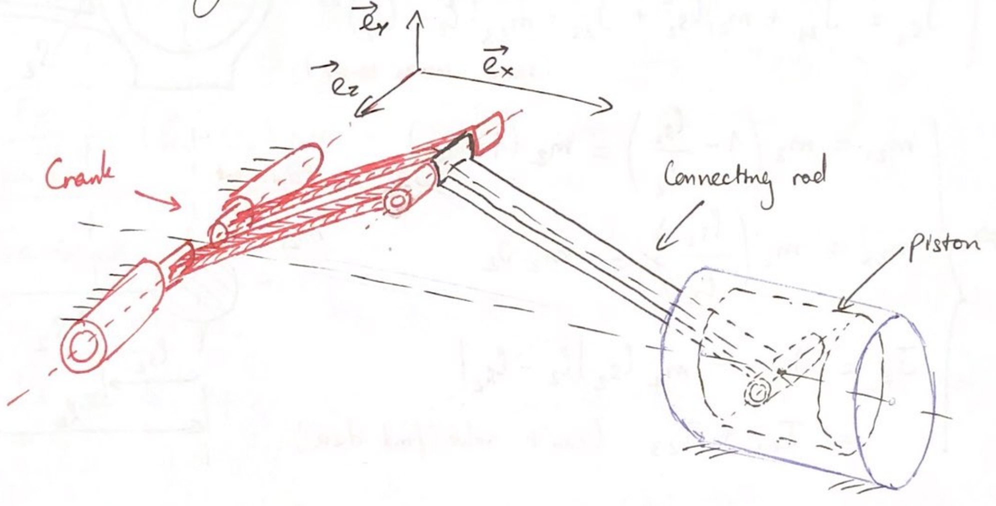
[Balancing & Power Smoothing
[Loading depends on forces & moments from outside/inside

1) Mass balance < determination of inertia forces & moments
investigating the possibility of mass balancing

for { Single cylinder piston engines
Multi cylinder piston engines

2) Power Smoothing
- Setting up equation of motion
- Solution of equation of motion
- Non-uniformity factor
- Determination of the fly-wheel

Single cylinder engine



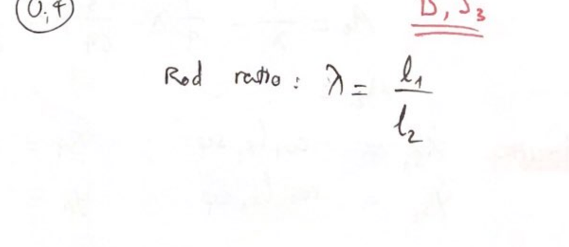
L1)

Crank:

$l_1 = \overline{A_0A} = \lambda l_2$

$l_{S_1} = \overline{A_0S_1} = \nu_1 l_1$

$\dot{\varphi} = \dfrac{d\varphi}{dt} = \omega_1 \; ; \; \ddot{\varphi} = \dfrac{d^2\varphi}{dt^2} = \alpha_1$



Rod ratio : $\lambda = \dfrac{l_1}{l_2}$

Connecting rod (coupler):

$l_2 = \overline{AB}$

$l_{S_2} = \overline{AS_2} = \nu_2 l_2$

$\dot{\psi} = \dfrac{d\psi}{dt}$

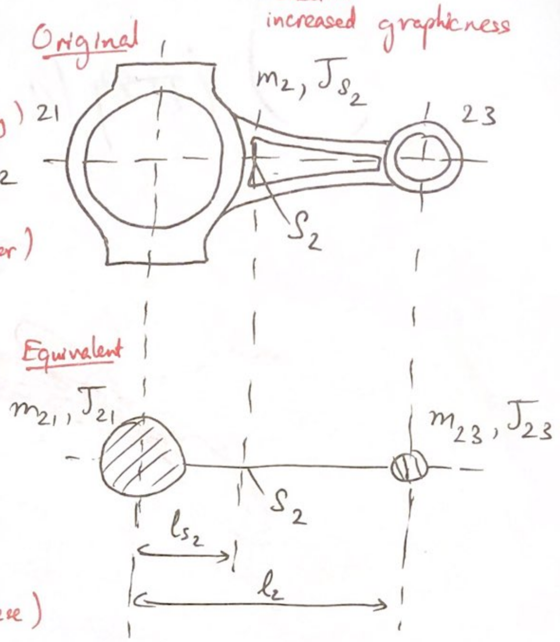
$\ddot{\psi} = \dfrac{d^2\psi}{dt^2}$

Piston: $x_B = \overline{A_0B}$

+) Dynamically equivalent system for the connecting rod $\Rightarrow$ [better / easier / clearer] for dynamics, increased graphicness

3 equations, 4 unknowns

$$\begin{cases} m_{21} + m_{23} = m_2 & \text{(same mass)} \\ m_{21}\, l_{s_2} = m_{23}(l_2 - l_{s_2}) & \text{(same centre of gravity)} \\ J_{s_2} = J_{21} + m_{21} l_{s_2}^2 + J_{23} + m_{23}(l_2 - l_{s_2})^2 & \text{(same inertia tenser)} \end{cases}$$



$$\Leftrightarrow \begin{cases} m_{21} = m_2\left(1 - \dfrac{l_{s_2}}{l_2}\right) = m_2(1 - \vartheta_2) \\ m_{23} = m_2\left(\dfrac{l_{s_2}}{l_2}\right) = m_2 \vartheta_2 \\ J_2 = J_{s_2} - m_2 l_{s_2} |l_2 - l_{s_2}| \\ \quad = J_{21} + J_{23} \quad \text{(can't solve/find these)} \end{cases}$$

(*)

L2

+) Kinematics of slider-crank mechanism: calculate all velocity & acceleration given $\varphi, \dot{\varphi}, \ddot{\varphi}$

Distances

$x_{s_1} = l_{s_1} . c\varphi$; $x_A = l_1 c\varphi$

$y_{s_1} = l_{s_1} . s\varphi$ $\quad y_A = l_1 s\varphi$

$l_1 s\varphi = l_2 s\psi$

$$x_B = l_1 c\varphi + l_2 c\psi = l_1 c\varphi + l_2\sqrt{1 - \lambda^2 s^2\varphi}$$

$$= l_1\left(A_0 + c\varphi + \frac{1}{4}A_2 c2\varphi - \frac{1}{16}A_4 c4\varphi + \frac{1}{36}A_6 c6\varphi - + \ldots\right)$$

$$A_0 = \frac{1}{\lambda} - \frac{1}{4}\lambda - \frac{3}{64}\lambda^3 - \frac{5}{256}\lambda^5 - \ldots$$

$$A_2 = \ldots$$

Velocities

$\dot{x}_{s_1} = -\omega_1 l_{s_1} s\varphi$; $\dot{x}_A = -\omega_1 l_1 s\varphi$; $\dot{x}_B = \omega_1 l_1\left(-s\varphi - \frac{1}{2}A_2 s2\varphi + \ldots\right)$

$\dot{y}_{s_1} = \omega_1 l_{s_1} c\varphi$ $\quad \dot{y}_A = \omega_1 l_1 c\varphi$

Acceleration:

$\ddot{x}_{s_1} = -\omega_1^2 l_{s_1} c\varphi - \alpha_1 l_{s_1} s\varphi$; $\ddot{x}_A = -\omega_1^2 l_1 c\varphi - \alpha_1 l_1 s\varphi$

$\ddot{y}_{s_1} = -\omega_1^2 l_{s_1} s\varphi + \alpha_1 l_{s_1} c\varphi$ $\quad \ddot{y}_A = -\omega_1^2 l_1 s\varphi + \alpha_1 l_1 c\varphi$

$\ddot{x}_B = -\omega_1^2 l_1 (c\varphi + A_2 c2\varphi - \ldots) - \alpha_1 l_1\left(s\varphi + \frac{1}{2}A_2 s2\varphi - \ldots\right)$ 1, 2, 4, 6, ..

Angle of connecting rod ψ

$\dot{\psi} = \omega_1 \lambda\left(C_1 c\varphi - \frac{1}{3}C_3 c3\varphi + \frac{1}{5}C_5 c5\varphi - + \ldots\right)$

$\ddot{\psi} = -\omega_1^2 \lambda\left(C_1 s\varphi - C_3 s3\varphi + C_5 s5\varphi - + \ldots\right) + \alpha_1 \lambda\left(C_1 c\varphi - \frac{1}{3}C_3 c3\varphi + \ldots\right)$

1, 3, 5, ..

+) Inertia Forces   if $\underline{\omega_1 = const}$ $(\dot{\omega}_1 = \ddot{\omega}_1 = 0)$

- Longitudinal force:

$$F_x = -m_1\ddot{x}_{s_1} - m_{21}\ddot{x}_A - (m_{23}+m_3)\ddot{x}_B$$

ignore α_1..

$\alpha_1 = \dot{\omega}_1$

$$\approx \omega_1^2 l_1 \underbrace{(m_1\upsilon_1 + m_{21})}_{Q_1} c\varphi + \omega_1^2 l_1 \underbrace{(m_3 + m_{23})}_{Q_3 = m_2\upsilon_2 + m_3} (c\varphi + A_2 c2\varphi - A_4 c4\varphi + ..)$$

$$\Leftrightarrow \boxed{\frac{F_x}{\omega_1^2 l_1} \approx (Q_1+Q_3)c\varphi + Q_3(A_2 c2\varphi - A_4 c4\varphi + ...)}$$

- Tranverse force:

ignore α_1..

$$F_y = -m_1\ddot{y}_{s_1} - m_{21}\ddot{y}_A \approx m_1\omega_1^2 l_1 \upsilon_1 s\varphi + m_{21}\omega_1^2 l_1 s\varphi$$

$$\Leftrightarrow \boxed{\frac{F_y}{\omega_1^2 l_1} \approx Q_1 s\varphi}$$

$$m_1\upsilon_1 + m_{21} = Q_1 = m_1\upsilon_1 + m_2(1-\upsilon_2)$$

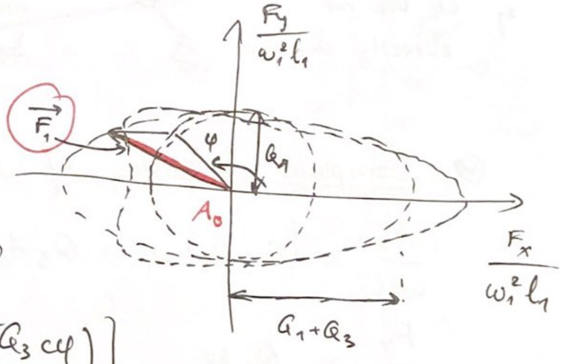
when ω is very big, $\omega^2 >> \alpha_1 \Rightarrow$ don't need to consider $\omega_1 \neq const$

+) Polar plot (of inertia force):

$$\vec{F}_1 = \vec{F}_{x_1} + \vec{F}_{y_1}$$

$$\frac{F_{y_1}}{\omega_1^2 l_1} \approx Q_1 s\varphi \; ; \; \frac{F_{x1}}{\omega_1^2 l_1} \approx (Q_1+Q_3)c\varphi$$

$$\vec{F}_1 = \vec{F}_{x_1} + \vec{F}_{y_1} = \omega_1^2 l_1 \left[\begin{pmatrix} Q_1 c\varphi \\ Q_1 s\varphi \end{pmatrix} + \begin{pmatrix} Q_3 c\varphi \\ 0 \end{pmatrix}\right]$$



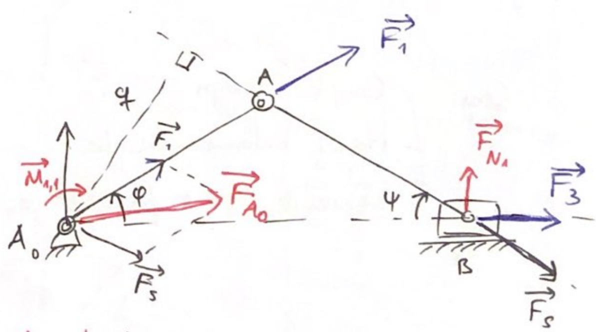
+) Inertia moment: (= 0 on frame) ⊗

$$\vec{F}_3 = \vec{F}_s + \vec{F}_{N_1}$$

To move $\vec{F}_s$ to A_0, need to add:

Torque acting on crank-shaft $\Rightarrow |\vec{M}_{1,1}| = |\vec{F}_s q| = |F_s x_B s\psi|$

which is equal but with inverse sign of the moment from $\vec{F}_{N_1}$: $|\vec{M}_{G,1}| = |\vec{F}_{N_1} x_B|$



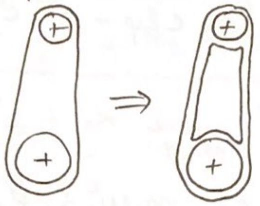
$\Rightarrow$ ⊗ No frame moment because of displacement

$$\vec{M}_G(F) = 0$$

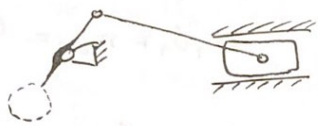
L3) Balancing of inertia/shaking forces

General rule: Shaking forces acting on the frame can be balanced exclusively by shaking forces

Change mass parameters	Additional links or mechanism(s)
+) Reduce link mass or Increase frame mass	Ex: Identical mechanism in anti symmetrical arrangement



+) Balancing mass at link directly connected with the frame



+) at link not directly connected

⊗ Complete balancing of inertia forces:

$$\frac{F_x}{\omega_1^2 l_1} \simeq (Q_1 + Q_3)\, c\varphi + Q_3 A_2\, c2\varphi - Q_3 A_4\, c4\varphi + \dots$$

$$\frac{F_y}{\omega_1^2 l_1} \simeq Q_1\, s\varphi$$

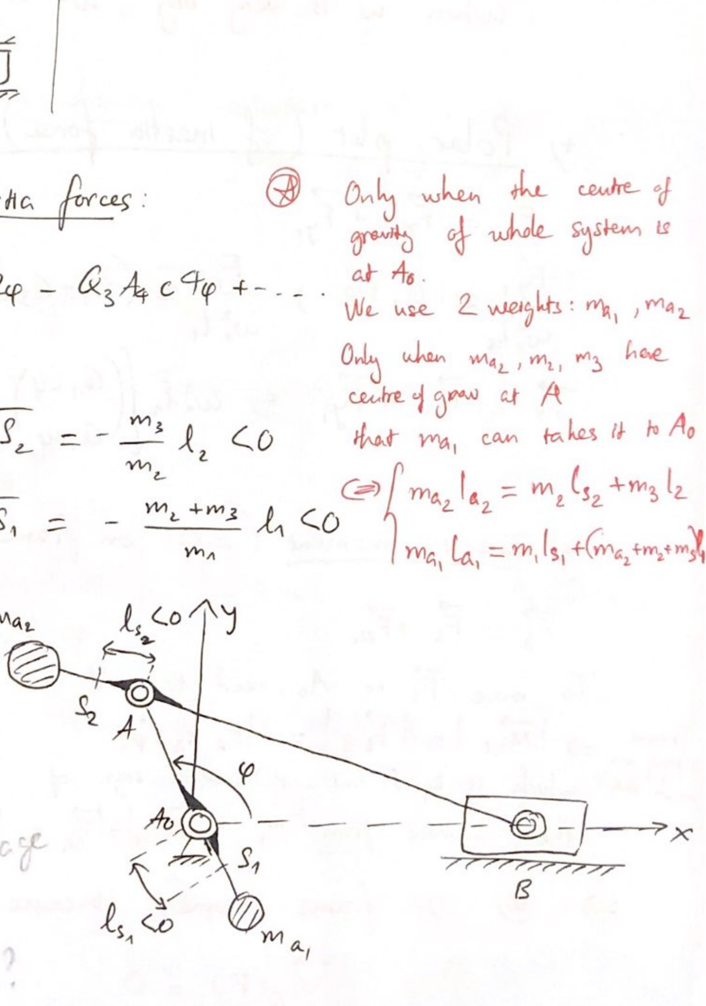
$$\begin{cases} F_x = 0 \\ F_y = 0 \end{cases} \Leftrightarrow \begin{cases} Q_3 = 0 \\ Q_1 = 0 \end{cases} \Leftrightarrow \begin{cases} l_{s_2} = \overline{AS_2} = -\dfrac{m_3}{m_2} l_2 < 0 \\ l_{s_1} = \overline{A_0 S_1} = -\dfrac{m_2 + m_3}{m_1} l_1 < 0 \end{cases}$$

Ⓐ Only when the centre of gravity of whole system is at A_0.
We use 2 weights: m_{a_1}, m_{a_2}
Only when m_{a_2}, m_2, m_3 have centre of grav at A that m_{a_1} can takes it to A_0

$$\Leftrightarrow \begin{cases} m_{a_2} l_{a_2} = m_2 l_{s_2} + m_3 l_2 \\ m_{a_1} l_{a_1} = m_1 l_{s_1} + (m_{a_2} + m_2 + m_3) l_1 \end{cases}$$

Cons: Complex design
Heavy balancing-weight
Increase joint force/torque
...

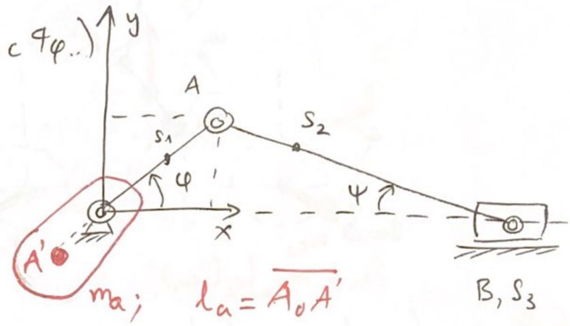
? ⇒ The design in next page complex? heavy? increased force/torque?



Balancing by counter weight/mass at the crank

$$F_x^* \approx (Q_1+Q_3)\,\omega_1^2 l_1 c\varphi + Q_3\,\omega_1^2 l_1 \left(A_2 c2\varphi - A_4 c4\varphi \ldots\right) + m_a \omega_1^2 l_a c(\varphi+\pi)$$

$$\Rightarrow \frac{F_{x1}^*}{\omega_1^2 l_1} \approx (Q_1+Q_3)\,c\varphi - m_a \frac{l_a}{l_1} c\varphi$$



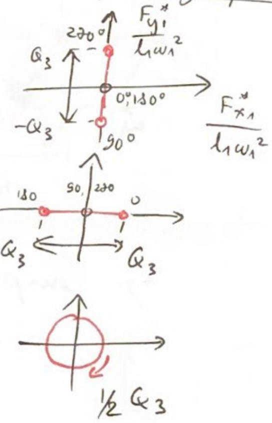
$$F_y^* = Q_1\omega_1^2 l_1 s\varphi + m_a\omega_1^2 l_a s(\varphi+\pi)$$

$$\Rightarrow \frac{F_y^*}{\omega_1^2 l_1} = Q_1 s\varphi - m_a \frac{\cancel{\omega_1}\, l_a}{l_1} s\varphi$$

F_{x1} means: F_x with only 1st Harmonic

$\Rightarrow$ Balancing:

Polar plot/graph



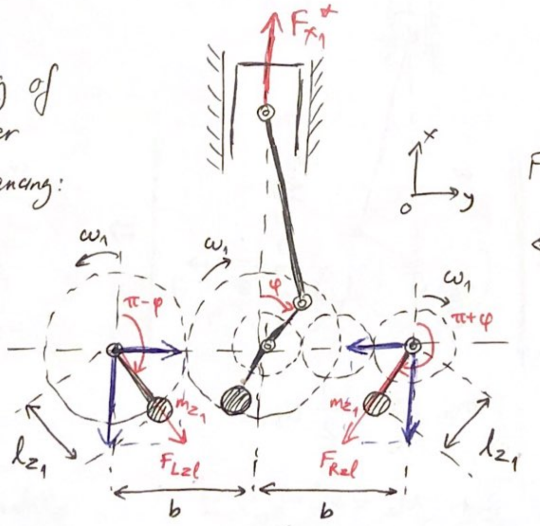
+) Longitudinal force: $\begin{cases} F_{x1}^* = 0 \\ F_{y1}^* = -Q_3 l_1\omega_1^2 s\varphi \end{cases} \Leftrightarrow m_a\frac{l_a}{l_1} = Q_1 + Q_3$

+) Transverse force: $\begin{cases} F_{x1}^* = Q_3 l_1\omega_1^2 c\varphi \\ F_{y1}^* = 0 \end{cases} \Leftrightarrow m_a\frac{l_a}{l_1} = Q_1$

+) Mean force: $\begin{cases} F_{x1}^* = \frac{1}{2} Q_3 l_1\omega_1^2 c\varphi \\ F_{y1}^* = \frac{-1}{2} Q_3 l_1\omega_1^2 s\varphi \end{cases} \Leftrightarrow m_a\frac{l_a}{l_1} = Q_1 + \frac{Q_3}{2}$

+) Complete balancing of first harmonic after tranverse force balancing:

$$\begin{cases} F_{x1}^* = Q_3 l_1 \omega_1^2 c\varphi \\ F_{y1}^* = 0 \end{cases}$$

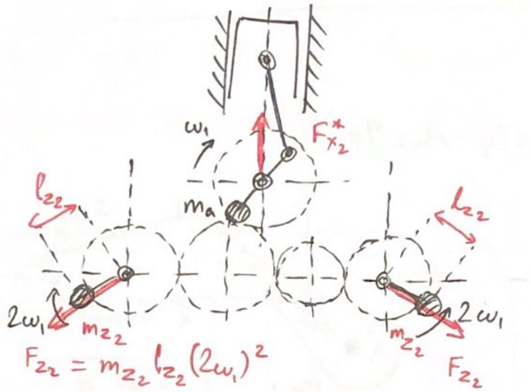


$$F_{x1}^* + F_{Lz1x} + F_{Rz1x} = 0$$

$$\Leftrightarrow m_{z1} l_{z1}\omega_1^2\left[c(-\varphi+\pi) + c(\varphi+\pi)\right] + Q_3 l_1\omega_1^2 c\varphi = 0$$

$$\Leftrightarrow \boxed{m_{z1}\frac{l_{z1}}{l_1} = \frac{Q_3}{2}}$$

+) Complete balancing of 2nd harmonic after tranverse force balancing:

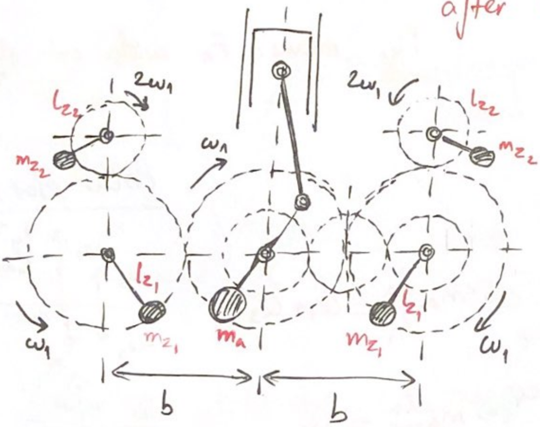


$$\begin{cases} F^*_{x_2} = Q_3 A_2 l_1 \omega_1^2 c2\varphi \\ F^*_{y_2} = 0 \end{cases}$$

$$2 m_{z_2} l_{z_2} (2\omega_1)^2 c2\varphi = A_2 Q_3 l_1 \omega_1^2 c2\varphi$$

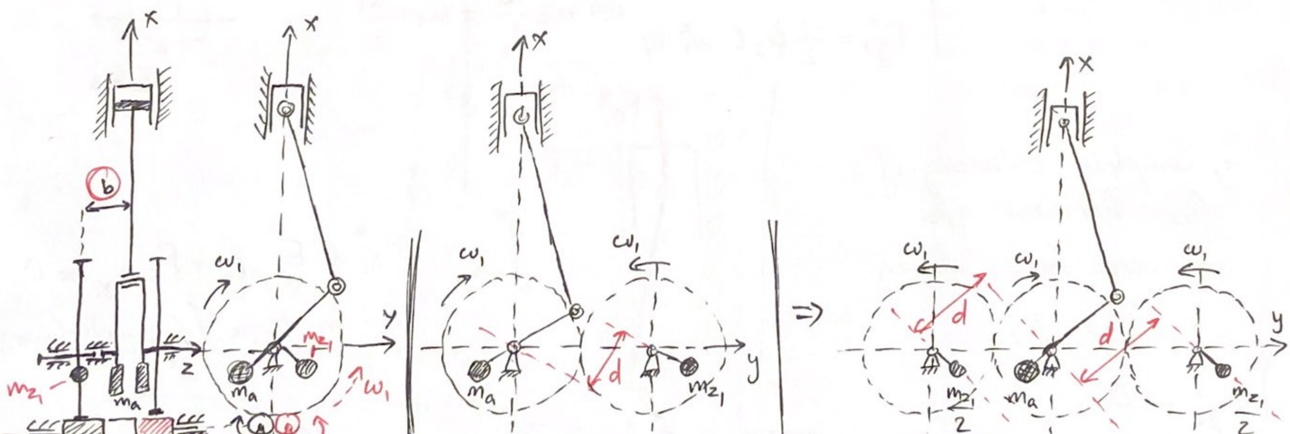
$$\Leftrightarrow \quad \boxed{m_{z_2} \frac{l_{z_2}}{l_1} = \frac{A_2 Q_3}{8}}$$

+) <u>Lanchester balancing:</u> Complete balancing of 1st & 2nd harmonic after tranverse force balancing



$$\begin{cases} m_{z_1} \dfrac{l_{z_1}}{l_1} = \dfrac{Q_3}{2} \\ m_{z_2} \dfrac{l_{z_2}}{l_1} = \dfrac{A_2 Q_3}{8} \end{cases}$$

+) Complete balancing of 1st harmonic after mean force balancing



$m_{z_1} \frac{l_{z_1}}{l_1} = \frac{1}{2} Q_3$ — <u>But</u>: $M \approx \omega_1^2 m_{z_1} l_{z_1} \underline{b}$

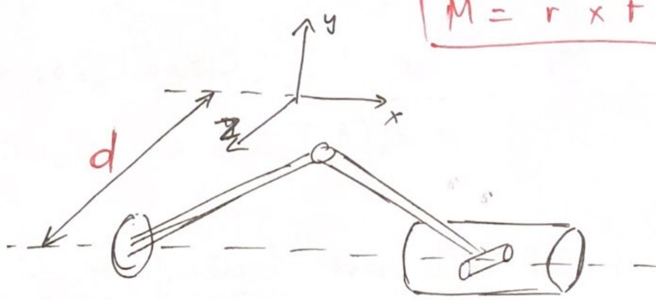
$m_{z_1} \frac{l_{z_1}}{l_1} = \frac{1}{2} Q_3$ — <u>But</u>: $M_z = \omega_1^2 m_{z_1} l_{z_1} \underline{d}$

$m_{z_1} \frac{l_{z_1}}{l_1} = \frac{1}{2} Q_3$ — And $\underline{M_z = 0}$

(4)

$\boxed{\vec{M} = \vec{r} \times \vec{F}}$

+) Inertia (Shaking) Torque (Moment):



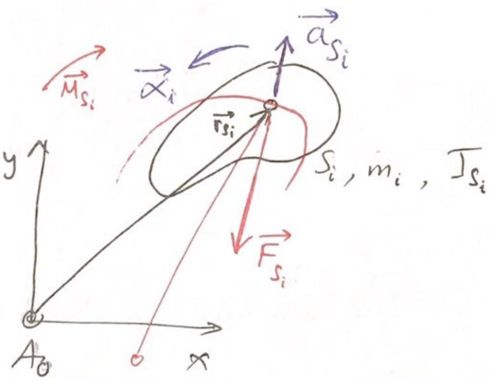
- $F_y \approx Q_1 l_1 \omega_1^2 s\varphi$, $M_x = -d . F_y$ $\Rightarrow M_x \approx -d(m_1 v_1 + m_{21}) l_1 \omega_1^2 s\varphi$

longitudinal torque M_x

- $F_x \approx (Q_1 + Q_3) l_1 \omega_1^2 c\varphi + \ldots$, $M_y = d . F_x$ $\Rightarrow M_y \approx d \omega_1^2 l_1 \left[(Q_1 + Q_3) c\varphi + Q_3 (A_2 c2\varphi - A_4 c4\varphi + \ldots) \right]$

transverse torque M_y

- Frame torque M_z:



Each link

$$\boxed{\vec{M}_z = -J_{S_i} \vec{\alpha}_i + \vec{r}_{S_i} \times \vec{F}_{S_i}} \quad \star$$

$$= -J_{S_i} \vec{\alpha}_i - m_i \left[\vec{r}_{S_i} \times \vec{a}_{S_i} \right]$$

S_{i_0}: centre of curvature

$$\vec{a}_{S_i} = \frac{d\vec{v}_{S_i}}{dt} = \frac{d(v_{S_i} \vec{e}_t)}{dt}$$

$$= v_{S_i} \frac{d\vec{e}_t}{dt} + \frac{dv_{S_i}}{dt} \vec{e}_t$$

If $A_0 \equiv S_{i_0}$ (or in general, S_{i_0} is fixed):

$$\vec{a}_{S_i} = -\omega_i^2 \overrightarrow{S_{i_0} S_i} + \vec{\alpha}_i \times \overrightarrow{S_{i_0} S_i} \Longrightarrow \vec{a}_{S_1} \ \& \ \vec{a}_A$$

$$\Rightarrow \vec{M}_z = -\sum_{i=1}^{n} J_{S_i} \vec{\alpha}_i + \sum_{i=1}^{n} \left(-m_i \left[\vec{r}_{S_i} \times \vec{a}_{S_i} \right] \right)$$

all links (in one-cylinder engine)

$$= -J_{S_1} \vec{\alpha}_1 + \boxed{J_2 \ddot{\vec{\psi}}} - m_1 \left[\overrightarrow{A_0 S_1} \times \vec{a}_{S_1} \right] - m_{21} \left[\overrightarrow{A_0 A} \times \vec{a}_A \right]$$

$\rightarrow \vec{M}_{2,z}$

Since:
$$\begin{cases} \vec{a}_{S_1} = -\omega_1^2 \overrightarrow{A_0 S_1} + \vec{\alpha}_1 \times \overrightarrow{A_0 S_1} \\ \vec{a}_A = -\omega_1^2 \overrightarrow{A_0 A} + \vec{\alpha}_1 \times \overrightarrow{A_0 A} \end{cases}$$

$$\Rightarrow \vec{M}_z = -J_{S_1} \vec{\alpha}_1 + J_2 \ddot{\vec{\psi}} - m_1 \overline{A_0 S_1}^2 \vec{\alpha}_1 - m_{21} \overline{A_0 A}^2 \vec{\alpha}_1$$

$$\boxed{M_z = -(J_{S_1} + m_1 l_{S_1}^2 + m_{21} l_1^2) \alpha_1 + J_2 \ddot{\psi}}$$

$$M_Z = -\omega_1^2 \lambda J_2 \left(C_1 s\varphi - C_3 s 3\varphi + C_5 s 5\varphi - + .. \right)$$
$$+ \alpha_1 \left[\lambda J_2 \left(C_1 c\varphi - \tfrac{1}{3} C_3 c 3\varphi + \tfrac{1}{5} C_5 c 5\varphi - + .. \right) - \left(J_{S_1} + m_1 l_{S_1}^2 + m_{21} l_1^2 \right) \right]$$

+) If $\omega = \text{const}$ (even if $\alpha_1 \ll \omega_1^2$) then:

$$M_Z \approx -\omega_1^2 \lambda J_2 \left(C_1 s\varphi - C_3 s 3\varphi + C_5 s 5\varphi - + .. \right) \Rightarrow$$

so basically: $M_Z \approx M_{2Z}$

the frame torque on link 2 (the connecting rod)

$$\boxed{M_Z = |\vec{F}_{N,2}| . l_2 . c\psi}$$

⊛ Effect of frame torque M_Z acting on the frame ..

⊛ Balancing of the inertia torque

+) Balancing of the frame torque M_Z:

$$M_Z = -\left(J_{S_1} + m_1 l_{S_1}^2 + m_{21} l_1^2 \right) \alpha_1 + J_2 \ddot{\psi} \overset{!}{=} 0$$

Consider $\alpha \ll \ddot{\psi}$, thus, we try to set $J_2 \overset{!}{=} 0$:

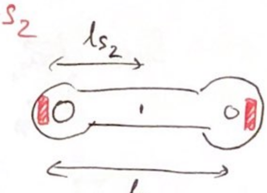
$$J_2 = J_{S_2} - m_2 l_{S_2} (l_2 - l_{S_2}) = 0 \; ; \quad i_2 = \sqrt{\frac{J_{S_2}}{m_2}} \Leftrightarrow J_{S_2} = m_2 i_2^2$$

radius of gyration

$$\Leftrightarrow \quad l_2 = \frac{i_2^2 + l_{S_2}^2}{l_{S_2}} = \frac{i_2^2 + (l_2 - l_{S_2})^2}{l_2 - l_{S_2}}$$

Very look proof .. $J_2 = 0 \Leftrightarrow l_2 = 2 l_{S_2}$

$\Rightarrow$ Add masses to connecting rod so that $l_{S_2} = \frac{l_2}{2}$

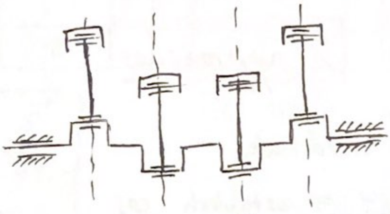


Multi cylinder engine

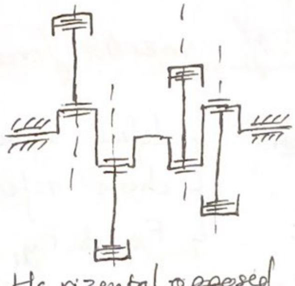
↳,

- Different structures:

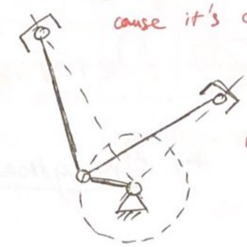
Usually found in (car / motorcycle
cause it's compact
more than in-line engines



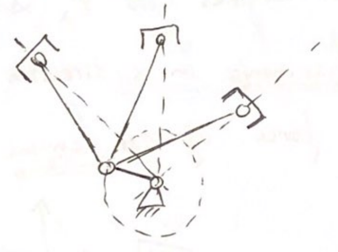
In-line Engine



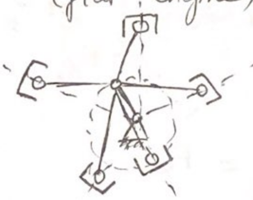
Horizental opposed engine
(flat engine)



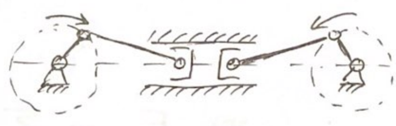
V-engine



W-engine



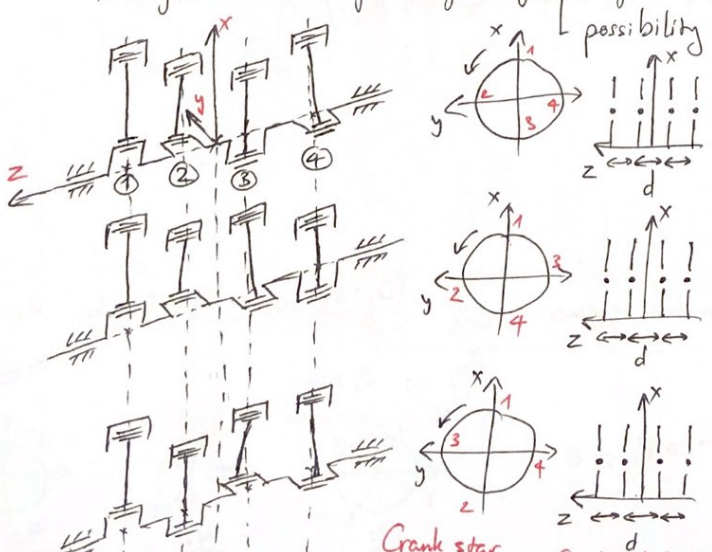
Radial engine



Opposite Piston Engine

+) In-line engines: most famous & common in the industry

- Design crank-shaft regarding: [regular firing order
  possibility of balancing inertia forces & torques



Crank star

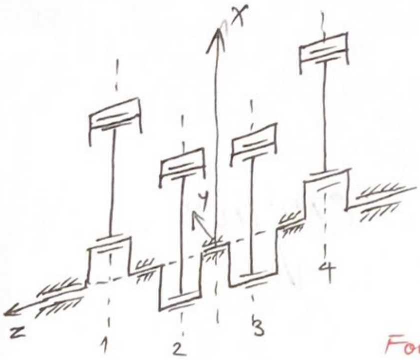
Graph of distance

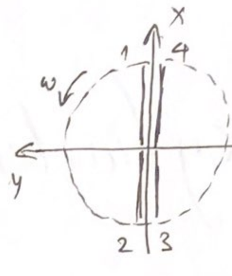
⊕ Four-cylinder two-stroke in-line engines

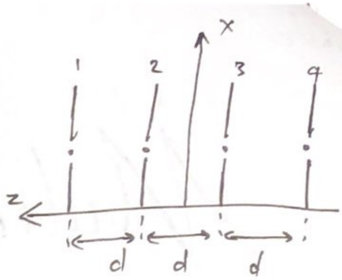
- With different configs, no difference in term of forces

BUT different in term of (as d_i changes) SHAKING TORQUES

Relocation angle of crank:
$2\pi/4 = \pi/2$







Four-cylinder four-stroke in-line Engines

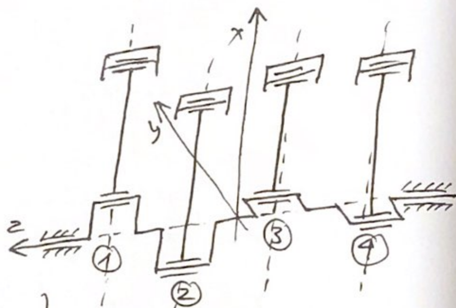
Relocation angle of crank $4\pi/4 = \pi$

Determination of inertia forces: $\boxed{\text{2 approaches}}$

+) Analytical approach:
- define system coordinates
- choose reference $\varphi \Rightarrow$ establish φ_i
- F_{x_i} & F_{y_i} each engines $\Rightarrow F_x$ & F_y

Note: $F_{x1} \neq F_{x_1}$:
- F_{x1} : 1st harmonic in x-direction of over-all frame force
- F_{x_1} : frame force of 1st engine in x-direction

$$\Rightarrow F_{x1} \approx \sum F_{x_i} = F_x$$



Example: Two-stroke 4-cylinder in-line engines

$$\frac{F_x}{l\omega^2} = -4Q_3\left[A_4 c4\varphi + A_8 c8\varphi + A_{12} c12\varphi + ..\right]$$

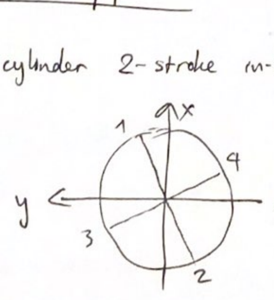
$$\frac{F_y}{l\omega^2} = 0$$

+) Graphical approach:

Example: 4-cylinder 2-stroke in-line engines: $\frac{F_{x_i}}{\omega^2 l_1} = (Q_1 + Q_3)c\varphi_i + Q_3(A_2 c2\varphi_i - A_4 c4\varphi_i \pm ..)$

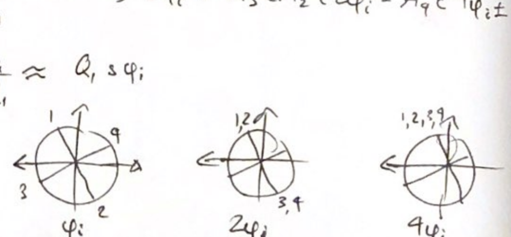
$\frac{F_{y_i}}{\omega^2 l_1} \approx Q_1 s\varphi_i$

$$\begin{cases} \varphi_1 = \varphi \\ \varphi_2 = \varphi + \pi \\ \varphi_3 = \varphi + \frac{1}{2}\pi \\ \varphi_4 = \varphi + \frac{3}{2}\pi \end{cases}$$



$\Rightarrow F_y \approx 0$

y-direction



x-direction

Unbalanced F and M_z: Order of cosin of inertia forces & torque

Number of Cylinder

	1	2	3	4	5	6	7	8	9	10	11	12
1	F, M_z	F	M_z	F	M_z	F	M_z	F	M_z	F	M_z	F
2		F		F		F		F		F		F
3			M_z			F			M_z			F
4				F				F				F
5					M_z					F		
6						F						F
7							M_z					
8								F				
9									M_z			
10										F		
11											M_z	
12												F

2-stroke in-line engines

	1	2	3	4	5	6	7	8	9	10	11	12
1	F, M_z	F	M_z	F	M_z	F	M_z	F	M_z	F	M_z	F
2	F, M_z	F	M_z	F	M_z	F	M_z	F	M_z	F	M_z	F
3			M_z			F			M_z			F
4		F		F		F		F		F		F
5					M_z					F		
6			M_z			F			M_z			F
7							M_z					
8				F				F				F
9									M_z			
10					M_z					F		
11											M_z	
12						F						F

4-stroke in-line engines

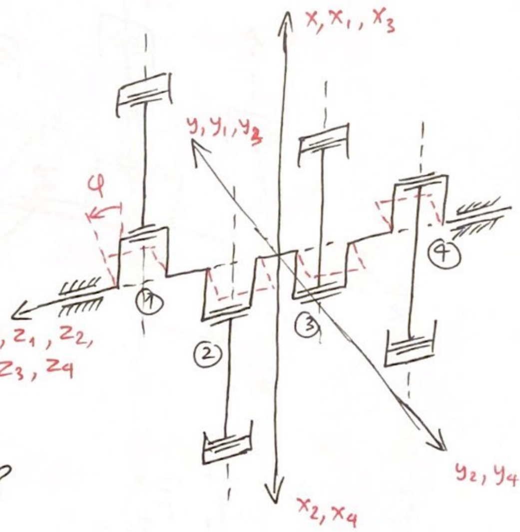
● $\vec{F}$
● $\vec{M_z}$

- The 3-cylinder has 6^{th} order which is < 4^{th} order of the 4-cylinder
- The 2-stroke engines has better natural-balancing than 4-stroke eng. (example: in 4, 6, 8 .. -cylinder model

+) **Analytical approach**: horizontally opposed engine (flat-engine)

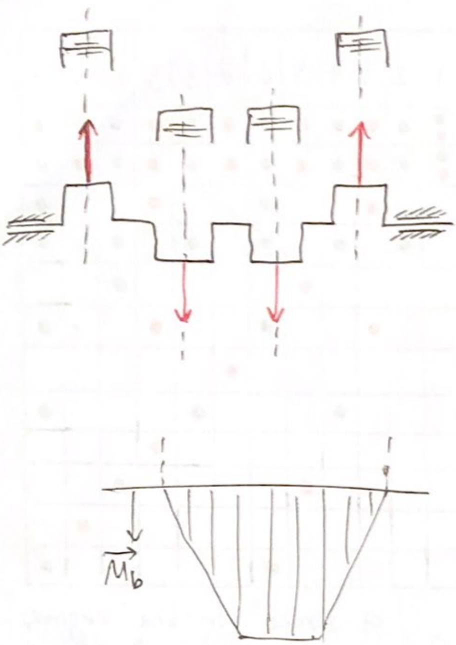
$$- \begin{cases} \varphi_1 = \varphi_2 = \varphi \\ \varphi_3 = \varphi_4 = \varphi + \pi \end{cases}$$

$$- \begin{cases} F_{x_1} = -F_{x_2} \\ F_{x_3} = -F_{x_4} \end{cases} \Leftrightarrow \begin{cases} F_x \approx 0 \\ F_y \approx 0 \end{cases}$$

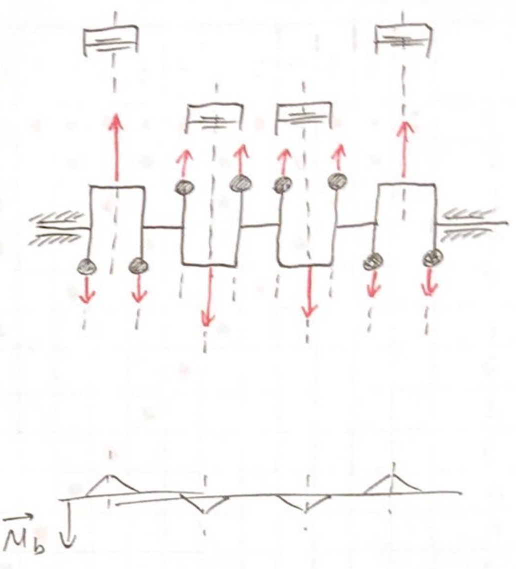


⊗ There is "natural inertia force balancing" do we still NEED COUNTERWEIGHT ?

YES, cause bending torque



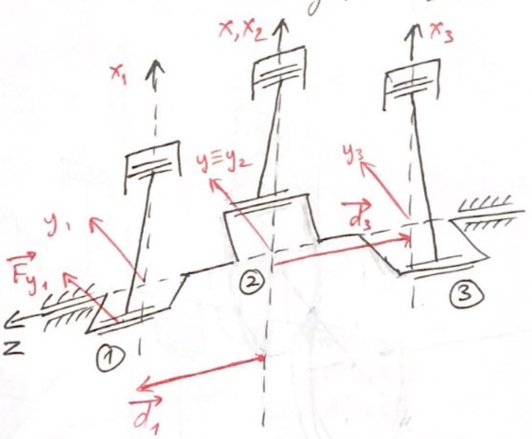
$\Rightarrow$ less bending torque

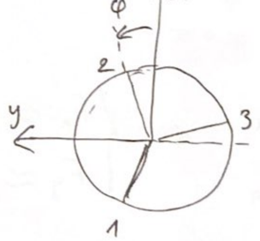


46)

⊗ 3-Cylinder in-line engine (either 2 or 4 stroke)

- For 2k-Cylinder 2 or 4-stroke in-line engine, there is no inertia frame torque M_z to consider
  $\Rightarrow$ We investigate 2k+1-Cylinder engine (3-Cylinder)

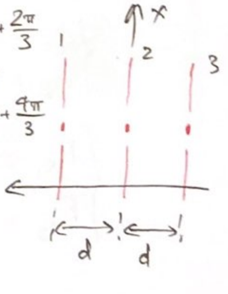




$\varphi_1 = \varphi + \frac{2\pi}{3}$

$\varphi_2 = \varphi$

$\varphi_3 = \varphi + \frac{4\pi}{3}$



$$\vec{M}_x = \sum \vec{d}_i \, \vec{F}_{y_i}$$

$$M_x = -d.F_{y_1} + d.\,F_{y_3} \quad (\vec{d}_2 = \vec{0})$$

$$= -dl\omega^2 Q_1 (s\varphi_1 - s\varphi_3)$$

$$\boxed{M_x = -\sqrt{3}\, dl\omega^2 Q_1\, c\varphi}$$

$$M_y = dl\omega^2 \left[(Q_1 + Q_3)(c\varphi_1 - c\varphi_3) + Q_3 A_2 (c2\varphi_1 - \ldots \right]$$

$$\boxed{M_y = -\sqrt{3}\, dl\omega^2 \left[(Q_1 + Q_3) s\varphi - Q_3 (A_2 s2\varphi + A_4 s4\varphi \pm \ldots) \right]}$$

$$M_{z_j} = -\omega_1^2 \lambda J_2 \left(C_1 s\varphi - C_3 s3\varphi + C_5 s5\varphi - + ..\right)$$

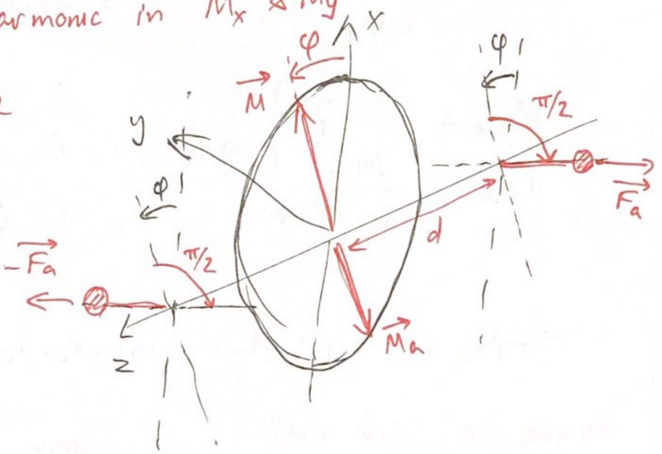
$$\Rightarrow M_z = -J_2 \omega^2 \lambda \left[C_1 (s\varphi_1 + s\varphi_2 + s\varphi_3) - C_3 (s3\varphi_1 + s3\varphi_2 + s3\varphi_2) + - ..\right]$$

$$\Leftrightarrow \boxed{M_z = 3 J_2 \omega^2 \lambda \left[C_3 s3\varphi \mp C_9 s9\varphi + C_{15} s15\varphi \mp ..\right]}$$

M_z has lower order than M_x & M_y

$\Leftrightarrow$ we will try to balance 1st harmonic in M_x & M_y

Intuition: to balance torque



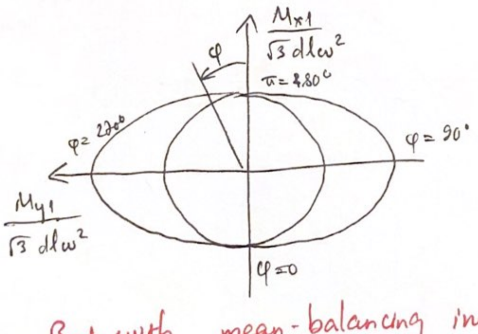
to balance $\vec{M}$

$\Rightarrow$ we need $\vec{M_a}$

$\Rightarrow$ add 2 counter weights perpendicular to φ

$$2\vec{d} \cdot \vec{F_a} = \vec{M_a}$$

+) However, M_{x1} & M_{y1} have different magnitude:



$$\frac{M_{x1}}{\sqrt{3} d l \omega^2} = -Q_1 c\varphi$$

$$\frac{M_{y1}}{\sqrt{3} d l \omega^2} = -(Q_1 + Q_3) s\varphi$$

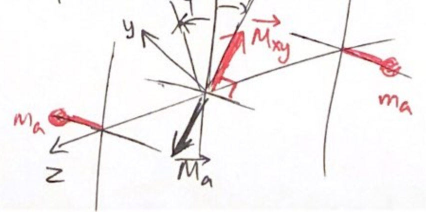
which is ...impossible to balance both to 0

But with mean-balancing inertia force, the subsequent forces have equal magn

With $m_a l_a = l\left(Q_1 + \frac{1}{2} Q_3\right)$

$$\Rightarrow \begin{cases} F^*_{y1_i} = \frac{-1}{2} Q_3 l \omega^2 s\varphi_i \\ F^*_{x1_i} = \frac{1}{2} Q_3 l \omega^2 c\varphi_i \end{cases} \Rightarrow \begin{cases} M^*_{x1} = \frac{\sqrt{3}}{2} d l \omega^2 Q_3 c\varphi \\ M^*_{y1} = -\frac{\sqrt{3}}{2} d l \omega^2 Q_3 s\varphi \end{cases} \Rightarrow$$

M in $-\varphi$ - direction



L7)

⊛ Additional Approach: Reducing the crank torque

We calculated frame torque before, but now, this just focus on the crank shaft

$$\left.\begin{aligned} F_3 &= -Q_3 \cdot \ddot{x}_B \\ F_s &= \frac{F_3}{c\psi} \end{aligned}\right\} \Rightarrow \begin{aligned} M_{1,1} &= -F_s \cdot x_B \cdot s\psi \\ &= Q_3 \cdot x_B \cdot \ddot{x}_B \cdot \tan\psi \end{aligned}$$

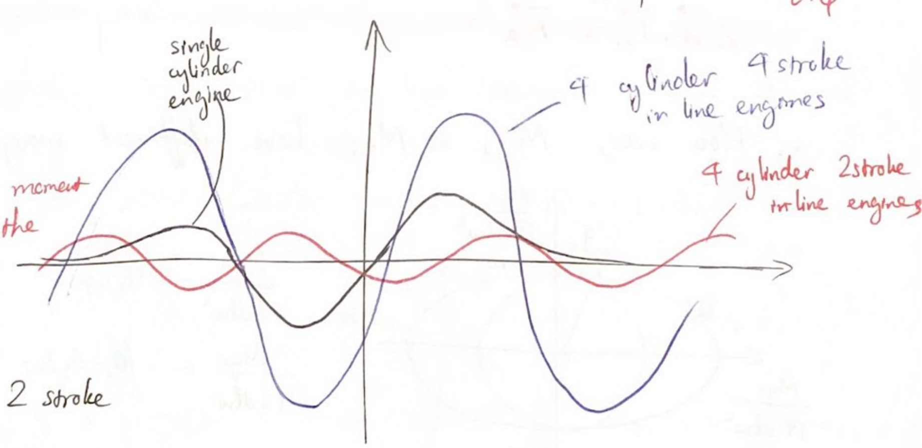
Moment on the crank (part 1) ~~from~~ (part 1)

$$\left.\begin{aligned} M_{2,2} &= J_2 \cdot \ddot{\psi} \\ F_{N,2} &= \frac{M_{2,2}}{l_2 \cdot c\psi} \end{aligned}\right\} \Rightarrow \begin{aligned} M_{1,2} &= -F_{N,2} \cdot l_1 \cdot c\varphi \\ &= -J_2 \cdot \ddot{\psi} \, \frac{l_1 c\varphi}{l_2 c\psi} \end{aligned}$$

Moment on the crank (part 1) ~~from~~ (part 2) ~~connecting rod~~

$$\Rightarrow -M_1 = M_{1,1} + M_{12} = Q_3 \cdot x_B \cdot \ddot{x}_B \cdot \tan\psi - J_2 \cdot \ddot{\psi} \cdot \frac{l_1 c\varphi}{l_2 c\psi} = \frac{dE_{kin}(\varphi)}{d\varphi}$$

torque on crank-shaft = driving torque



⊗ There is an effect of moment on the crank-shaft, from the inertia forces.

⊗ The 4 cylinder 2 stroke in line engines:

with irregular firing order (123 12 112)

We want combustion torque ≈ composite torque

⇒ use Cross-plane crankshaft to reduce inertia torque ≈ 0

Power Smoothing

7)

In mass balancing, we assume known ω (as a constant)

- Power: $P = M.\omega = F.v$

⇒ Aims: reducing non-uniformity, peaks ..

⇒ Strategy: 1) Determine motion/speed $\omega(\varphi)$
2) Reduce the alternation of it

But in here, we can not, we must find it and consider try to reduce its alternation through power P

- Power balance:

$$\boxed{P = \sum P_{in} + \sum P_{out} + \sum P_{loss} = \frac{d(E_{kin} + E_{pot})}{dt} = M_{red}(\varphi) \cdot \omega(\varphi) = M_{red}(\varphi) \cdot \frac{d\varphi}{dt}}$$

$$\Leftrightarrow M_{red}(\varphi) = \frac{d E_{kin}(\varphi)}{d\varphi} + \frac{d E_{pot}(\varphi)}{d\varphi}$$ Differential form

$$\Leftrightarrow \int_{\varphi_0}^{\varphi} M_{red}(\varphi) d\varphi = E_k(\varphi) - E_k(\varphi_0) + E_p(\varphi) - E_p(\varphi_0)$$ Integral form

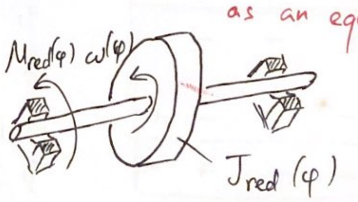
$\boxed{\text{Find } \omega \text{ with this}}$

$$\Rightarrow \omega(\varphi) = \pm \sqrt{\frac{2}{J_{red}(\varphi)} \left[\Delta W(\varphi) + E_k(\varphi_0) - E_p(\varphi) + E_p(\varphi_0) \right]}$$ ⇒ We need ΔW, $E_k(\varphi_0)$, E_p ... to find $\omega(\varphi)$

Cause $E_k(\varphi) = \frac{1}{2} \cdot J_{red}(\varphi) \cdot \omega^2(\varphi)$ & Set $\Delta W(\varphi) = \int_{\varphi_0}^{\varphi} M_{red}\, d\varphi$

We will REDUCE the whole engine system to a simple fly-wheel as an equipvalent system

+) Mred (φ): represents all external forces, torques acting on the machine ..



4 Important Parameters

⊗) **Mred (φ)** – External forces & Torques:

① $P = M_{red}\,\omega_1 = F_3 \dot{x}_B + M_1 \omega_1 = \dfrac{d}{dt}(E_k + E_p)$

$\Rightarrow \quad M_{red} = F_3 \dfrac{\dot{x}_B}{\omega_1} + M_1$

$= \quad M_t \quad - M_{load}$

$M_{eff} = -M_1$

M_t: tangential torque

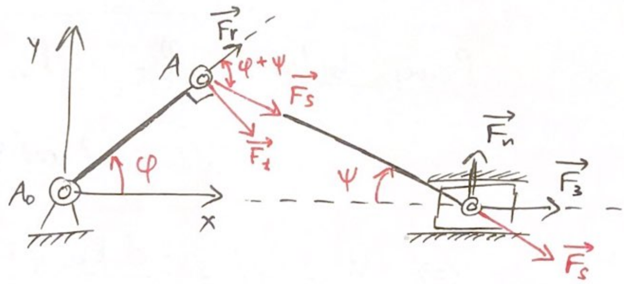
M_{load}: load torque (effective output torque aka crank torque)

$\Rightarrow M_1^{*} = -Q_3 . x_B . \ddot{x}_B . \tan\psi + J_2 . \ddot{\psi} . \dfrac{l_1 c\varphi}{l_2 s\varphi} = \dfrac{dE_k}{d\varphi}$

+) To find $M_t(\varphi)$ – Tangential ~~force~~ torque:

$$\begin{cases} x_B = l_1 c\varphi + l_2 c\psi \\ l_1 s\varphi = l_2 \cancel{s\varphi}\, s\psi \end{cases}$$

$$\Rightarrow \begin{cases} \dot{x}_B = -l_1 \omega_1 s\varphi - l_2 \dot{\psi} s\psi \\ l_1 \omega_1 c\varphi = l_2 \dot{\psi} c\psi \end{cases}$$



$\Rightarrow \quad \dfrac{\dot{x}_B}{\omega_1} = -l_1 \dfrac{s(\varphi+\psi)}{c\psi}$

$\Rightarrow \quad M_t = F_3 \dfrac{\dot{x}_B}{\omega_1} = F_t\, l_1 \quad \Rightarrow \quad F_t = -F_s\, s(\varphi+\psi) = -F_3 \dfrac{s(\varphi+\psi)}{c\psi}$

Find M_t without ω_1 but from φ & ψ

L8)

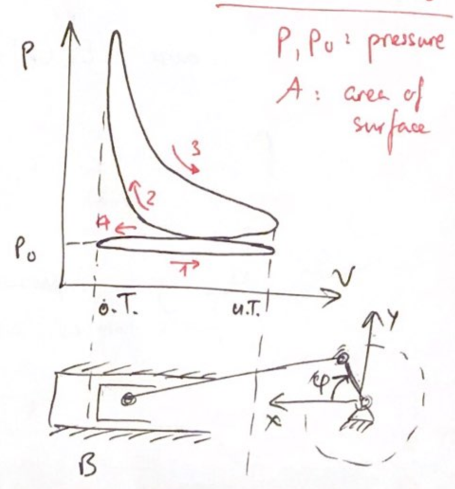
$\omega_1 = \dfrac{d\varphi}{dt}$, because we have rigid system

⊗ x_B actually just depends only on φ, we don't need $t \Rightarrow$ don't need ω

+) $M_t = F_3 \dfrac{\dot{x}_B}{\omega_1}$

$F_3 = -(p - p_0) A$

$\dot{x}_B = \omega_1 l_1 \left(-s\varphi - \frac{1}{2} A_2 s2\varphi + \frac{1}{4} A_4 s4\varphi \dots\right)$



p, p_0 : pressure

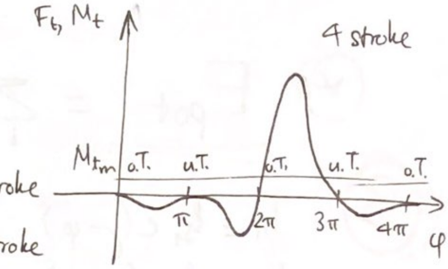
A : area of surface

stroke	1	2	3	4
F_3	+	–	–	–
$\dot{x}_B$	–	+	–	+
M_t	–	–	+	–

1) Intake stroke
2) Compression stroke
3) Power stroke
4) Exhaust stroke

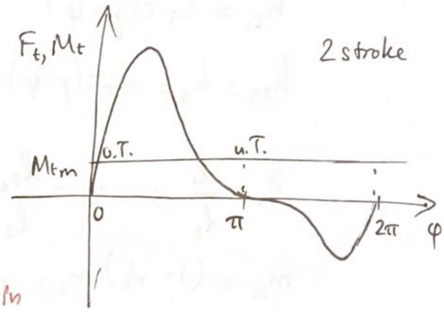
+) <u>Mean tangential torque M_{tm}</u>

$$M_{tm} = \frac{1}{2\pi q} \int_{\varphi_0}^{\varphi_0 + 2\pi q} M_t(\varphi)\, d\varphi \; ; \quad q = \begin{cases} 1 & \text{- 2 stroke} \\ 2 & \text{- 4 stroke} \end{cases}$$



+) <u>Mean shaft torque M_{1m}:</u>

$$M_{1m} = \frac{1}{2\pi q} \int_{\varphi_0}^{\varphi_0 + 2\pi q} M_1(\varphi)\, d\varphi$$



For steady state operation: after a period, we don't put in any work

$$\int M_{red} \cdot d\varphi = 0 \Rightarrow M_{1m} = -M_{tm}$$

⊗ <u>$J_{red}(\varphi)$</u>

$$E_{kin} = \frac{1}{2} J_{red}\, \omega^2 = \sum_i E_{kin,i}$$

2

+) Crank: $E_{k1} = \frac{1}{2} m_1 l_{S_1}^2 \omega_1^2 + \frac{1}{2} J_{S_1} \omega_1^2$

$$= \frac{1}{2} J_1 \omega_1^2$$

+) Connecting rod: $E_{k2} = \frac{1}{2} m_{21} l_1^2 \omega_1^2 + \frac{1}{2} J_{21} \dot{\psi}^2 + \frac{1}{2} m_{23} \dot{x}_B^2 + \frac{1}{2} J_{23} \dot{\psi}^2$

$$= \frac{1}{2} m_2 \left[(1 - \nu_2) l_1^2 \omega_1^2 + \nu_2 \dot{x}_B^2 \right] + \frac{1}{2} J_2 \dot{\psi}^2$$

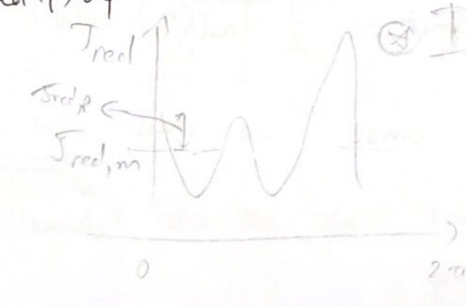
+) Piston: $E_{k3} = \frac{1}{2} m_3 \dot{x}_B^2$

$$\Leftrightarrow J_{red}(\varphi) = J_1 + m_2 (1 - \nu_2) l_1^2 + (m_3 + \nu_2 m_2) x_B'^2 + J_2 \psi'^2$$

$$= J_{red,m} + J_{red,p}(\varphi)$$

$$J_{red,m} = \frac{1}{2\pi} \int_0^{2\pi} J_{red}(\varphi)\, d\varphi$$ mean

$$\int_0^{2\pi} J_{red,p}(\varphi) \cdot d\varphi = 0$$

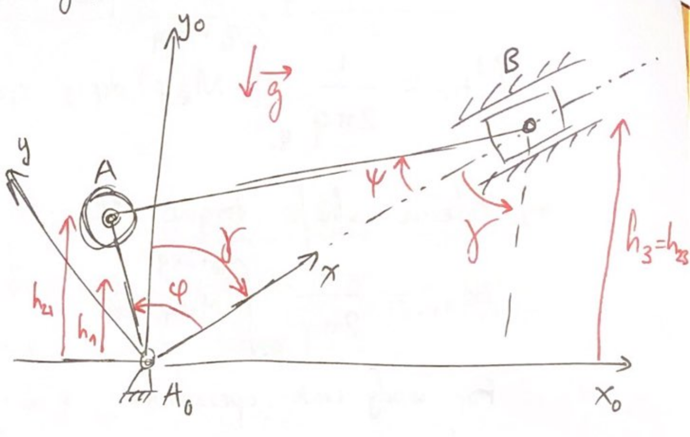


(*) $\underline{E_{pot}} = \sum_i E_{p_i} = \sum_i m_i g h_i$

(3) $h_1 = l_{S_1} c(\gamma - \varphi)$

$h_{21} = l_1 c(\gamma - \varphi)$

$h_{23} = h_3 = l_1 \cdot c(\gamma - \varphi) + l_2 \cdot c(\psi + \gamma)$



$\lambda_1 = \frac{l_1}{l_2}$; $\quad \upsilon_2 = \frac{l_{S_2}}{l_2}$

$m_{21} = (1 - \upsilon_2) m_2$; $\quad m_{23} = \upsilon_2 m_2$

$\Rightarrow E_{pot} = g l_1 \left[(\upsilon_1 m_1 + m_2 + m_3) c(\gamma - \varphi) + \frac{1}{\lambda} (\upsilon_2 m_2 + m_3)\, c(\psi + \gamma) \right]$

(4) Equation of motion: $P = M_{red}(\varphi) . \omega(\varphi) = \sum P$

$= M_{red} \frac{d\varphi}{dt} = \frac{d(E_k + E_p)}{dt}$

(*) $\underline{\text{Question}}$: How does the mechanism move

when $\left[\begin{array}{l} \text{there are no external forces } \& \text{ moments (no losses = frictionless)} \\ \Delta E_p = 0 \end{array} \right.$

$\underline{\text{Answer:}}$ $\left. \begin{array}{l} M_{red} = 0 \;;\; \sum P = 0 \\ \Delta E_p = 0 \end{array} \right\} \Rightarrow \frac{dE_k}{d\varphi} = 0$

$\Rightarrow E_k = \text{const} = \frac{1}{2} J_{red}(\varphi) . \omega^2(\varphi)$

J_{red} is not const $\Rightarrow$ thus $\underline{\omega \neq \text{const}}$

For $J_{red} = \text{const}$, we will add balancing weight with specific radius with each φ

L_9/

- Steady & periodically operation:

$$\Delta W(2q\pi) = \int_{\varphi_0}^{\varphi_0+2q\pi} M_{red}(\varphi)\, d\varphi = 0$$

$M_{red} \neq 0$; but its effect in a period: $= 0$

Solution for the equation of Motion

+) General case: $M_{red}(\varphi) = \frac{dE_k}{d\varphi} + \frac{dE_p}{d\varphi}$

1 $$\omega(\varphi) = \sqrt{\frac{2}{J_{red}(\varphi)}\left[\Delta W(\varphi) + E_k(\varphi_0) - E_p(\varphi) + E_p(\varphi_0)\right]}$$

In high speed engine: $\Delta E_k \gg \Delta E_p \Leftrightarrow \Delta E_p \approx 0$

$$\Leftrightarrow \quad M_{red} \approx \frac{dE_k}{d\varphi}$$

$$\omega(\varphi) \approx \sqrt{\frac{2}{J_{red}(\varphi)}\left[\Delta W(\varphi) + E_k(\varphi_0)\right]}$$

5 cases

+) $J_{red}(\varphi) = const$ (Constant reduced moment of inertia)

2 $$\Rightarrow \omega(\varphi) \approx \sqrt{\frac{2 \cdot \Delta W(\varphi)}{J_{red}} + \omega_0^2}$$

+) $\omega = const$ (Constant angular crank speed)

3 $$M_{red}(\varphi) = M_t + M_1 = \frac{dE_k}{d\varphi} = \frac{d}{d\varphi}\left(\frac{1}{2} J_{red}(\varphi) \cdot \omega^2\right) = \frac{1}{2} J'_{red}(\varphi) \cdot \omega^2$$

$$\Rightarrow M_{eff}(\varphi) = -M_1(\varphi) = M_t - \frac{1}{2} J'_{red}(\varphi) \cdot \omega^2$$

(4) +) ω & α are given (given motion)

$$M_{red} = \frac{dE_k}{d\varphi} = \frac{d}{d\varphi}\left(\frac{1}{2} J_{red}(\varphi) \,.\, \omega^2(\varphi)\right) = \frac{1}{2} J_{red}' \omega^2 + \frac{1}{2} J_{red} \cdot 2\omega \cdot \frac{d\omega}{d\varphi}$$

$\frac{d\varphi}{dt}$

$$= \frac{1}{2} J_{red}'(\varphi) \,.\, \omega^2(\varphi) + J_{red}(\varphi) \,.\, \alpha(\varphi)$$

Extended Law of Conservation of Angular Momentum

$\Rightarrow$ Given motion, we can find the torque M_{red}

(5) +) $E_k = const$ (constant energy)

$$E_k(\varphi) = E_k(\varphi_0) \Leftrightarrow \frac{1}{2} J_{red}(\varphi) \,.\, \omega^2(\varphi) = \frac{1}{2} J_{red}(\varphi_0) \,.\, \omega^2(\varphi_0)$$

$$\Rightarrow \omega^2(\varphi) = \frac{J_{red}(\varphi_0)}{J_{red}(\varphi)} \cdot \omega_0^2 \Rightarrow \frac{\omega_{max}}{\omega_{min}} = \sqrt{\frac{J_{red,max}}{J_{red,min}}}$$

Use fly wheel to reduce that ratio

$$\frac{\omega_{max}}{\omega_{min}} = \sqrt{\frac{J_{red,max} + J_{SR}}{J_{red,min} + J_{SR}}} < \sqrt{\frac{J_{red,max}}{J_{red,min}}}$$

Example: $1{,}32 = \sqrt{\frac{4+3}{1+3}} < \sqrt{\frac{4}{1}} = 2$

$\Rightarrow$ This is also a way to achieve $\omega \approx const$ as $\frac{\omega_{max}}{\omega_{min}} \to 1$

+) Fluctuation of Angular Speed:

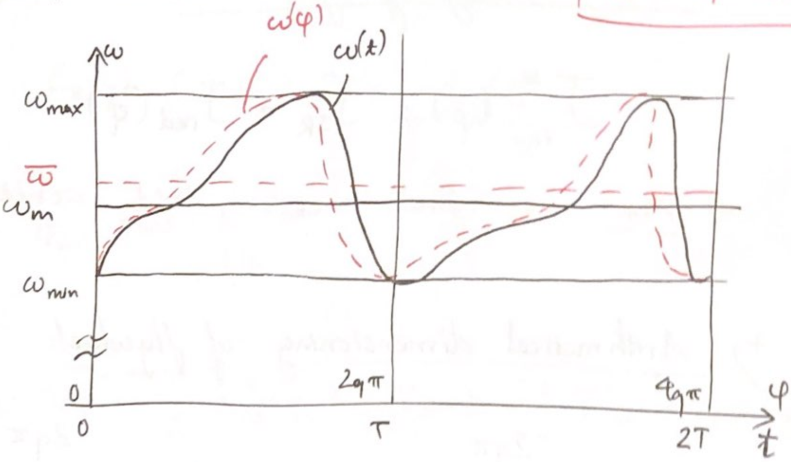
$$\boxed{\omega(t) \neq \omega(\varphi)}$$

- Temporal mean:

$$\omega_m = \frac{1}{T}\int_0^T \omega(t)\,dt$$

- Arithmetic mean:

$$\overline{\omega} = \frac{1}{2}(\omega_{max} + \omega_{min})$$



- 1 cycle correspond to q revolution of driving shaft

$$\omega_m = \frac{2q\pi}{T}$$

$$\omega(\varphi) = \frac{d\varphi}{dt} \Rightarrow dt = \frac{d\varphi}{\omega(\varphi)} \Rightarrow \int_{\varphi_0}^{\varphi} dt = t(\varphi) - t(\varphi_0) = \int_{\varphi_0}^{\varphi} \frac{d\varphi}{\omega(\varphi)}$$

$$\Rightarrow T = \int_0^{2q\pi} \frac{d\varphi}{\omega(\varphi)}$$

$$\omega_m = \frac{2q\pi}{\int_0^{2q\pi} \frac{d\varphi}{\omega(\varphi)}}$$

Note: what we have, the solution from equation of motion is $\omega(\varphi)$

We don't have $\omega(t)$

+) Non-uniformity factor:

$$\delta = \frac{\omega_{max} - \omega_{min}}{\omega_m}$$

$$\overline{\delta} = \frac{\omega_{max} - \omega_{min}}{\overline{\omega}} = 2\,\frac{\omega_{max} - \omega_{min}}{\omega_{max} + \omega_{min}}$$

$$\frac{\overline{\delta}}{\delta} = \frac{\omega_m}{\overline{\omega}}$$

If $\delta \ll 1$

$\Rightarrow \omega_m \approx \overline{\omega}$ & $\delta \approx \overline{\delta}$

$$\delta \approx \overline{\delta}\left(1 \pm \frac{1}{2}\overline{\delta}\right) \quad \text{with } \delta \ll 1$$

⊗ Dimensioning of flywheel:

$$J^{*}_{red}(\varphi) = J_{SR} + J_{red}(\varphi)$$

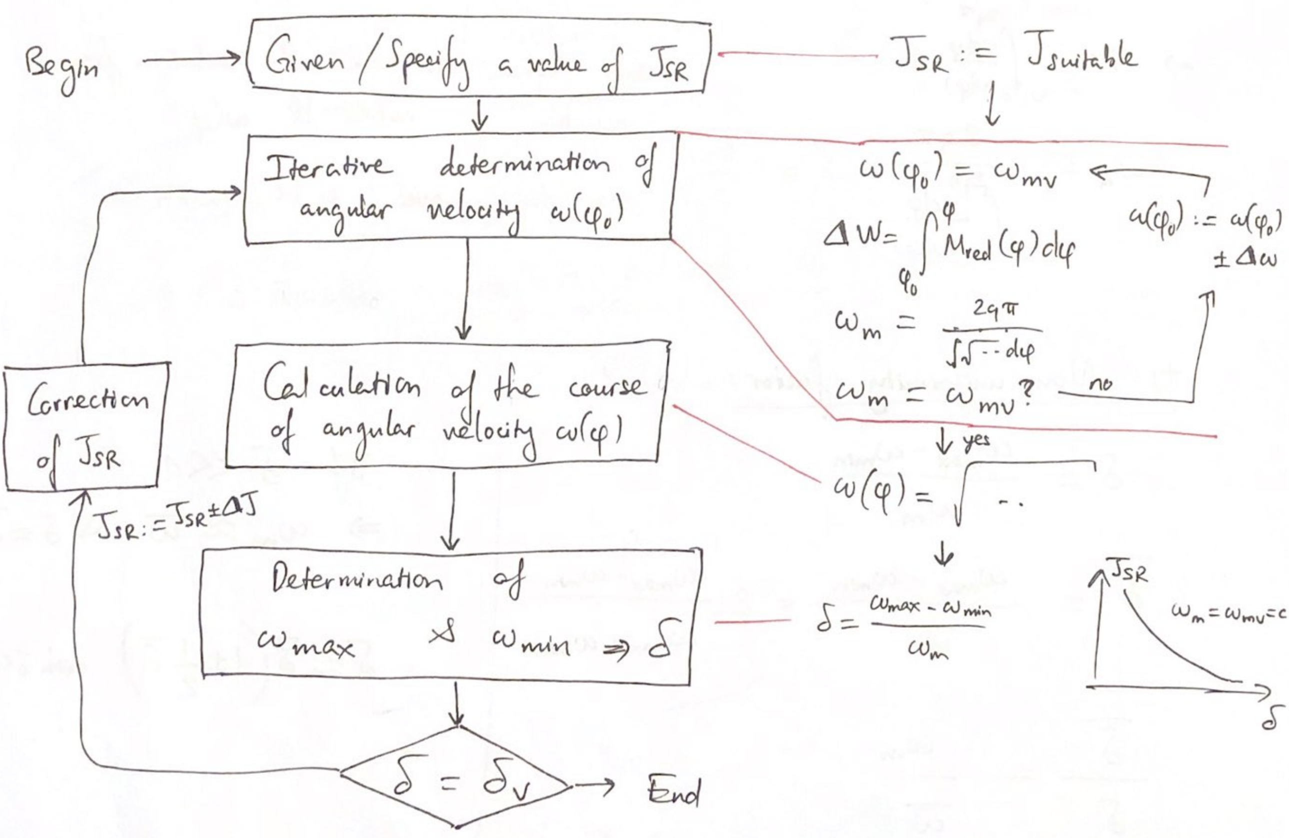
⇒ Aim: to find J_{SR}, that achieve a value of δ_V

① +) Arithmetical dimensioning of flywheel:

$$\omega_m = \frac{2q\pi}{\int_0^{2q\pi} \frac{d\varphi}{\omega(\varphi)}} = \frac{2q\pi}{\int_0^{2q\pi} \sqrt{\frac{J^{*}_{red}(\varphi)}{2\Delta W(\varphi) + J^{*}_{red}(\varphi_0)\cdot \omega^2(\varphi_0)}}\, d\varphi}$$

The main problem is that $\omega(\varphi_0)$ is unknown

- Given: mean angular velocity ω_{mv}, non-uniformity factor δ_V
  $J_{red}(\varphi)$, $M_{red}(\varphi)$, $E_\varphi = const$



② +) Direct (graphical) determination of J_{SR}:

$$\int_{\varphi_0}^{\varphi} M_{red}(\varphi).d\varphi = E_k(\varphi) - E_k(\varphi_0) + \underbrace{E_p(\varphi) - E_p(\varphi_0)}_{0}$$

Consider φ_n & φ_m that $\begin{cases} \omega_{min} = \omega(\varphi_n) \\ \omega_{max} = \omega(\varphi_m) \end{cases}$

$$\Rightarrow \Delta W_{max} = \int_{\varphi_n}^{\varphi_m} M_{red}(\varphi).d\varphi = \frac{1}{2} J^*_{red}(\varphi_m).\omega^2(\varphi_m) - \frac{1}{2} J^*_{red}(\varphi_n).\omega^2(\varphi_n)$$

$$= \frac{1}{2} J_{SR}(\omega^2_{max} - \omega^2_{min}) + \frac{1}{2} J_{red}(\varphi_m).\omega^2_{max} - \frac{1}{2} J_{red}(\varphi_n).\omega^2_{min}$$

$$\Rightarrow \boxed{J_{SR} = \frac{2\Delta W_{max} + J_{red}(\varphi_n).\omega^2_{min} - J_{red}(\varphi_m).\omega^2_{max}}{\omega^2_{max} - \omega^2_{min}}}$$

- With: $\begin{cases} \omega_{max} = \overline{\omega}\left(1 + \frac{1}{2}\overline{\delta}\right) \\ \omega_{min} = \overline{\omega}\left(1 - \frac{1}{2}\overline{\delta}\right) \end{cases} \Rightarrow \omega^2_{max} - \omega^2_{min} = 2\overline{\delta}\,\overline{\omega}^2$

and $\overline{\omega} \approx \omega_{mv}$

$\overline{\delta} \approx \delta_v$

- $\begin{cases} E_k(\varphi_m) \approx E_{k\,max} \\ E_k(\varphi_n) \approx E_{k\,min} \end{cases} \Leftrightarrow \begin{cases} \overline{\varphi}_m = \varphi(E_{k\,max}) \\ \overline{\varphi}_n = \varphi(E_{k\,min}) \end{cases}$ use as substitution for φ_m & φ_n

- Use M_{red} plot to find $\overline{\varphi}_m$ & $\overline{\varphi}_n$

One among position where $M_{red} = 0$

Cause $M_{red}(\varphi) = \dfrac{dE^*_k(\varphi)}{d\varphi}$

$\Rightarrow E_k$ reach extremem when $M_{red} = 0$

$$\Rightarrow \boxed{J_{SR} \approx \frac{\Delta W_{max,k}}{\delta_v\,\omega^2_{mv}} - \frac{J_{red}(\overline{\varphi}_m).\omega^2_{max} - J_{red}(\overline{\varphi}_n).\omega^2_{min}}{2.\delta_v.\omega^2_{mv}}}$$

- Even can approximate even more, to be more simple.

$$J_{SR} = \frac{\Delta W_{max,k}}{\delta_v \cdot \omega^2_{mv}} - \frac{J_{red,min}(\omega^2_{max} - \omega^2_{min})}{2\delta_v \omega^2_{mv}} - \underbrace{\frac{J_{red,v}(\bar{\varphi}_m)\omega^2_{max} - J_{red,v}(\bar{\varphi}_n)\omega^2_{min}}{2\delta_v \cdot \omega^2_{mv}}}_{\text{Ignore this}}$$

$$\Rightarrow J_{SR} \approx \frac{\Delta W_{max,k}}{\delta_v \cdot \omega^2_{mv}} - J_{red,min}$$

Approximate formular of "Radinger"

(3) +) Exact graphical determination of J_{SR}:

Find intersection of $M_{red}(\varphi)$ plot with $M_{red\,\varphi_n}(\varphi) = \frac{1}{2} J'_{red}(\varphi) \cdot \omega^2_{min}$

and $M_{red\,\varphi_m}(\varphi) = \frac{1}{2} J'_{red}(\varphi) \cdot \omega^2_{max}$

/ MDcR3

Exercise & Exam Prep

Ng Huu Duc

1, Kinematics

- Careful with direction coordination $\uparrow^{x} \rightarrow y$ $\quad \uparrow \curvearrowleft^{\varphi} \rightarrow$
- Calculate $x \to \dot{x} \to \ddot{x}$

2, Inertia Forces:

$\vec{F} = -m\vec{a}$ $\quad \vec{F}_x = -\sum_i m_i \ddot{x}_i$

$|\vec{F}| = m.\omega^2.l$ $\quad \vec{F}_y = -\sum_i m_i \ddot{\vec{y}}_i$

3, Dynamic Equipvalent System of Connecting Rod:

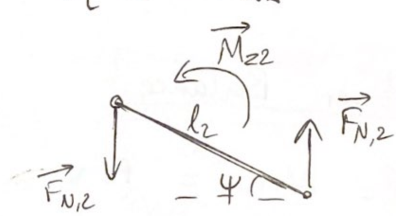
- mass $\quad m_{21} + m_{23} = m_2$
- center of gravity $\quad m_{21} l_{s_2} = m_{23}(l_2 - l_{s_2})$
- inertia tensor $\quad J_{s_2} = J_{21} + m_{21} l_{s_2}^2 + \ldots$

4, Frame Torque:

$$\vec{M}_z = \vec{M}_\alpha + \vec{M}_a$$
$$= -J_{s_i}.\vec{\alpha}_i + \vec{r}_{s_i} \times \vec{F}_{s_i}$$
$$= -J_{s_i}.\vec{\alpha}_i - m_i\left[\vec{r}_{s_i} \times \vec{a}_{s_i}\right]$$

5, Transform torque to force:

$\vec{M} \to \vec{F}$



$$F_{N,2} . l_2 . c\psi = \vec{M}_{z,2}$$

6, Inertia forces of single cylinder engine:

$$\frac{F_x}{\omega_1^2 l_1} = (Q_1 + Q_3)c\varphi + Q_3(A_2 c2\varphi - A_4 c4\varphi + A_6 c6\varphi - + ..)$$

$$\frac{F_y}{\omega_1^2 l_1} = Q_1 \, s\varphi$$

2

7) Moment of single cylinder engine:

$$\lambda = \frac{l_1}{l_2}$$

$$M_z \approx J_2 \ddot{\psi} = M_{2,z} = -\omega^2 \lambda J_2 \left(C_1 s\varphi - C_3 s3\varphi + C_5 s5\varphi - + .. \right)$$

$$M_x = \pm d . F_{y_i} \quad ; \quad M_y = \pm d . F_{x_i}$$

- To balance inertia torque : frame torque M_z: $J_2 = 0 \Leftrightarrow l_2 = 2 . l_{s_2}$

8) Multi Cylinder Engine:

- Pay attention to other order, not just the 1st order. harmonic.
  2nd, 4th ..
- When balance, prioritize lower order harmonic first

9) Torque on the crank shaft = Driving torque:

F_1 : none

F_3 : $\vec{M}_{1,1} = \vec{F_s} . q \Rightarrow M_{1,1} = Q_3 . x_B . \ddot{x}_B . \tan\varphi$

J_2 : $\vec{M}_{1,2} = -F_{N,2} . l_1 . c\varphi = -J_2 . \ddot{\psi} \dfrac{l_1 c\varphi}{l_2 c\psi}$

$$\Rightarrow M_1 = -\left(M_{1,1} + M_{1,2} \right) = - Q_3 . x_B . \ddot{x}_B . tg\varphi + J_2 \ddot{\psi} \lambda \frac{c\varphi}{c\psi}$$

10) Power Balance:

$$P = M . \omega = F . v$$

$$P = \Sigma P_{in} + \Sigma P_{out} + \Sigma P_{loss} = \frac{d(E_{kin} + E_{pot})}{dt}$$

$$= M_{red}(\varphi) . \omega(\varphi) = M_{red}(\varphi) . \frac{d\varphi}{dt}$$

$$\Leftrightarrow M_{red} = \frac{dE_{kin}}{d\varphi} + \frac{dE_{pot}}{d\varphi}$$

$$\Leftrightarrow \Delta W = \int_{\varphi_0}^{\varphi} M_{red} . d\varphi = E_k(\varphi) - E_k(\varphi_0) + E_p(\varphi) - E_p(\varphi_0)$$

$$\Leftrightarrow \omega(\varphi) = \sqrt{\frac{2}{J_{red}(\varphi)} \left[\Delta W(\varphi) + E_k(\varphi_0) - E_p(\varphi) + E_p(\varphi_0) \right]}$$

$M_{red}\,\omega_1 = P = F_3 \cdot \dot{x}_B + M_1\,\omega_1$

$M_{1,1} = -F_3 \cdot x_B \cdot S\psi$

$M_{1,2} = -F_{N,12}\, l_1\, c\varphi$

$$\Rightarrow \boxed{M_{red} = F_3 \frac{\dot{x}_B}{\omega_1} + M_1}$$

$-M_1 = M_{eff}$

$M_1 = -M_{1,1} - M_{1,2}$

$= \ldots$

$M_{red} = M_t + M_1$

*) Mean torque: $M_{tm} = \dfrac{1}{2\pi q}\displaystyle\int_{\varphi_0}^{\varphi_0+2\pi q} M_t(\varphi)\,d\varphi$ $\qquad q = \begin{cases} 1 - 2\text{ stroke} \\ 2 - 4\text{ stroke}\end{cases}$

$$M_{1m} = \frac{1}{2\pi q}\int_{\varphi_0}^{\varphi_0+2\pi q} M_1(\varphi)\,d\varphi$$

*) Steady state operation: $\displaystyle\int_{\varphi_0}^{\varphi_0+2\pi\ldots} M_{red}\,d\varphi = 0 \Leftrightarrow M_{1m} = -M_{tm}$

J_{red}: $\quad E_{kin} = \frac{1}{2} J_{red} \cdot \omega^2$

Find J_{red} from $E_{kin} = \Sigma \frac{1}{2} m v^2 + \frac{1}{2} J \omega^2$

$m . l^2 . \omega^2 \ldots$

II) Fluctuation of Angular Speed:

$$\omega_m = \frac{2q\pi}{\displaystyle\int_0^{2q\pi} \frac{d\varphi}{\omega(\varphi)}} = \frac{2q\pi}{T}$$

non-uniformity factor:

$$\delta = \frac{\omega_{max} - \omega_{min}}{\omega_m}$$

$$\bar{\delta} = \frac{\omega_{max} - \omega_{min}}{\bar{\omega}} = 2\frac{\omega_{max} - \omega_{min}}{\omega_{max} + \omega_{min}}$$

Temporal mean: ω_m

Arithmetic mean: $\bar{\omega} = \frac{1}{2}(\omega_{max} + \omega_{min})$

$$\cancel{J_{SR}}\ \Delta W_{max} = \int_{\varphi_n}^{\varphi_m} M_{red}(\varphi)\,d\varphi = \frac{1}{2}\left(J_{SR} + J_{red}(\varphi)\right) . \omega(\varphi)^2 \Big|_{\varphi_n}^{\varphi_m}$$

$$= \frac{1}{2} J_{SR}(\omega_{max}^2 - \omega_{min}^2) + \frac{1}{2} J_{red}(\varphi_m) . \omega_{max}^2 - \frac{1}{2} J_{red}(\varphi_n) . \omega_{min}^2$$

$$\Leftrightarrow J_{SR} = \frac{2\Delta W_{max} + J_{red}(\varphi_n) \cdot \omega^2_{min} - J_{red}(\varphi_m) \cdot \omega^2_{max}}{\omega^2_{max} - \omega^2_{min}}$$

$$J_{SR} \approx \frac{\Delta W_{max,k}}{\delta_v \cdot \omega^2_{mv}} - J_{red,\,min}$$ ⊛ Radinger.