Ecological Vehicle Control on Roads With Up-Down Slopes

M. A. S. Kamal, *Member, IEEE*, Masakazu Mukai, *Member, IEEE*, Junichi Murata, *Member, IEEE*, and Taketoshi Kawabe, *Member, IEEE*

***Abstract*—This paper presents a novel development of an ecological (eco) driving system for running a vehicle on roads with up–down slopes. Fuel consumed in a vehicle is greatly influenced by road gradients, aside from its velocity and acceleration characteristics. Therefore, optimum control inputs can only be computed through anticipated rigorous reasoning using information concerning road terrain, model of the vehicle dynamics, and fuel consumption characteristics. In this development, a nonlinear model predictive control method with a fast optimization algorithm is implemented to derive the vehicle control inputs based on road gradient conditions obtained from digital road maps. The fuel consumption model of a typical vehicle is formulated using engine efficiency characteristics and used in the objective function to ensure fuel economy driving. The proposed eco-driving system is simulated on a typical road with various shapes of up–down slopes. Simulation results reveal the ability of the eco-driving system in significantly reducing fuel consumption of a vehicle. The fuel saving behavior is graphically illustrated, compared, and analyzed to focus on the significance of this development.**

***Index Terms*—Driver-assistance system, eco-driving, model predictive control.**

I. Introduction

ENERGY-SAVING strategic technologies are highly demanding in intelligent transportation systems (ITS) to reduce both the dependency on fuel and the impact on the environment such as air pollution and global warming. It is expected that ITS through the advancement of new sensing and information technologies may lead to the realization of the least-fuel-consuming vehicle in the 21st century. There has been significant progress in realizing fuel economy transportation systems through the development of hybrid cars, advanced engine control systems, solar- or electric-power-driven cars, road infrastructure, and many other aspects to reduce the impact on the ecological balance of the earth. Research related to the ideas of controlling vehicle speed for fuel economy has a long history. Some simplified approaches were proposed to investigate the optimal cruising velocity of a vehicle based only on its internal operating characteristics [1], [2]. The effect of other traffic or road terrain is not considered. Fuel economy for urban driving can be improved by the use of an optimal hybrid power train and the use of intelligent technologies using traffic flow information [3]. Fuel savings associated with recent hybrid vehicles come with the tradeoff of a significantly increased initial vehicle cost due to the increased complexity of the power train. A model predictive controller utilizing integrated traffic data can be used to predict an optimum velocity profile and required engine torque to improve fuel economy [4]. The system provides satisfactory results using periodically updated traffic information on flat highways, as it computes and provides optimal engine torque every 40 m. The effect of road gradient on fuel economy while driving over varying road terrains is also an ongoing research problem. A feedback algorithm that allows the driver to choose an arbitrary steady-state velocity for a level of road but modifies the speed for optimal fuel consumption on various grades was proposed [5]. Optimal driving for single-vehicle fuel economy was proposed to determine the optimal way to accelerate from rest to cruising speed, to drive a block between stop signs, and to cruise on hilly terrain while maintaining a given average speed [6]. A look-ahead controller using dynamic programming is developed for a heavy diesel truck that utilizes information about the road topography ahead of the vehicle when the route is known [7]. A similar approach over an entire drive cycle has been used to derive the optimal switching strategy when the vehicle speed is fixed to the drive cycle [8]. These dynamic programming approaches solved for the global optimal power split strategy over an entire drive cycle. While useful in identifying the global optimum, these approaches are not suited to real-time implementation as they require knowledge of the full drive behavior before the trip, which is clearly infeasible. All the aforementioned optimal driving approaches mainly focus on automatic driving of a vehicle without addressing human issues in driving.

Eco-driving is a way of maneuvering a vehicle with a human driver that is intended to minimize fuel consumption while coping with varying and uncertain road traffic by trading off the most efficient driving point of the vehicle whenever necessary. Aside from various physical factors, driving style has a great influence on vehicle emissions and energy consumption [9]. Driving style has the capacity to significantly improve the driving efficiency of a vehicle. A recent experiment conducted on urban roadways showed that the reduction in fuel consumption can be as high as 25% [10]. Generally, fuel economy is maximized when accelerating and braking events are minimized [11], [12]. Therefore, a fuel-efficient or ecological

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strategy is to anticipate what is happening ahead, drive with acceleration and braking as little as possible, cruise at the optimal velocity, and slowly decelerate at stops. A driver can easily maneuver a car on a road tackling uncertain, discontinuous, and complex traffic situations using his perception of the environment. For optimal eco-driving, the same driver needs to be more anticipative on road-traffic situations with perfect knowledge of the engine dynamics of his car, which is hardly attainable by a human driver. Therefore, the driver can be technologically assisted to ecologically drive his car.

Recently, various assistance schemes for eco-driving have emerged. Speculative features of ecological driving are available in the form of driving tips [11], [12]. Some recently manufactured cars have an ecological indicator that shows a green "ECO" mark to a driver when it consumes little or no fuel. A driver would find his driving as ecological only when he maintains a steady velocity at a reasonable level or brakes the car. Nissan launched an off-board eco-driving support service for some users in which, after the driving record is sent to a telemetric data center for offline analysis, advice is sent to the driver to improve his driving style the next time. Based on past performance, they have proposed an onboard assist system to motivate the driver to ecologically drive the car by showing his comparative driving efficiency, his position in fuel composition ranking, etc. [13]. Recent work in determining ecological strategy uses an optimal control approach in which only the model of the engine in terms of velocity, gear ratio, and load is considered [14]. A more realistic approach to assisting a driver uses information of traffic signal, jams, road gradient, and distance between cars, and the advice is given in a very rough form such as "keep driving" or "reduce pressure on pedal," depending on the motivation of the driver [15]. However, existing approaches to eco-driving assistance are very superficial; they do not provide concrete information such as the level of velocity or acceleration required for long-term fuel-efficient driving by analyzing current vehicle–road–traffic situation and its trend.

Recently, an ecological driver-assistance system (EDAS) that can guide a driver for fuel-efficient driving on a flat urban road has been proposed [16]. Using current road–traffic information, the EDAS anticipates the future states of vehicles using their dynamic models, and based on the anticipated future states and the fuel consumption model of the engine, it calculates the optimum vehicle control input required for ecological driving. This unique approach is found to be very promising on a flat urban road. Countries such as Japan have many hilly areas, where many roads have many sections with up–down slopes. Fuel consumption significantly varies on such a road with up–down slopes [17]. Some earlier approaches relating to the optimal driving on roads with varying grades mainly used dynamic programming to solve a known drive cycle [7], [8], which are not suitably applicable with immediately perceived road grade information. This paper extends the scope of the EDAS to roads with up–down slopes that are neither crowded nor have too many traffic signals, which is a scenario typically found in non-urban areas. Using information about road gradients and anticipation of the vehicle's future state on the road, it generates control inputs that are required for optimally fuel-efficient driving. The road gradient angle is computed using road altitude data obtained from the digital road map and introduced in the optimization process in a simple way. Once the optimal control input becomes available, a suitable human interface may convey it to the driver in the case of an assistance system. For simplicity in this paper, however, any interface component is omitted, and the control input generated by the system is directly fed to the host vehicle, focusing mainly on the vehicle control aspects on hilly roads.

The aim of this study is to present the concept of eco-driving system on hilly roads, implement an efficient way of using data from the digital road map, formalize the control algorithm, and evaluate the developed system through simulations. Performance of the proposed system in terms of velocity characteristic and comparative fuel consumption on typical roads will be observed. Obtained results will be analyzed to evaluate computational soundness and the ability of the algorithm to ecologically drive a car.

The rest of this paper is organized as follows: The fundamental concept of an eco-driving system on a hilly road, modeling of the vehicle, and the model predictive control algorithm are described in Section II. Section III describes simulation results on roads with various shapes, followed by a discussion and conclusions in Section IV. The formulation of the fuel consumption model and the optimal control problem are described in the Appendices.

II. Ecological Driving on Hilly Roads

A. Fundamental Concept

Energy consumption and emissions of a vehicle depend on various physical factors such as engine condition, road condition, weather, etc. Driving style also has a great influence on energy consumption. Aggressive driving, with speeding, rapid acceleration, and braking, considerably wastes energy. On a road with up–down slopes, it is necessary to anticipate future states of the vehicle and information of the gradient of the road ahead to drive the car in an optimally ecological way. Therefore, an assistance system could be very effective to support a driver to ecologically drive his car on the road with up–down slopes.

The concept of an eco-driving system on a free roadway containing up–down slopes is shown in Fig. 1. The system uses the model of vehicle dynamics, the fuel consumption model of the engine, and information of road gradient and generates the control input that maximizes fuel economy over the traveling distance. The Global-Positioning-System-based navigation system can be used with the digital road map to obtain the position of the car and road gradient in the case of real implementation.

The control input is generated by anticipating future conditions of the road and the car through a rigorous reasoning approach based on optimization of a relevant performance index. Details of the problem formulation including modeling, the performance index, and the control method are described in the succeeding sections.

B. Modeling of the Vehicle

Only the longitudinal motion control of a vehicle on a freeway is considered, assuming no interaction with other vehicles

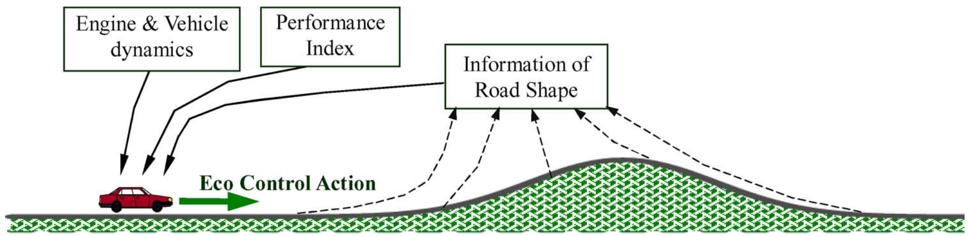

Fig. 1. Fundamental concept of the proposed eco-driving system using model-based prediction and road shape information.

or traffic signals. It is assumed that the state equation of the vehicle at any instant t can be represented by

$$\dot{x}(t) = f\left(x(t), u(t)\right) \tag{1}$$

where $x(t) = [x_1(t), x_2(t)]^T$ denotes the state vector representing location $(x_1(t))$ and velocity $(x_2(t))$ of the host vehicle, and $u(t)$ is the control input. The velocity of the vehicle at time t is subjected to the total forces acting on the vehicle, and it is related by

$$M\frac{dx_2(t)}{dt} = F_T(t) - F_R(t) \tag{2}$$

where M, $F_T(t)$), and $F_R(t)$ are the equivalent mass of the vehicle and its rotating part, the traction force, and the sum of all motion resistance forces, respectively. The resistance forces include aerodynamic, rolling, and slope forces, which can be represented by

$$F_R = \frac{1}{2}C_D\rho_a A_V x_2^2 + \mu M g\cos\left(\theta(x_1)\right) + Mg\sin\left(\theta(x_1)\right) \tag{3}$$

where C_D, ρ_a, A_V, μ, and $\theta(x_1)$ are the drag coefficient, the air density, the frontal area of the vehicle, the rolling resistance coefficient, and the road slope angle as a function of location x_1, respectively.

The traction force is related by the mass of the vehicle and control input as $F_T(t) = Mu(t)$. Therefore, the state equation (1) of the vehicle can be rewritten as

$$f(x,u) = \begin{bmatrix} x_2 \\ \left(-\frac{1}{2M}C_D\rho_a A_V x_2^2 - \mu g\cos(\theta(x_1)) - g\sin(\theta(x_1)) + u\right)\end{bmatrix}. \tag{4}$$

This states that the apparent acceleration of the vehicle is $a_V = -(1/2M)C_D\rho_a A_V x_2^2 - \mu g\cos(\theta(x_1)) - g\sin(\theta(x_1)) + u$. The maximum vehicle control input $u(t)$ is bounded by $u_{\min} \leq u(t) \leq u_{\max}$ to meet the physical limits of the accelerator and the brake, which are usually subjected to variation in engine torque. The road altitude R_{alt} (in meters) information from the digital road map is used to calculate the gradient angle $\theta(x_1)$ at location x_1 as follows:

$$\theta(x_1) = \tan^{-1}\left(\frac{R_{\text{alt}}(x_1 + \Delta x_1) - R_{\text{alt}}(x_1 - \Delta x_1)}{2\Delta x_1}\right). \tag{5}$$

Fuel consumption in a vehicle depends on various factors related to the engine speed, torque, gear ratio, temperature, calorific value of the fuel, efficiency, etc. An exact derivation of the fuel consumption equation could be very complex and is not the main objective here. Instead, an approximate and differentiable function of velocity and control input is enough to develop the algorithm for ecological driving. For typical vehicles, fuel consumption in milliliters per second is estimated as

$$f_V = \left(b_0 + b_1x_2 + b_2x_2^2 + b_3x_2^3 + \hat{a}\left(c_0 + c_1x_2 + c_2x_2^2\right)\right) \tag{6}$$

where $\hat{a} = a_V + a_\theta$ is the sum of the apparent acceleration of the vehicle a_V and the acceleration internally required to counteract decelerating force due to the road slope $(a_\theta = g\sin\theta(x_1))$. It is assumed that, if $u < 0$, then no fuel is consumed in the engine. Here, at the idling condition, while there exists a constant consumption, the vehicle is not controlled. Details of vehicle specification and the formation of fuel consumption estimation equation are described in Appendix A.

C. Model Predictive Control

The vehicle acceleration and deceleration, and their rates are directly related to the input force and variation of engine torque. Therefore, the control input is subject to both min-max and rate constraints. The inequality constraint relating to the control input is redefined in the form of equality constraint using a dummy input u_d for computational simplicity and expressed as

$$C\left(x(t), u(t)\right) = \left(u^2(t) + u_d^2(t) - u_{1max}^2\right)/2 = 0. \tag{7}$$

Although a vehicle is capable, a very high value of input is not desirable at all. Instead of choosing the maximum possible limit of the vehicle input, a moderate and symmetric value is chosen just to keep the searching space of the optimizer limited. The input rate constraint and state constraints are not incorporated in this problem formulation for simplicity, assuming that the rate of input will remain within reasonable limit in the driving condition and the driver can handle any abnormal state situations. Such a simplification ensures that the optimization problem solved at each step is tractable and can be implemented in real time.

In the model predictive control process, for the given input constraint (7), the following optimal control problem is solved at each time t with the current state $x(t)$ used as the initial state and a prediction horizon from current time t to T ahead:

$$\min_u J = \int_t^{t+T} L\left(x(t'), u(t')\right)dt'. \tag{8}$$

In this formulation, the cost function L is considered to have the following form:

$$L = w_1 \left[\frac{b_0 + b_1 x_2 + b_2 x_2^2 + b_3 x_2^3}{x_2} \right] + w_2 \left[\frac{1}{2} \left(a_V + g \sin\left(\theta(x_1)\right) \right)^2 \right] + w_3 \left[\frac{1}{2} (x_2 - V_d)^2 \right]. \tag{9}$$

The cost function consists of three terms, each of which is multiplied by a constant weight w_1, w_2, and w_3, respectively. The first term defines the cost in terms of the cruising fuel consumption rate (in milliliters per minute), which is obtained from (6) as f_V/x_2 and setting $\hat{a} = 0$. Instead of the total fuel consumption, the cruising consumption rate is chosen to ensure the vehicle at steady condition runs at the velocity that maximizes the distance per unit of fuel, i.e., mileage. The second term is the cost that corresponds to the acceleration force on the car, considering the counteracting effect of gravitational force on the car due to the road slope. This term is to ensure that, at varying situations, acceleration and braking magnitudes are kept as minimum as possible. The last term is the cost for not running at the specified velocity on the road V_d. The weights are chosen in such a way that the usual magnitudes of the cost terms are balanced with respect to others. Finally, the weights were fine tuned through observation of simulation results to maximize the ultimate fuel efficiency.

To derive the optimal control input, the Hamiltonian function is formed using (1), (7), and (9) as follows:

$$H(x, \lambda, u, \psi) = L(x, u) + \lambda^T f(x, u) + \psi^T C(x, u) \tag{10}$$

where λ denotes the co-state, and ψ denotes the Lagrange multiplier associated with the constraint [18], [19]. A brief description of the optimal control problem and its solution scheme is included in Appendix B. Since the road gradient angle $\theta(x_1)$ is the function of only x_1, for this specific problem of deriving optimal control input, it is required to compute the following:

$$\begin{aligned} H_{x_1} &= \frac{\partial}{\partial x_1} H(x, \lambda, u, \psi) \\ &= \frac{\partial}{\partial x_1} \left(L(x, u) + \lambda^T f(x, u) + \psi^T C(x, u) \right) \\ &= -\lambda_2 g cos\theta(x_1) \frac{d}{dx_1} \theta(x_1). \end{aligned} \tag{11}$$

Therefore, it is necessary to compute $(d/dx_1)\theta(x_1)$ using the data from the digital road map. The value of $\theta(x_1)$ using (5) is used for this purpose as follows:

$$\frac{d}{dx_1} \theta(x_1) = \left(\frac{\theta(x_1 + \Delta x_1) - \theta(x_1 - \Delta x_1)}{2\Delta x_1} \right). \tag{12}$$

Using the given performance index (8), for a prediction horizon T discretized into N steps of size h, from virtual time $t' = nh$ to $t' = nh + T$, the instant and future vehicle control inputs $\{u_{nh}(t')\}_{t'=nh}^{t'=nh+T}$ are optimized. The C/GMRES method is used to generate the sequence of control inputs required in the prediction horizon. Since this method does not require iterative searches, it requires less computational power and can be implemented for the proposed vehicle control problem in real time [20]. Therefore, a set of optimum actions $\{u_{nh}^{\mathrm{opt}}(t')\}_{t'=nh}^{t'=nh+T}$ for the instant and future time is available after optimization. The optimal input corresponding to the current time is used to control the vehicle (or to assist a human driver, in the case of a real driver assistance system) up to the next sampling instant, i.e.,

$$u(t) = u_{nh}^{\mathrm{opt}}(nh), \qquad nh \le t < (n+1)h. \tag{13}$$

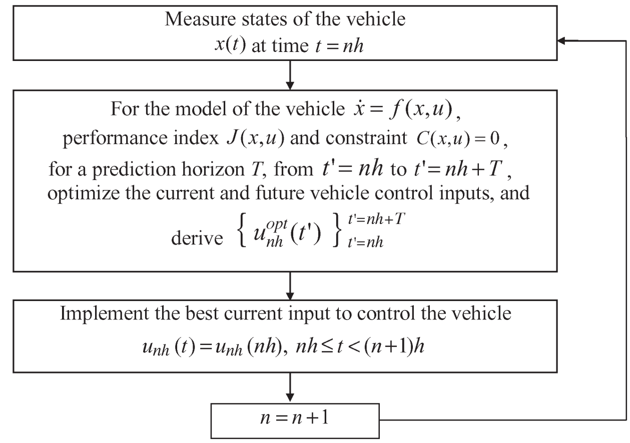

Fig. 2. Flowchart of the control procedure of the proposed eco-driving system.

At each time step, the immediate input calculated by this way is fed to the vehicle, and the whole process is repeated throughout the driving course, except at idling time. The flowchart of the whole procedure is shown in Fig. 2. Repeating the whole process and renewing the control input at each sampling time are required to overcome the influence of varying traffic and modeling errors in computation. Only the initial state value corresponding to time t is used as the actual feedback signal to the system on which the optimal control input is determined over the horizon.

III. Simulation Results

Simulation of the proposed eco-driving system has been conducted by choosing suitable values of the parameters as $u_{1max} = 2.75$ and $V_d = 13.89$ m/s. The weights in the cost function are set at $w_1 = 230.0$, $w_2 = 22.0$, and $w_3 = 0.80$. The prediction horizon $T = 10$ s is split into $N = 100$ steps of size $h = 0.1$ s. The simulation step is set at 0.1 s. The road slope angle is calculated using $\Delta x_1 = 20$ m. The iteration number in the C/GMRES method is set at $k_{\max} = 8$. The simulation is conducted in two steps. At first eco-driving strategies derived by the method on four typical road sections are observed. Then, a simulation is conducted on a virtually real road of about 2.5 km in length using the road gradient data from the digital road map. For the purpose of comparison, the vehicle is also separately driven using two other hypothetical methods, i.e., fixed speed drive (FSD) and automatic speed control drive (ASCD). The

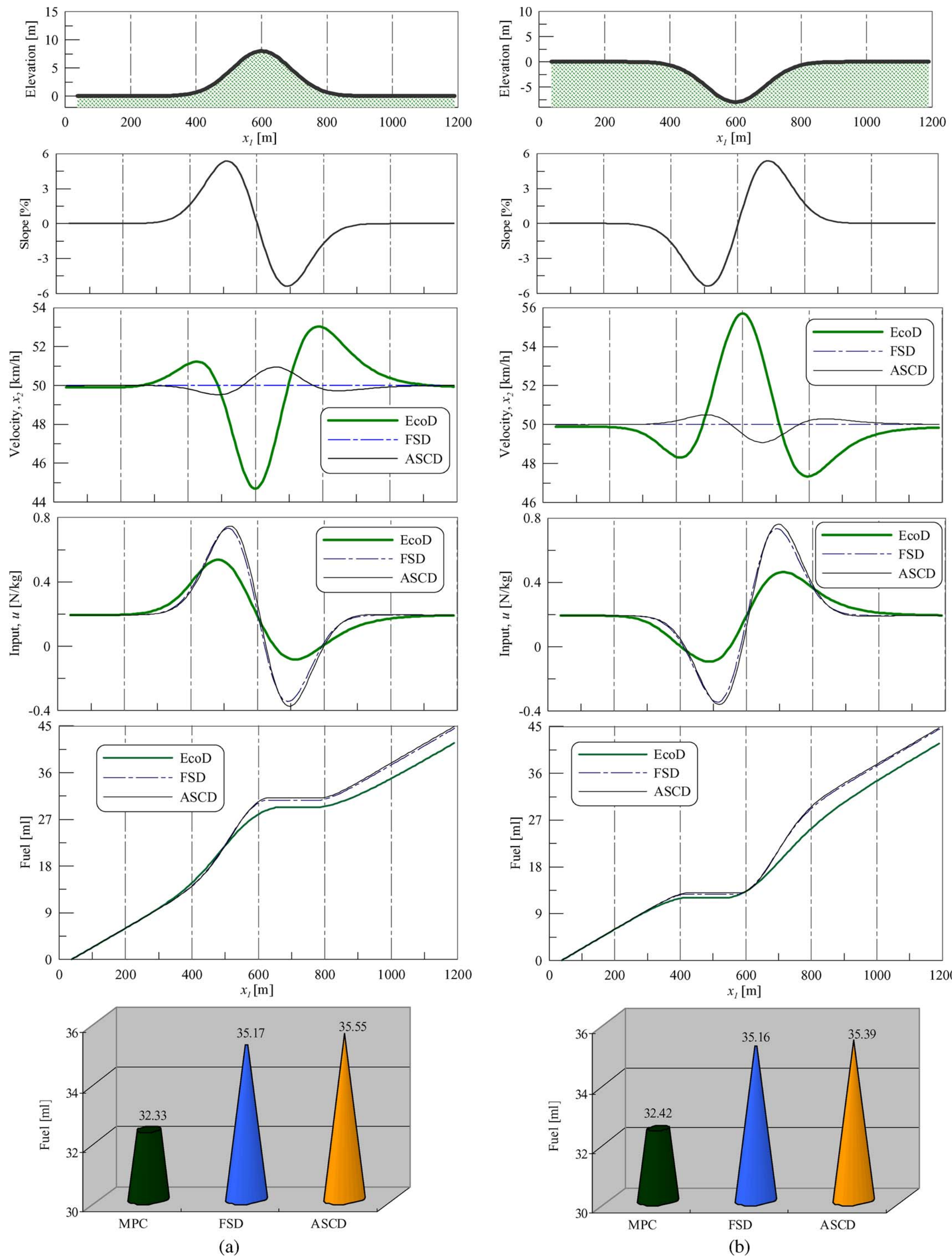

Fig. 3. Driving over the road section having (a) an up–down hill and (b) a down–up hill. Both (a) and (b) contain, from top to bottom, a graph of road elevation, slope percentage, velocities, control inputs, instantaneous fuel, and total fuel consumption by EcoD, ASCD, and FSD vehicles.

FSD method uses the information of road and generates the control input so that the velocity remains constant, regardless of the road gradient. ASCD uses the proportional–integral control algorithm without using the information of road gradient. For simplicity in representation, here, we use a notation "EcoD vehicle" to mean a vehicle that is controlled by the proposed MPC algorithm for eco-driving; the same applies to "ASCD vehicle" and "FSD Vehicle."

Fig. 3(a) shows typical driving scenarios over a road having an up-slope section, immediately followed by a down-slope section. The vehicle entered in the simulating environment with a velocity of 50 km/h, and then, it is controlled by the proposed eco-driving system. The first and second graphs (from the top) show the road elevation and percentage of road gradient. The next three graphs show comparative velocities, vehicle control inputs, and instantaneous (cumulative) fuel consumption.

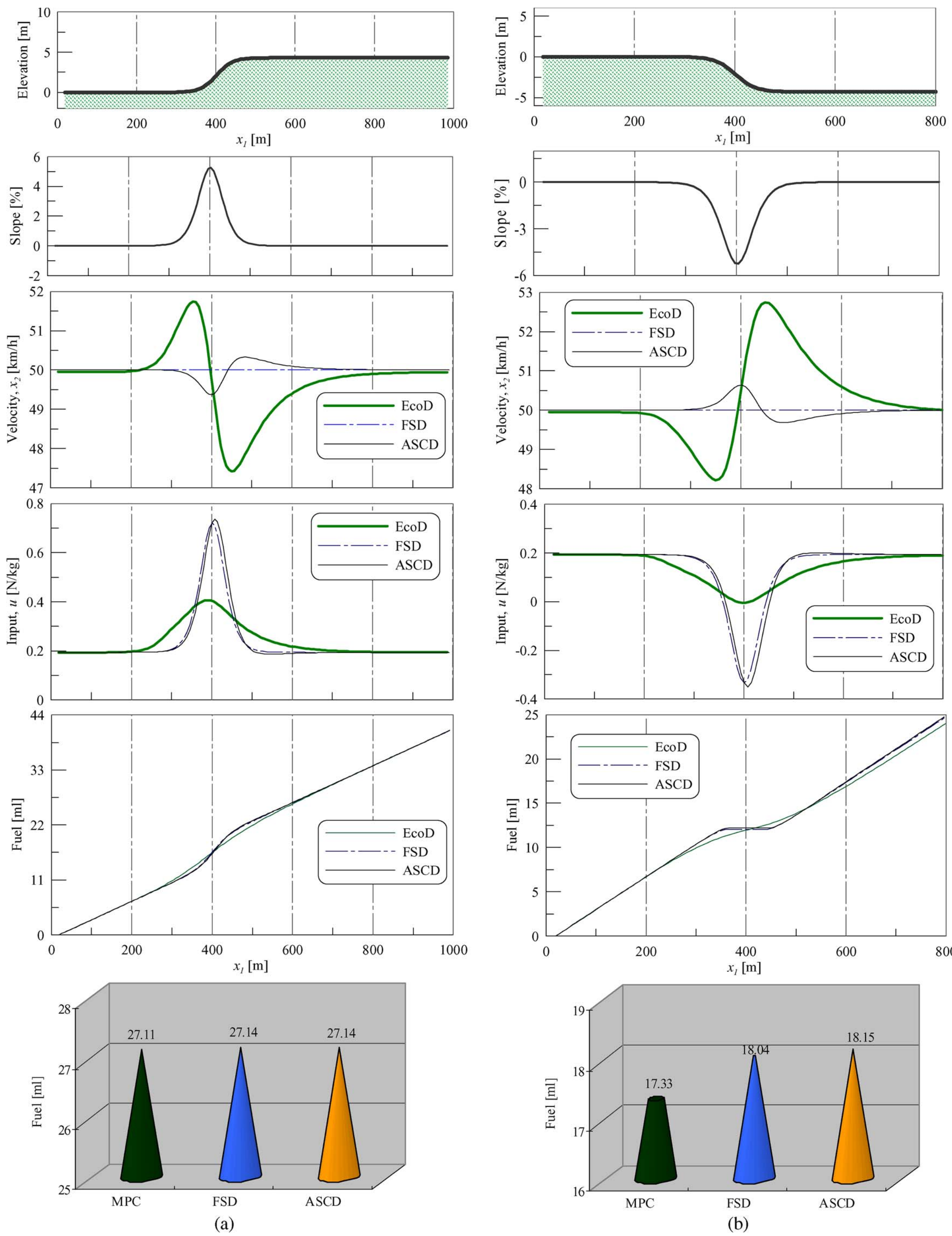

Fig. 4. Driving over the road having only (a) an up- and (b) a down-going section. Both (a) and (b) contain, from top to bottom, a graph of road elevation, slope percentage, velocities, control inputs, instantaneous fuel, and total fuel consumption by EcoD, ASCD, and FSD vehicles.

The EcoD vehicle increased the velocity before entering the uphill in a preplanned manner so that very high input can be avoided during going up. At down slope, it utilized the advantage of down slope, and without any braking, the velocity is allowed to increase to some extent and finally settled at a specified speed. The bar graph at the bottom shows the fuel consumed by three vehicles while traveling a distance from $x_1 = 250$ m to $x_1 = 1150$ m. It is found that the FSD vehicle and the ASCD vehicle required 8.77% and 9.96% of extra fuel, compared with the EcoD vehicle, respectively. This achievement in fuel saving reveals the creditability of such ecological driving based on road shape prediction. Fig. 3(b) shows the similar scenarios while driving over a road with a down-slope section, immediately followed by an up-slope section. It is found that, for a travel from $x_1 = 250$ m to $x_1 = 1150$ m, the FSD vehicle and the ASCD vehicle required 8.44% and 9.15% of extra fuel, compared with the EcoD vehicle.

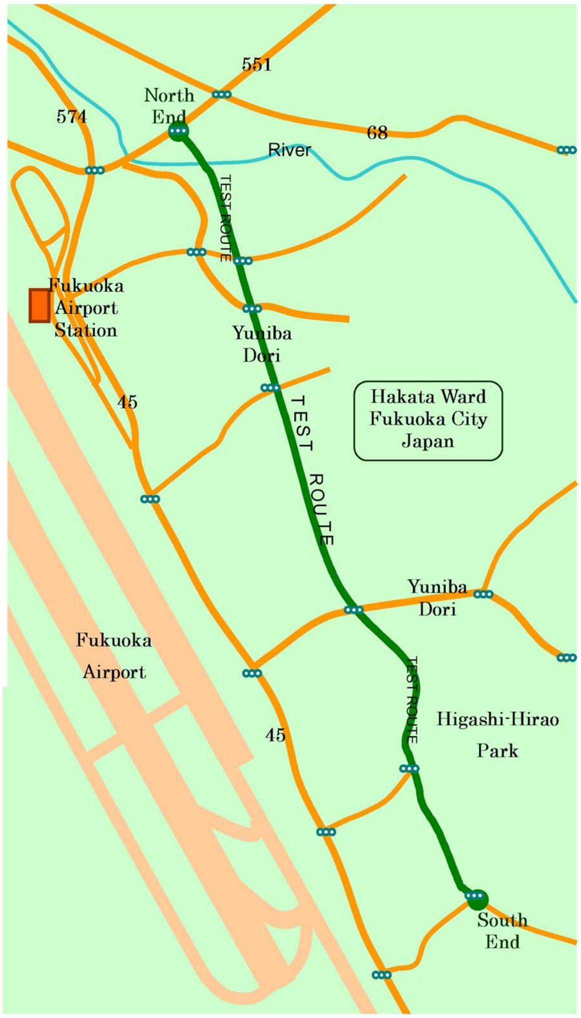

Fig. 5. Model test route from the north end to the south end in the map of Fukuoka City, Japan.

Fig. 4(a) and (b) shows typical driving scenarios on a road with only an up-slope section and a down-slope section, respectively. In the case of only an up-slope section, i.e., Fig. 4(a), all vehicles consumed almost the same amount of fuel. Since there is no necessity of braking, there is no waste of kinetic energy by the vehicle. Only negligible differences in fuel consumption are due to different engine efficiency for different levels of control inputs. On the other hand, in the case of only a down-slope section [see Fig. 4(b)], the EcoD vehicle could avoid braking, resulting in a fuel savings of about 4.73% and 4.03%, compared with the FSD vehicle and the ASCD vehicle, respectively.

Finally, the effectiveness of the proposed eco-driving system is evaluated on a virtually real road. A road of about 2.5 km in length called Yuniba Dori and its extension, which was located in Hakata Ward of Fukuoka City, Japan, is taken as the testing route (see Fig. 5). The road elevation information are taken from the digital map of road–land elevation with a resolution of 5-m grid [21]. The north end of the route has an altitude of about 6 m, and the south end of the test route has an altitude of about 25 m. There are up–down hilly areas with various shapes in the test route. The road altitude data taken from the digital map for the test route is plotted in Fig. 6. A sudden drop in the road elevation close to the north end is due to a river over which there

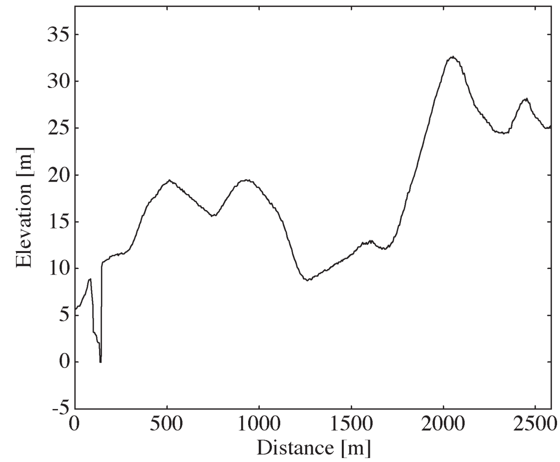

Fig. 6. Road elevation of the test route from the north end to the south end obtained from digital road maps of Fukuoka City.

is a bridge, and the actual elevation on the bridge is estimated from the value from its two ends. Fig. 7(a) and (b) shows driving scenarios on the test route for traveling from the north to the south and from the south to the north ends, respectively. The bar graphs show the comparative fuel consumption for a complete travel on the test route. In the case of driving from the north end to the south end of the test route, the EcoD vehicle could save about 5.0% and 4.45% of fuel, compared with the ASCD vehicle and FSD vehicle, respectively. On the other hand, traveling from the south end to the north end, the EcoD vehicle managed to save about 7.04% and 5.70% of fuel, compared with the ASCD vehicle and FSD vehicle. Since the elevation of the south end is higher than the north end, traveling from the south yields higher fuel savings by the EcoD vehicle.

It is necessary to choose a suitable time horizon for implementing model predictive control algorithm. A long horizon may provide better results, but it obviously increases computation burden. For implementing it on a real time, it is necessary to keep the computation time much shorter than the sampling time, considering sensing and executing delays. The effect of variation in the time horizon on the average fuel consumption tested for the proposed eco-driving system is shown in Fig. 8. The average fuel consumption in two complete trips in the both directions (from the north end to the south end and the other way round) on the test route is taken in this consideration. It suggests that a horizon over at least 5 s can be used for satisfactory results. In this paper, a horizon of 10 s that ensures satisfactory performance of the eco-driving system, keeping the computational burden manageable, is used.

The proposed method is found to be very robust against slope-sensing error. Fig. 9 shows the deviation of the velocity profiles due to error in slope sensor while traveling the route from the north end. An error of $+25\%$ means that the sensor provides 1.25 times the actual value, whereas -25% error means that the sensor provides 0.75 times the actual value of the slope with which the vehicle has to actually cope. Without any slope error, the EcoD vehicle could save 5.0% fuel, compared with the ASCD vehicle. Due to a sensing error of $+25\%$, the controller generated a bit higher input during up slope, which caused the vehicle to complete the route about 2 s earlier,

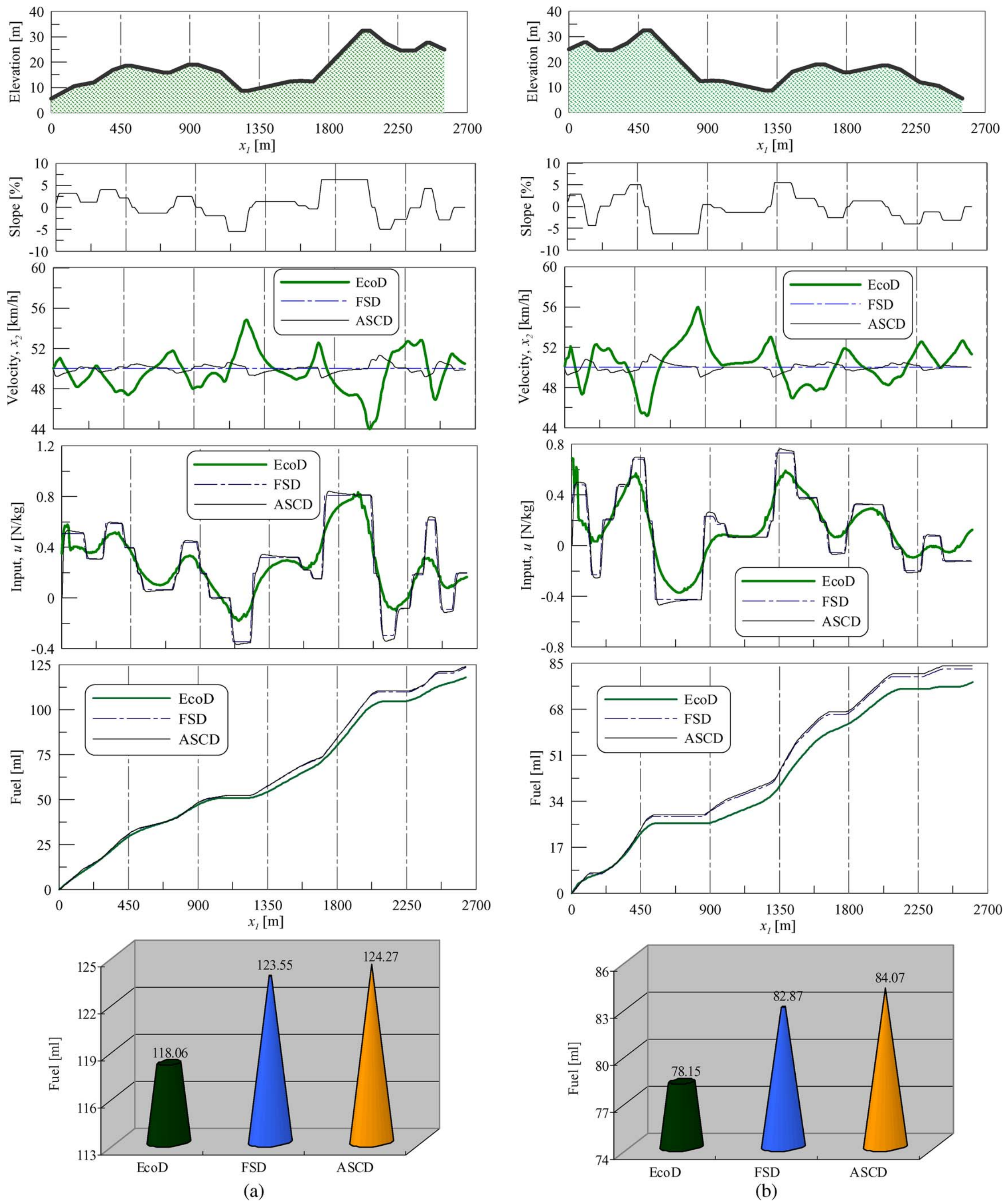

Fig. 7. Driving on the test route (a) from the north end to the south end and (b) from the south end to the north end. Both (a) and (b) contain, from top to bottom, a graph of road elevation, slope percentage, velocities, control inputs, instantaneous fuel, and total fuel consumption by EcoD, ASCD, and FSD vehicles.

and compared with the ASCD vehicle, fuel savings dropped to 4.0%. Similarly, due to a sensing error of -25%, the controller generated a bit smaller input during up slope, which caused the vehicle complete the route about 2 s later, and compared with the ASCD vehicle, fuel savings dropped to 4.2%. Therefore, it can be concluded that, within a reasonable sensing error, the system is able to maintain its ecological track without significant deviation.

The proposed computational algorithm is very fast, and it can be implemented in real time. For the simulation shown in Fig. 7(a), the vehicle required 191.5 s to complete traveling the route. The total computational time required for this in a typical personal computer is 8.59 s. Since simulation was conducted with a sampling interval of 100 ms, the computational time per sampling interval was only 4.49 ms. Therefore, it is worth stating that this algorithm can be implemented for real-time vehicle control.

IV. Discussion and Conclusion

A novel development of ecological driving system using model predictive control has been presented in this paper. Based on information of road gradient, model of vehicle dynamics, and engine fuel consumption characteristics, a model predictive

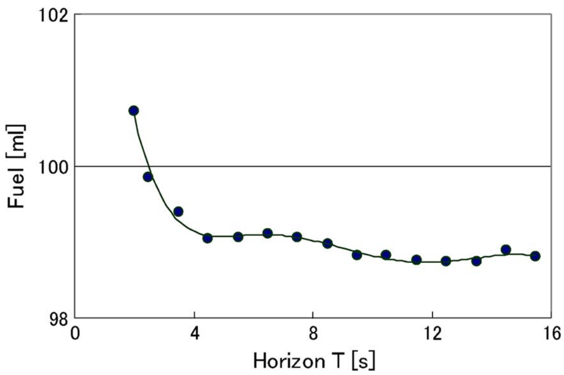

Fig. 8. Effect of prediction horizon T on the average fuel consumption.

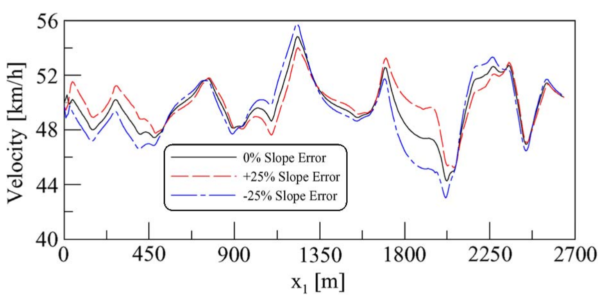

Fig. 9. Velocity deviation due to the slope-sensing error, traveling from the north end.

control algorithm has been implemented to generate control inputs required for ecological driving. A realistic method has been derived to incorporate the road gradient data in the optimization process after computing it using the road altitude data from a digital road map. The system could generate appropriate vehicle control inputs to avoid braking and high control inputs in a well-prearranged way in the course of driving over hilly roads. Compared to the other control methods, the vehicle driven using the proposed method required much less amount of fuel.

The proposed eco-driving is suitable for a non-urban road where traffic is not crowded. Although a very simplified control problem was formulated, the rate of the control input was kept within acceptable values, and modeling or sensor errors have very little effect on its performance. The computational time is found to be very short, and it can be easily implemented in real time. The proposed eco-driving control system can be used for realizing a driver assistance system, an automatic driving system, or an adaptive cruise control system, by introducing a human interface or some appropriate technologies, respectively.

In the future, the interactions with other vehicles and traffic control signals will be incorporated to realize a comprehensive eco-driving assistance system.

APPENDIX A
FUEL CONSUMPTION ESTIMATION

Fuel consumption of a vehicle at any driving instant depends on the torque and rotational speed of the engine, as illustrated in [22]. A typical car is chosen in this study; it has an engine size of about 1.3 L and a fuel consumption rate of about

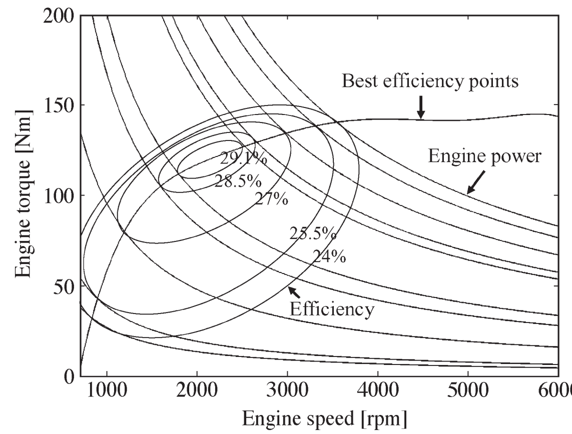

Fig. 10. Engine characteristic map including constant power and efficiency curves based on torque and engine speed.

17.2 km/L as per 10–15-mode fuel tests from the model catalog of the car [23]. The typical engine torque–speed characteristic curves of the vehicle, as shown in Fig. 10, were constructed in such a way that the estimated fuel consumption on 10–15-mode matches the catalog rate of 17.2 km/L. The constant power curve is determined by the torque and the rotational speed. It is assumed that the gear ratio is maintained using the continuous vector transmission system by maintaining maximum possible efficiency for the required driving power. This curve is obtained by connecting the intersections of maximum efficiency points on the power curves.

The total resistance force F (in Newtons) acting on the vehicle in terms of velocity v and acceleration a is given by

$$F = \frac{1}{2}C_D\rho_a A_V v^2 + \mu M g \cos(\theta(x_1)) + Mg\sin(\theta(x_1)) + Ma. \quad (14)$$

The model vehicle has the following parameters: $M = 1200$ kg, $A_V = 2.5$ m^2, $\rho = 1.184$ kg/m^3, $C_D = 0.32$, and $\mu = 0.015$. The energy consumption per second or the power required to overcome the resistance forces can be written in the following form:

$$P = Fv + P_c \quad (15)$$

where P_c is the power required to run the engine when the vehicle is idling. This way, for any velocity and acceleration, the required power and corresponding engine efficiency η is obtained from the engine characteristic map. Using the calorific value of the gasoline $C_f = 34.5 \times 10^6$ J, the consumption per second at any time is estimated from η and P as

$$W = \frac{P}{\eta}. \quad (16)$$

It is assumed that, at braking, no fuel is consumed in the engine. Therefore, for any vehicle state, the fuel consumption rate can be manually obtained from the map in Fig. 10. For computational approach in determining fuel-efficient driving, it is necessary to have a differentiable function to estimate fuel consumption.

To form such a fuel estimation equation, fuel consumption data f_V (in milliliters per second) are obtained for sufficient sampled values of velocity (ranging from 0 to 16 m/s) and acceleration (from 0 to 4.0 m/s^2). These manually obtained data from the engine map are plotted against velocity and acceleration. Next, from the fuel consumption graph for a cruising vehicle (without acceleration or braking), an estimation function is approximated by the curve-fitting process as the polynomial of velocity as

$$f_{\mathrm{cruise}} = b_0 + b_1 v + b_2 v^2 + b_3 v^3. \tag{17}$$

Additional fuel consumption (in milliliters per second) due to the presence of positive acceleration is approximated by a similar curve-fitting process as

$$f_{\mathrm{accel}} = \hat{a}(c_0 + c_1 v + c_2 v^2) \tag{18}$$

where $\hat{a} = a_v + a_\theta$ is the sum of the apparent acceleration of the vehicle $a_v = -(1/2M)C_D \rho_a A_V v^2 - \mu g - g\theta + u$ and the acceleration internally required to counteract the decelerating force due to the road gradient $(a_\theta = g\theta))$. Therefore, combining (17) and (18), fuel consumption in the vehicle at any velocity and acceleration can be obtained as

$$f_V = b_0 + b_1 v + b_2 v^2 + b_3 v^3 + \hat{a}(c_0 + c_1 v + c_2 v^2). \tag{19}$$

It is assumed that no fuel is consumed during braking with a control input $u \leq 0$. The consumption parameters in (17) and (18) obtained through the curve-fitting process are $b_0 = 0.1569$, $b_1 = 2.450 \times 10^{-2}$, $b_2 = -7.415 \times 10^{-4}$, $b_3 = 5.975 \times 10^{-5}$, $c_0 = 0.07224$, $c_1 = 9.681 \times 10^{-2}$, and $c_2 = 1.075 \times 10^{-3}$. At the idling condition, while there exists a constant consumption, fuel consumption is approximated from (17), with $x_2 = 0$. This way, the fuel consumption of any vehicle can be estimated by forming the torque–speed characteristic map of the engine and approximating parameters from the standard 10–15-mode fuel consumption rate. Fig. 11 shows fuel consumption data obtained from the engine map by bullet markers at various values of velocity and acceleration, and corresponding approximation curves obtained using (19). Each curve represents constant-acceleration ($a = 0.0, 0.4, \ldots, 2.4$ m/s^2) fuel consumption rate (in milliliters per second) with respect to the velocity. Approximation is slightly deviated at high values of acceleration at high velocities, which is usually avoided in normal driving condition, and very high acceleration at high velocity requires driving power beyond the limit of the vehicle. Finally, the fuel consumption rate evaluated in 10–15 mode is estimated using (19) as 17.16 km/L, which is very close to the catalog rate of 17.2 km/h. Therefore, the approximation can be considered satisfactorily fitted.

Appendix B
Optimal Control Problem

For optimization of the vehicle control inputs, the prediction horizon T is divided into N steps with a step size of $\Delta\tau = T/N$, and using $x_0 = x(t)$, the discretized optimal control problem can be stated as follows [20]:

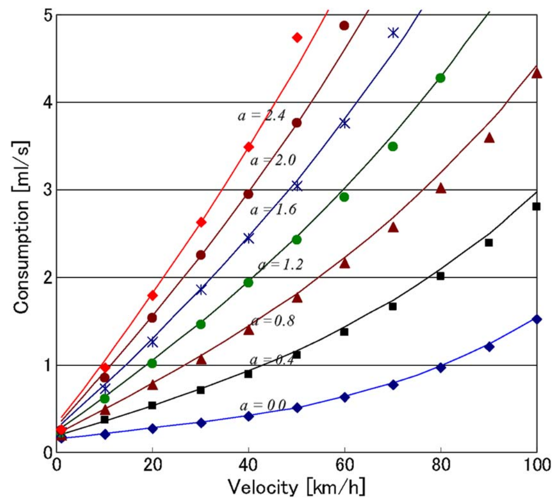

Fig. 11. Fuel consumption values at various velocities and accelerations and corresponding approximation curves.

$$\text{Minimize} : J = \sum_{i=0}^{N-1} L\left(x_i(t), u_i(t)\right) \Delta\tau(t) \tag{20}$$

$$\text{Subject to} : x_{i+1}(t) = x_i(t) + f\left(x_i(t), u_i(t)\right) \Delta\tau(t) \tag{21}$$

$$C\left(x_i(t), u_i(t)\right) = 0 \tag{22}$$

where $i = 0 \ldots N$ indicates the respective values at the ith instance in the descritized prediction horizon. Given the Hamiltonian function (10) satisfying (20) and (21), the first-order necessary conditions for the sequences of the optimal control input $\{u_i(t)\}_{i=0}^{N-1}$, Lagrange multiplier $\{\psi_i(t)\}_{i=0}^{N-1}$, and costate $\{\lambda_i(t)\}_{i=0}^{N}$ are obtained by the calculus of variation as

$$H_u\left(x_i(t), \lambda_{i+1}(t), u_i(t), \psi_i(t)\right) = 0 \tag{23}$$

$$\lambda_i(t) = \lambda_{i+1}(t) + H_x^T\left(x_i(t), \lambda_{i+1}(t), u_i(t), \psi_i(t)\right) \tag{24}$$

where $\lambda_N(t) = 0$ for this problem. The term H_x is called Jacobian; it is defined as $[(\partial/\partial x_1)H, (\partial/\partial x_2)H]^T$ in this problem with state variable $x = [x_1, x_2]^T$, and the same applies for H_u.

For N-step horizon with only one input variable and one equality constraint for this problem, the necessary condition of optimality can be expressed by the following $2N$-dimensional nonlinear equation:

$$F\left(U(t), x(t), t\right) := \begin{bmatrix} H_u\left(x_0(t), \lambda_1(t), u_0(t), \psi_0(t)\right) \\ C\left(x_0(t), u_0(t)\right) \\ \vdots \\ H_u\left(x_{N-1}(t), \lambda_N(t), u_{N-1}(t), \psi_{N-1}(t)\right) \\ C\left(x_{N-1}(t), u_{N-1}(t)\right) \end{bmatrix} = 0 \tag{25}$$

where $U(t) := [u_0(t), \psi_0(t), \ldots, u_{N-1}(t), \psi_{N-1}(t)]$. At each sampling time, $F(U(t), x(t), t) = 0$ needs to be solved for the optimum $U(t)$ with the measured state $x(t)$. Instead of using a time-consuming iterative algorithm such as the Newton method, the C/GMRES algorithm can derive the solution without any iterative searches. $F(U(t), x(t), t)$ can be identically zero if the following conditions hold:

$$F(U(0), x(0), 0) = 0 \tag{26}$$

$$\frac{d}{dt} F(U(t), x(t), t) = -\zeta F(U(t), x(t), t) \tag{27}$$

where ζ is a positive constant to stabilize $F(U(t), x(t), t) = 0$ against numerical errors and unmodeled disturbances. The condition (27) can be written as

$$F_U \dot{U} = -F_x - F_t - \zeta F \tag{28}$$

which can be regarded as a linear algebraic equation for $\dot{U}$ and can be efficiently solved by the linear solver GMRES [24]. The solution $U(t)$ can be found by integrating $\dot{U}$ in real time.

The Combination of the Continuation and GMRES (C/GMRES) method is used to numerically optimize the sequence of control inputs, as previously stated. From the viewpoint of real-time control, it solves the differential equation only once at each sampling time and therefore requires much less computational burden. Detailed description on the C/GMRES method can be found in [20].

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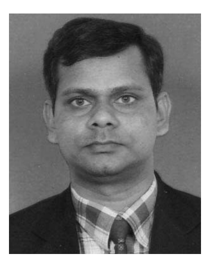

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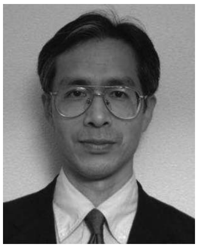

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