$$L = \sum_i y_i \log\left(\frac{1}{1+\exp(-w^T x_i)}\right) + (1-y_i)\log\left(\frac{\exp(-w^T x_i)}{1+\exp(-w^T x_i)}\right)$$

$$= \sum_i y_i \log\left(\frac{1}{1+\exp(-w^T x_i)}\right) + (1-y_i)\log\left(1 - \frac{1}{1+\exp(-w^T x_i)}\right)$$

---

let $z_i = w^T x_i$ and $\sigma(x) = \frac{1}{1+\exp(-x)} = [1+\exp(-x)]^{-1}$

$$\frac{d}{dx}\sigma(x) = (-1)(1+\exp(-x))^{-2}\exp(-x)(-1) = \frac{1}{1+\exp(-x)} \cdot \frac{\exp(-x)}{1+\exp(-x)} = \sigma(x)(1-\sigma(x))$$

$$L = \sum_i y_i \log(\sigma(z_i)) + (1-y_i)\log(1-\sigma(z_i))$$

$$\frac{dL}{dw} = \sum_i y_i \frac{1}{\sigma(z_i)}\sigma(z_i)(1-\sigma(z_i))x_i + (1-y_i)\frac{1}{1-\sigma(z_i)}(-1)\sigma(z_i)(1-\sigma(z_i))x_i$$

$$= \sum_i y_i(1-\sigma(z_i))x_i - (1-y_i)\sigma(z_i)x_i$$

$$= \sum_i [y_i - y_i\sigma(z_i) - \sigma(z_i) + y_i\sigma(z_i)]\cdot x_i$$

$$= \sum_i [y_i - \sigma(z_i)]x_i$$

$$= -\sum_i \left(\frac{1}{1+\exp(-w^T x_i)} - y_i\right)x_i$$