AST5220, Milestone 1

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1 Introduction

During the next four milestones we are going to calculate the temperature of the photons of the *Comic Microwave Background* [1]. There are many hurdles to overcome on this journey, but the first is the expansion of the space-time in which the photons exist.

The photons we are looking at lives in the same Universe as we do, and from theoretical predictions and observations we know that this Universe is expanding, and has been doing so from the start time, thought the decoupling for matter and photons and all the way to today. As the space and time the photons occupy is expanding the photons will redshift. Heuristically this is because the photons dissipates energy to overcome this expansion . This loss of energy leads to a decrease in the photon energy and therefore a change in the wavelength towards the redder part of the spectrum [2]. We need a way of quantifying this expansion, to be ables to calculate this change in temperature. We are going to do this by solving the *Friedmann equation*(2) to get the Hubble parameter

Look over and rewrite this explanation

$$H = \frac{a}{\dot{a}}, \tag{1}$$

where a is the scale factor and $\dot{a}$ its time derivative. The Hubble parameters is an observable quantity which describes the expansion of the Universe. From this we are going to calculate the *Conformal Time* η, which is a convenient way of representing cosmic time in many of the equations we are going to solve in later milestones.

2 Theory

The main parameters we are looking for is the Hubble parameter (1) H and the conformal time η. From Einsteins field equation we can obtain equations for finding H, namely the Friedmann equations. We only need the first of these,

$$H = H_0\sqrt{(\Omega_{b,0} + \Omega_{m,0})a^{-3} + (\Omega_{r,0} + \Omega_{\nu,0})a^{-4} + \Omega_{\Lambda,0}}, \tag{2}$$

where $\Omega_{b,0}, \Omega_{m,0}, \Omega_{r,0}, \Omega_{\nu,0}, \Omega_{\Lambda,0}$ are the *Density Parameters* for respectively baryons, (cold) dark matter, neutrinos and dark energy. These parameters are given as

$$\Omega_{x,0} = \frac{\rho_{x,0}}{\rho_{crit,0}} = \rho_{crit,0} \cdot \left[\frac{3H_0^2}{8\pi G}\right]^{-1}, \tag{3}$$

where $\rho_{x,0}$ is the density and $\rho_{crit,0} = \frac{3H_0^2}{8\pi G}$ is the critical density. Notice that the subscript 0 means that these are the parameters at present time. If we instead calculate the density and the critical density at a given time, we obtain the density parameter at this time as well. We are going to assume that there are no neutrinos, and set $\Omega_{nu} = 0$ for all times.

To find the Ω_x throughout time, we need the densities. These are gives as

$$\rho_m = \rho_{m,0}a^{-3}, \quad \rho_b = \rho_{b,0}a^{-3}, \quad \rho_4 = \rho_{4,0}a^{-4}, \quad \rho_\Lambda = \rho_{\Lambda,0}. \tag{4}$$

Before going on to look at the conformal time, we are going to define some variables to make our life easier. First, we are not going to have (2) as a function of time or the scale factor, but rather over the logarithmic scale factor $x \equiv \ln a = -\ln(1+z)$, where z is the red shift. We are also going to introduce the scaled Hubble parameter $\mathcal{H} \equiv aH$.

[1]Or to be more precise, the power spectrum of the CMB
[2]In other words longer wavelength.

We can now find the conformal time η. From the FRW-line element it is easy to show that one can introduce a new time variable, defines as

$$\frac{d\eta}{dt} = \frac{c}{a}. \tag{5}$$

It can be shown that we can rewrite this in another variable

$$\frac{d\eta}{da} = \frac{c}{a\mathcal{H}}. \tag{6}$$

Since we have introduces the time parameter x, we can rewrite this again

$$\frac{d\eta}{da} = \frac{d\eta}{dx}\frac{dx}{da} = \frac{d\eta}{dx}\frac{d\ln a}{da} = \frac{d\eta}{dx}\frac{1}{a} \Rightarrow \frac{d\eta}{dx} = \frac{c}{a\mathcal{H}} \cdot a = \frac{c}{\mathcal{H}}. \tag{7}$$

This makes it easy to integrate to find η.

3 Method

We are going to calculate (2) by saying that we are going to look at the universe from the start of the recombination, $z = 1630.4$, to the end of recombination, $z = 614.2$, and from this to today, $z = 0$. We are then going to convert these red shifts into the logarithmic scale factor, and make a grid with 200 points during recombination and 300 points after. With this grid we can calculate H from (2).

To find the the conformal time η we need a new grid. This time we start with a grid from $a = 10^{-10}$ to $a = 1$ (today), with 1000 points. To be able to integrate (7) we need initial condition for η at the start of recombination. We can do this be noting that $a\mathcal{H}(a) \to H_0\sqrt{\Omega_r}$ as $a \to 0$ *Cite Callin*. This means that we can make the integral from the start of time, to recombination

$$\eta(a = a_{recombination}) = \int_0^{a_{recombination}} \frac{c}{a\mathcal{H}} da \approx \int_0^{a_{recombination}} \frac{c}{H_0\sqrt{\Omega_r}} da = \frac{c \cdot a_{recombination}}{H_0\sqrt{\Omega_r}}. \tag{8}$$

Thus we have an initial value for η. To integrate until today, we can just use a simple Euler scheme

$$\eta_{i+1} = \eta_i + \frac{d\eta}{dx} \cdot dx. \tag{9}$$

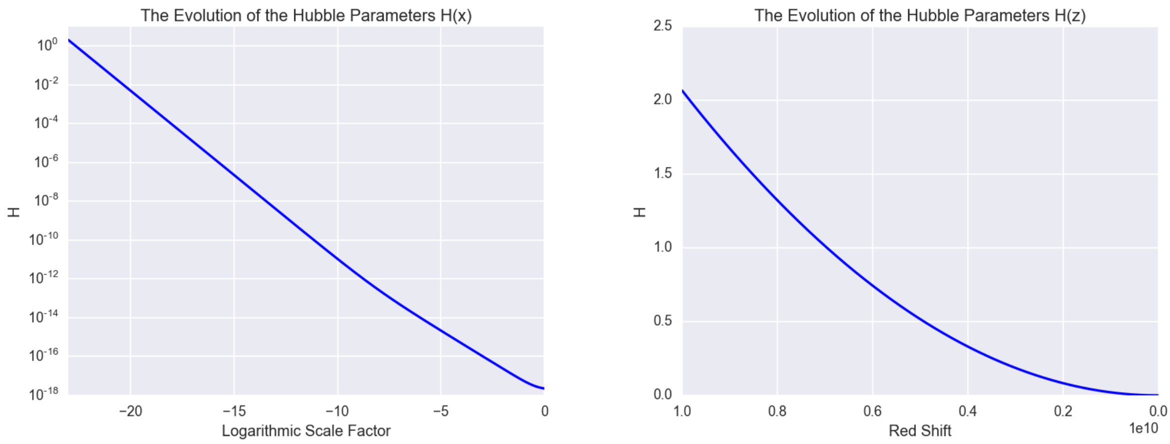
We want to know η at all possible time, not just the ones used here. Thus we use a *spline* on the calculated dataset to find get a continuous approximation to η.

We also want to find an expression for $d\mathcal{H}/dx$. This is simply done with our definition of $\mathcal{H}$ and H

$$\frac{d\mathcal{H}}{dx} = \frac{d}{dx}e^x H = e^x\left(H + \frac{dH}{dx}\right) = e^x\left(H + \frac{d}{dx}\left[H_0\sqrt{(\Omega_{b,0} + \Omega_{m,0})\exp(-3x) + (\Omega_{r,0} + \Omega_{\nu,0})\exp(-4x) + \Omega_{\Lambda,0}}\right]\right)$$

$$= e^x\left(H + \frac{H_0^2}{H}\left[-3(\Omega_{b,0} + \Omega_{m,0})\exp(-3x) - 4(\Omega_{r,0} + \Omega_{\nu,0})\exp(-4x)\right]\right) \tag{10}$$

4 Results



(a) This shows the Hubble Parameter as a function of the logarithmic scale factor x.

(b) This shows the Hubble Parameter as a function red shift z

Figur 1: These plots show the evolution of the Hubble parameter through the lifetime of the Universe. Notice that for 1a the y-axis is logged, due to the x-axis being a logarithmic variable, which makes H close to 0 for x all values except for those to the around -20 and below. Since 1b isn't plotted against a logarithmic variable, no logging of the y-axis is required. This unfortunately makes a direct comparison between the plots less trivial.

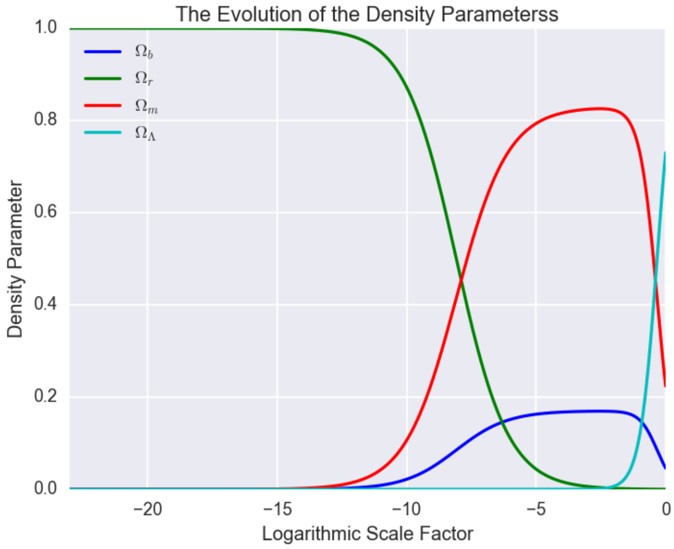
Maybe rewrite the explanation of the logarithmic y-axis in fig:Hs

In fig. 1 we see the evolution of the Hubble parameter. From (1) we see that H is proportional to a and inversely proportional to $\dot{a}$. This means that in the early Universe, where $\dot{a}$ is small, the expansion of the Universe $\dot{a}$ was even smaller. In the late Universe, where $a = a_0 = 1$, the expansion rate must be very large, so that $H \approx 0$.

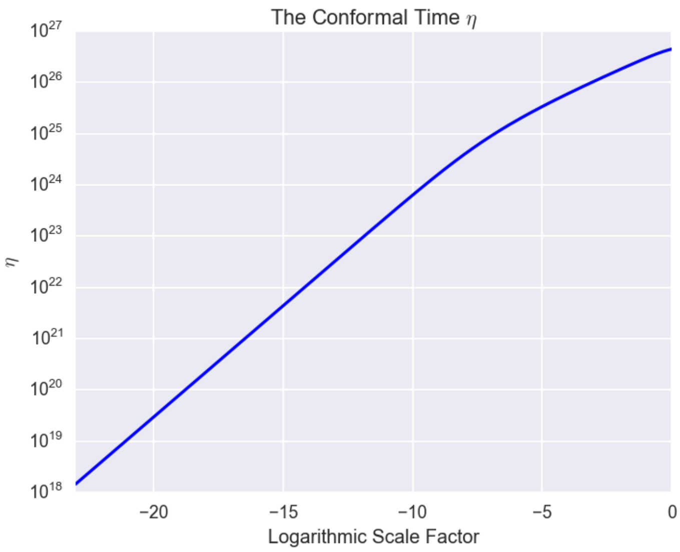
Not completely sure of this, so double check the reasoning and results

Fig. 2 shows the evolution of the density fraction. From 4 we see that the different densities have different dependencies on the scale factor a. These dependencies also extends to the density fractions 3. Since radiation is dependent on a^{-4}, we expect that to be dominant in the early Universe with $a << 1$. This is what we see from fig. 2. At some point matter (baryons and dark matter) will be more dominant than due to the fact that $\Omega_{r,0} \approx 10^{-6}$, so even though its dependence on a^{-4} makes it larger that matter which has a dependence on a^{-3} if $a < 1$ this only holds as long as $a < 10^{-6}$, so around this point matter starts to dominate (dark matter in this case, since $\Omega_{b,0}$ is only 0.046). This is also what we see from the plot. At present time, when $a = 1$, we expect that $\Omega_{x,0}$ is the determiner of the values of the densities, which we can see is correct from the figure.

Can maybe be explained easier...



Figur 2: Here we see the evolution of the four density fractions of the Universe. Ω_r is the density fraction for *radiation*, Ω_b for *baryons*, Ω_b for *dark matter*, and Ω_Λ for *dark energy*.



Figur 3: Here we can see the conformal time η through the lifetime of the Universe. The slope of this plot is determined by the type of energy that dominates the Universe. This will be described more in the text.

Rewrite eta caption!

Fig. 3 shows the main result of this article. We can try to explain and justify the plot shown in 3 from what we know about cosmology. We see that the plot seems to be three (more or less) linear functions (which means power law before taking the logarithm) sewn together. This is because the slope of the function goes as $1/aH$ [3], and since H and a goes as power laws when dominated by radiation or matter, and a exponential when dominated by dark energy in the very late Universe.

[3] This is the derivative of η

So if we compare fig. 2 and fig. 3 we see that when radiation dominates the density parameters we have a step slope of $\log \eta$, but when matter starts to dominate in 2, the slope in 3 becomes less step. During the late Universe dark matter starts to dominate, and in the very last part of the plot of η, we can see that the slope starts to become even less step.

Look through this expla-nation!

5 Conclusion

Our main result, shown in fig. 3, gave us a way of finding a value for the conformal time η for a given logarithmic scale factor $x = \log(a)$. We used calculations of the density parameters, fig. 2, to see that our model looked like what we expect from out Universe.

We also made functions to be able to get values for the Hubble parameter, fig. 1, and for $\mathcal{H}$ and $\frac{d\mathcal{H}}{dx}$, which are not shown in this article.