CSCI 301, Winter 2017
Math Exercises # 5

YOUR NAME HERE

Due date:

Construct a context-free grammar for each of the languages in questions 1 to 7.

1. $\{0^{2n}1^n : n \geq 0\}$
2. $\{w : w$ containst at least three 1s$\}$
3. $\{w :$ the length of w is odd and its middle symbol is $0\}$
4. $\{w : w$ is a palindrome$\}$
5. $\{w : w$ starts and ends with the same symbol$\}$
6. $\{w : w$ starts and ends with different symbols$\}$
7. $\{a^m b^n : 0 \leq m \leq n \leq 2m\}$
8. Let G be the grammar:

$$\begin{aligned} S &\rightarrow aB \mid bA \\ A &\rightarrow a \mid aS \mid bAA \\ B &\rightarrow b \mid bS \mid aBB \end{aligned}$$

   For the string *aaabbabbba*, find a

   (a) leftmost derivation,
   (b) rightmost derivation,
   (c) parse tree.

9. Convert the following grammar to Chomsky normal form:

$$\begin{aligned} S &\rightarrow bA \mid aB \\ A &\rightarrow bAA \mid aS \mid a \\ B &\rightarrow aBB \mid bS \mid b \end{aligned}$$

   Follow the steps documented in my notes and the text, and show the resulting grammar after each step.

   Step 1 Eliminate the start variable from the right-hand side of rules.

   Step 2 Eliminate ϵ-rules.

   Step 3 Eliminate unit-rules.

   Step 4 Eliminate all rules having more than two symbols on the right-hand side.

   Step 5 Eliminate all rules of the form $A \rightarrow u_1 u_2$ where u_1 and u_2 are not both variables.