# Robotics Notes

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# Contents

# Abbreviations

| | |
|---|---|
| **prob.** | probability |
| **a.k.a.** | also known as |
| **func.** | function |
| **vs.** | versus |
| **KF** | Kalman Filter |
| **EKF** | Extended Kalman Filter |
| **IF** | Information Filter |
| **EIF** | Extended Information Filter |
| **MHEKF** | Multi-Hypothesis Extended Kalman Filter |
| **MDP** | Markov Decision Process |
| **POMDP** | Partially Observable Markov Decision Process |

# 1 Probabilistic Robotics

Reference from the great book: [**thrun2006probalistic**].

## 1.1 State Estimation

### 1.1.1 Bayes Filters

$$\begin{aligned} bel(x_t) &= p(x_t|z_{1:t}, u_{1:t}) && \text{belief over a state} \\ \overline{bel}(x_t) &= p(x_t|z_{1:t-1}, u_{1:t}) && \text{a posterior (before adapt to } z_t) \\ \Rightarrow \text{Calculating } & bel(x_t) \text{ from } \overline{bel}(x_t) && \text{correction/measurement update} \end{aligned}$$

**Bayes Filter Algorithm:**

2 for all $x_t$ do:

3 $\overline{bel}(x_t) = \int p(x_t|u_t, x_{t-1}) bel(x_{t-1}) dx$ prediction step

4 $bel(x_t) = \eta \, p(z_t|x_t) \, \overline{bel}(x_t)$ update step

$\Rightarrow$ Can only be implemented for very simple estimation problems, finite state space

**Important assumption: Markov property** (each state s a complete summary of the past)

**Problem:** can not be implemented on digital computers

The next subsections describe two Gaussian filters (Kalman Filter (KF) and Information Filter (IF)) and two extensions of them (Extended Kalman Filter (EKF) and Extended Information Filter (EIF)). The Gaussian filters have major advantage in computational cost, with the disadvantage of having assumption on uni-model distribution.

### 1.1.2 The Kalman Filter (KF)

- ***Learning Resources:*** www.bzarg.com
- For continuous space, not discrete or hybrid

- **Assumption:** posterior are Gaussians and Markov property

| | |
|---|---|
| $p(x_t \vert u_t, x_{t-1})$ | must be linear func. (linear system dynamics) |
| $p(z_t, x_t)$ | also linear |
| $bel(x_0)$ | initial belief must be Gaussian |

$$\begin{aligned}
x_t &= A_t x_{t-1} + B_t u_t + \varepsilon_t \\
p(x_t|u_t, x_{t-1}) &= \det(2\pi\Sigma_t)^{-\frac{1}{2}} \exp\left[-\frac{1}{2}(x_t - A_t x_{t-1} - B_t u_t)^T R_t^{-1} (x_t - A_t x_{t-1} - B_t u_t)\right] \\
z_t &= C_t x_t + \delta_t \quad (= y) \\
p(z_t|x_t) &= \det(2\pi Q_t)^{-\frac{1}{2}} \exp\left[-\frac{1}{2}(z_t - C_t x_t)^T Q_t^{-1} (z_t - C_t x_t)\right] \\
bel(x_0) = p(x_0) &= \det(2\pi\Sigma_0)^{-\frac{1}{2}} \exp\left[-\frac{1}{2}(x_t - \mu_0)^T \Sigma_0^{-1} (x_t - \mu_0)\right]
\end{aligned}$$

**Kalman Filter Algorithm:** $(\mu_{t-1}, \Sigma_{t-1}, u_t, z_t)$

$$\begin{array}{lll}
2 & \overline{\mu}_t = A_t \mu_{t-1} + B_t u_t & \\
3 & \overline{\Sigma}_t = A_t \Sigma_{t-1} A_t^T + R_t & \Big\} \text{ incorporate } u_t \text{ - prediction step - } \mathcal{O}(n^2) \\
4 & K_t = \overline{\Sigma}_t C_t^T \left(C_t \overline{\Sigma}_t C_t^T + Q_t\right)^{-1} \quad \text{(Kalman gain)} & \\
5 & \mu_t = \overline{\mu}_t + K_t(z_t - C_t \overline{\mu}_t) & \Bigg\} \text{ incorporate } z_t \text{ - correction step - } \mathcal{O}(n^{2,8}) \\
6 & \Sigma_t = (I - K_t C_t)\overline{\Sigma}_t & \\
7 & \text{return } \mu_t, \Sigma_t & \Rightarrow \text{ belief at time } t
\end{array}$$

$\Rightarrow$ quite computationally expensive, **everything are Gaussians**

### 1.1.3 Extended Kalman Filter (EKF)

Overcome the assumption on linearity by only approximate by Gaussians

$$x_t = g(u_t, x_{t-1}) + \varepsilon_t$$
$$z_t = h(x_t) + \delta_t$$

$$
\begin{aligned}
g(u_t, x_{t-1}) &\approx g(u_t, \mu_{t-1}) + g'(u_t, \mu_{t-1})(x_t - \mu_{t-1}) \\
&= g(u_t, \mu_{t-1}) + G_t(x_t - \mu_{t-1}) \\
& g' \text{ is the Jacobian of state } (n \times n \text{ matrix}) \\
p(x_t|u_t, x_{t-1}) &\approx \det(2\pi R_t)^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}[x_t - g(u_t, x_{t-1})]^T R_t^{-1}[x_t - g(u_t, x_{t-1})]\right\} \\
h(t) &\approx h(\overline{\mu}_t) + h'(\mu_t)(x_t - \mu_{t-1}) \\
&= h(\overline{\mu}_t) + H_t(x_t - \mu_{t-1}) \\
p(z_t|x_t) &= \det(2\pi Q_t)^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}[z_t - h(x_t)]^T Q_t^{-1}[z_t - h(x_t)]\right\}
\end{aligned}
$$

**Extended Kalman Filter Algorithm:** $(\mu_{t-1}, \Sigma_{t-1}, u_t, z_t)$

2 $\overline{\mu}_t = g(u_t, \mu_{t-1})$

3 $\overline{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + R_t$

4 $K_t = \overline{\Sigma}_t H_t^T \left(H_t \overline{\Sigma}_t H_t^T + Q_t\right)^{-1}$

5 $\mu_t = \overline{\mu}_t + K_t[z_t - h(\overline{\mu}_t)]$

6 $\Sigma_t = (I - K_t H_t)\overline{\Sigma}_t$

7 return $\mu_t, \Sigma_t$

- Can extend EKF $\Rightarrow$ Multi-Hypothesis Extended Kalman Filter (MHEKF)
- EKF's performance depends on degree of nonlinearities and uncertainty
- Unscented KF and moments matching KF are better

### 1.1.4 Information Filter (IF)

- Moment representation: $\mu$ & $\Sigma$
- Canonical representation: $\xi$ & $\Omega$

  Information precision matrix: $\Omega = \Sigma^{-1}$; $\Sigma = \Omega^{-1}$

  Information vector: $\xi = \Sigma^{-1}\mu$; $\mu = \Omega^{-1}\xi$

$$
\begin{aligned}
p(x) &= \det(2\pi\Sigma)^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}(x-\mu)^T \Sigma^{-1}(x-\mu)\right\} \\
&= \eta \exp\left\{-\frac{1}{2}x^T \Omega x + x^T \xi\right\}
\end{aligned}
$$

**Information Filter Algorithm:** $(\xi_{t-1}, \Omega_{t-1}, u_t, z_t)$

$$\left.\begin{aligned} &\overline{\Omega}_t = (A_t \Omega_{t-1}^{-1} A_t^T + R_t)^{-1} \\ &\overline{\xi}_t = \overline{\Omega}_t (A_t \Omega_{t-1}^{-1} \xi_{t-1} + B_t u_t) \end{aligned}\right\} \mathcal{O}(n^{2,8})$$

$$\left.\begin{aligned} &\Omega_t = C_t^T Q_t^{-1} C_t + \overline{\Omega}_t \\ &\xi_t = C_t^T Q_t^{-1} z_t + \overline{\xi}_t \end{aligned}\right\} \mathcal{O}(n^2)$$

return $\xi_t, \Omega_t$

### 1.1.5 Extended Information Filter (IF)

$$\begin{aligned} x_t &= g(u_t, x_{t-1}) + \varepsilon_t \\ z_t &= h(x_t) + \delta_t \\ G_t &= g'(u_t, \mu_{t-1}) \\ H_t &= h'(\mu_t) \end{aligned}$$

**Extended Information Filter Algorithm:** $(\xi_{t-1}, \Omega_{t-1}, u_t, z_t)$

$\mu_{t-1} = \Omega_{t-1}^{-1} \xi_{t-1}$

$\overline{\Omega}_t = (G_t \Omega_{t-1}^{-1} G_t^T + R_t)^{-1}$

$\overline{\xi}_t = \overline{\Omega}_t \, g(u_t, \mu_{t-1})$

$\overline{\mu}_t = g(u_t, \mu_{t-1})$

$\Omega_t = \overline{\Omega}_t + H_t^T Q_t^{-1} H_t$

$\xi_t = \overline{\xi}_t + H_t^T Q_t^{-1} \left[ z_t - h(\overline{\mu}_t) - H_t \overline{\mu}_t \right]$

- ***Global uncertainty:*** set $\Omega = 0$ is better than set $|\Sigma| = \infty$
- IF tends to be numerically more stable than KF
- IF is better for multi-robot problems
- For high dimensional state, EKF is computational better than EIF

## 1.2 Measurements

### 1.2.1 Map Representation

Maps: $m = \{m_1, m_2, \dots, m_N\}$

**There are 2 ways to represent a map:**

[**TODO: Add images**]

| feature-based | location-based |
|---|---|
| $m_n$: properties of a feature and location of feature | a specific location |
| **only the shape** of the environment **at the specific locations** | volumetric: **label for any location** in the world |
| easy to adjust positions of objects $\Rightarrow$ popular in the robotic mapping field | occupancy grid map |

### 1.2.2 Measurement Noise

The 4 types of measurement noise:

- Correct range with local measurement noise
  With $z_t^{k*}$ as the correct distance
  $$p_{hit}(z_t^k|x_t, m) = \begin{cases} \eta\, \mathcal{N}(z_t^k|z_t^{k*}, \sigma_{hit}^2) & \text{if } 0 \leq z_t^k \leq z_{\max} \\ 0 & \text{otherwise} \end{cases} \tag{1.1}$$
- Unexpected object
  $$p_{short}(z_t^k|x_t, m) = \begin{cases} \eta \lambda_{short} e^{-\lambda_{short} z_t^k} & \text{if } 0 \leq z_t^k \leq z_t^{k*} \\ 0 & \text{otherwise} \end{cases} \tag{1.2}$$
- Failures
  $$p_{\max}(z_t^k|x_t, m) = I(z = z_{\max}) \tag{1.3}$$
- Random measurements
  $$p_{rand}(z_t^k|x_t, m) = \begin{cases} \frac{1}{z_{\max}} & \text{if } 0 \leq z_t^k < z_{\max} \\ 0 & \text{otherwise} \end{cases} \tag{1.4}$$

[**TODO: Add image, plot**]

$$p(z_t^k|x_t, m) = \begin{bmatrix} z_{hit} \\ z_{short} \\ z_{max} \\ z_{rand} \end{bmatrix}^T \cdot \begin{bmatrix} p_{hit}(z_t^k|x_t, m) \\ p_{short}(z_t^k|x_t, m) \\ p_{max}(z_t^k|x_t, m) \\ p_{rand}(z_t^k|x_t, m) \end{bmatrix} \tag{1.5}$$

## 1.3 Robot Motion

Pose: $[x, y, \theta]^T$ at location $[x, y]^T$ and orientation $\theta$

### 1.3.1 Motion Model

Motion Model, also known as (a.k.a.) Probabilistic Kinematic Model: $p(x_t, u_t, x_{t-1})$

| Velocity commands | Odometry<br>(distance traveled, angle turned, etc.) |
|---|---|
| Use for Probabilistic motion planning | **more accurate**<br>**but post-the-fact**<br>**(not for motion planning)**<br>Use for estimation |

Each has closed form calculation and sampling algorithm.

### 1.3.2 Velocity Motion Model

Assuming we can control a robot through velocities:

$$u_t = \begin{bmatrix} v_t \\ \omega_t \end{bmatrix}; \qquad x_{t-1} = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}; \qquad x_t = \begin{bmatrix} x' \\ y' \\ \theta' \end{bmatrix}$$

**Motion Model Velocity Algorithm:** $(x_t, u_t, x_{t-1})$

$$\left.\begin{aligned}
&\mu = \frac{1}{2}\frac{(x - x')\cos\theta + (y - y')\sin\theta}{(y - y')\cos\theta - (x - x')\sin\theta} \\
&x^* = \tfrac{x+x'}{2} + \mu(y - y') \\
&y^* = \tfrac{y+y'}{2} + \mu(x' - x) \\
&r^* = \sqrt{(x - x^*)^2 + (y - y^*)^2}
\end{aligned}\right\}\text{Invert the motion model}$$

$$\Delta\theta = \operatorname{atan2}(y' - y^*, x' - x^*) - \operatorname{atan2}(y - y^*, x - x^*)$$

$$\left.\begin{aligned}
&\hat{v} = \tfrac{\Delta\theta}{\Delta t} r^* \\
&\hat{\omega} = \tfrac{\Delta\theta}{\Delta t} \\
&\hat{\gamma} = \tfrac{\theta' - \theta}{\Delta t} - \hat{\omega}
\end{aligned}\right\}\text{compared actual velocities with the desired}$$

return $p(v - \hat{v}, \alpha_1|v| + \alpha_2|\omega|) \cdot p(\omega - \hat{\omega}, \alpha_3|v| + \alpha_4|\omega|) \cdot p(\hat{\gamma}, \alpha_5|v| + \alpha_6|\omega|)$

**Sample Motion Model Velocity Algorithm:** $(u_t, x_{t-1})$

$$\hat{v} = v + sample(\alpha_1|v| + \alpha_2|\omega|)$$

$$\hat{\omega} = \omega + sample(\alpha_3|v| + \alpha_4|\omega|)$$

$$\hat{\gamma} = sample(\alpha_5|v| + \alpha_6|\omega|)$$

$$x' = x - \frac{\hat{v}}{\hat{\omega}} \sin\theta + \frac{\hat{v}}{\hat{\omega}} \sin(\theta + \hat{\omega}\Delta t)$$

$$y' = y + \frac{\hat{v}}{\hat{\omega}} \cos\theta - \frac{\hat{v}}{\hat{\omega}} \cos(\theta + \hat{\omega}\Delta t)$$

$$\theta' = \theta + \hat{\omega}\Delta t + \hat{\gamma}\Delta t$$

return $x_t = [x', y', \theta']^T$

- Probability normal distribution(a, b): return $\frac{1}{\sqrt{2\pi b}} e^{-\frac{a^2}{2b}}$
- Probability triangular distribution(a, b): return $\begin{cases} 0 & \text{if } |a| > \sqrt{6b} \\ \frac{\sqrt{6b}-|a|}{6b} & \text{else} \end{cases}$
- Sample normal distribution(b): return $\frac{b}{6}\sum_{i=1}^{12} rand(-1,1)$
- Sample triangle distribution(b): return $b.rand(-1,1).rand(-1,1)$

### 1.3.3 Odometry Motion Model

Only available after the robot has moved
$\Rightarrow$ only use for filter algorithm
not for accurate motion planning and control

**Motion Model Odometry Algorithm:** $(x_t, u_t, x_{t-1})$

$$\delta_{rot1} = \text{atan2}(\bar{y}' - \bar{y}, \bar{x}' - \bar{x}) - \bar{\theta}$$

$$\delta_{trans} = \sqrt{(\bar{x} - \bar{x}')^2 + (\bar{y} - \bar{y}')^2}$$

$$\delta_{rot2} = \bar{\theta}' - \bar{\theta} - \delta_{rot1}$$

$$\left.\begin{aligned} &\hat{\delta}_{rot1} = \text{atan2}(y' - y, x' - x) - \theta \\ &\hat{\delta}_{trans} = \sqrt{(x - x')^2 + (y - y')^2} \\ &\hat{\delta}_{rot2} = \theta' - \theta - \hat{\delta}_{rot1} \end{aligned}\right\} \Rightarrow \text{inverse motion model}$$

$$p_1 = prob(\delta_{rot1} - \hat{\delta}_{rot1}, \alpha_1\hat{\delta}_{rot1} + \alpha_2\hat{\delta}_{trans})$$

$$p_2 = prob(\delta_{trans} - \hat{\delta}_{trans}, \alpha_3\hat{\delta}_{trans} + \alpha_4(\hat{\delta}_{rot1} + \hat{\delta}_{rot2}))$$

$$p_3 = prob(\delta_{rot2} - \hat{\delta}_{rot2}, \alpha_1\hat{\delta}_{rot2} + \alpha_2\hat{\delta}_{trans})$$

return $p_1.p_2.p_3 \qquad (= p(x_t|u_t, x_{t-1}))$

**NOTE:**

- Bar $\Leftrightarrow$ measurements

$$\bar{x}_{t-1} = [\bar{x} \quad \bar{y} \quad \bar{\theta}]^T$$
$$\bar{x}_t = [\bar{x}' \quad \bar{y}' \quad \bar{\theta}']^T$$

- Hat $\Leftrightarrow$ estimations
- No bar and hat $\Leftrightarrow$ hypothesized final pose $x, y$

**Sample Motion Model Odometry Algorithm:** $(u_t, x_{t-1})$

$\delta_{rot1} = \text{atan2}(\bar{y}' - \bar{y}, \bar{x}' - \bar{x}) - \bar{\theta}$

$\delta_{trans} = \sqrt{(\bar{x} - \bar{x}')^2 + (\bar{y} - \bar{y}')^2}$

$\delta_{rot2} = \bar{\theta}' - \bar{\theta} - \delta_{rot1}$

$\hat{\delta}_{rot1} = \delta_{rot1} - sample(\alpha_1 \delta_{rot1} + \alpha_2 \delta_{trans})$

$\hat{\delta}_{trans} = \delta_{trans} - sample(\alpha_3 \delta_{trans} + \alpha_4(\delta_{rot1} + \delta_{rot2}))$

$\hat{\delta}_{rot2} = \delta_{rot2} - sample(\alpha_1 \delta_{rot2} + \alpha_2 \delta_{trans})$

$x' = x + \hat{\delta}_{trans} \cos(\theta + \hat{\delta}_{rot1})$

$y' = y + \hat{\delta}_{trans} \sin(\theta + \hat{\delta}_{rot1})$

$\theta' = \theta + \hat{\delta}_{rot1} + \hat{\delta}_{rot2}$

return $\quad x_t = [x' \quad y' \quad \theta']^T$

### 1.3.4 Map-based Motion Model

Map-based Motion Model: $p(x_t|u_t, x_{t-1}, m)$

- Occupancy maps: $p(x_t|m) = 0 \Leftrightarrow$ the robot collides
- If the distance from $x_{t-1} \to x_t$ is small enough ($<$ half the robot's diameter), we can estimate the probability (prob.) $p(x_t|u_t, x_{t-1}, m) \approx \eta p(x_t|u_t, x_{t-1}) p(x_t|m)$, which discards the info relating the robot's path to $x_t$

**Motion Model with Map Algorithm:** $(x_t, u_t, x_{t-1}, m)$
return $p(x_t|u_t, x_{t-1}).p(x_t|m)$

**Sample Motion Model with Map Algorithm:** $(u_t, x_{t-1}, m)$

$$
\begin{aligned}
&do:\\
&\quad x_t = sample\_motion\_model(u_t, x_{t-1})\\
&\quad \pi = p(x_t|m)\\
&until \quad \pi > 0\\
&return < x_t, \pi >
\end{aligned}
$$

# 2 Markov Decision Process

## 2.1 Definitions

### 2.1.1 Markov Chain

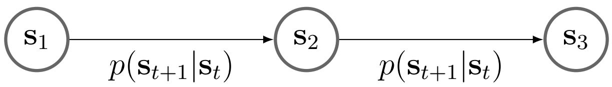
A Markov Chain $\mathcal{M}$ is a tuple consisting of:

$$\mathcal{M} = \{\mathcal{S}, \mathcal{T}\}$$

$\mathcal{S}$ – state space $\quad s \in \mathcal{S}$ (discrete or continuous)

$\mathcal{T}$ – transition operator $\quad p(s_{t+1}|s_t)$ or $\vec{\mu}_{t+1} = \mathcal{T}\vec{\mu}_t$ ($\mathcal{T}$ is a matrix)

Markov chain is a process without memories. In other words, the next state $s_{t+1}$ depends only on the current state $s_t$, not the previous state $s_{t-1}$.




### 2.1.2 Markov Decision Process (MDP)

A Markov Decision Process (MDP) $\mathcal{M}$ is a tuple consisting of:

$$\mathcal{M} = \{\mathcal{S}, \mathcal{A}, \mathcal{T}, r, \gamma\}$$

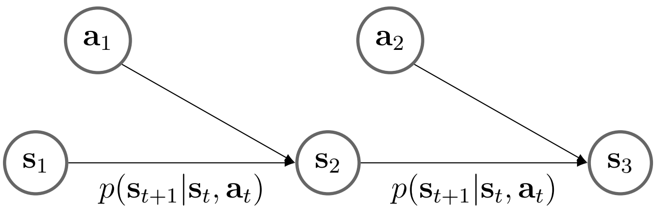
$\mathcal{S}$ – state space $\quad s \in \mathcal{S}$ (discrete or continuous)

$\mathcal{A}$ – action space $\quad a \in \mathcal{A}$ (discrete or continuous)

$\mathcal{T}$ – transition operator $\quad p(s_{t+1}|s_t)$ (now a tensor)

$r$ – reward function $\quad r : \mathcal{S} \times \mathcal{A} \to \mathbb{R}, \quad r(s_t, a_t)$

$\gamma$ – discount factor $\quad \gamma \in [0, 1]$ (optional)




There are definitions relating to MDP:

- Policy: choice of action (at each state): $\pi_\theta(\mathbf{a}_t|\mathbf{s}_t)$
- Utility: sum of (discounted) rewards

A MDP can also be considered as a Markov chain on the augmented state $(\mathbf{s},\ \mathbf{a})$, a.k.a. Q-state. Knowing the state transition $p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t)$ and policy $\pi_\theta(\mathbf{a}_t|\mathbf{s}_t)$, the transition of these Q-states can be derived as follows:

$$p((\mathbf{s}_{t+1}, \mathbf{a}_{t+1})|(\mathbf{s}_t, \mathbf{a}_t)) = p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t)\pi_\theta(\mathbf{a}_{t+1}|\mathbf{s}_{t+1}) \tag{2.1}$$

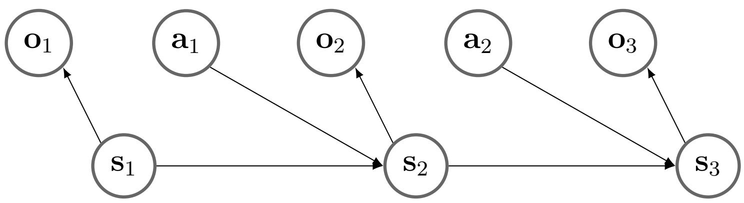
MDP state projects an expectimax-like search tree [**TODO: add image**]

### 2.1.3 Partially Observable Markov Decision Process (POMDP)

A Partially Observable Markov Decision Process (POMDP) $\mathcal{M}$ is a tuple consisting of:

$$\mathcal{M} = \{\mathcal{S}, \mathcal{A}, \mathcal{O}, \mathcal{T}, \mathcal{E}, r, \gamma\}$$

| | |
|---|---|
| $\mathcal{S}$ – state space | $s \in \mathcal{S}$ (discrete or continuous) |
| $\mathcal{A}$ – action space | $a \in \mathcal{A}$ (discrete or continuous) |
| $\mathcal{O}$ – observation space | $o \in \mathcal{O}$ (discrete or continuous) |
| $\mathcal{T}$ – transition operator | $p(s_{t+1}|s_t)$ |
| $\mathcal{E}$ – emission prob. | $p(o_t|s_t)$ |
| $r$ – reward function | $r : \mathcal{S} \times \mathcal{A} \to \mathbb{R}$ |
| $\gamma$ – discount factor | $\gamma \in [0, 1]$ (optional) |




Finite versus (vs.) infinite horizon: [**TODO:** ]

***<u>To solve infinite utilities:</u>***

- Finite horizon
- Discounting
- Absorbing state (like the fire hole, overheating)

## 2.2 Bellman equations

- $T(s,a,s') = P(s'|s,a)$ the prob. of reaching state $s'$ from taking action $a$ at state $s$
- $R(s,a,s')$ the reward of making the transition
- $Q^*(s,a)$: expected utility starting in state $s$ and having taken action $a$, then act optimally

$$Q^*(s,a) = \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V^*(s')\right] \tag{2.2}$$

  It is the sum over possible next state $s'$, because there is uncertainty of which state $s'$ will be reached, even with the same starting state $s$ and action $a$.

- $V^*(s)$: value of a state - expected utility starting in state $s$ and acting optimally

$$V^*(s) = \max_a Q^*(s,a) \tag{2.3}$$

$$\Rightarrow \quad V^*(s) = \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V^*(s')\right] \tag{2.4}$$

- $\pi^*(s)$: optimal action / policy from state $s$

[**TODO: Explain this??**]

$$\mathbf{V(s)} = \mathbb{E}[\mathbf{G_t}|\mathbf{S_t} = \mathbf{s}] = \mathbb{E}\left[\mathbf{R_{t+1}} + \gamma \mathbf{V(S_{t+1})}|\mathbf{S_t} = \mathbf{s}\right] = \mathbf{R_s} + \gamma \sum_{\mathbf{s'}} \mathbf{P_{ss'}} \mathbf{V(s')} \tag{2.5}$$

## 2.3 Partially Observable Markov Decision Process

POMDP is defined with a tuple: $< S, A, O, P, R, Z, \gamma >$:

- $S$: state
- $A$: action
- $O$: **observation**
- $P$: transition matrix
- $R$: reward
- $Z$: **observation func.**
- $\gamma$: discount factor (optional)

$$Z^a_{s'o} = P\left[O_{t+1} = o | S_{t+1} = s', A_t = a\right] \tag{2.6}$$

$$\Rightarrow \quad \begin{cases} V^*(b) \leftarrow \max_a \left[R^a_b + \gamma \sum_{s'} P^a_{bb'} V(b')\right] \\ \pi^*(b) = \arg\max_a \left[R^a_b + \gamma \sum_{s'} P^a_{bb'} V(b')\right] \end{cases} \tag{2.7}$$