Initial results

April 21, 2017

1 Model details

The model has 3 cars (Figure 1) running in the same lane on a highway. We choose the time window as $\Delta t = 0.1s$ and 10000 iterations. This means we consider a platooning system running for 1000 seconds. For each car, we consider 2 future steps, which means we consider up to time $t + 2$ to determine the optimized decision at time t. Notations are in Table 1.

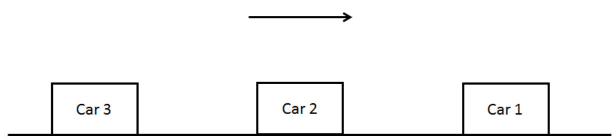

Figure 1: 3-car Model

Table 1: Notations

Notation	Representation
$v_i(t)$	velocity of the ith car at time t
$a_i(t)$	acceleration of the ith car at time t (from time t to time $t + 1$)
$s_i(t)$	position of the ith car at time t
$d_i(t)$	distance from ith car to the front car at time t
V_{min}	minimum velocity allowed
V_{max}	maximum velocity allowed
a_{min}	minimum acceleration allowed
a_{max}	maximum acceleration allowed

The platooning problem is formulated as follows. Given the velocity ($v_1(t)$, $v_2(t)$, $v_3(t)$), position ($s_1(t)$, $s_2(t)$, $s_3(t)$) of 3 cars and distance to the front car ($d_2(t)$, $d_3(t)$) at time t, how do car2 and car3 decide the acceleration ($a_2(t)$, $a_3(t)$) at time t?

2 Original solutions

2.1 Equations and parameters

In order to find the 'best' acceleration for the ith car at time t, the goal is to minimize the cost function. Since we make decisions at time t based on our prediction of the future (time $t + 1$), so every parameter of the cost function should be related to time t to $(t + 1)$.

There are 2 different cost functions:

(1) The cost function of the ith car (except for the last car because it doesn't have $v_{i+1}(t)$) at time t (from t to $t + 1$) is:

$$L_i(t) = \omega_1 \underbrace{\int_{\Delta_t} \frac{Fuel}{v_i(t)}}_{\text{Fuel consumption}} + \underbrace{\omega_2 R_{error}^2 + \omega_6 R_{error'}^2}_{\text{Distance to the front car}} + \underbrace{\omega_3 (v_i(t+1) - v_{i-1}(t+1))^2}_{\text{velocity difference between the (i+1)th car and the ith car}} + \underbrace{\omega_4 \hat{a}_i(t)^2}_{\text{minimize the acceleration}} + \underbrace{\omega_5 (v_i(t+1) - v_{i+1}(t+1))^2}_{\text{velocity difference between the ith car and the (i+1)th car}} \tag{1}$$

where $Fuel$ is the Fuel consumption at velocity $v_i(t)$.

$$Fuel = m_1 * v_i(t)^2 + m_2 * \hat{a}_i(t)^2 + m_3 * v_i(t)^2 * \hat{a}_i(t) + m_4 * v_i(t) * \hat{a}_i(t)^2 + m_5 * v_i(t) * \hat{a}_i(t) + m_6 * v_i(t) + m_7 * \hat{a}_i(t) + m_8 \tag{2}$$

$\hat{a}$ is the sum of the apparent acceleration (as a_V) and the acceleration internally required to counteract decelerating force due to the road slope (as $a_\theta = gsin\theta(s_i(t))$).

$$\begin{aligned} \hat{a} &= a_V + a_\theta \\ &= [-\frac{1}{2M}\Phi(d_i(t))C_D\rho_a A_v v_i(t)^2 - \mu gcos(\theta(s_i(t))) - gsin\theta(s_i(t)) + a_i(t)] + gsin\theta(s_i(t)) \\ &= -\frac{1}{2M}\Phi(d_i(t))C_D\rho_a A_v v_i(t)^2 - \mu gcos(\theta(s_i(t))) + a_i(t) \end{aligned} \tag{3}$$

(2) The cost function of the last car at time t (from t to $t+1$) is:

$$L_i(t) = \underbrace{\omega_1 \frac{Fuel}{v_i(t)}}_{\text{Fuel consumption}} + \underbrace{\omega_2 R_{error}^2 + \omega_6 R_{error'}^2}_{\text{Distance to the front car}} + \underbrace{\omega_3 (v_i(t+1) - v_{i-1}(t+1))^2}_{\text{velocity difference between the (i+1)th car and the ith car}} + \underbrace{\omega_4 \hat{a}_i(t)^2}_{\text{minimize the acceleration}} \tag{4}$$

Coefficient ω_2 is

$$\omega_2 = \gamma e^{-\alpha d_i(t+1)} \tag{5}$$

$$R_{error} = R + h_d * v_i(t+1) - d_i(t+1) \tag{6}$$

where R is the pre-defined critical distance.

The value of the parameters (from literature) are as follows. $\omega_1 = 4$, $\alpha = 0.2$, $m_1 = 0.442 \times 10^{-6}$, $m_2 = -5.67 \times 10^{-6}$, $m_3 = 1.166 \times 10^{-6}$, $m_4 = 39.269 \times 10^{-6}$, $m_5 = 58.284 \times 10^{-6}$, $m_6 = 19.279 \times 10^{-6}$, $m_7 = 82.426 \times 10^{-6}$, $m_8 = 185.360 \times 10^{-6}$,$M = 1200\,kg$, $A_v = 2.5\,m^2$, $\rho_a = 1.184\,kg/m^3$, $C_D = 0.32$, $\mu = 0.015$.

Our defined parameters or parameters different from literature are as follows. $R = 30\,m$, $h_d = 3$, $\gamma = 3$, $\omega_3 = 125$, $\omega_4 = 2$, $\omega_5 = 1.25$, $V_{min} = 28\,mile/hour(12.5\,m/s)$, $V_{max} = 60\,mile/hour(26.8\,m/s)$, $a_{min} = -1.5\,m/s^2$, $a_{max} = 1\,m/s^2$.

For the first car, the driving pattern (velocity and acceleration) is randomly generated (fulfilling $v_1(t) \in [V_{min}, V_{max}]$ and $a_1(t) \in [a_{min}, a_{max}]$). The road condition is in Figure 2 and the air drag reduction ratio is in Figure 3. We plot the data from the literature and get the fitted line. The fitted elevation y in Figure 2 is

$$\begin{aligned} y = 2.62 \times 10^{-17}s^6 - 2.1 \times 10^{-13}s^5 + 6.2 \times 10^{-10}s^4 - 8.27 \times 10^{-7}s^3 \\ +4.61 \times 10^{-4}s^2 - 6.46 \times 10^{-2}s + 7.05857862 \end{aligned} \tag{7}$$

By taking the derivative, we get the slope $\theta \in [-2\pi, 2\pi]$ as

$$\begin{aligned} \theta = tan^{-1}(\frac{dy}{ds}) = tan^{-1}(15.72 \times 10^{-17}s^5 - 10.5 \times 10^{-13}s^4 + 24.8 \times 10^{-10}s^3 \\ -24.81 \times 10^{-7}s^2 + 9.22 \times 10^{-4}s - 0.0646) \end{aligned} \tag{8}$$

The fitted air drag reduction ratio $\Phi(d)$ in Figure 3 is

$$\Phi(d) = 0.7 \times (1 - e^{-0.03d}) + 0.3 \tag{9}$$

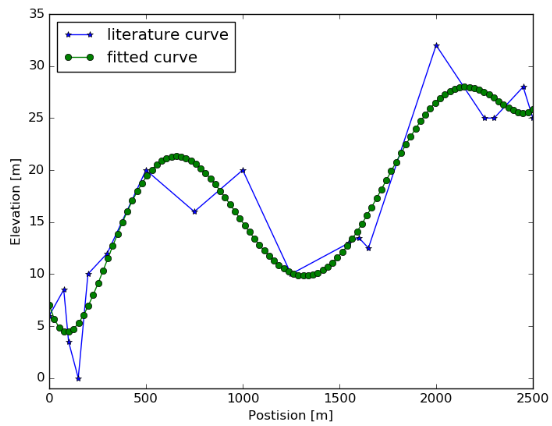

Figure 2: Road condition

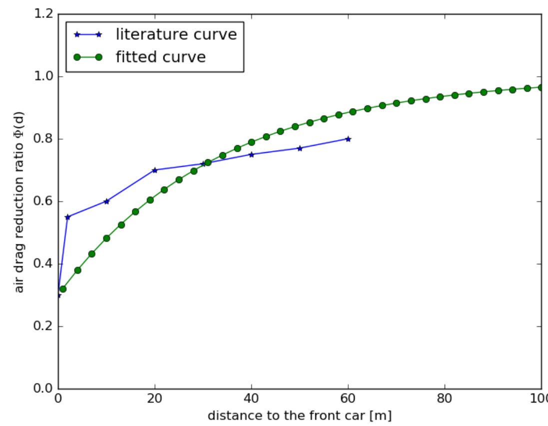

Figure 3: Air drag reduction ratio

2.2 Optimization constrains

The optimization goal is to minimize $L_i(t)$. The control variable is $x = \begin{bmatrix} a_i(t) \\ a_{i-1}(t) \\ a_{i+1}(t) \end{bmatrix}$. The constrains are:

$$V_{min} \leq v_{i-1}(t+1), v_i(t+1), v_{i+1}(t+1) \leq V_{max} \tag{10}$$

$$a_{min} \leq a_{i-1}(t), a_i(t), a_{i+1}(t) \leq a_{max} \tag{11}$$

$$s_{i+1}(t+1) \leq s_i(t+1) \leq s_{i-1}(t+1) \tag{12}$$

Similarly for the last car (the third car in this example), the steps are the same, except for using Eq (4) as the cost function in step (1) and (3). The control variable is $x = \begin{bmatrix} a_i(t) \\ a_{i-1}(t) \end{bmatrix}$.

For optimizing Q, we notice that it is a non-linear programming problem with inequality constrains. We use the Matlab solver 'fmincon' with Sequential Quadratic Programming (SQP) method.

3 Kalman Filter

3.1 Kalman Filter formulation

The notation used in Kalman Filter formulation is:

- X_t is the state variable at time t;
- Y_t is the observation at time t;
- $\hat{X}_t$ is the estimation at time t, also used as $\hat{X}_{t,t}$;
- μ_t is the control variable at time t;
- process noise is q, which forms a white noise distribution as $Q_t \sim N(0, q)$;
- measurement noise is r, which forms a white noise distribution as $R_t \sim N(0, r)$;
- P is the state covariance matrix;
- K_t is the Kalman gain.

Kalman Filter equations are:

- State update equation:

$$X_t = A \cdot X_{t-1} + B \cdot \mu_{t-1} + Q_t \tag{13}$$

- Observation equation:

$$Y_t = C \cdot X_t + R_t \tag{14}$$

- Predict:

$$\hat{X}_{t,t-1} = A \cdot \hat{X}_{t-1,t-1} + B \cdot \mu_{t-1} \tag{15}$$

$$P_{t,t-1} = A \cdot P_{t-1,t-1} \cdot A^T + Q_t \tag{16}$$

- Update:

$$K_t = P_{t,t-1} \cdot C^T \cdot (C \cdot P_{t,t-1} \cdot C^T + R_t)^{-1} \tag{17}$$

$$\hat{X}_{t,t} = \hat{X}_{t,t-1} + K_t \cdot (Y_t - C \cdot \hat{X}_{t,t-1}) \tag{18}$$

$$P_{t,t} = (I - K_t \cdot C) \cdot P_{t,t-1} \tag{19}$$

3.2 Applying Kalman Filter to platooning models

For our platooning problem, if we consider the ith car at time t, common notations are:

- $s_i(t)$ represents the position of the ith car at time t;
- $v_i(t)$ represents the velocity of the ith car at time t;
- $a_i(t)$ represents the acceleration of the ith car at time t;
- $d_i(t)$ represents the distance from the ith car to $i-1$th car at time t.
- Δ_t represents the time interval.

3.2.1 Decentralized model

For the ith car (middle car) at time t, the state variable $X_i(t)$ is $[s_i(t), v_i(t), d_i(t), s_{i-1}(t), v_{i-1}(t), v_{i+1}(t)]^T$. The control variable is $a_i(t)$. The Kalman filter parameters are calculated.

For updating state variable equation, we have

$$s_i(t) = s_i(t-1) + v_i(t-1) \cdot \Delta_t + \frac{1}{2} \cdot a_i(t-1)\Delta_t^2 \tag{20}$$

$$v_i(t) = v_i(t-1) + a_i(t-1) \cdot \Delta_t \tag{21}$$

$$d_i(t) = d_i(t-1) + (v_{i-1}(t-1) - v_i(t-1)) \cdot \Delta_t + \frac{1}{2} \cdot \Delta_t^2 \cdot (a_{i-1}(t-1) - a_i(t-1)) \tag{22}$$

$$s_{i-1}(t) = s_{i-1}(t-1) + v_{i-1}(t-1) \cdot \Delta_t + \frac{1}{2} \cdot a_{i-1}(t-1)\Delta_t^2 \tag{23}$$

$$v_{i-1}(t) = v_{i-1}(t-1) + a_{i-1}(t-1) \cdot \Delta_t \tag{24}$$

$$v_{i+1}(t) = v_{i+1}(t-1) + a_{i+1}(t-1) \cdot \Delta_t \tag{25}$$

We calculate A and B with

$$\begin{aligned} \begin{bmatrix} s_i(t) \\ v_i(t) \\ d_i(t) \\ s_{i-1}(t) \\ v_{i-1}(t) \\ v_{i+1}(t) \end{bmatrix} = \begin{bmatrix} 1 & \Delta_t & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -\Delta_t & 1 & 0 & \Delta_t & 0 \\ 0 & 0 & 0 & 1 & \Delta_t & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} s_i(t-1) \\ v_i(t-1) \\ d_i(t-1) \\ s_{i-1}(t-1) \\ v_{i-1}(t-1) \\ v_{i+1}(t-1) \end{bmatrix} + \\ \begin{bmatrix} 0 & \frac{1}{2}\Delta_t^2 & 0 \\ 0 & \Delta_t & 0 \\ \frac{1}{2}\Delta_t^2 & -\frac{1}{2}\Delta_t^2 & 0 \\ \frac{1}{2}\Delta_t^2 & 0 & 0 \\ \Delta_t & 0 & 0 \\ 0 & 0 & \Delta_t \end{bmatrix} \cdot \begin{bmatrix} a_{i-1}(t-1) \\ a_i(t-1) \\ a_{i+1}(t-1) \end{bmatrix} + Q_t \end{aligned} \tag{26}$$

For Q_t, it is a matrix of 6×1, representing the plant noise of the system. We suppose the standard deviation of the white noise Q_t is $[\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6]^T$. They should satisfy the following requirements:

$$\sigma_3 < \sigma_1 = \sigma_4 < \sigma_2 = \sigma_5 = \sigma_6 \tag{27}$$

For observation equation, the observation $Y_i(t)$ is defined as $[s'_i(t), v'_i(t), d'_i(t), s''_{i-1}(t), v'_{i-1}(t), v''_{i-1}(t), v'_{i+1}(t), v''_{i+1}(t)]^T$, which includes all of the raw inputs into the ith car at time t. Note that the variables with '$'$' are obtained by direct measurement from the ith car, and the variables with '$''$' are obtained through information sharing from the $(i-1)$th or $(i+1)$th car.

The observation update equations can be built from the relations between $X_i(t)$ and $Y_i(t)$ (ignoring noise for the moment).

$$s'_i(t) = s_i(t) \tag{28}$$

$$v'_i(t) = v_i(t) \tag{29}$$

$$d'_i(t) = d_i(t) \tag{30}$$

$$s''_{i-1}(t) = s_{i-1}(t) \tag{31}$$

$$v'_{i-1}(t) = v_{i-1}(t) \tag{32}$$

$$v''_{i-1}(t) = v_{i-1}(t) \tag{33}$$

$$v'_{i+1}(t) = v_{i+1}(t) \tag{34}$$

$$v''_{i+1}(t) = v_{i+1}(t) \tag{35}$$

We calculate C with:

$$\begin{bmatrix} s_i'(t) \\ v_i'(t) \\ d_i'(t) \\ s_{i-1}''(t) \\ v_{i-1}'(t) \\ v_{i-1}''(t) \\ v_{i+1}'(t) \\ v_{i+1}''(t) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} s_i(t) \\ v_i(t) \\ d_i(t) \\ s_{i-1}(t) \\ v_{i-1}(t) \\ v_{i+1}(t) \end{bmatrix} + R_t \tag{36}$$

For R_t, it is a matrix of 8×1, representing the sensor noise of the system. Suppose the standard deviation of the white noise R_t is $[\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6, \sigma_7, \sigma_8]^T$, which satisfies

$$\sigma_3 < \sigma_1 = \sigma_4 < \sigma_2 = \sigma_6 = \sigma_8 < \sigma_5 = \sigma_7 \tag{37}$$

3.2.2 Centralized model

We take a 3-car model as an example. The state variable $X_i(t)$ is $[s_1(t), v_1(t), s_2(t), v_2(t), s_3(t), v_3(t)]^T$.

For updating state variable equation (i = 2,3), we have

$$s_i(t) = s_i(t-1) + v_i(t-1) \cdot \Delta_t + \frac{1}{2} \cdot a_i(t-1)\Delta_t^2 \tag{38}$$

$$v_i(t) = v_i(t-1) + a_i(t-1) \cdot \Delta_t \tag{39}$$

We calculate A and B with:

$$\begin{bmatrix} s_1(t) \\ v_1(t) \\ s_2(t) \\ v_2(t) \\ s_3(t) \\ v_3(t) \end{bmatrix} = \begin{bmatrix} 1 & \Delta_t & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & \Delta_t & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \Delta_t \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} s_1(t-1) \\ v_1(t-1) \\ s_2(t-1) \\ v_2(t-1) \\ s_3(t-1) \\ v_3(t-1) \end{bmatrix} + \begin{bmatrix} \frac{1}{2}\Delta_t^2 & 0 & 0 \\ \Delta_t & 0 & 0 \\ 0 & \frac{1}{2}\Delta_t^2 & 0 \\ 0 & \Delta_t & 0 \\ 0 & 0 & \frac{1}{2}\Delta_t^2 \\ 0 & 0 & \Delta_t \end{bmatrix} \begin{bmatrix} a_1(t-1) \\ a_2(t-1) \\ a_3(t-1) \end{bmatrix} \tag{40}$$

For observation equation, the observation $Y_i(t)$ is defined as $[s_2'(t), d_2'(t), v_2'(t), v_2''(t), s_3'(t), d_3'(t), v_3'(t)]$.
We calculate C with:

$$\begin{bmatrix} s_2'(t) \\ d_2'(t) \\ v_2'(t) \\ v_2''(t) \\ s_3'(t) \\ d_3'(t) \\ v_3'(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} s_1(t) \\ v_1(t) \\ s_2(t) \\ v_2(t) \\ s_3(t) \\ v_3(t) \end{bmatrix} \tag{41}$$