

10/22/20

Quiz #1

- 2)  $h_i$  = leverage w/  $i=1,2,3,\dots,n$  associated with  
 $y = X\beta + \varepsilon$  .  $h_i$  = main diagonal of  
 $H = X(X^T X)^{-1} X^T$

a) Idempotency:

$$\begin{aligned} H^2 &= H H = (X(X^T X)^{-1} X^T) (X(X^T X)^{-1} X^T) \\ &= X(X^T X)^{-1} \cancel{(X^T X)} \cancel{(X^T X)^{-1}} X^T \\ &= X(X^T X)^{-1} X^T = H \end{aligned}$$

Symmetry:

$$H^T = [X(X^T X)^{-1} X^T]^T = X[(X^T X)^{-1}]^T X^T = H$$

since  $[(X^T X)^{-1}]^T = [(X^T X)^T]^{-1}$   
 $= (X^T X)^{-1}$

c) Show  $0 \leq h_{ii} \leq 1$

PF: Knowing  $H$  is idempotent & symmetric.

$$h_{ii} = h_{ii}^2 + \sum_{j \neq i} h_{ij}^2 \geq 0$$

and

$$h_{ij} \geq h_{ii}^2 \Rightarrow \boxed{h_{ii} \leq 1}$$