DHS CS Club Problem Set 0

DHS CS Officers

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1 USACO March 2020/G2, William Lin and Benjamin Qi

Each of Farmer John's N cows $(1 \leq N \leq 2 \cdot 10^5)$ has a favorite color. The cows are conveniently labeled $1 \ldots N$ (as always), and each color can be represented by an integer in the range $1 \ldots N$. There exist M pairs of cows (a, b) such that cow b admires cow $a (1 \leq M \leq 2 \cdot 10^5)$. It is possible that $a = b$, in which case a cow admires herself. For any color c, if cows x and y both admire a cow with favorite color c, then x and y share the same favorite color.

Given this information, determine an assignment of cows to favorite colors such that the number of distinct favorite colors among all cows is maximized. As there are multiple assignments that satisfy this property, output the lexicographically smallest one (meaning that you should take the assignment that minimizes the colors assigned to cows $1 \ldots N$ in that order).[1]

2 USACO March 2020/G3, Benjamin Qi

Farmer John has come up with a new morning exercise routine for the cows (again)! As before, Farmer John's N cows $(1 \leq N \leq 7500)$ are standing in a line. The ith cow from the left has label i for each $1 \leq i \leq N$. He tells them to repeat the following step until the cows are in the same order as when they started.

- Given a permutation A of length N, the cows change their order such that the ith cow from the left before the change is A_ith from the left after the change.

For example, if $A = (1, 2, 3, 4, 5)$ then the cows perform one step and immediately return to the same order. If $A = (2, 3, 1, 5, 4)$, then the cows perform six steps before returning to the original order. The order of the cows from left to right after each step is as follows:

- 0 steps: (1,2,3,4,5)
- 1 step: (3,1,2,5,4)
- 2 steps: (2,3,1,4,5)
- 3 steps: (1,2,3,5,4)
- 4 steps: (3,1,2,4,5)

[1] Input and output format can be found here.

- 5 steps: (2,3,1,5,4)
- 6 steps: (1,2,3,4,5)

Compute the product of the numbers of steps needed over all $N!$ possible permutations A of length N.

As this number may be very large, output the answer modulo M ($10^8 \leq M \leq 10^9 + 7$, M is prime).[2]

[2]Input and output format can be found here.