AST4320 Oblig2

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1 Exercise 2

We would like to find the optical depth τ_e of the IGM as a function of redshift z. The optical depth is given as

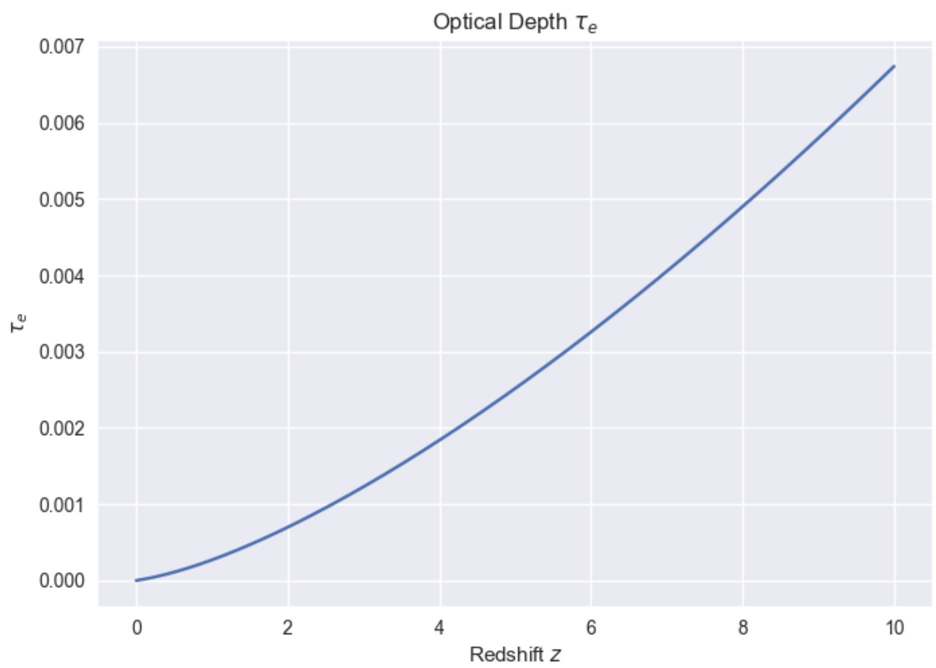
$$\tau_e(z) = c\int_0^z \frac{n_e(z)\sigma_T dz}{H(z)(1+z)}, \tag{1}$$

where σ_t is the Thompson cross section, n_e the electron density and $H(z)$ the Hubble parameter. Since the Universe is taken to be completely ionized, he assume that $n_e \approx \bar{n_H} = 1.9\cdot 10^{-7}(1+z)^3$ cm^{-3}. We get the Hubble parameter from the Friedmann equation

$$H(z) = H_0\sqrt{\Omega_m(1+z)^3 + \Omega_r(1+z)^4 + \Omega_\Lambda}, \tag{2}$$

where $\Omega_\Lambda = 0.692$, $\Omega_m = 0.308$ and $\Omega_r = 0$.

We will integrate (1) from $z = 0$ to 10. We assume that $\tau_e(0) \approx 0$.



Figur 1: The optical depth of the intergalactic medium. We see that for larger redshift the optical depth increases.

2 Exercise 3

2.1 a)

We have the differential equation for an isothermal halo

$$-\frac{k_bT}{m_{DM}r^2}\frac{d}{dr}r^2\frac{d}{dr}\ln\rho = 4\pi G\rho. \tag{3}$$

We have an ansatz that

$$\rho(r) = \frac{A}{r^2}, \qquad A = \frac{k_b T}{2\pi G m_{DM}}. \tag{4}$$

To see that this is a solution, we put (4) into (3). Looking at the RHS of (3) we get

$$-\frac{k_b T}{m_{DM} r^2}\frac{d}{dr}r^2\frac{d}{dr}\ln\rho = -\frac{k_b T}{m_{DM} r^2}\frac{d}{dr}r^2\frac{r^2}{A}\cdot\left(-\frac{2A}{r^3}\right) \tag{5}$$

$$2\frac{k_b T}{m_{DM} r^2}\frac{d}{dr}r = 2\frac{k_b T}{m_{DM} r^2}. \tag{6}$$

Thus we get

$$2\frac{k_b T}{m_{DM} r^2} = 4\pi G\frac{A}{r^2} \Rightarrow A = \frac{k_b T}{2\pi G m_{DM}}. \tag{7}$$

Thus (4) solves (3).

2.2 b)

We have that a gas in hydrostatic equilibrium

$$\frac{dp}{dr} = -\frac{GM(<r)\rho}{r^2}, \tag{8}$$

where $M(<r)$ is the mass within a radius r and p is the pressure. We can see that our isothermal gas, with the density defined in (4), behaves in the similar way. We start by finding the mass, which for a spherical symmetric mass is given as

$$M(<r) = 4\pi\int_0^r \rho(r')r'^2 dr' = 4\pi\int_0^r \frac{A}{r'^2}r'^2 dr' = 4\pi\int_0^r A dr' = 4\pi A r. \tag{9}$$

In out expression of A we have the dark matter mass m_{DM}. Since we now have a gas, we let $m_{MD} \to m_p$, which is the proton mass. Thus we have

$$\rho = \frac{A}{r^2} = \frac{k_b T}{2\pi G m_p r^2}. \tag{10}$$

We when use that for an isothermal gas the pressure is given as

$$p = \frac{k_b T}{m_p}\rho = \frac{k_b T A}{m_p r^2}. \tag{11}$$

We can now take the differentiation of this with respect to r and use (9) and (4) to find

$$\frac{dp}{dr} = -\frac{2k_b T A}{m_p r^3} = -\frac{2k_b T}{m_p r^3}\cdot\frac{M(<r)}{4\pi r} = -\frac{GM(<r)}{r^2}\frac{k_b T}{2\pi G m_p r^2} = -\frac{GM(<r)\rho}{r^2}. \tag{12}$$

This is the same as for the gas in hydrostatic equilibrium from (8).