Homework 20200930

Due Date: 20201012, 10 P.M.

Give asymptotic upper and lower bounds for $T(n)$ in each of the following recurrences. Assume that $T(n)$ is constant for $n \leq 2$. Make your bounds as tight as possible, and justify your answers.

a. $T(n) = 2T(n/2) + n^4$.

b. $T(n) = T(7n/10) + n$.

c. $T(n) = 16T(n/4) + n^2$.

d. $T(n) = 7T(n/3) + n^2$.

e. $T(n) = 7T(n/2) + n^2$.

f. $T(n) = 2T(n/4) + \sqrt{n}$.

g. $T(n) = T(n-2) + n^2$.

Given an array A=<5, 13, 2, 25, 37, 17, 20, 8, 24, 9, 7>, please show: 1) A after the BuildHeap(A) process; 2) A right after heap_size(A) turns into 8; 3) A right after heap_size(A) turns into 3.

What is the running time of HEAPSORT on an array A of length n that is already sorted in increasing order? What about decreasing order?

Give an $O(n \lg k)$-time algorithm to merge k sorted lists into one sorted list, where n is the total number of elements in all the input lists. (*Hint:* Use a min-heap for k-way merging.)