

Question 1

3. Let $G = (V, E)$ be a directed graph, with source $s \in V$, sink $t \in V$, and nonnegative edge capacities $\{c_e\}$. Give a polynomial-time algorithm to decide whether G has a unique minimum $s - t$ cut (i.e., an $s - t$ of capacity strictly less than that of all other $s - t$ cuts).

Let G be an arbitrary flow network, with a source s , a sink t , and a positive integer capacity c_e on every edge e ; and let (A, B) be a minimum $s - t$ cut with respect to these capacities $\{c_e : e \in E\}$. Now suppose we add 1 to every capacity; then (A, B) is still a minimum $s - t$ cut with respect to these new capacities $\{1 + c_e : e \in E\}$.