Calculating Spin-flavor Operator relations

$$\boxed{\begin{aligned}(GS^4G)^{ijab}_{I=0,J=0} &= \delta_s^{ij}\delta_s^{ab}\Big(\frac{1}{12}N_c(4+N_c)+\frac{1}{36}(-24+5N_c(4+N_c))S^2\\ &\quad+\frac{1}{144}(-112+3N_c(4+N_c))S^4-\frac{1}{18}S^6\Big)\end{aligned}}$$

$$\begin{aligned}(GS^4G)^{ijab}_{I=0,J=1} &= i(-1)^k\delta_s^{ab}\epsilon_s^{ij-k}\Big(-\frac{1}{8}N_c(4+N_c)S^k\\ &\quad+\frac{1}{12}(2-N_c)(6+N_c)S^kS^2+\frac{11}{24}S^kS^4\Big)\end{aligned}$$

$$\begin{aligned}(GS^4G)^{ijab}_{I=1,J=1} &= (-1)^{k+c}\epsilon_s^{ij-k}\epsilon_s^{ab-c}\Big(\frac{1}{8}(2-N_c(4+N_c))S^kI^c+\frac{5}{4}S^kI^cS^2\\ &\quad+\frac{1}{8}S^kI^cS^4-\frac{1}{8}(2+Nc)\{S^2,G^{kc}\}-\frac{1}{16}(2+Nc)\{S^4,G^{kc}\}\Big)\end{aligned}$$

$$\begin{aligned}(GS^4G)^{ijab}_{I=0,J=2} &= \delta_s^{ab}\Big(-\frac{1}{24}N_c(4+N_c)\left\{S^i,S^j\right\}|_{J=2}\\ &\quad+\frac{1}{3}S^2\left\{S^i,S^j\right\}|_{J=2}+\frac{1}{24}S^4\left\{S^i,S^j\right\}|_{J=2}\Big)\end{aligned}$$

$$\begin{aligned}(GS^4G)^{ijab}_{I=1,J=2} &= i(-1)^c\epsilon_s^{ab-c}\Big(\frac{1}{8}(2+N_c)\{S^2,\{S^i,G^{jc}\}|_{J=2}\}\\ &\quad-\frac{1}{4}I^c\left\{S^i,S^j\right\}|_{J=2}-\frac{1}{4}S^2I^c\left\{S^i,S^j\right\}|_{J=2}\Big)\end{aligned}$$

$$\begin{aligned}(GS^4G)^{ijab}_{I=2,J=2} &= -\{G^{ia},G^{jb}\}|_{I=2,J=2}+\frac{1}{2}\{S^2,\{G^{ia},G^{jb}\}|_{I=2,J=2}\}\\ &\quad+\frac{1}{2}S^2\{G^{ia},G^{jb}\}|_{I=2,J=2}\,S^2-\frac{1}{2}(2+N_c)\{S^iI^a,G^{jb}\}|_{I=2,J=2}\end{aligned}$$

$$\underbrace{GS^4G^{ijab}_{I=0,J=0}}_{f_1} =$$

$$(0)\times \underbrace{\delta^{ij}\delta^{ab}}_{g_1} +$$

$$(3)\times \underbrace{N_c\,\delta^{ij}\delta^{ab}}_{g_2} +$$

$$\left(\frac{1}{12}\right)\times \underbrace{N_c^2\,\delta^{ij}\delta^{ab}}_{g_3} +$$

$$\left(-\frac{2}{3}\right) \underbrace{\delta^{ij}\delta^{ab}\,S^2}_{g_4} +$$

$$\left(\frac{5}{9}\right) N_c\,\delta^{ij}\delta^{ab}\,S^2 +$$

$$\left(\frac{5}{36}\right) N_c^2\,\delta^{ij}\delta^{ab}\,S^2 +$$

$$\left(\frac{9}{7}\right) \delta^{ij}\delta^{ab}\,S^4 +$$

$$\left(\frac{1}{12}\right) N_c\,\delta^{ij}\delta^{ab}\,S^4 +$$

$$\left(\frac{1}{48}\right) N_c^2\,\delta^{ij}\delta^{ab}\,S^4 +$$

$$\left(\frac{-1}{18}\right) \delta^{ij}\delta^{ab}\,S^6$$

rational numbers.

$$\underbrace{GS^2G^{ijab}_{I=0,J=0}}_{f_2}$$

$$GS^4G^{ij\,ab}_{I=0,J=0} = f_1(i,j,a,b,\overbrace{S,S_3,I_3}^{\text{in state}},\overbrace{S',S'_3,I'_3}^{\text{out state}},N_c)$$

$$\delta^{ij}\delta^{ab} = g_1(i,j,a,b,S,S_3,I_3,S',S'_3,I'_3,N_c)$$

$$N_c\,\delta^{ij}\delta^{ab} = g_2(\qquad\qquad)$$

$$N_c^2\,\delta^{ij}\delta^{ab} = g_3(\qquad\qquad)$$

$\vdots$

$\boxed{\text{11 discrete variables}}$

$$i,j,a,b = -1,0,1$$

$$N_c = 3,5,7,\ldots$$

$$S,S' = 1/2,\ 3/2 \ldots N_c/2$$

$$S_3,\ I_3 = -S,\ -S+1,\ -S+2,\ \ldots\ +S$$

$$S'_3\ I'_3 = -S',\ -S'+1,\ -S'+2,\ \ldots\ +S'$$

$$f_1(\ldots) = c_1 g_1(\ldots) + c_2 g_2(\ldots) + c_3 g_3(\ldots) + c_4 g_4(\ldots) + \ldots$$

3 – 30 terms

⊛ How we find c_i

bunch of random : $i\,j\,a\,b,\ N_c,\ S, S_3, I_3, S', S_3', I_3'$

$$\left\{ \begin{array}{l} 3 = c_1 + 2c_2 - c_3 + 5c_4 + \ldots \\ 10 = 2c_1 - 2c_2 + c_3 + \ldots \\ \vdots \end{array} \right.$$

$$\left(\begin{array}{ll} N_c = 3-15 & = 5 \\ i\,j\,a\,b & = 2^4 \\ S, S' & = 3\times 3 \\ S_3 I_3 & = 2\times 2 \\ S_3' I_3' & = 2\times 2 \end{array} \right) \times 4 \text{ sets}$$

$\downarrow$

difficult ~ 40 000 equations

→ Solve linear equations to find c_i

- → unique c_i ✓ good
- → non unique
- → no solutions

non-unique

① Ther exist some dependencis between g^s

ex: $g_1(\ ) + 2g_3(\ ) = 7' g_{15}(\ )$

② we should try more radom values $ij\,ab\ N_c, S\, S', S_3 S_3'\ I_3 I_3'$

No Solutions

we guess where to en

$$f_1 = c_1 g_1 + c_2 g_2 + c_3 g_3 + \dots + c_{20}\, g_{20}$$

we are missing $g_{21}, g_{22}, \dots$
(higher order terms)

⊛ Ther are | 50-60 fs to calculate.
200 g^s

but we can categorize thes in to 6 categories

∴ we have ($10 \to f$ and $30 \to g$) × 6 sets.