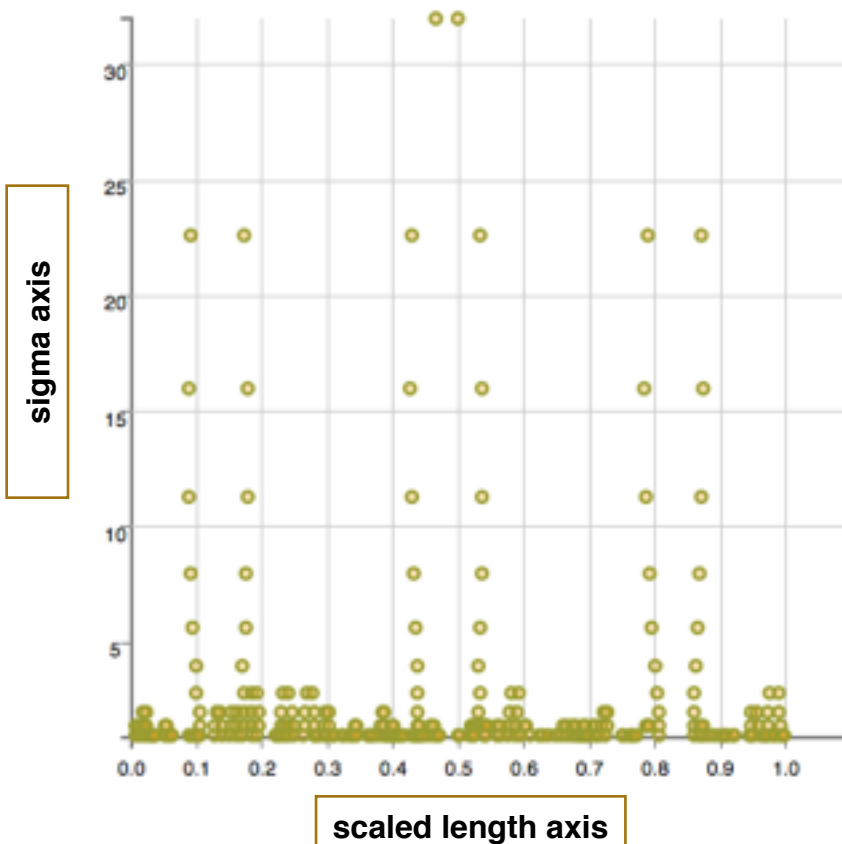


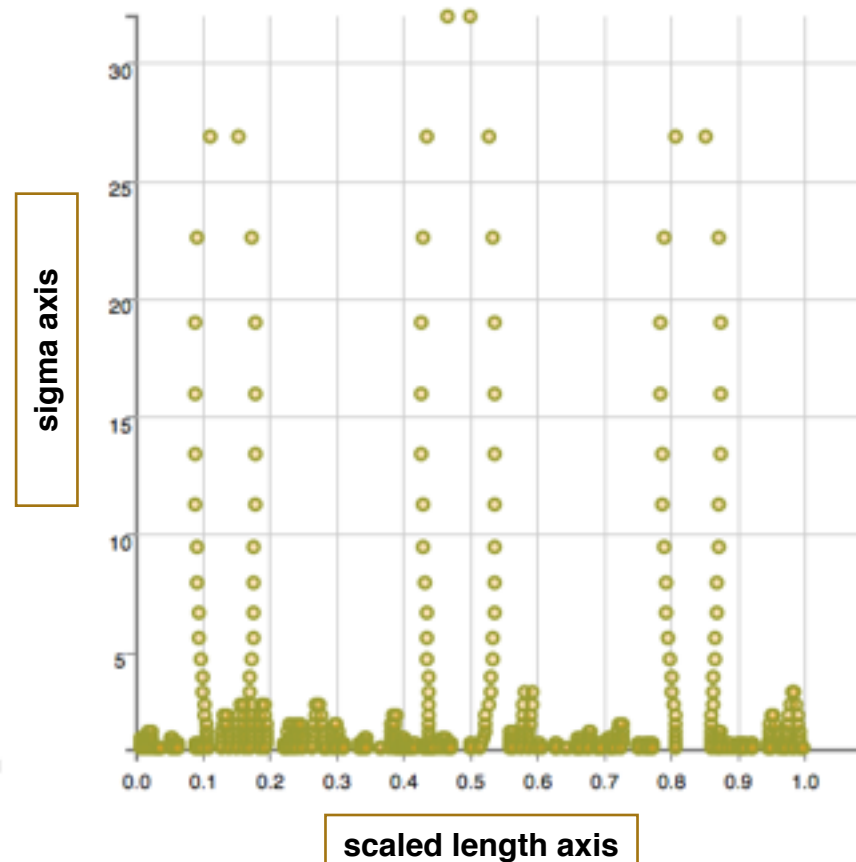
contour finder



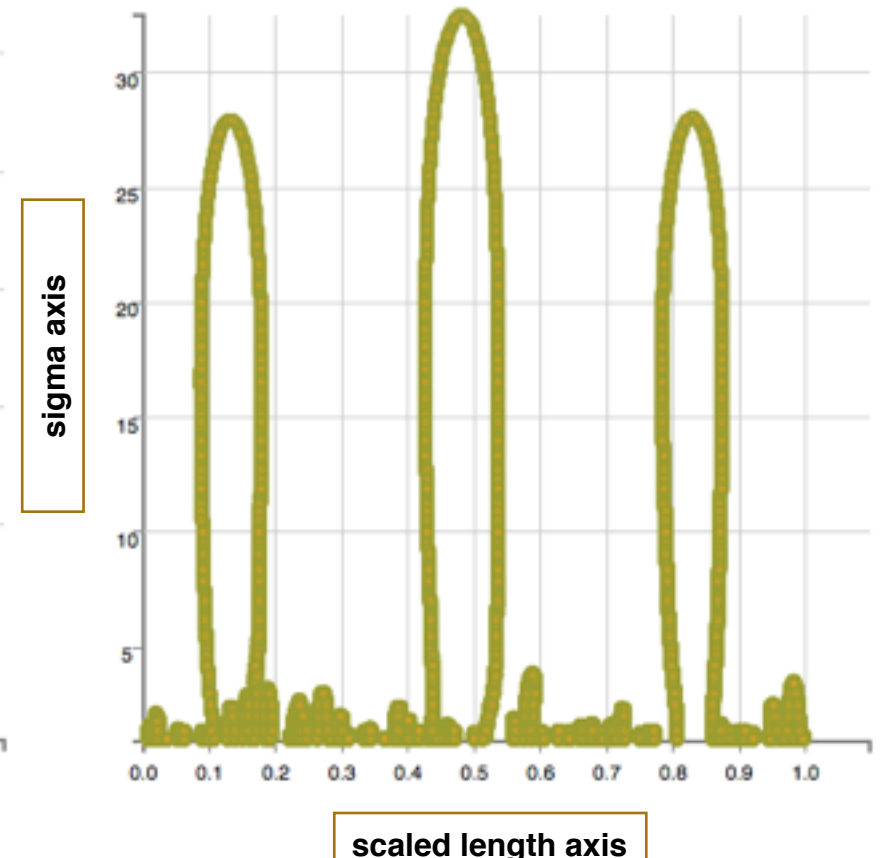
scale space image produced
for sigma factors of $\sqrt{2}$



scale space image produced
for sigma factors of $2^{1/8}$



scale space image produced
for sigma factors of $2^{1/128}$



There is an error in estimating the peak of a contour for fastest creation of scale space images is $\leq \sqrt{2}$. That error can be reduced overall, by having more contours in the final solution. For an error $< 10\%$ in determining a contour's peak height, one should choose a sigma factor of $2^{1/8}$. It takes 2^3 more convolutions if the smaller sigma factor of $2^{1/8}$ is used instead of $2^{1/2}$.

Inflection points for $\sigma > 0$

Scale-Based Description and Recognition of Planar Curves and Two-Dimensional Shapes

FARZIN MOKHTARIAN AND ALAN MACKWORTH

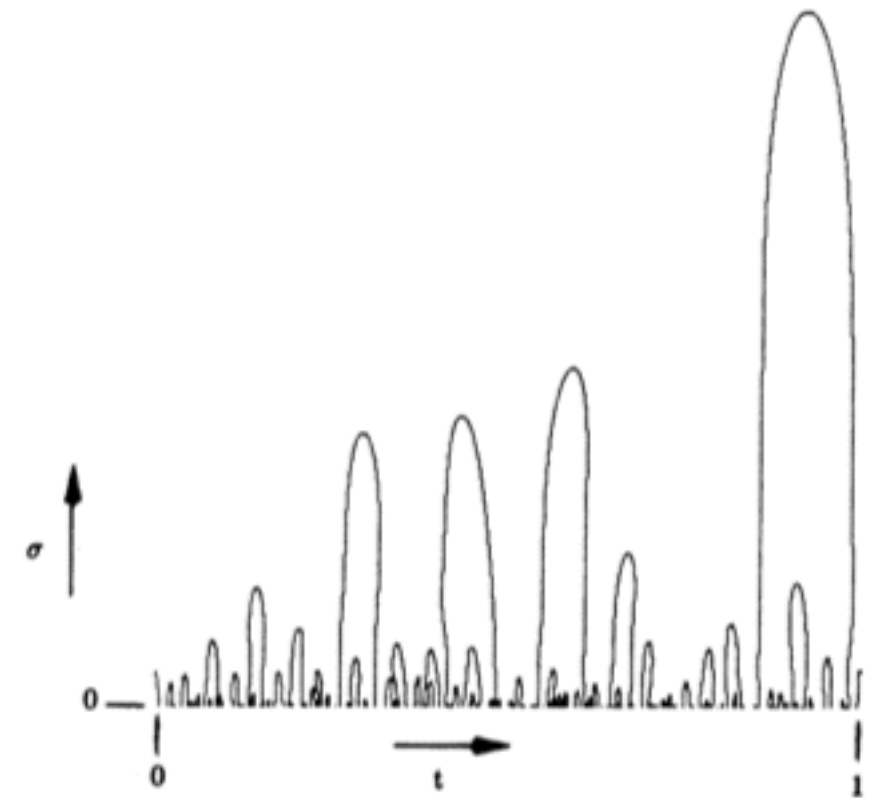
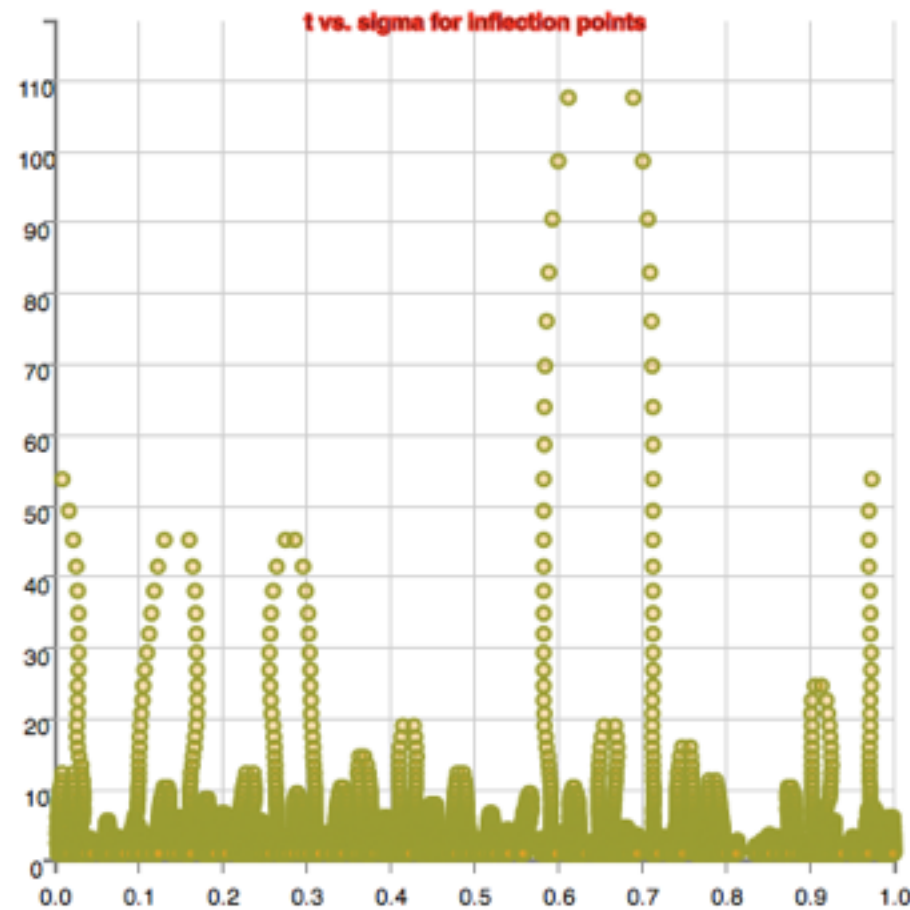
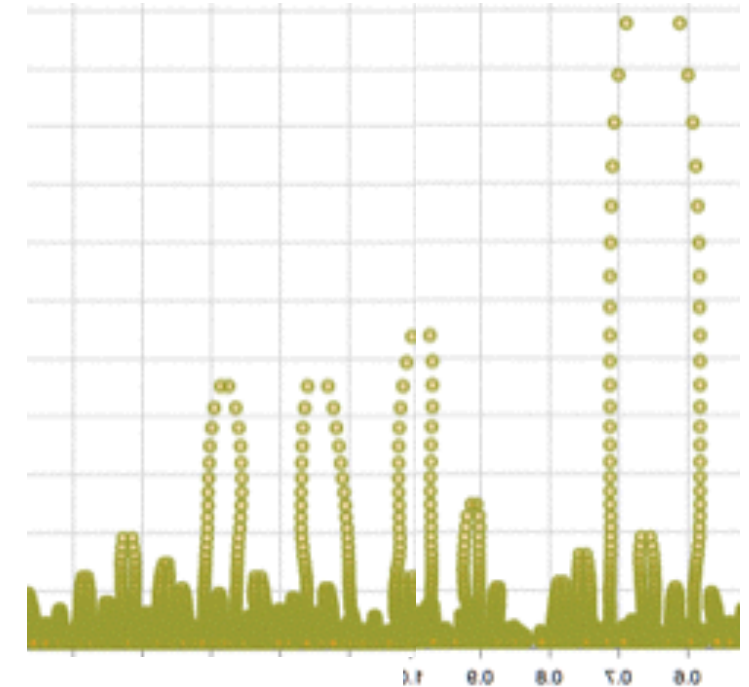


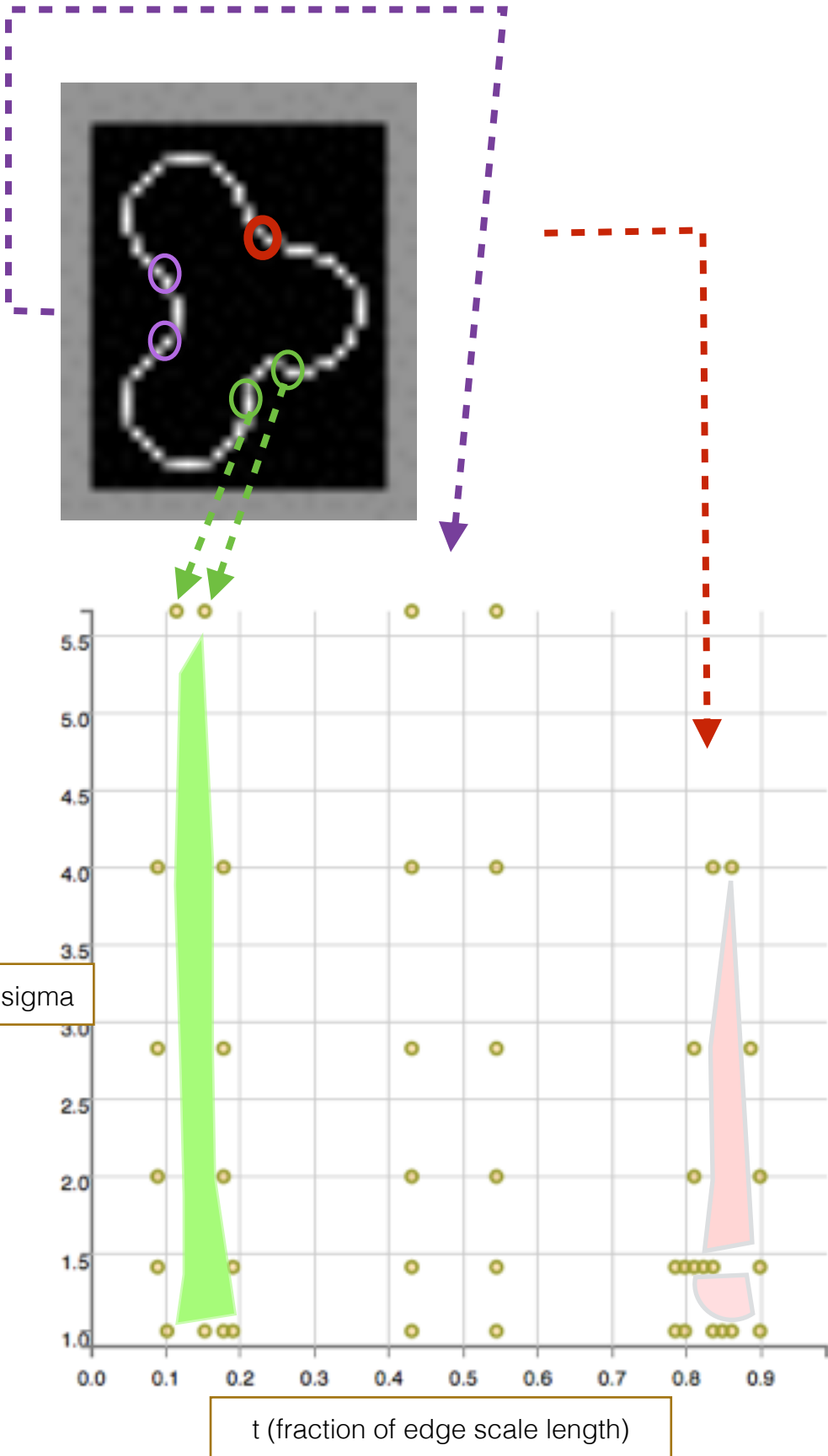
Fig. 3. Generalized scale space image of Africa.



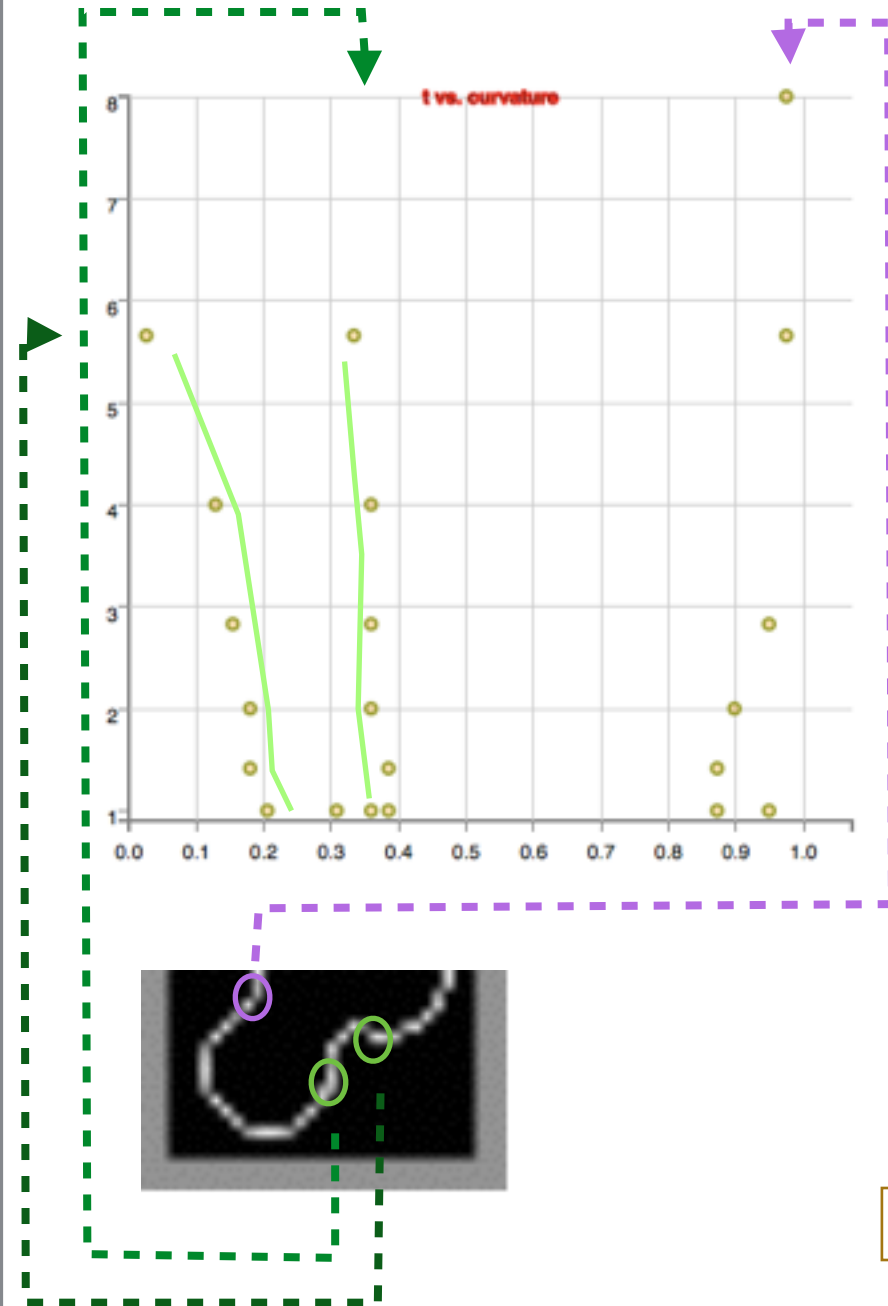
my scale space image
agrees with theirs

contour finder

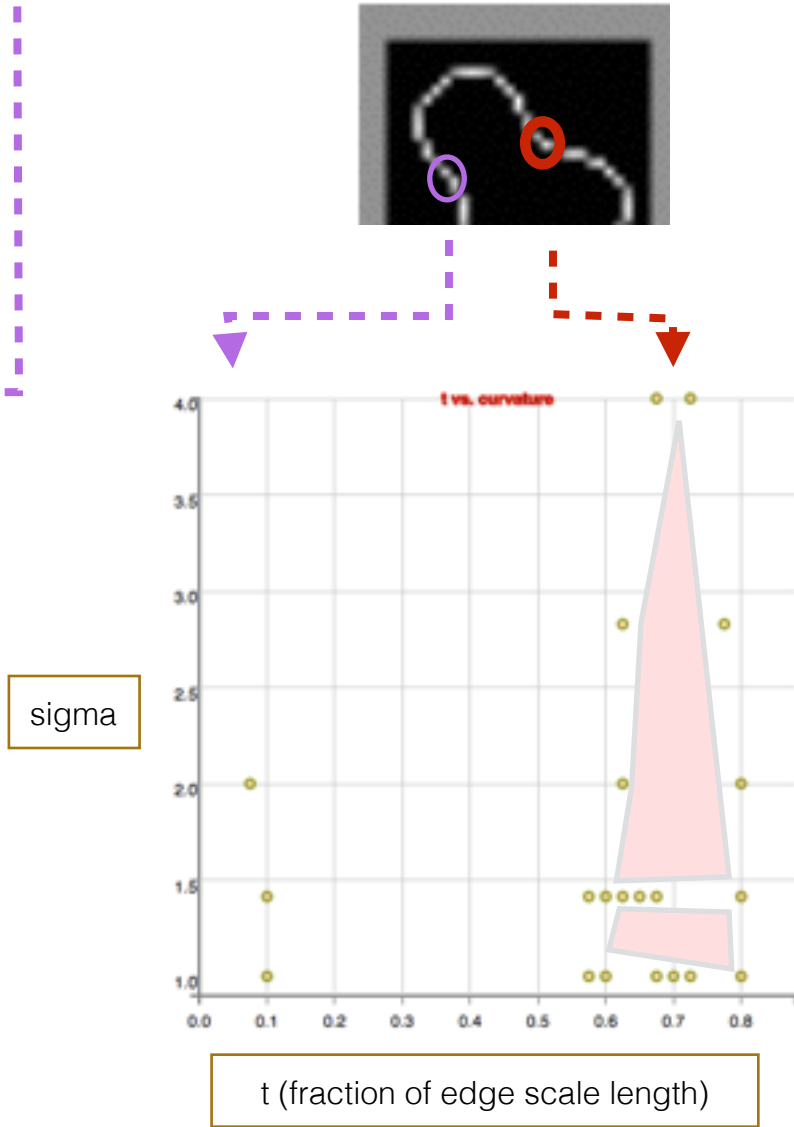
single closed curve's scale space image:



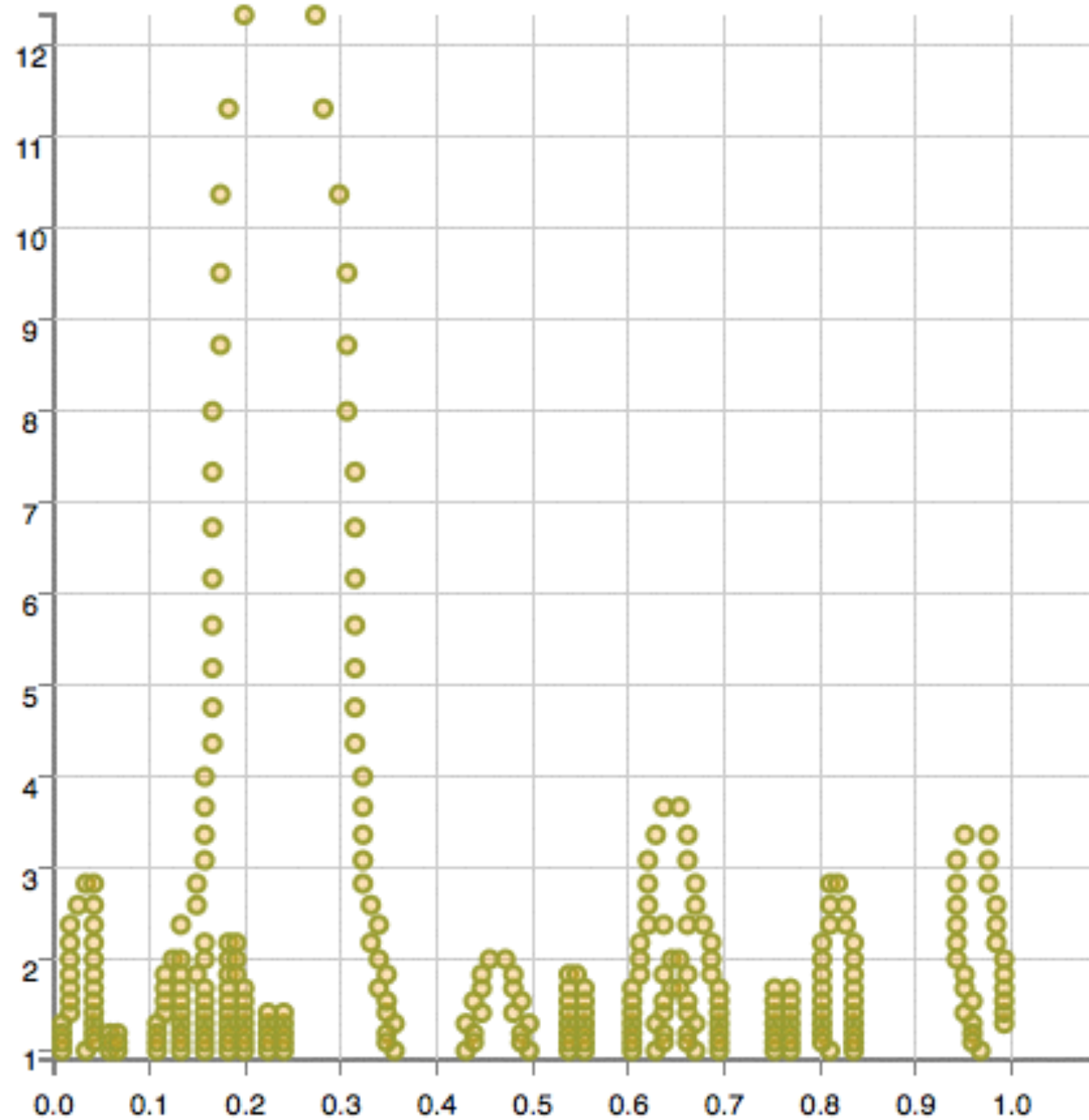
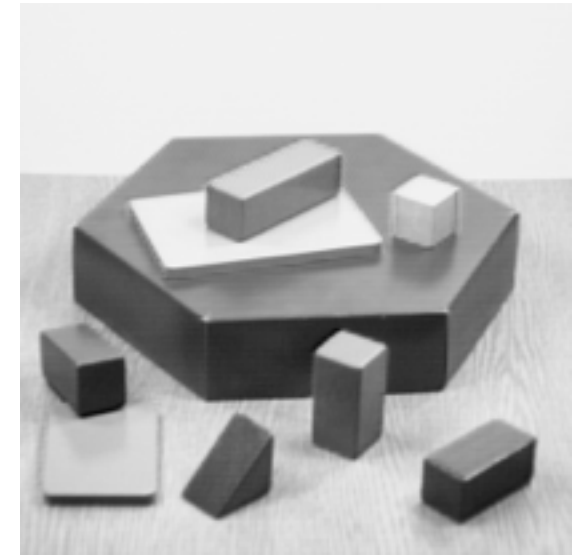
divided into 2 curves manually, then made into scale images (that is, 2 open curves possibly create open contours):



∴ open contours are hard to match in another image's scale images



contour finder



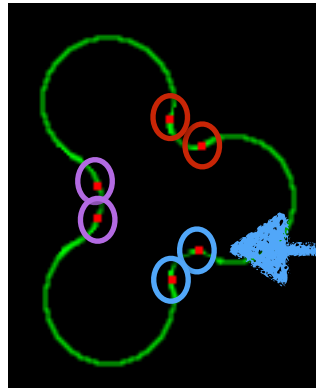
The contour finder looks for the peaks at the highest sigma and then follow the left and right branches down, subtracting that contour from the scale space image. each contour is found that way and subtracted to a lower threshold.

Then contours from one image are matched with the contours of another image having the same content.

Then euclidean transformation parameters rotation, scale, and translation are calculated from the matched contour peak coordinates.

Then, the parameters are refined with small changes and applied to the edges in image 1. The transformed closed curve edges from image 1 are compared to the closest matches in image 2 to find the best fitting transformation parameters.

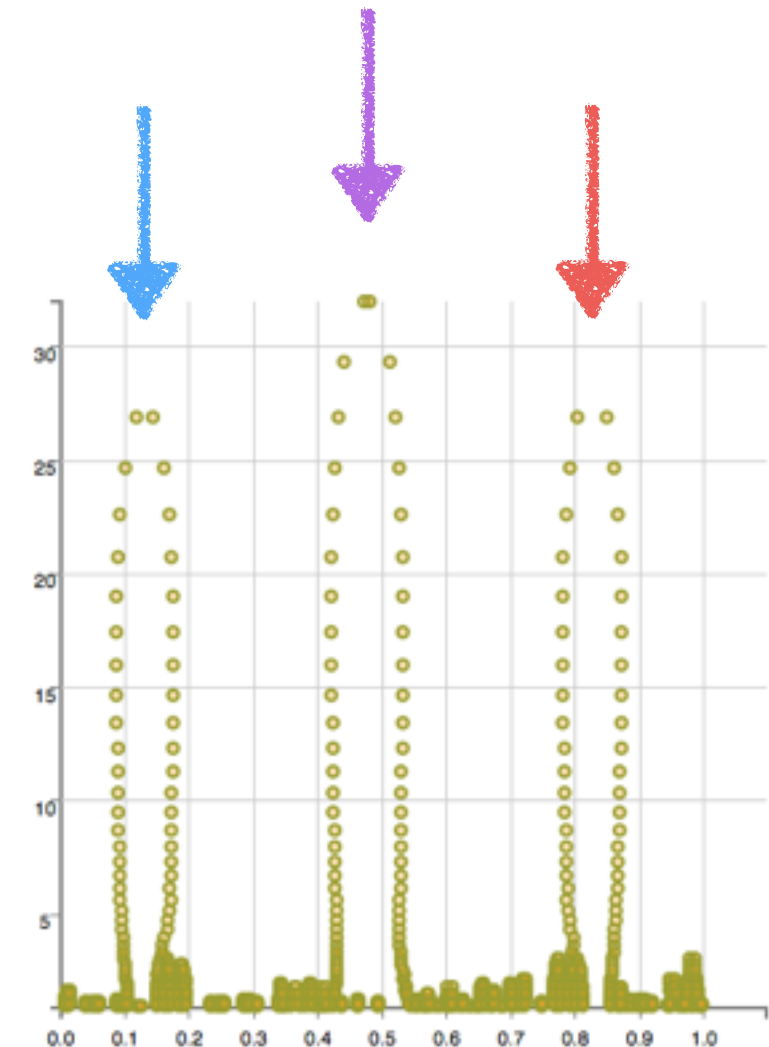
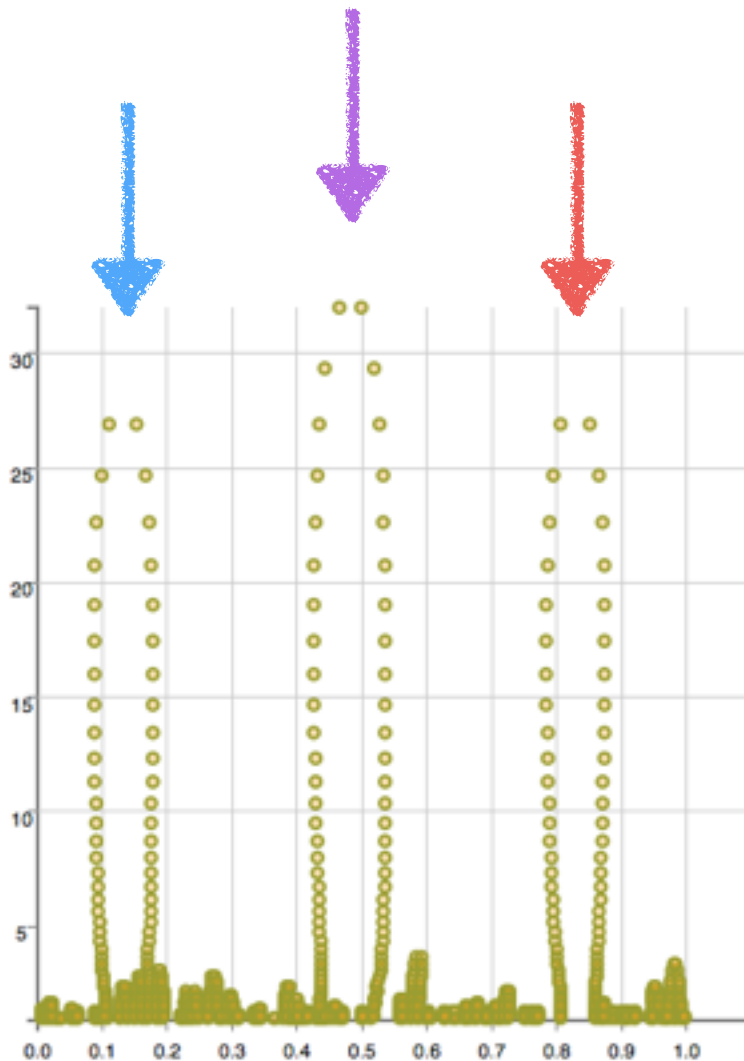
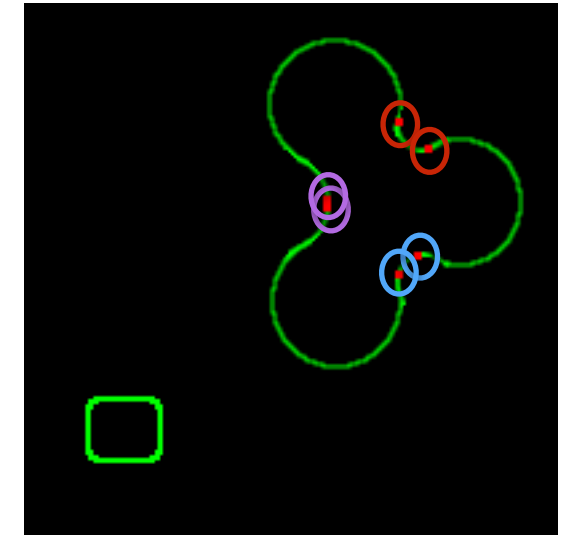
coordinate transformation, after matching contours



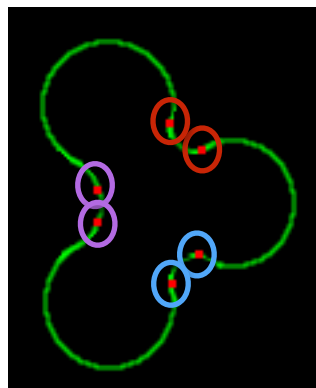
(71,92)

```
centroidX1=53.41090539097786 centroidY1=73.55932730436325
centroidX2=132.28379026055336 centroidY2=73.8728848695755
x1 y1 x2 y2 len1 len2 theta1 theta2 (x,y) (x,y)
-19.6 5.2 -19.3 2.1 20.3 19.4 165 174 ( 33, 79) ( 113, 76)
-18.7 -3.4 -19.3 -0.9 19.0 19.3 190 183 ( 34, 70) ( 113, 73)
15.0 17.7 13.7 20.1 23.2 24.4 50 56 ( 69, 92) ( 146, 94)
7.3 27.3 7.7 26.1 28.3 27.2 75 74 ( 61, 102) ( 140, 100)
6.3 -27.5 7.7 -27.9 28.2 28.9 283 285 ( 60, 45) ( 140, 46)
16.9 -19.8 16.7 -19.9 26.0 26.0 311 310 ( 71, 53) ( 149, 54)
translationX=81.774155 translationY=0.025433423
scaleX=0.96064633 scaleY=0.9625189 (0.9615826)
rotation=4.473403395193617
```

translation should be 80, 0
scale should be 1
rotation should be 0

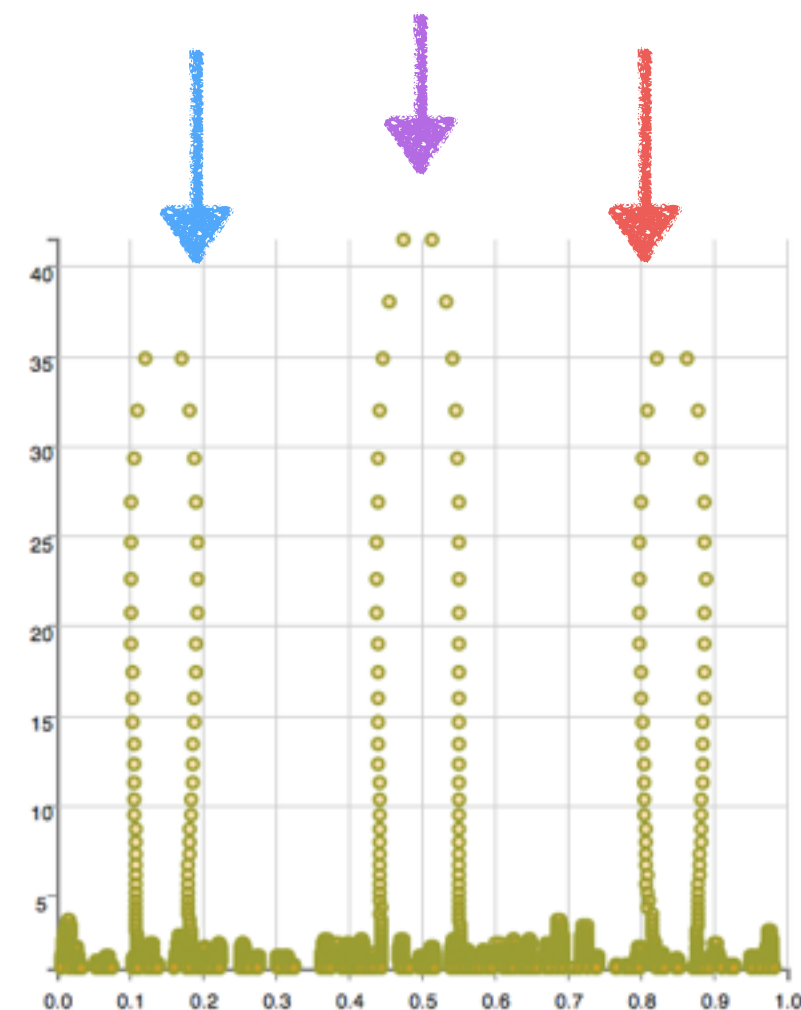
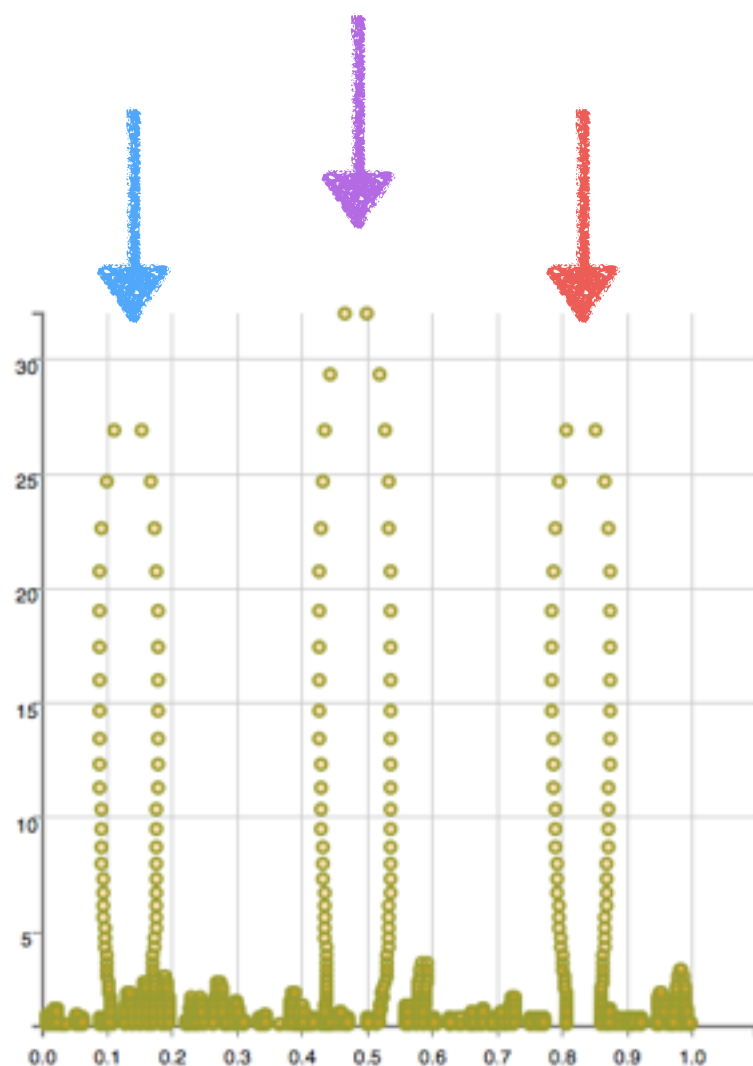
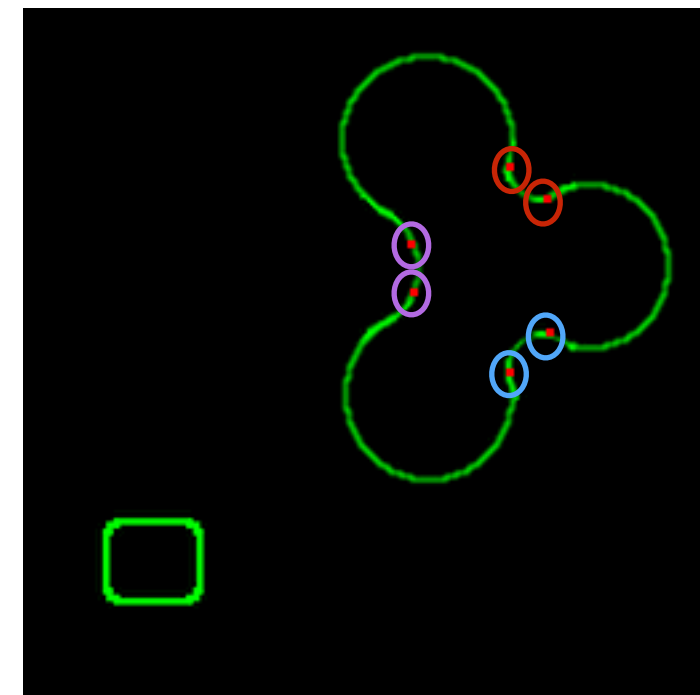


coordinate transformation, after matching contours

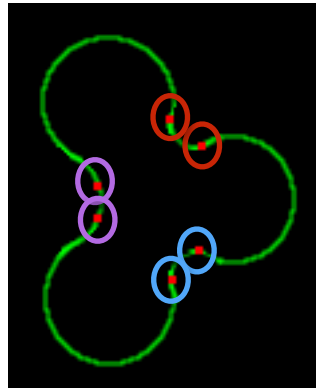


```
centroidX1=53.41090539097786 centroidY1=73.55932730436325
centroidX2=172.30917616188526 centroidY2=97.1567816734314
x1 y1 x2 y2 len1 len2 theta1 theta2 (x,y) (x,y)
-27.2 7.2 -26.3 8.8 28.1 27.8 165 161 ( 33, 79) ( 146, 106)
-25.9 -4.7 -27.3 -9.2 26.3 28.8 190 199 ( 34, 70) ( 145, 88)
20.8 24.6 22.7 24.8 32.2 33.6 50 48 ( 69, 92) ( 195, 122)
10.1 37.9 9.7 37.8 39.2 39.1 75 76 ( 61, 102) ( 182, 135)
8.8 -38.1 9.7 -36.2 39.1 37.4 283 285 ( 60, 45) ( 182, 61)
23.4 -27.4 21.7 -26.2 36.1 34.0 311 310 ( 71, 53) ( 194, 71)
translationX=89.11711 translationY=0.89937526
scaleX=1.3439497 scaleY=1.3209674 (1.3324585)
rotation=2.910978424383675
```

translation should be 80, 0
scale should be 1.3
rotation should be 0

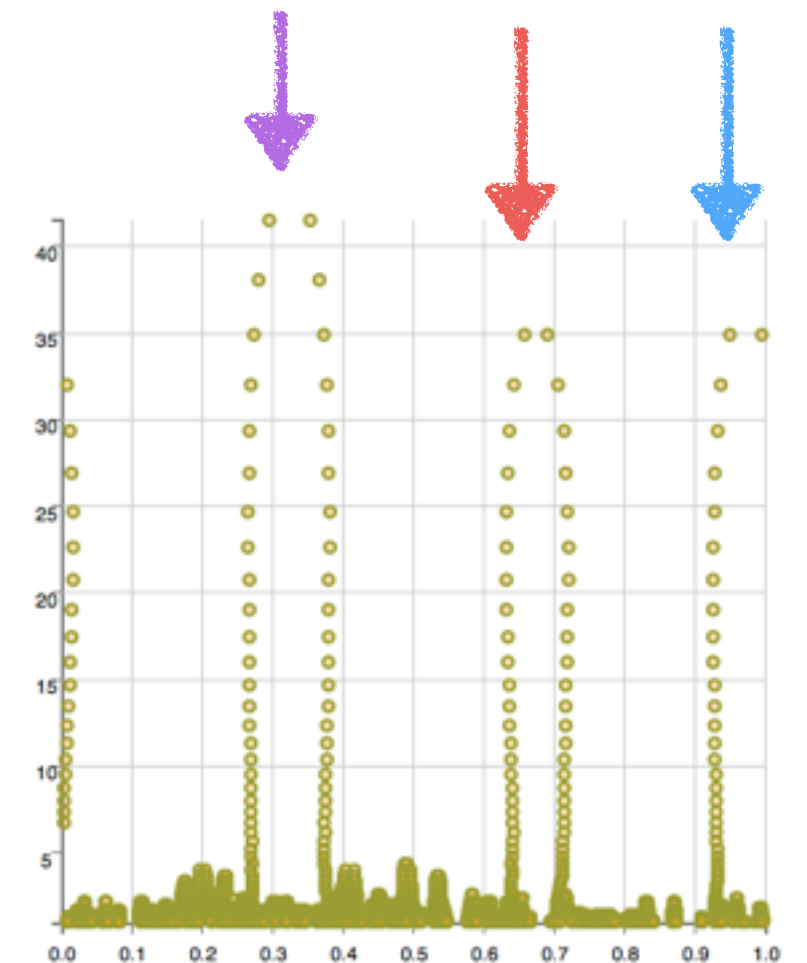
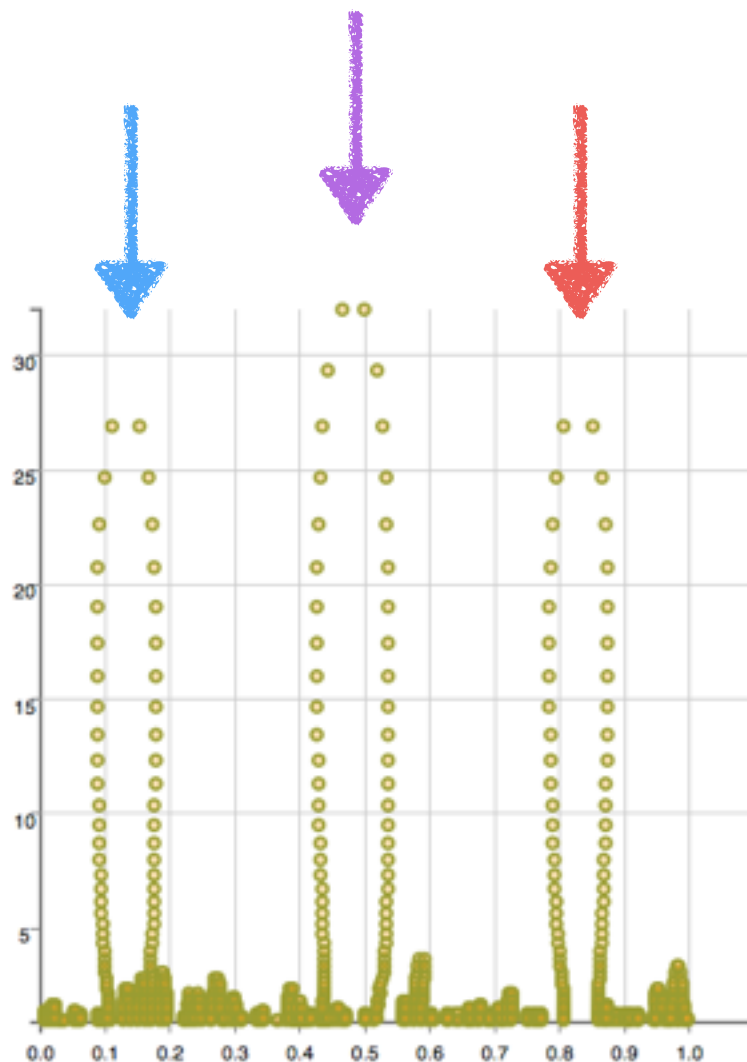
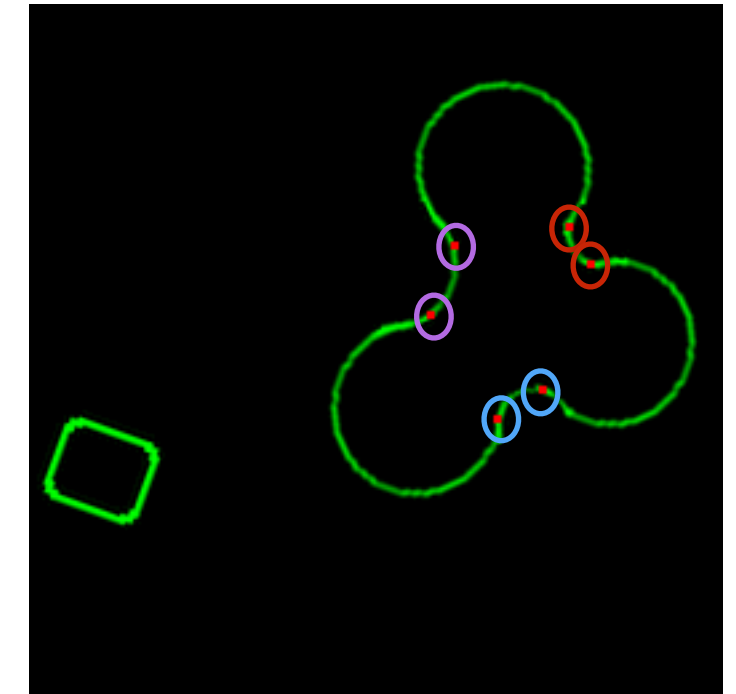


coordinate transformation, after matching contours

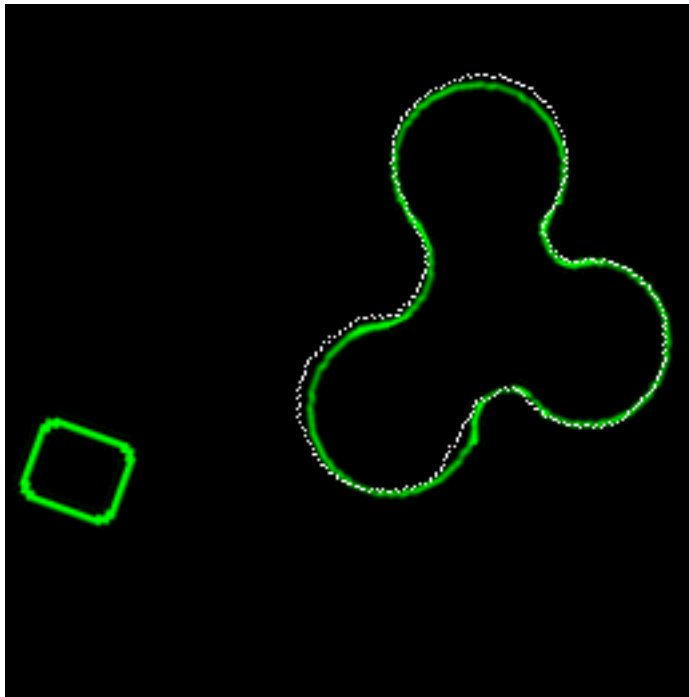


```
centroidX1=53.41090539097786 centroidY1=73.55932730436325
centroidX2=179.92783105373383 centroidY2=113.09317249059677
x1 y1 x2 y2 len1 len2 theta1 theta2 (x,y) (x,y)
-27.2 7.2 -28.9 2.9 28.1 29.1 165 174 ( 33, 79) ( 151, 116)
-25.8 -4.7 -20.9 -21.1 26.3 29.7 190 225 ( 34, 70) ( 159, 92)
20.8 24.5 10.1 30.9 32.1 32.5 50 72 ( 69, 92) ( 190, 144)
10.1 37.9 -2.9 36.9 39.2 37.0 75 95 ( 61, 102) ( 177, 150)
8.8 -38.0 22.1 -30.1 39.0 37.3 283 306 ( 60, 45) ( 202, 83)
23.4 -27.4 30.1 -16.1 36.0 34.1 311 332 ( 71, 53) ( 210, 97)
translationX=150.1189 translationY=-4.185816
scaleX=1.3642461 scaleY=1.2980525 (1.3311493)
rotation=-21.72218537902866
```

scale should be 1.3
rotation should be 20



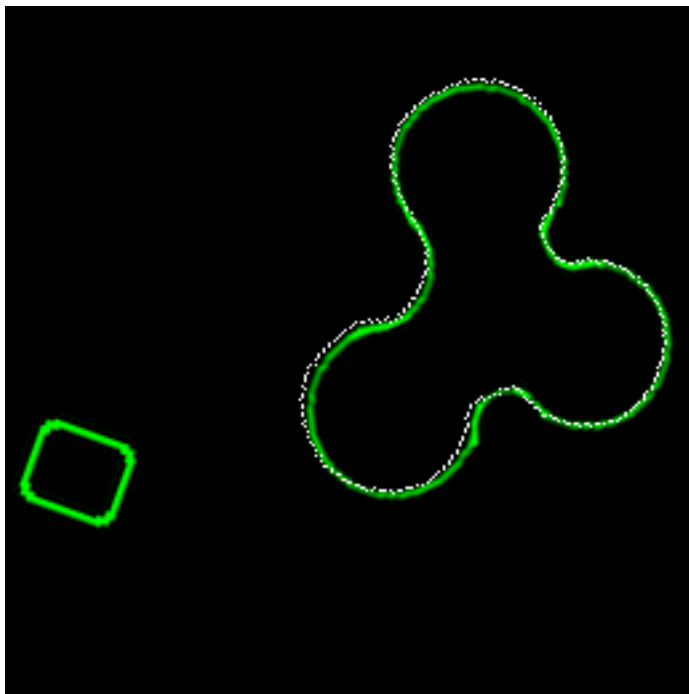
apply coordinate transformation



```
centroidX1=53.41090539097786 centroidY1=73.55932730436325
centroidX2=179.92783105373383 centroidY2=113.09317249059677
  x1    y1    x2    y2  len1  len2 theta1 theta2 (x,y) (x,y)
-27.2   7.2  -28.9   2.9  28.1  29.1   165   174 (  33,   79) (  151,  116)
-25.8  -4.7  -20.9 -21.1  26.3  29.7   190   225 (  34,   70) (  159,   92)
 20.8  24.5   10.1  30.9  32.1  32.5    50    72 (  69,   92) (  190,  144)
 10.1  37.9   -2.9  36.9  39.2  37.0    75    95 (  61,  102) (  177,  150)
   8.8 -38.0   22.1 -30.1  39.0  37.3   283   306 (  60,   45) (  202,   83)
 23.4 -27.4   30.1 -16.1  36.0  34.1   311   332 (  71,   53) (  210,   97)
translationX=150.1189 translationY=-4.185816
scaleX=1.3642461 scaleY=1.2980525 (1.3311493)
rotation=-21.72218537902866
```

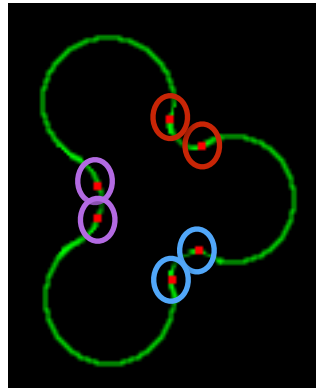
scale should be 1.3
rotation should be 20

After Refinement



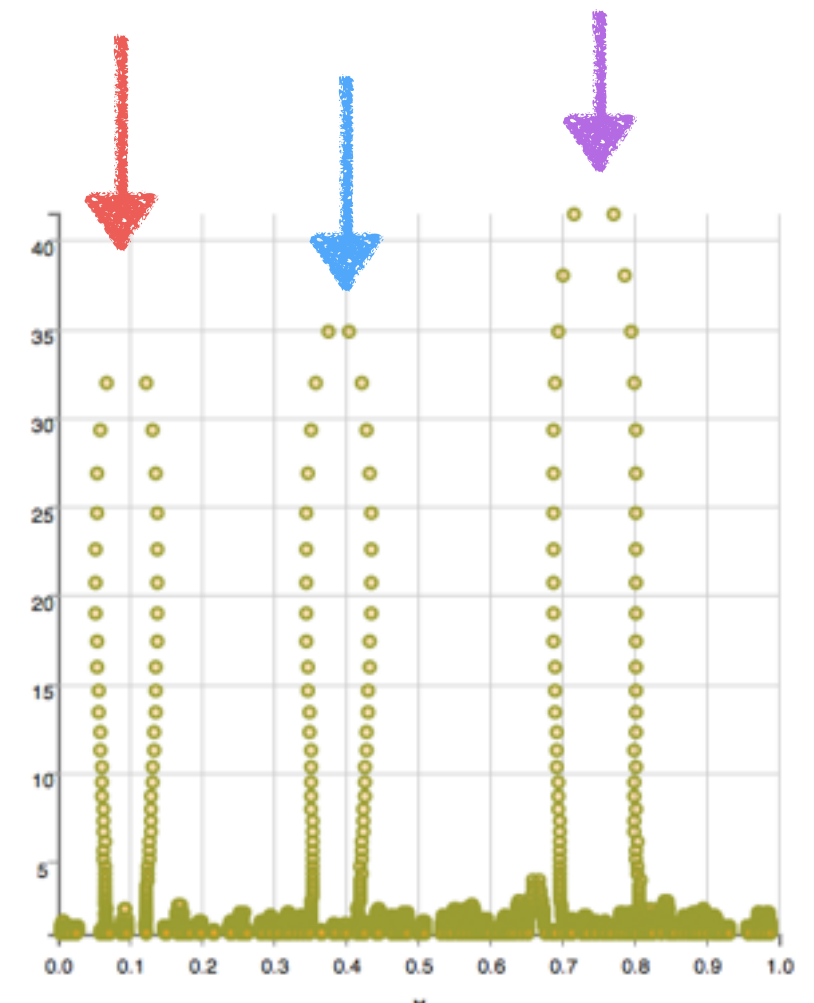
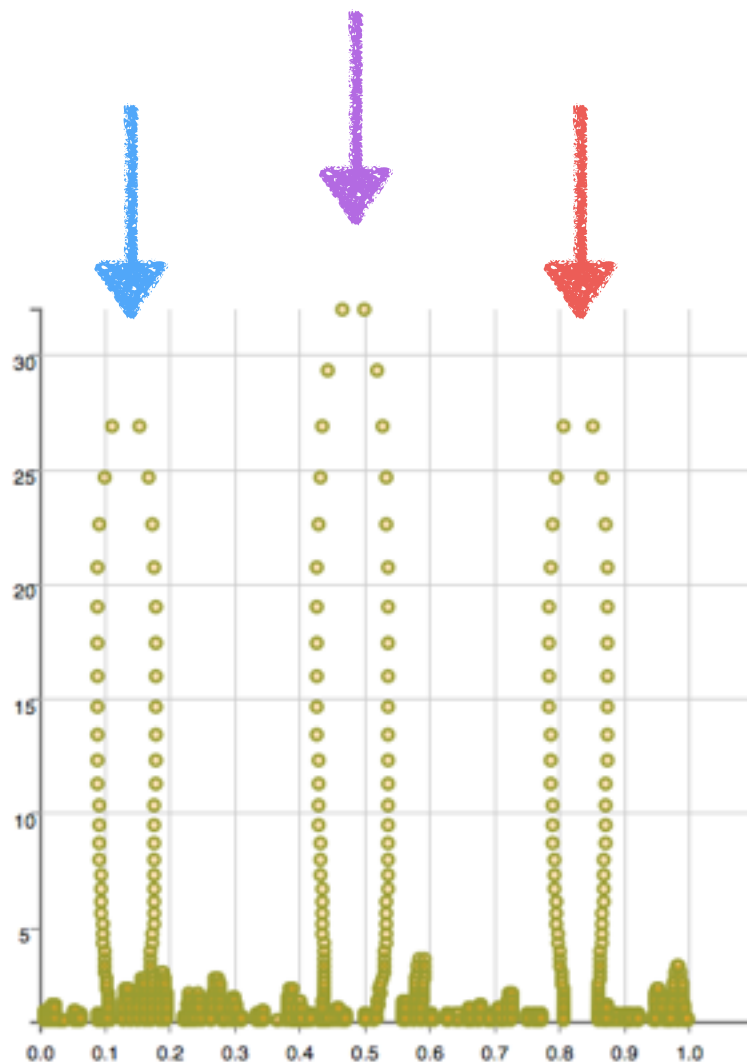
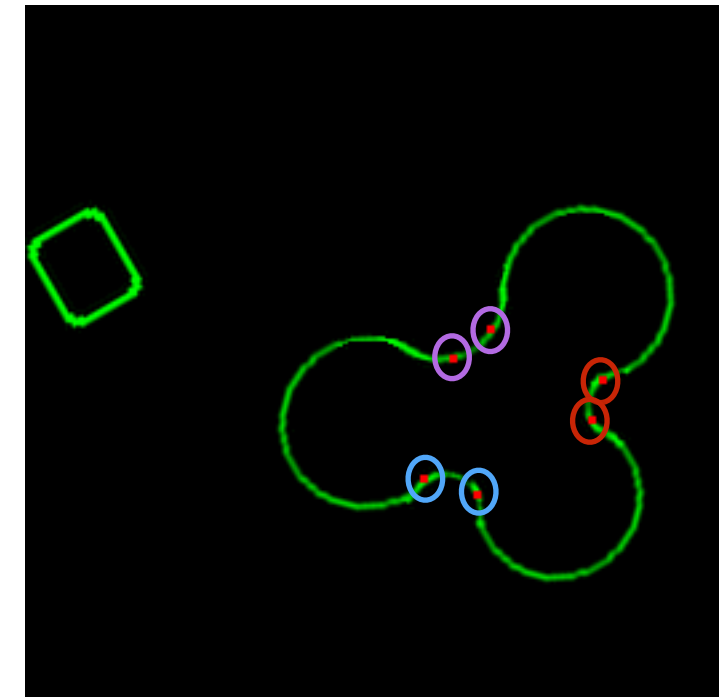
```
rotationInRadians=-0.3687062
rotationInDegrees=-21.125308942438586
scale=1.3146491
translationX=149.12663
translationY=-2.6055665
```


coordinate transformation, after matching contours

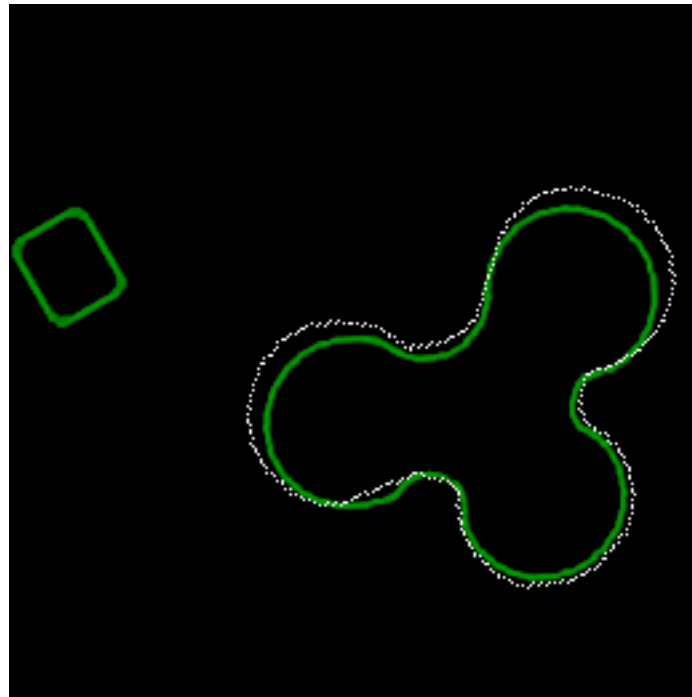


```
centroidX1=53.41090539097786 centroidY1=73.55932730436325
centroidX2=179.24156767129898 centroidY2=148.5439845919609
x1 y1 x2 y2 len1 len2 theta1 theta2 (x,y) (x,y)
-28.7 7.7 -25.2 -16.5 29.7 30.2 165 213 ( 33, 79) ( 154, 132)
-27.3 -5.0 -2.2 -30.5 27.8 30.6 190 266 ( 34, 70) ( 177, 118)
21.9 25.9 -12.2 31.5 34.0 33.8 50 111 ( 69, 92) ( 167, 180)
10.7 40.0 -25.2 26.5 41.4 36.6 75 134 ( 61, 102) ( 154, 175)
9.3 -40.2 39.8 -11.5 41.2 41.4 283 344 ( 60, 45) ( 219, 137)
24.7 -28.9 36.8 9.5 38.1 38.0 311 14 ( 71, 53) ( 216, 158)
translationX=234.14415 translationY=33.017292
scaleX=1.4073112 scaleY=1.164903 (1.2861071)
rotation=-61.401943693218776
```

scale should be 1.3
rotation should be 60



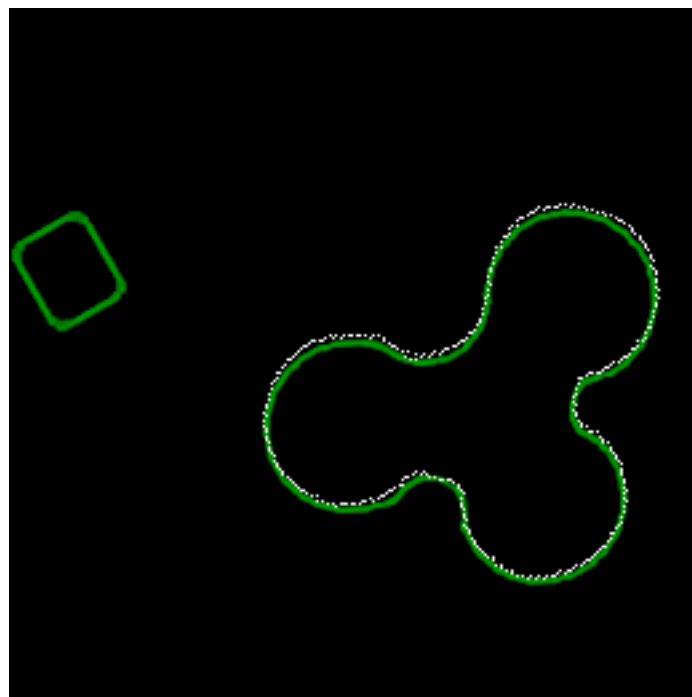
apply coordinate transformation



```
centroidX1=53.41090539097786 centroidY1=73.55932730436325
centroidX2=179.24156767129898 centroidY2=148.5439845919609
x1 y1 x2 y2 len1 len2 theta1 theta2 (x,y) (x,y)
-28.7 7.7 -25.2 -16.5 29.7 30.2 165 213 ( 33, 79) ( 154, 132)
-27.3 -5.0 -2.2 -30.5 27.8 30.6 190 266 ( 34, 70) ( 177, 118)
21.9 25.9 -12.2 31.5 34.0 33.8 50 111 ( 69, 92) ( 167, 180)
10.7 40.0 -25.2 26.5 41.4 36.6 75 134 ( 61, 102) ( 154, 175)
9.3 -40.2 39.8 -11.5 41.2 41.4 283 344 ( 60, 45) ( 219, 137)
24.7 -28.9 36.8 9.5 38.1 38.0 311 14 ( 71, 53) ( 216, 158)
translationX=234.14415 translationY=33.017292
scaleX=1.4073112 scaleY=1.164903 (1.2861071)
rotation=-61.401943693218776
```

scale should be 1.3
rotation should be 60

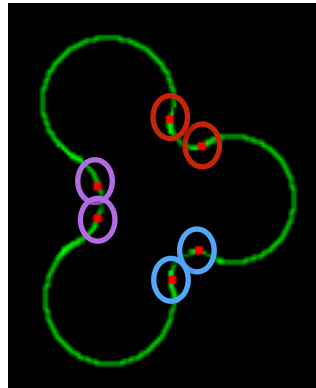
This shows that it's difficult to estimate scale unless some of the inflection points are further from the center of the shape



After Refinement

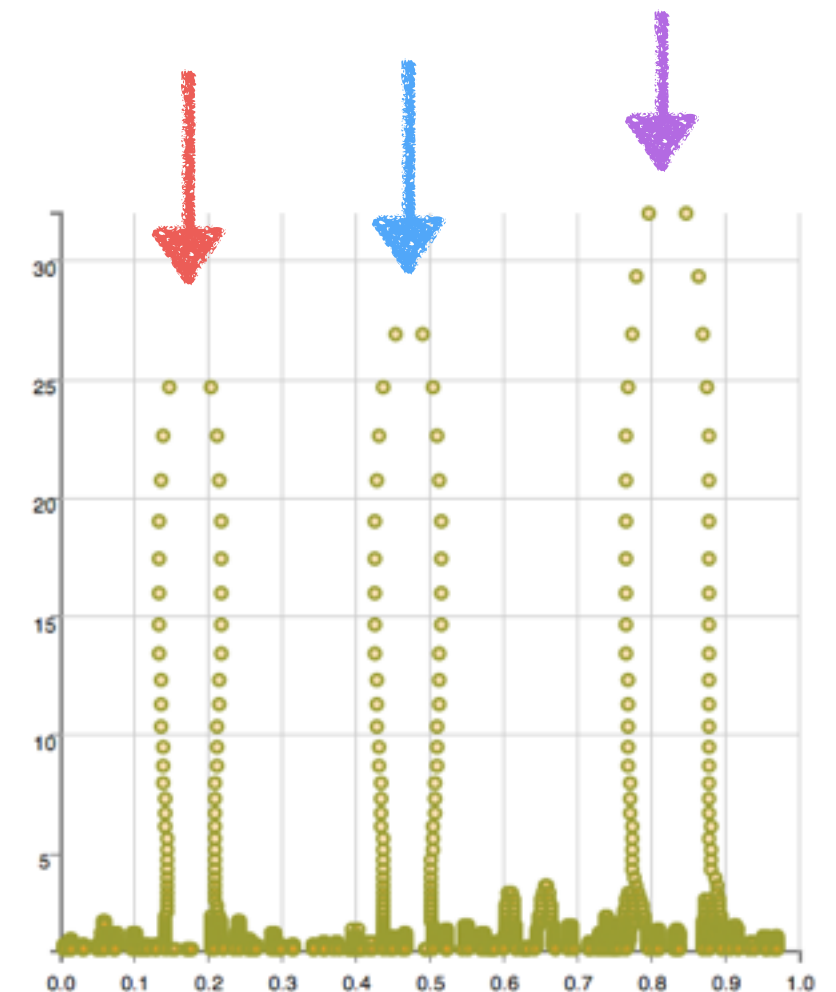
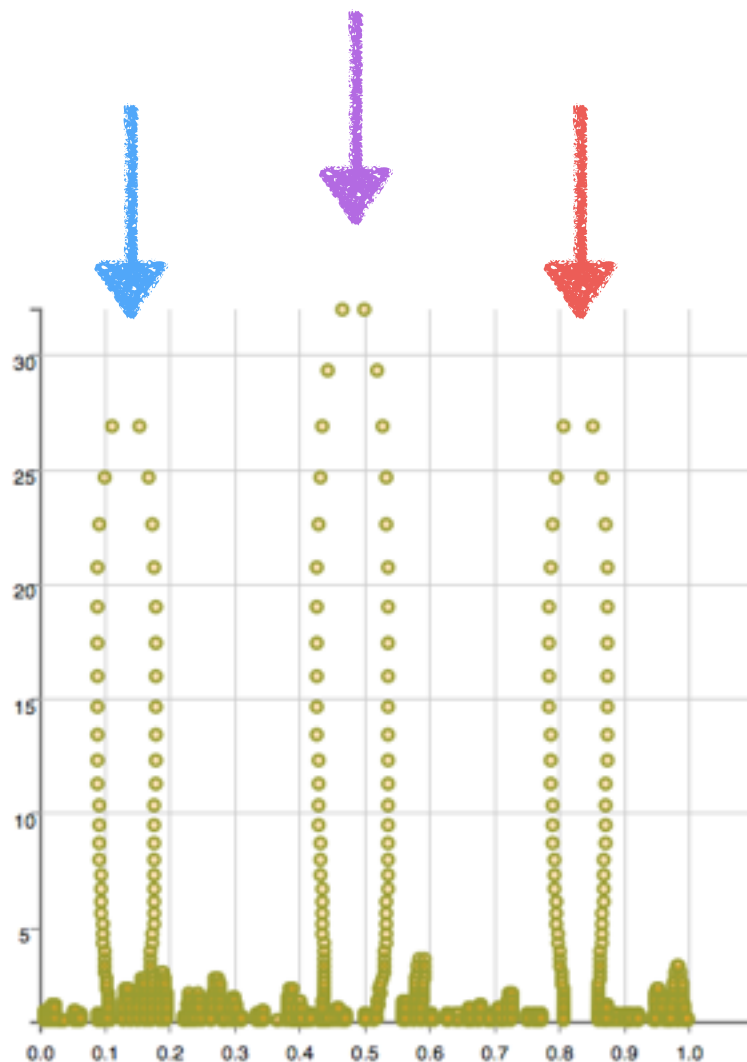
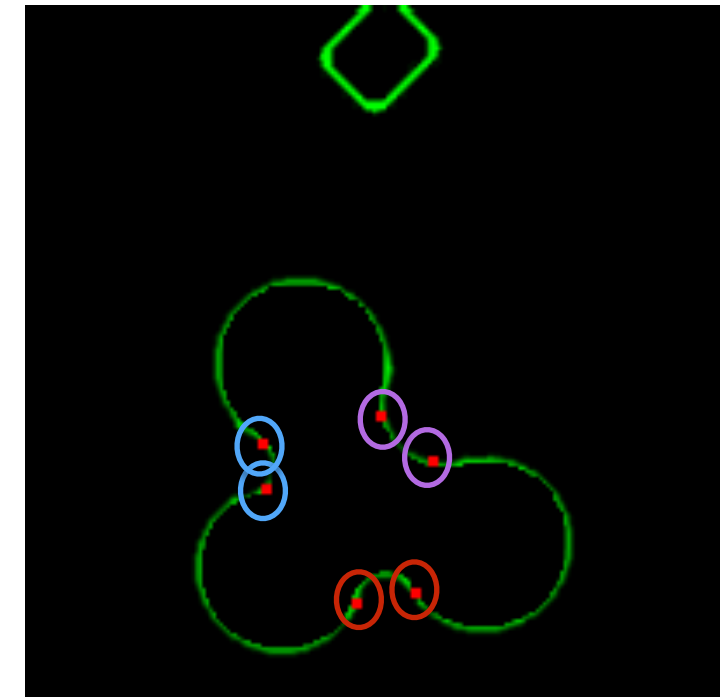
```
rotationInRadians=-1.0527769
rotationInDegrees=-60.31967501337622
scale=1.3123527
translationX=228.40427
translationY=39.845562
```

coordinate transformation, after matching contours

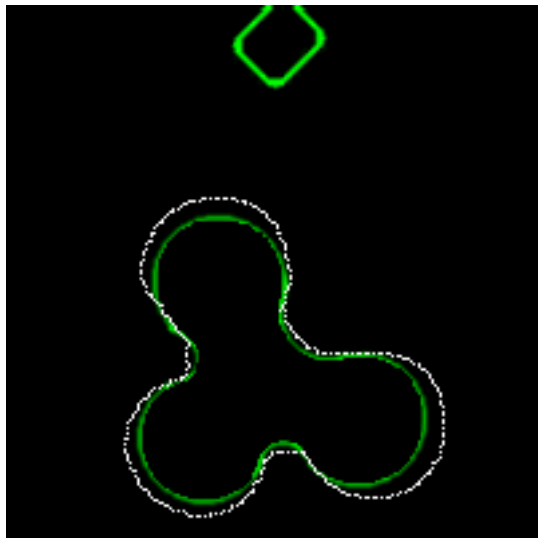


```
centroidX1=53.41090539097786 centroidY1=73.55932730436325
centroidX2=94.52934184670448 centroidY2=140.6554330587387
x1    y1    x2    y2    len1  len2  theta1 theta2 (x,y) (x,y)
-22.9  6.1    7.5  -22.7  23.8  23.9   165   288 ( 33,  79) ( 102, 118)
-21.8 -4.0   22.5  -9.7  22.2  24.5   190   337 ( 34,  70) ( 117, 131)
 17.5 20.7  -25.5  -1.7  27.1  25.6    50   184 ( 69,  92) ( 69, 139)
  8.5 32.0  -26.5 -14.7  33.1  30.3    75   209 ( 61, 102) ( 68, 126)
  7.4 -32.1  17.5  28.3  33.0  33.3   283    58 ( 60,  45) ( 112, 169)
 19.8 -23.1   0.5  31.3  30.4  31.3   311    89 ( 71,  53) ( 95, 172)
translationX=195.43027 translationY=156.95605
scaleX=1.195074 scaleY=1.0536311 (1.1243525)
rotation=-135.16002487520265
```

scale should be 1
rotation should be 135



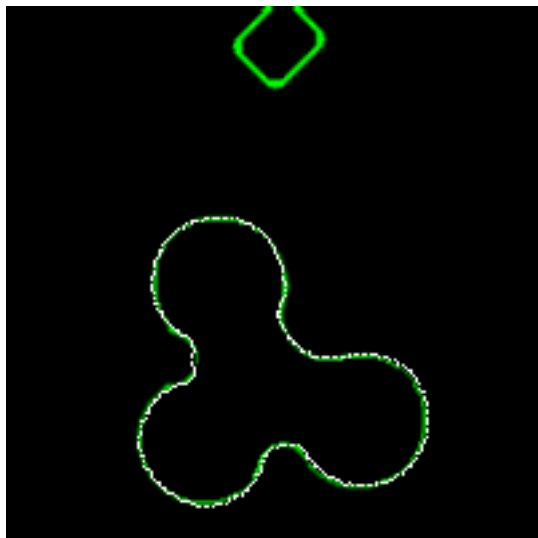
apply coordinate transformation



```
centroidX1=53.41090539097786 centroidY1=73.55932730436325
centroidX2=94.52934184670448 centroidY2=140.6554330587387
x1 y1 x2 y2 len1 len2 theta1 theta2 (x,y) (x,y)
-22.9 6.1 7.5 -22.7 23.8 23.9 165 288 ( 33, 79) ( 102, 118)
-21.8 -4.0 22.5 -9.7 22.2 24.5 190 337 ( 34, 70) ( 117, 131)
17.5 20.7 -25.5 -1.7 27.1 25.6 50 184 ( 69, 92) ( 69, 139)
8.5 32.0 -26.5 -14.7 33.1 30.3 75 209 ( 61, 102) ( 68, 126)
7.4 -32.1 17.5 28.3 33.0 33.3 283 58 ( 60, 45) ( 112, 169)
19.8 -23.1 0.5 31.3 30.4 31.3 311 89 ( 71, 53) ( 95, 172)
translationX=195.43027 translationY=156.95605
scaleX=1.195074 scaleY=1.0536311 (1.1243525)
rotation=-135.16002487520265
```

scale should be 1
rotation should be 135

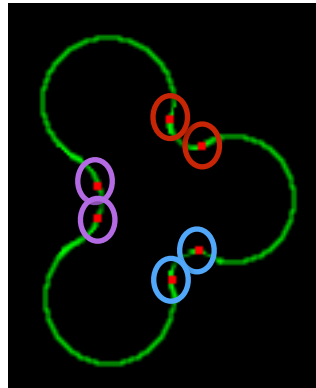
This shows that it's difficult to estimate scale unless some of the inflection points are further from the center of the shape



After Refinement

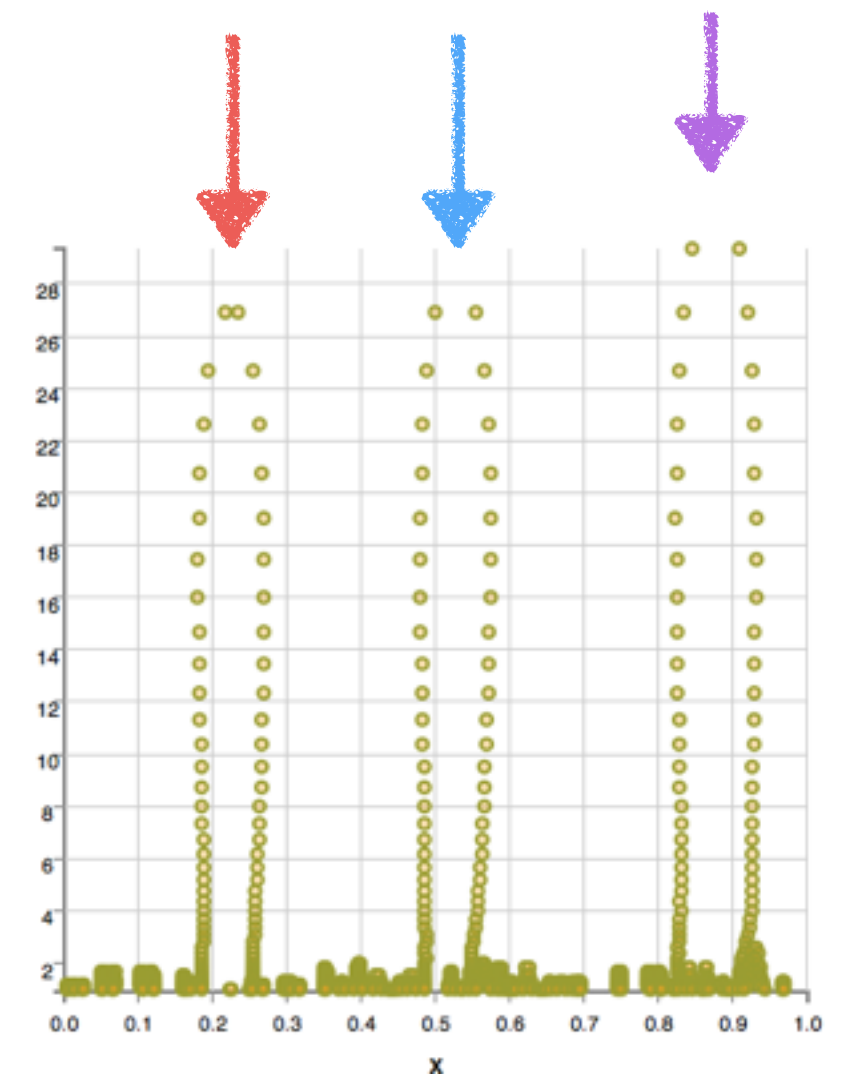
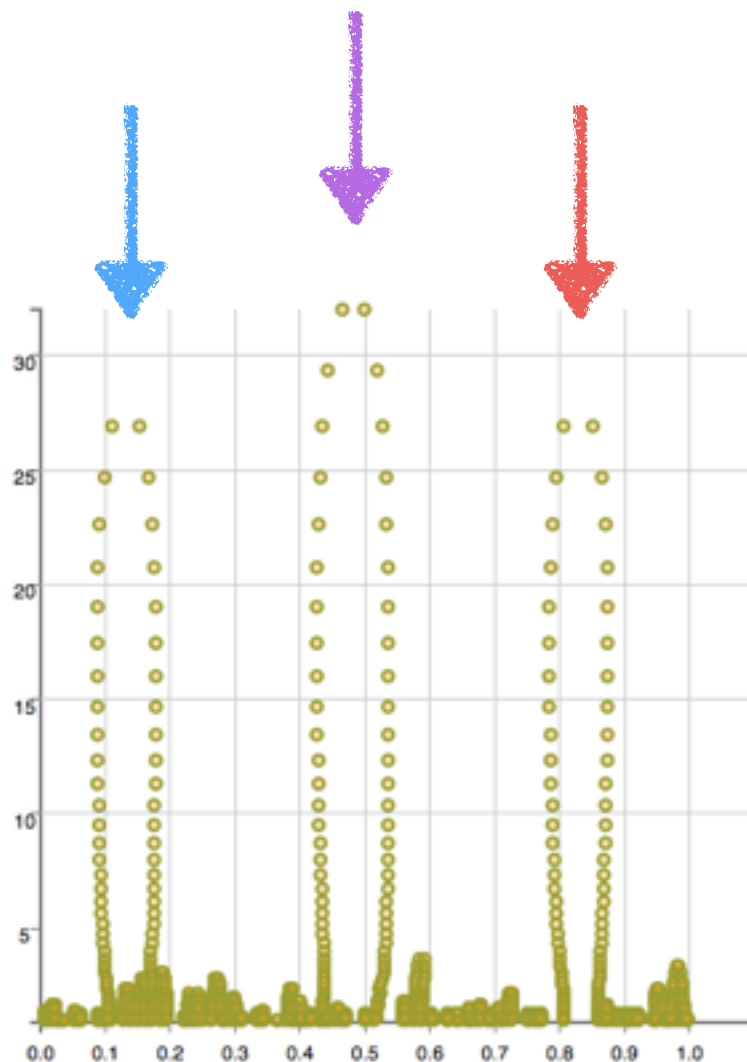
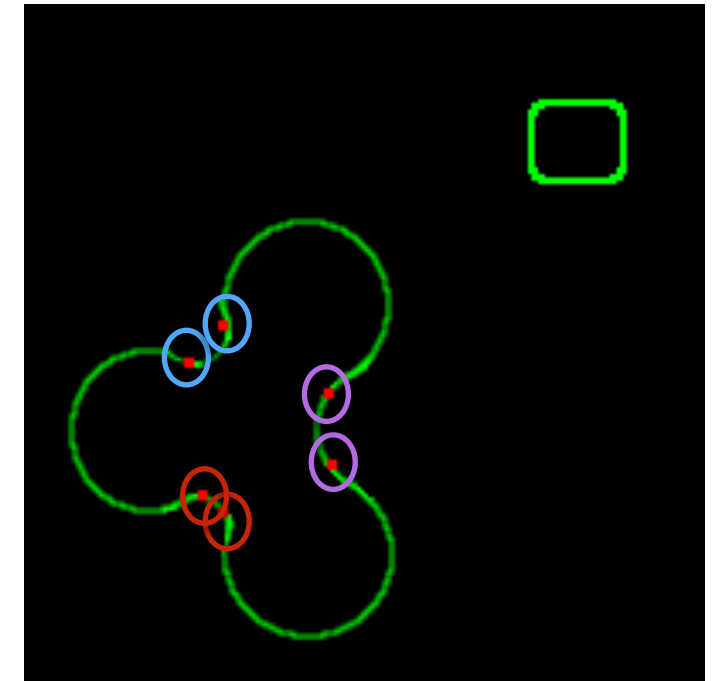
```
rotationInRadians=-2.3401618
rotationInDegrees=-134.08139453976932
scale=1.0155846
translationX=185.93031
translationY=153.66089
```

coordinate transformation, after matching contours

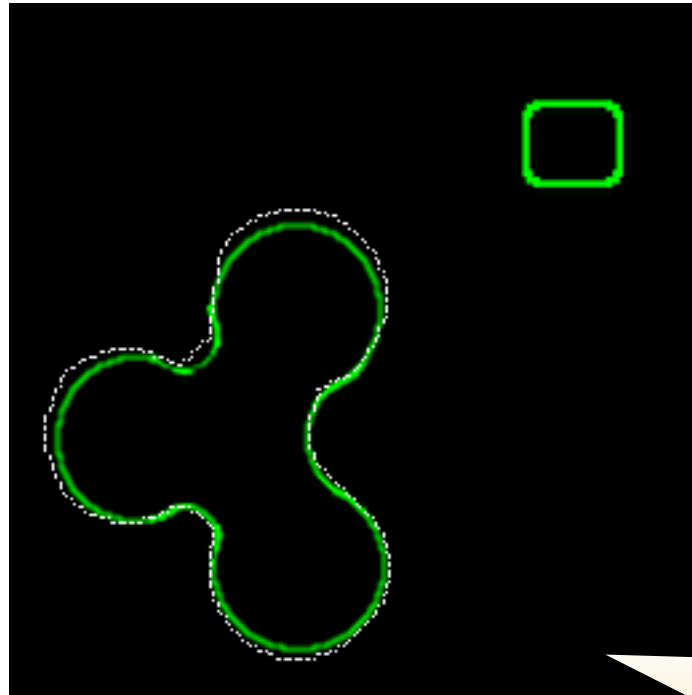


```
centroidX1=53.41090539097786 centroidY1=73.55932730436325
centroidX2=86.03405275940895 centroidY2=160.3728896677494
x1    y1    x2    y2    len1  len2  theta1 theta2 (x,y) (x,y)
-28.1  7.5    27.0  -8.4  29.1  28.2  165    343 ( 33,  79) ( 113, 152)
-26.7 -4.9    27.0   9.6  27.1  28.6  190    20 ( 34,  70) ( 113, 170)
21.4  25.4   -24.0 -24.4  33.2  34.2   50   225 ( 69,  92) (  62, 136)
10.4  39.1   -11.0 -42.4  40.5  43.8   75   255 ( 61, 102) (  75, 118)
 9.1 -39.3    -9.0  37.6  40.3  38.7  283   104 ( 60,  45) (  77, 198)
24.2 -28.3   -20.0  27.6  37.2  34.1  311   126 ( 71,  53) (  66, 188)
translationX=159.47272 translationY=261.5787
scaleX=1.3507035 scaleY=1.4003783 (1.375541)
rotation=-180.0171116418655
```

scale should be 1.3
rotation should be 180



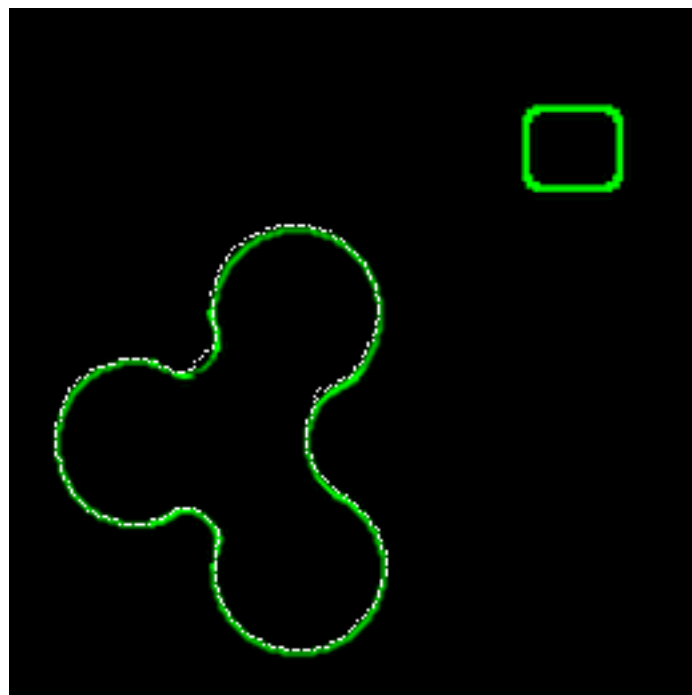
apply coordinate transformation



```
centroidX1=53.41090539097786 centroidY1=73.55932730436325
centroidX2=86.03405275940895 centroidY2=160.3728896677494
x1 y1 x2 y2 len1 len2 (theta1) (theta2)
21.4 25.4 -24.0 -24.4 33.2 34.2 ( 50) ( 225)
10.4 39.1 -11.0 -42.4 40.5 43.8 ( 75) ( 255)
9.1 -39.3 -9.0 37.6 40.3 38.7 ( 283) ( 104)
24.2 -28.3 -20.0 27.6 37.2 34.1 ( 311) ( 126)
-28.1 7.5 27.0 -8.4 29.1 28.2 ( 165) ( 343)
-26.7 -4.9 27.0 9.6 27.1 28.6 ( 190) ( 20)
translationX=159.47272 translationY=261.5787
scaleX=1.3507035 scaleY=1.4003783 (1.375541)
rotation=-180.0171116418655
```

scale should be 1.3
rotation should be 180

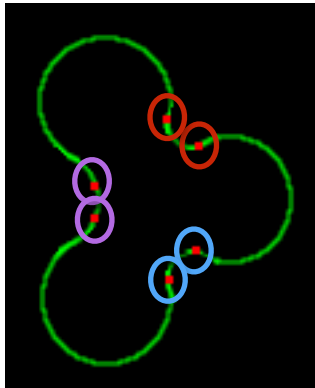
This shows that it's difficult to estimate scale unless some of the inflection points are further from the center of the shape



After Refinement

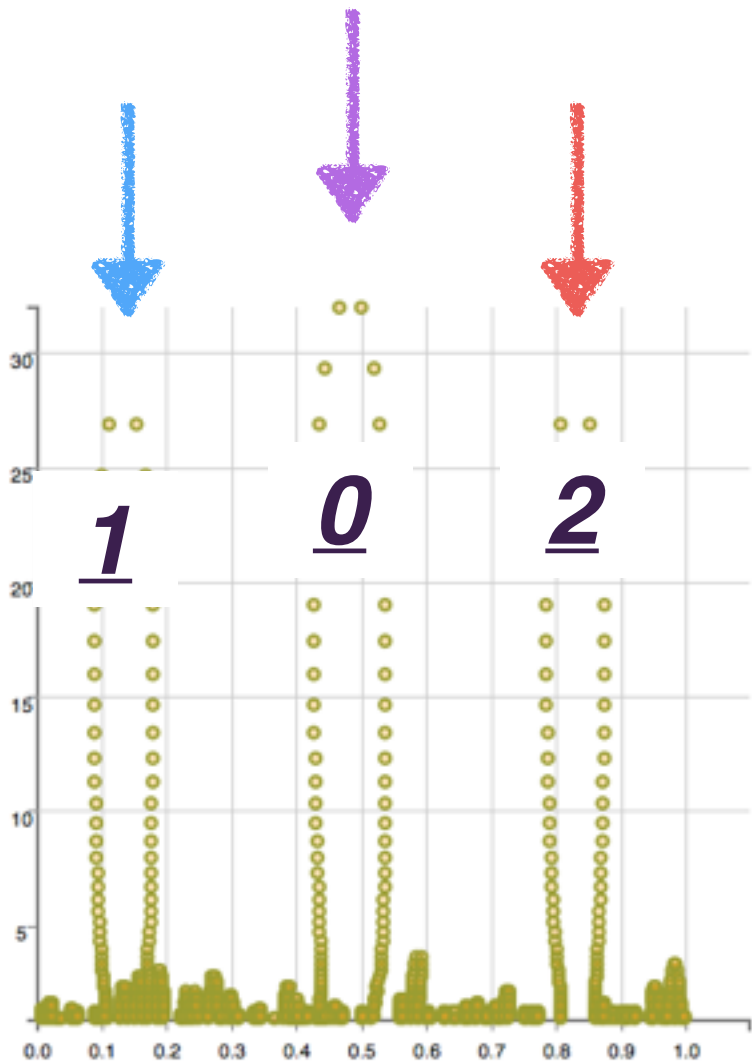
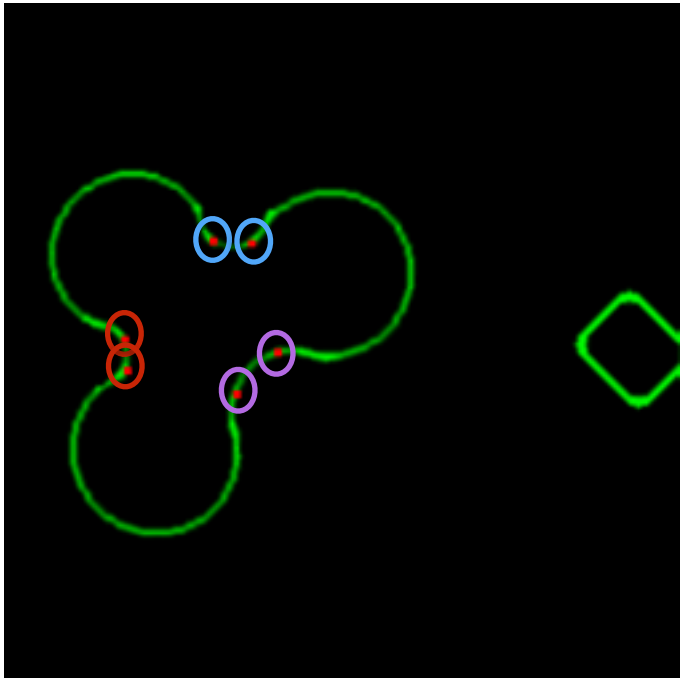
```
rotationInRadians=-3.1295626
rotationInDegrees=-179.31072963867533
scale=1.3046649
translationX=157.36684
translationY=255.68439
```

coordinate transformation, after matching contours

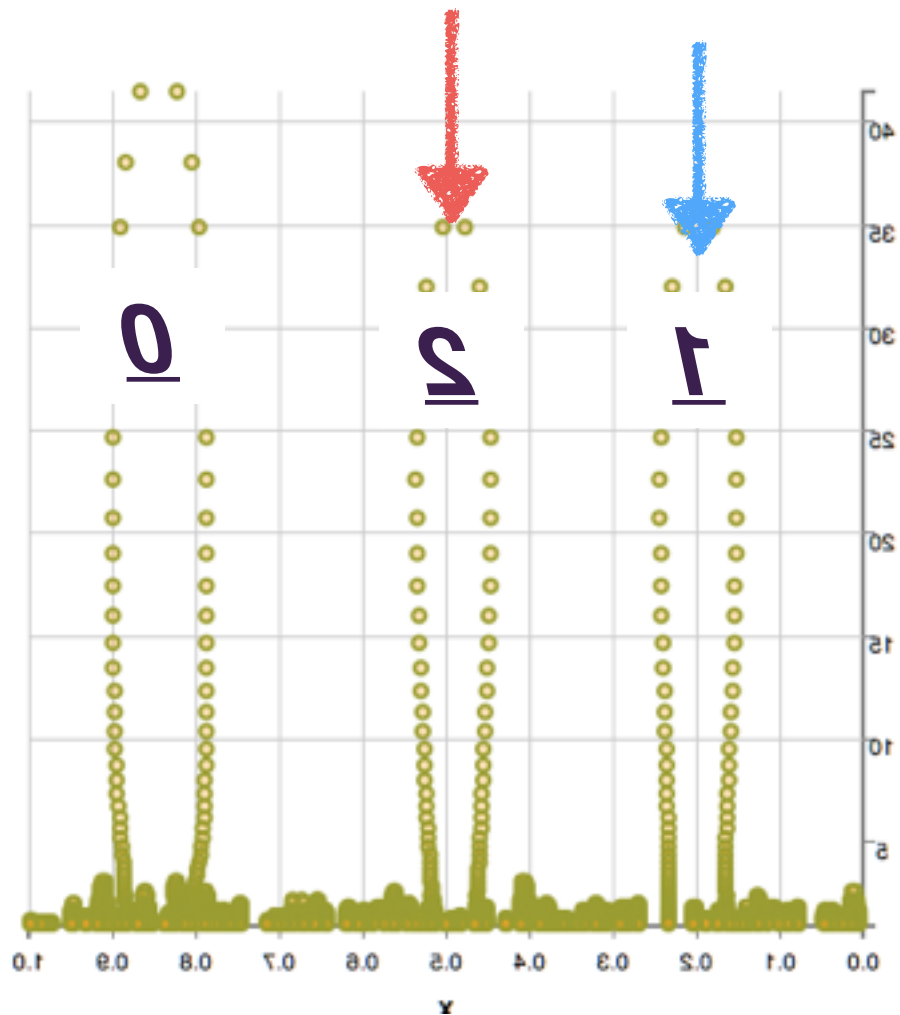


```
centroidX1=53.41090539097786 centroidY1=73.55932730436325
centroidX2=77.37299139797688 centroidY2=124.12722301483154
x1    y1    x2    y2    len1  len2  theta1 theta2 (x,y) (x,y)
-26.8  7.1    26.6  10.9  27.7  28.8   165    22 ( 33,  79) ( 104, 135)
-25.4 -4.7    11.6  24.9  25.9  27.5   190    65 ( 34,  70) (  89, 149)
 20.4 24.2     2.6 -33.1  31.7  33.2    50   275 ( 69,  92) (  80,  91)
  9.9 37.3    16.6 -32.1  38.6  36.2    75   297 ( 61, 102) (  94,  92)
  8.6 -37.4   -32.4  18.9  38.4  37.5   283   150 ( 60,  45) (  45, 143)
 23.1 -27.0   -32.4   3.9  35.5  32.6   311   173 ( 71,  53) (  45, 128)
translationX=59.192436 translationY=241.90477
scaleX=1.418664 scaleY=1.2032518 (1.3109579)
rotation=-224.75825264490877
```

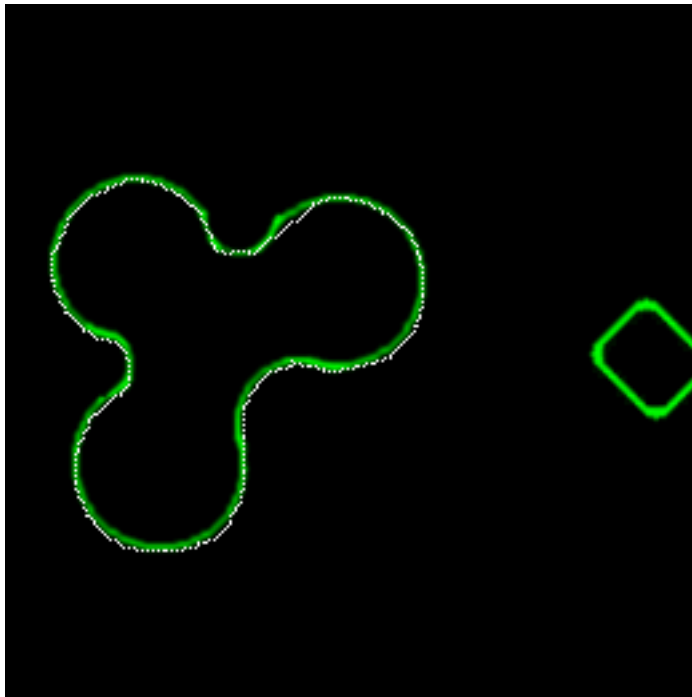
scale should be 1.3
rotation should be 225



reversed the scale
space image to have
CCW ordering of
points in the curve

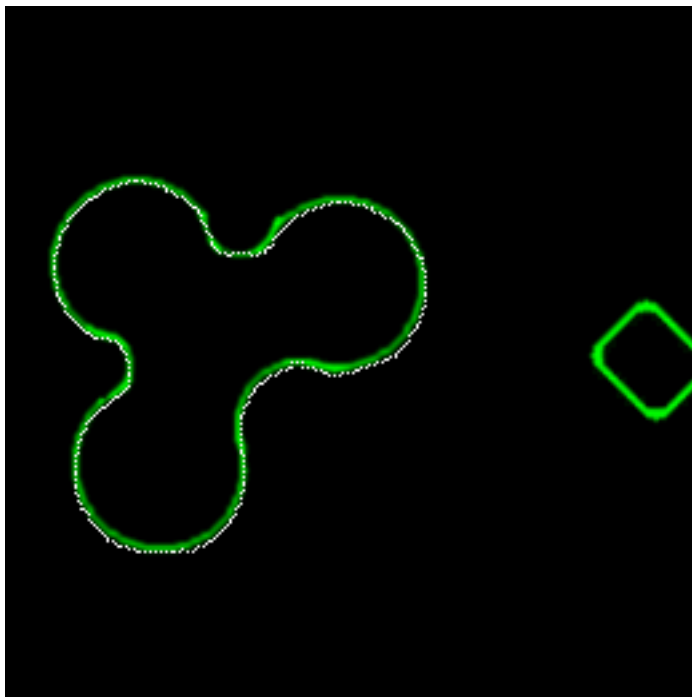


apply coordinate transformation



```
centroidX1=53.41090539097786 centroidY1=73.55932730436325
centroidX2=77.37299139797688 centroidY2=124.12722301483154
x1 y1 x2 y2 len1 len2 theta1 theta2 (x,y) (x,y)
-26.8 7.1 11.6 24.9 27.7 27.5 165 65 ( 33, 79) ( 89, 149)
-25.4 -4.7 26.6 10.9 25.9 28.8 190 22 ( 34, 70) ( 104, 135)
20.4 24.2 16.6 -32.1 31.7 36.2 50 297 ( 69, 92) ( 94, 92)
9.9 37.3 2.6 -33.1 38.6 33.2 75 275 ( 61, 102) ( 80, 91)
8.6 -37.4 -32.4 3.9 38.4 32.6 283 173 ( 60, 45) ( 45, 128)
23.1 -27.0 -32.4 18.9 35.5 37.5 311 150 ( 71, 53) ( 45, 143)
translationX=59.192436 translationY=241.90477
scaleX=1.418664 scaleY=1.2032518 (1.3109579)
rotation=-224.75825264490877
```

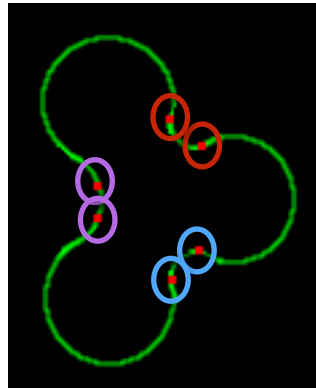
scale should be 1.3
rotation should be 225



After Refinement

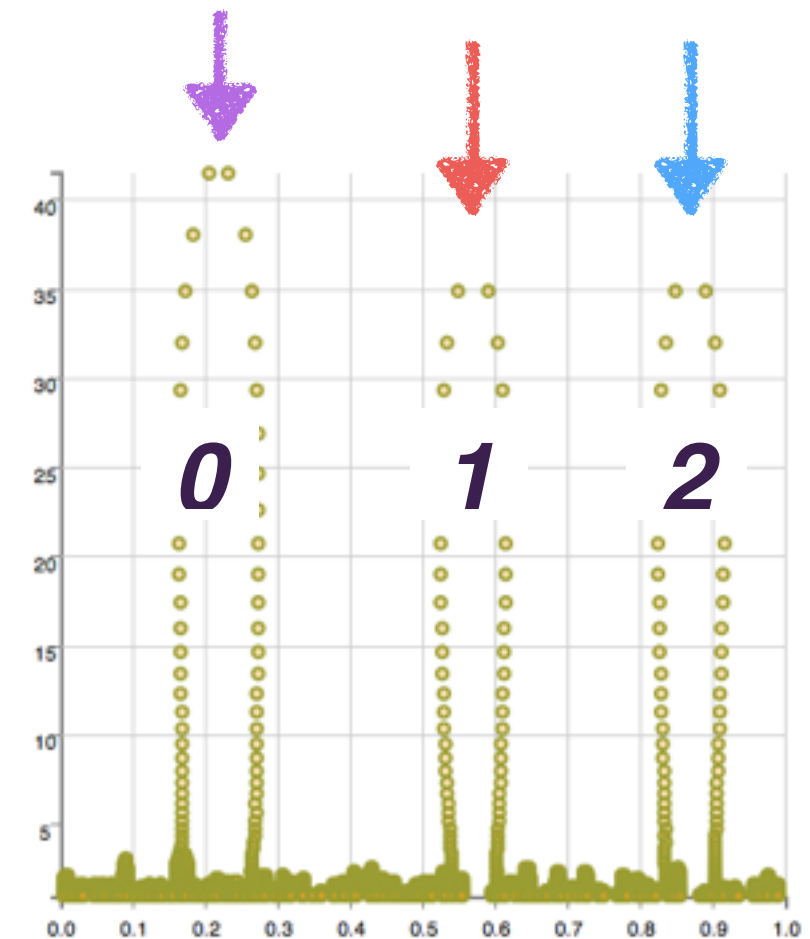
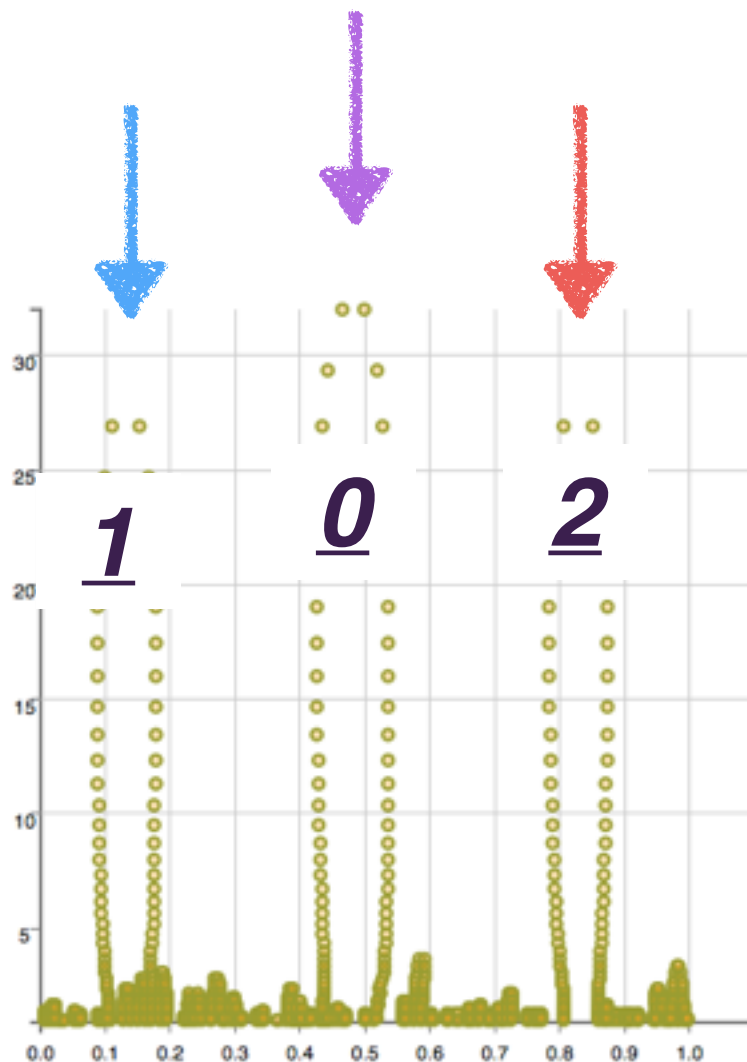
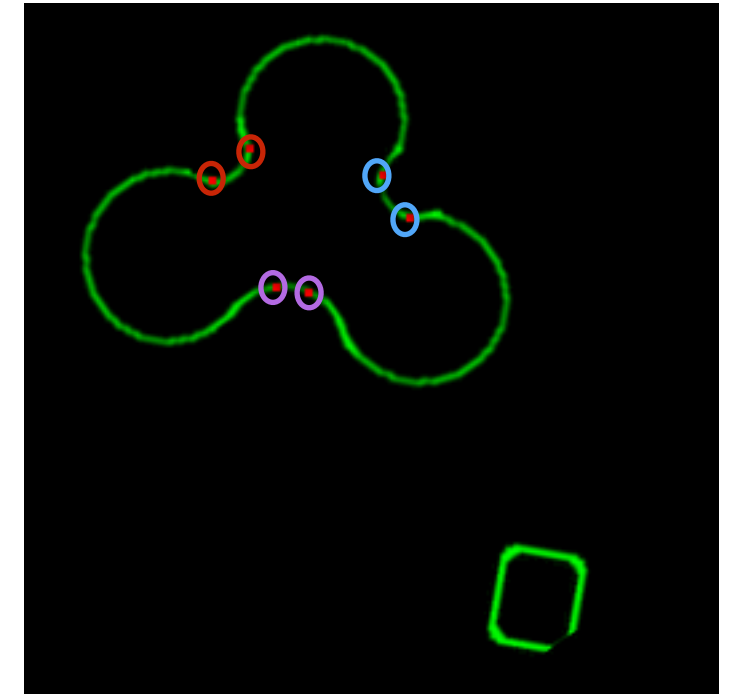
```
rotationInRadians=-3.9322567
rotationInDegrees=-225.3017127923078
scale=1.3120887
translationX=58.216263
translationY=241.98524
```

coordinate transformation, after matching contours

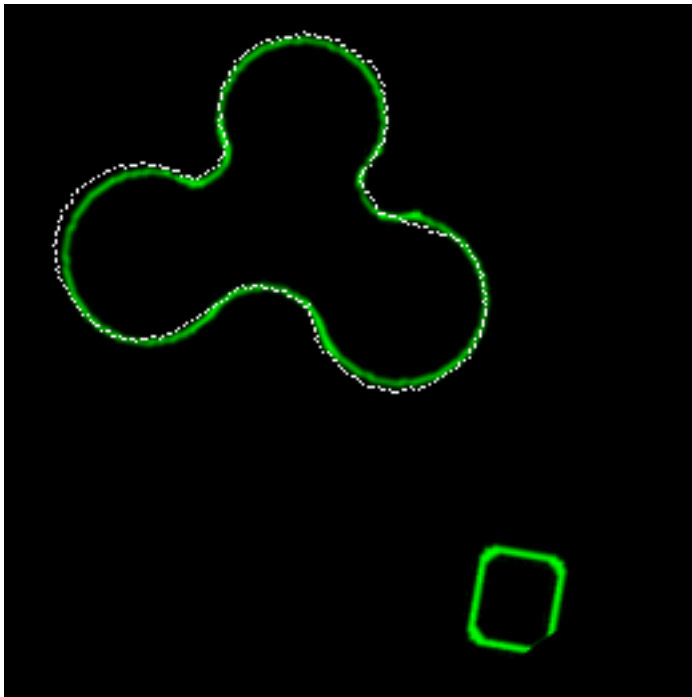


```
centroidX1=53.41090539097786 centroidY1=73.55932730436325
centroidX2=105.01692402362823 centroidY2=81.28828284144402
x1 y1 x2 y2 len1 len2 theta1 theta2 (x,y) (x,y)
-27.3 7.3 1.0 26.7 28.2 26.7 165 88 ( 33, 79) ( 106, 108)
-25.9 -4.8 -11.0 24.7 26.4 27.1 190 114 ( 34, 70) ( 94, 106)
20.8 24.6 29.0 -17.3 32.3 33.7 50 329 ( 69, 92) ( 134, 64)
10.1 38.0 39.0 -1.3 39.3 39.0 75 358 ( 61, 102) ( 144, 80)
8.8 -38.2 -35.0 -15.3 39.2 38.2 283 204 ( 60, 45) ( 70, 66)
23.5 -27.5 -21.0 -27.3 36.2 34.4 311 232 ( 71, 53) ( 84, 54)
translationX=-5.963674 translationY=130.73174
scaleX=1.5381713 scaleY=1.1348706 (1.3365209)
rotation=-281.9694727611886
```

scale should be 1.3
rotation should be 280

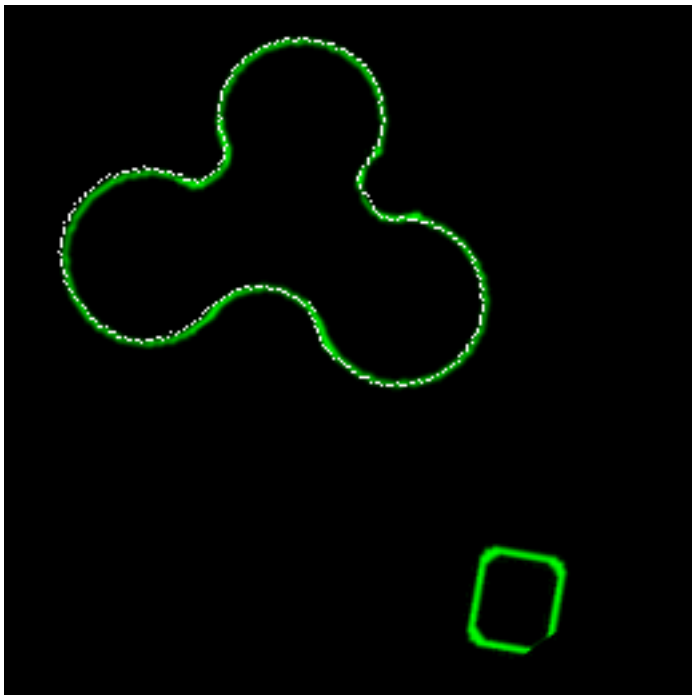


apply coordinate transformation



```
centroidX1=53.41090539097786 centroidY1=73.55932730436325
centroidX2=105.01692402362823 centroidY2=81.28828284144402
x1 y1 x2 y2 len1 len2 theta1 theta2 (x,y) (x,y)
-27.3 7.3 1.0 26.7 28.2 26.7 165 88 ( 33, 79) ( 106, 108)
-25.9 -4.8 -11.0 24.7 26.4 27.1 190 114 ( 34, 70) ( 94, 106)
20.8 24.6 29.0 -17.3 32.3 33.7 50 329 ( 69, 92) ( 134, 64)
10.1 38.0 39.0 -1.3 39.3 39.0 75 358 ( 61, 102) ( 144, 80)
8.8 -38.2 -35.0 -15.3 39.2 38.2 283 204 ( 60, 45) ( 70, 66)
23.5 -27.5 -21.0 -27.3 36.2 34.4 311 232 ( 71, 53) ( 84, 54)
translationX=-5.963674 translationY=130.73174
scaleX=1.5381713 scaleY=1.1348706 (1.3365209)
rotation=-281.9694727611886
```

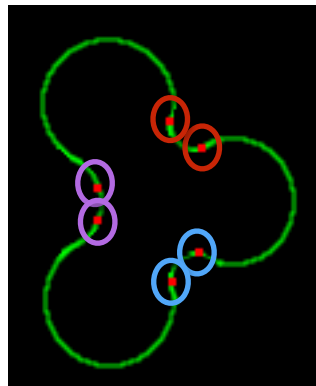
scale should be 1.3
rotation should be 280



After Refinement

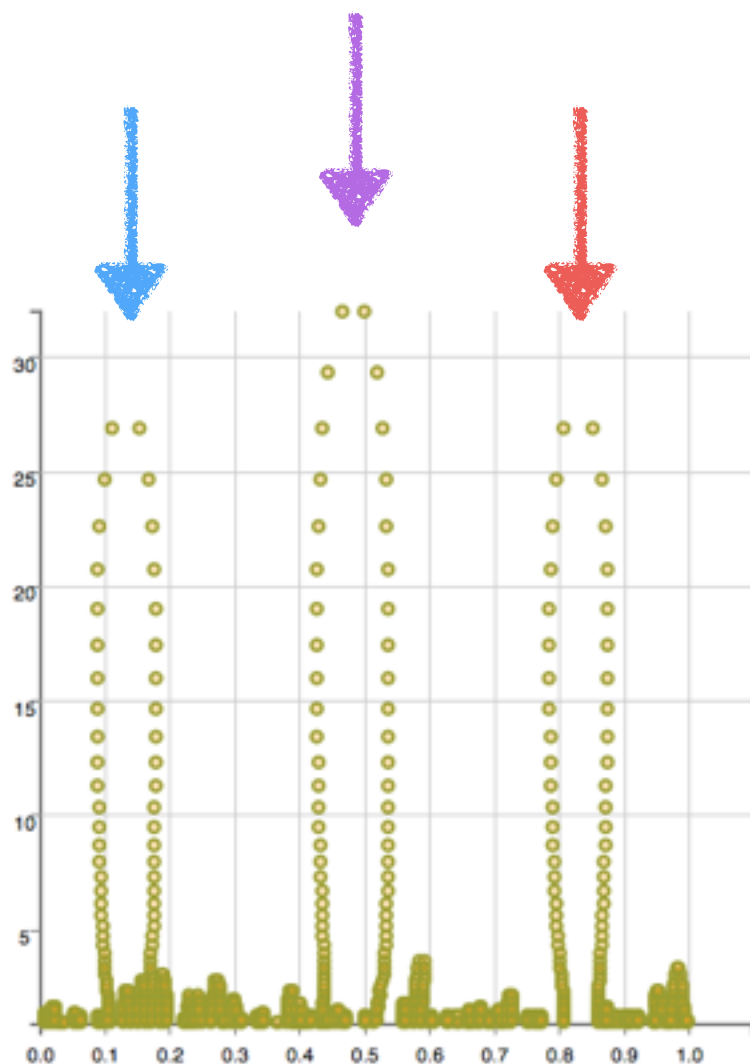
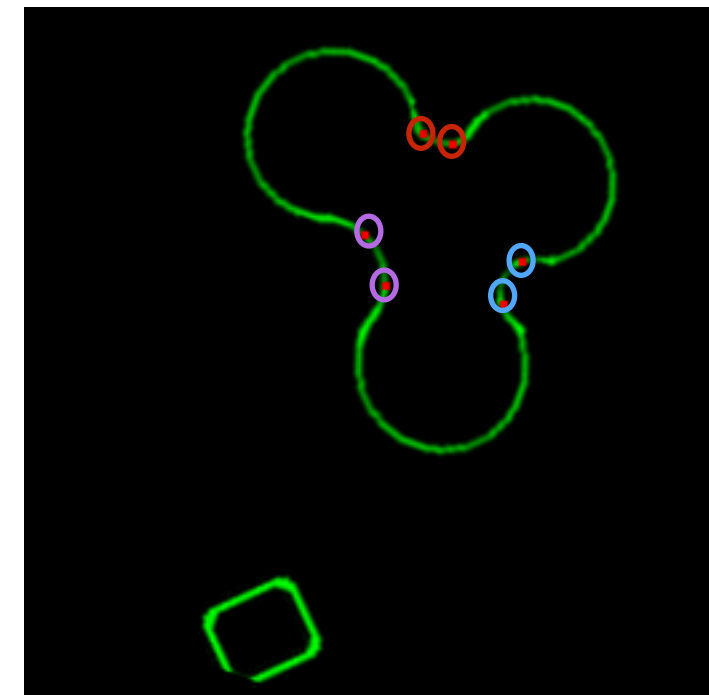
```
rotationInRadians=-4.90338
rotationInDegrees=-280.94297460160294
scale=1.3074504
translationX=-2.6658194
translationY=131.59349
```


coordinate transformation, after matching contours

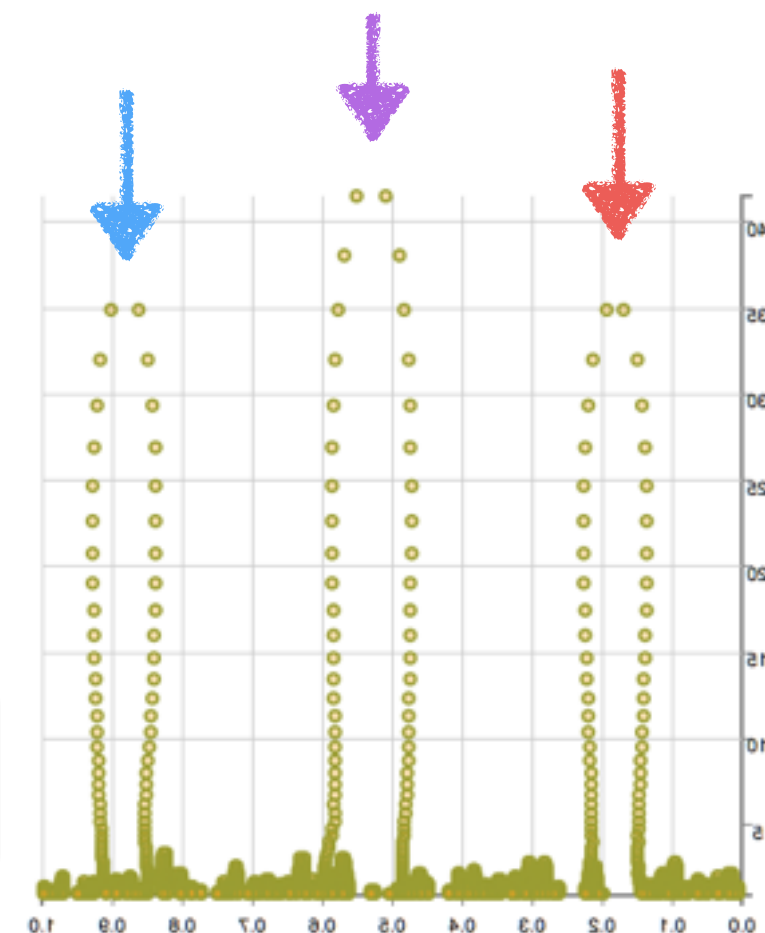


```
centroidX1=53.41090539097786 centroidY1=73.55932730436325
centroidX2=154.51682129502296 centroidY2=82.89837290346622
x1    y1    x2    y2    len1  len2  theta1 theta2 (x,y) (x,y)
-26.7  7.1   -19.5  21.1  27.6  28.7   165   133 ( 33, 79) ( 135, 104)
-25.4 -4.7   -27.5  2.1   25.8  27.6   190   176 ( 34, 70) ( 127, 85)
 20.4 24.1   31.5 12.1  31.6  33.7    50    21 ( 69, 92) ( 186, 95)
  9.9 37.2   24.5 28.1  38.5  37.3    75    49 ( 61, 102) ( 179, 111)
  8.6 -37.3  -5.5 -35.9  38.3  36.3   283   261 ( 60, 45) ( 149, 47)
 23.0 -26.9   5.5 -31.9  35.4  32.4   311   280 ( 71, 53) ( 160, 51)
translationX=50.000797 translationY=26.382126
scaleX=1.3551828 scaleY=1.258931 (1.3070569)
rotation=-334.38510452640554
```

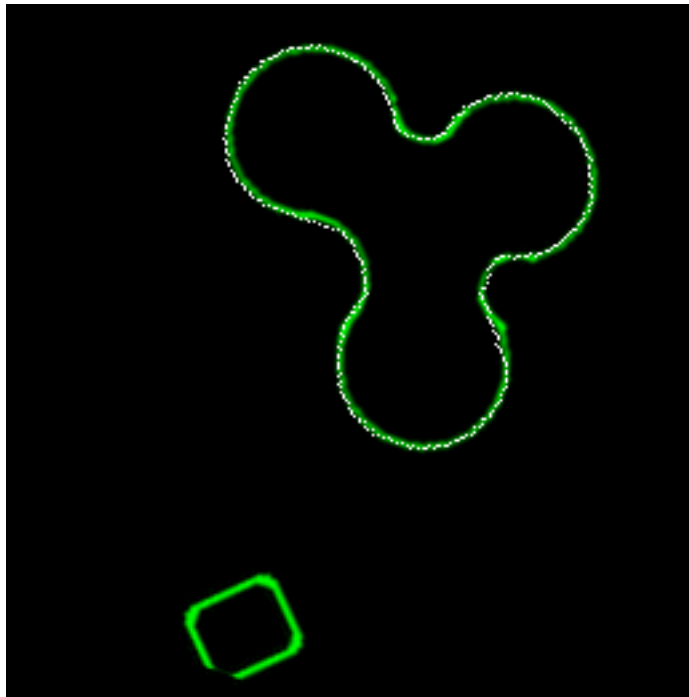
scale should be 1.3
rotation should be 335



reversed the scale
space image to have
CCW ordering

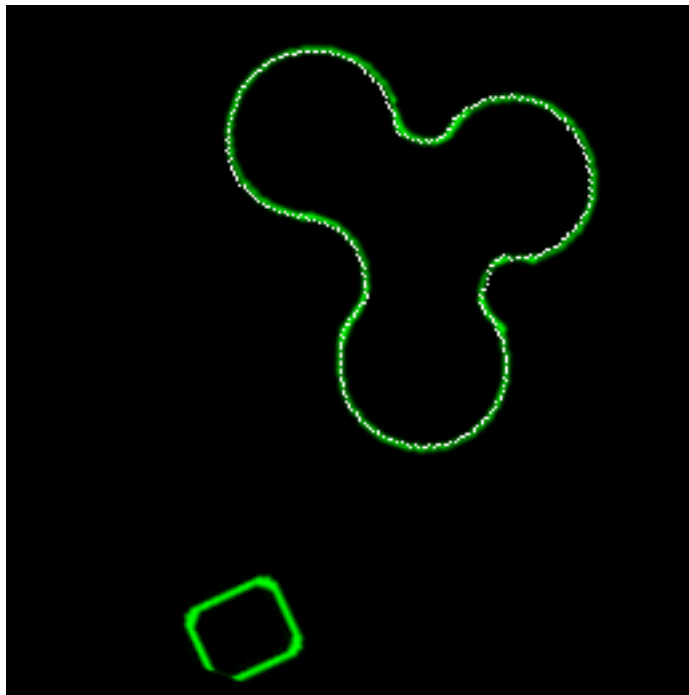


apply coordinate transformation



```
centroidX1=53.41090539097786 centroidY1=73.55932730436325
centroidX2=154.51682129502296 centroidY2=82.89837290346622
x1 y1 x2 y2 len1 len2 theta1 theta2 (x,y) (x,y)
-26.7 7.1 -19.5 21.1 27.6 28.7 165 133 ( 33, 79) ( 135, 104)
-25.4 -4.7 -27.5 2.1 25.8 27.6 190 176 ( 34, 70) ( 127, 85)
20.4 24.1 31.5 12.1 31.6 33.7 50 21 ( 69, 92) ( 186, 95)
9.9 37.2 24.5 28.1 38.5 37.3 75 49 ( 61, 102) ( 179, 111)
8.6 -37.3 -5.5 -35.9 38.3 36.3 283 261 ( 60, 45) ( 149, 47)
23.0 -26.9 5.5 -31.9 35.4 32.4 311 280 ( 71, 53) ( 160, 51)
translationX=50.000797 translationY=26.382126
scaleX=1.3551828 scaleY=1.258931 (1.3070569)
rotation=-334.38510452640554
```

scale should be 1.3
rotation should be 335



After Refinement

```
rotationInRadians=-5.83458
rotationInDegrees=-334.29680605786075
scale=1.2935956
translationX=50.991127
translationY=27.123665
```