## CSCI 480, Winter 2017 Math Exercises # 2

## YOUR NAME HERE

Due date: Wednesday, February 1, midnight.

Turn in both the tex and pdf files (not zipped): math02.tex and math02.pdf.

Exercises for Chapter 4 Use the method of direct proof to prove the following statements.

16. If two integers have the same parity, then their sum is even.

Exercises for Chapter 5 Use the method of contrapositive proof to prove the following statements.

**12.** Suppose  $a \in \mathbb{Z}$ . If  $a^2$  is not divisible by 4, then a is odd.

Exercises for Chapter 6 Use the method of proof by contradiction to prove the following statements.

**18.** Suppose  $a, b \in \mathbb{Z}$ . If  $4 \mid (a^2 + b^2)$ , then a and b are not both odd.

Exercises for Chapter 7 State clearly which method of proof you are using.

**24.** If  $a \in \mathbb{Z}$ , then  $4 \nmid (a^2 - 3)$ .

Exercises for Chapter 8

**20.** Prove that  $\{9^n : n \in \mathbb{Q}\} = \{3^n : n \in \mathbb{Q}\}.$ 

**Exercises for Chapter 9** Each of the following statements is either true or false. If a statement is true, prove it. If a statement is false, disprove it.

**18.** If  $a, b, c \in \mathbb{N}$ , then at least one of a - b, a + c, and b - c is even.

Exercises for Chapter 10

**2.** For every integer  $n \in \mathbb{N}$ , it follows that

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

**6.** For every natural number n, it follows that

$$\sum_{i=1}^{n} (8i - 5) = 4n^2 - n$$

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**10.** For every integer  $n \ge 0$ , it follows that  $3 \mid (5^{2n} - 1)$ .

**14.** Suppose  $a \in \mathbb{Z}$ . Prove that  $5 \mid 2^n a$  implies  $5 \mid a$  for any  $n \in \mathbb{N}$ .