CSCI 480, Winter 2016
Math Exercises # 4

YOUR NAME HERE

Due date:

- Build deterministic finite automata and/or regular expressions (as requested) for each of the languages in questions **??** to **??**. Create simple, meaningful automata and regular expressions (rather than, *e.g.*, using the algorithm to create a regular expression from a DFA) and explain how they work. In all cases the alphabet is $\Sigma = \{0, 1\}$.
- DFAs should be specified with pictures, preferablly typeset with **TikZ**, not tables (as tables are very hard to read). Try to typeset them so that the labels on the arcs are clear, *etc.*
- If you are having a difficult time with **TikZ**, clear, legible hand-drawn figures (or figures created with a drawing program) are acceptable as graphics inclusions into your LaTeX documents.

1. The language $\{100, 10, 011\}$.

   Regular expression:

2. The language $\{100, 10, 011\}$.

   DFA:

3. The set of all strings that begin or end with a doubled digit, either 11 or 00.

   Regular expression:

4. The set of all strings that begin or end with a doubled digit, either 11 or 00.

   DFA:

5. The set of all strings that have exactly one doubled digit in them. In other words, either 11 or 00 occurs in the string, but not both, and it only occurs once.

   Regular expression:

6. The set of all strings that have exactly one doubled digit in them. In other words, either 11 or 00 occurs in the string, but not both, and it only occurs once.

   DFA:

7. The set of all strings such that every block of three consecutive digits has at least two 0's in it. Note that we have to accept everything with less than three digits, such as 11 and 01.

   Hint: you may want to break the machine up into two parts, one for the first few letters and another for the rest, and then specify the complete machine as the union of the two machines.

   DFA:

8. The set of all strings beginning with a 1 such that, interpreted as a binary representation of an integer, it has a remainder of 1 when divided by 3. For example, the binary number 1010_b is decimal 10. When you divide 10 by 3 you get a remainder of 1, so 1010 is in the language. However, the binary number 1111_b is decimal 15. When you divide 15 by 3 you get a remainder of 0, so 1111 is not in the language.

Hint: if you have a binary string, such as 1100_b, which is 12 in decimal, what happens if you add a 0 to the right end? You get 11000_b which in decimal is 24. What happens if you add a 1 to the right end? You get 11001_b which is decimal 25.

DFA:

9. Give an example of a regular language R and a nonregular language N such that $R \cup N$ is regular. Describe all three languages in English and either prove they are regular/nonregular, or show that they are instances of languages with known regularity.

10. Give an example of a regular language R and a nonregular language N such that $R \cup N$ is nonregular. Describe all three languages in English and either prove they are regular/nonregular, or show that they are instances of languages with known regularity.