

CHAPMAN University
 Department of Computational and Data Sciences
 CS501 Introductory Computation for Scientists
 Fall 2019
 Homework#12

Date Given: Dec 4, 2019

Due Date: Dec 14, 2019

Download Julia software from www.julialang.org and install it on your computer.
 Install "IJulia" package and run Julia language using Jupyter IDE.

Solve the following problems using Julia language. There are 6 problems in this homework assignment. Make sure that the answers computed by your Julia-code matches with the given answers.

1. The *half-life* of a radioactive substance is the time it takes to decay by one-half. The half-life of carbon 14, which is used for dating previously living things is, is 5500 years. When an organism dies, it stops accumulating carbon 14. The carbon 14 present at the time of death decays with time. Let $C(t)/C(0)$ be the fraction of carbon 14 remaining at time t . In radioactive carbon dating, scientists usually assume that the remaining fraction decays exponentially according to the following formula.

$$\frac{C(t)}{C(0)} = e^{-bt}$$

- a. Use the half-life of carbon 14 to find the value of the parameter 'b' and plot the function.

Answer: $b = 0.000126$

- b. If 90% of the original carbon 14 remains, estimate how long ago the organism died.

Answer: 836.017 years

2. Given the matrices:

$$A = \begin{bmatrix} 4 & -2 & 1 \\ 6 & 8 & -5 \\ 7 & 9 & 10 \end{bmatrix} \quad B = \begin{bmatrix} 6 & 9 & -4 \\ 7 & 5 & 3 \\ -8 & 2 & 1 \end{bmatrix} \quad C = \begin{bmatrix} -4 & -5 & 2 \\ 10 & 6 & 1 \\ 3 & -9 & 8 \end{bmatrix}$$

Use Julia to

- a. Verify the associative property

$$\mathbf{A(B+C)} = \mathbf{AB + AC}$$

- b. Verify the distributive property

$$(\mathbf{AB})\mathbf{C} = \mathbf{A(BC)}$$

3. Suppose it is known that the graph of the following function

$$y = ax^3 + bx^2 + cx + d$$

Passes through four given points (x_i, y_i) where $i = 1, 2, 3, 4$.

Write a user defined function that accepts these four points as input and computes the coefficients a, b, c, d . The function should solve four linear equations in terms of the four unknowns a, b, c, d . Test your function for the case where $(x_i, y_i) = (-2, -20), (0, 4), (2, 68),$ and $(4, 508)$.

Answer: $a = 7, b = 5, c = -6, d = 4$.

4. Use a random number generator to produce 1,000 uniformly distributed numbers with a mean of 10, a minimum of 2, and a maximum of 18. Obtain the mean and the histogram of these numbers and discuss whether they appear uniformly distributed with the desired mean.
5. Use a random number generator to produce 1,000 normally distributed numbers with a mean of 20 and a standard deviation of 4. Obtain the mean, standard deviation, and the histogram of these numbers, and discuss whether they appear normally distributed with the desired mean and standard deviation.
6. The mean of the sum (or difference) of two independent random variables equals the sum (or difference) of their means, but the variance is always the sum of the two variances.

Use random number generation to verify this statement for the case where $z = x + y$, where 'x' and 'y' are independent and normally distributed random variables. The mean and the variance of 'x' and 'y' are given below.

$$\mu_x = 8, \sigma_x^2 = 2, \mu_y = 15, \sigma_y^2 = 4$$

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In [3]: using Statistics
println(Statistics.mean(X))
println(Statistics.var(X))

8.033498529299367
2.128885127222398

In [4]: using Statistics
println(Statistics.mean(Y))
println(Statistics.var(Y))

14.947237028629697
4.197148460558672

In [6]: Z1 = X + Y
using Statistics
println(Statistics.mean(Z1))
println(Statistics.var(Z1))

22.98073555792907
6.114000173925673

In [7]: Z2 = X - Y
using Statistics
println(Statistics.mean(Z2))
println(Statistics.var(Z2))

-6.913738499330334
6.538067001636152
```