Que-1

$$\begin{bmatrix} 4 & 1 & 2 \\ 2 & 4 & -1 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 9 \\ -5 \\ -9 \end{bmatrix} \quad -① $$

→ Any real no. in 2 digit floating pt. representation is written as $0.a_1a_2 e{+}m$

e.g. 10 is written as 0.10 e+1.

Note: I am doing calculating as we do for decimal no. analytically with ∞ pricition & then rounding off them to 2 digit.

So eq^n ① can be written in augmented form as-

$$(A|B) = \left[\begin{array}{ccc|c} 0.4e{+}1 & 0.10E1 & 0.20E1 & 0.90E1 \\ 0.20E1 & 0.40E1 & -0.10E1 & -0.50E1 \\ 0.10E1 & 0.10E1 & -0.30E1 & -0.90E1 \end{array}\right] \quad -②$$

$$R_1 \to \frac{1}{a_{11}} R_1 \Rightarrow \left[\begin{array}{ccc|c} 0.10E1 & 0.25E0 & 0.50E0 & 0.22E1 \\ 0.20E1 & 0.40E1 & -0.10E1 & -0.50E1 \\ 0.10E1 & 0.10E1 & -0.30E1 & -0.90E1 \end{array}\right] \quad -③$$

Note: $\frac{0.90E1}{0.40E1} = 0.22E1$ ← I did round offs here

$$R_2 \to R_2 - \frac{a_{21}}{a_{11}} R_1 \ \& \ R_3 \to R_3 - \frac{a_{31}}{a_{11}} R_1, \quad \left[\frac{a_{21}}{a_{11}} = 0.20E1, \ \frac{a_{31}}{a_{11}} = 0.10E1\right]$$

$$= \left[\begin{array}{ccc|c} 0.10E1 & 0.25E0 & 0.50E0 & 0.22E1 \\ 0.00E0 & 0.35E1 & -0.20E1 & -0.94E1 \\ 0.00E0 & 0.75E0 & -0.35E1 & -0.11E2 \end{array}\right]$$

$$R_2 \to \frac{1}{a_{22}} R_2, \text{ then } R_3 \to R_3 - \frac{a_{32}}{a_{22}} R_2$$

~~0.10E1 0.25E0 0.50E1~~

$$\left[\begin{array}{ccc|c} 0.10E1 & 0.25E0 & 0.50E0 & 0.22E1 \\ 0.00E0 & 0.10E1 & -0.57E0 & -0.27E1 \\ 0.00E0 & 0.00E0 & -0.31E1 & -0.90E1 \end{array}\right]$$

Now, by back-substitution, we have

$$-0.31E1 x_3 = -0.90E1 \Rightarrow \boxed{x_3 = 0.29E1 = 2.9} \quad -ⓐ$$

& $0.10E1\, x_2 - 0.57E0\, x_3 = -0.27E1$

using $x_3 = 0.29E1$

we get, $\boxed{x_2 = -0.11E1 = -1.1}$ — ⓑ

& from 1st row of augmented matrix, we get

$$0.1E0\, x_1 + 0.25E0\, x_2 + 0.50E0\, x_3 = 0.22E1$$

using eq^n ⓐ & ⓑ, I got

$\boxed{x_1 = 0.11E1 = 1.1}$ — ⓒ

So, I got $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1.1 \\ -1.1 \\ 2.9 \end{pmatrix}$ ← Solution obtained from 2 digit-float

<u>Note:</u> Each & individual operation is ~~do~~ calculated & then rounded off

e.g. $\underbrace{0.2\cancel{8}E1 \times 0.4\cancel{1}E1}_{0.10E2} + \underbrace{0.10E1 * 0.5E0}_{0.50E0}$ ← multiplication + Rounded off

1st multiplication + rounded off → $0.10E2$

$\downarrow$

$0.10E2$ ← Now addition + Rounded off

Que-3

Ans:

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & & \\ a_{21} & a_{22} & a_{23} & \cdots & \\ \vdots & a_{32} & a_{33} & a_{34} & \cdots \\ \vdots & \vdots & a_{43} & a_{44} & \cdots \\ & & \vdots & \vdots & a_{nn} \end{bmatrix}_{n\times n}$$ ← triadiagonal matrix

$$B = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

$\Rightarrow [A|B] = \left[A \,\middle|\, B \right]_{n\times(n+1)}$ ← Augmented form

→ 1st operation : On coloum 1

$a_{11} \to 1$: $R_1 \to \frac{1}{a_{11}} R_1 \Rightarrow 3$ - divisions $\left(\because \frac{a_{11}}{a_{11}}, \frac{a_{12}}{a_{11}}, \frac{b_1}{a_{11}}\right)$ ↳ $(1+1+1=3)$

$a_{21} \to 0$: $R_2 \to R_2 - \frac{a_{21}}{a_{11}} R_1 \Rightarrow$ a — ①

let $\frac{a_{21}}{a_{11}} = t \Rightarrow$ 1-division

① ⇒ 3-multiplication which are : $t\times a_{11}$, $t\times a_{12}$ & $t\times b_1$

3-Subtractions which are : $(a_{21} - ta_{11})$, $(a_{22} - ta_{12})$, $(b_2 - tb_1)$

So, total operation $= 4 + 3 + 3$ ← subtraction
↑ division ↑ multiplication

→ Now, operation of 2 coloum:

$a_{22} \to 1$; $R_2 \to \frac{1}{a_{22}} R_2 \Rightarrow$ 3-divisions again which are $\left(\left(\frac{a_{22}}{a_{22}}\right), \left(\frac{a_{23}}{a_{22}}\right), \left(\frac{b_2}{a_{22}}\right)\right)$

$a_{32} \to 0$; $R_3 \to R_3 - tR_2$

$t = a_{32}/a_{22}$

⇒ 1-division (i.e t)

3-multiplication i.e. $[ta_{22}, ta_{23}, b_2 t]$

3 - subtraction again.

Total operation = $4 + 3 + 3$ ← subtraction
↑ division ↑ multiplication

Similarly continuing for (n) coloumn, In total we get

$$= (4n + 3n + 3n) - (1 + 1 + 1)$$

This is subtracted because in last coloumn we have 1 less, multiplication, subtraction & addition, since no. element as $a_{n+1,n} = 0$.

$\Rightarrow$ Total operation $= (10n - 3)$

Back substitution:

- $x_n = b'_n \quad \leftarrow n^{th}$ eq^n

$x_{n-1} + a'_{(n-1),n} x_n = b'_{n-1} \quad \leftarrow (n-1)^{th}$ eq^n

$x_{n-2} + a'_{(n-2)(n-1)} x_{n-1} + \cancel{a'_{(n-2)n} x_n} = b'_{n-2} \quad \leftarrow (n-2)^{th}$ eq^n

$x_{n-3} + a'_{(n-3)(n-2)} x_{n-2} + 0 + 0 = b'_{n-3}$ & so on

$\vdots$

$\rightarrow n^{th}$ eq^n $\Rightarrow$ no-operation

$\rightarrow (n-1)^{th}$ eq^n $\Rightarrow$ 1 multiplication + 1 subtraction

$\rightarrow (n-2)^{th}$ eq^n $\rightarrow$ 1 " + 1 "

$\vdots$

$\rightarrow$ 1st eq^n = 1 " + 1 "

$\Downarrow$

Total operation required $= 2(n-1) = (2n-2)$

$\Rightarrow$ Total operation = Guass elimination + Backsubstitution

$$= (10n - 3) + (2n - 2)$$

$$= \boxed{12n - 5} = O(n)$$

Hence Proved that no. of operation required to solve a tridiagonal system by guass elimination is order of $O(n)$.