

AST5220 Milestone4

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1 Introduction

So far we have found out how the Universe expands, found the number density of free electrons and how this impact the mean free path of the photons. We have then used this to calculate how initial perturbations after inflation would have evolved through till today. We are now ready to use all of this to find what we are after: The power spectrum of the Cosmic Microwave Background (CMB).

2 Theory

We are after the power spectrum

$$C_l = \langle |a_{l,m}|^2 \rangle = \int \frac{d^3k}{(2\pi)^3} |a_{l,m}|^2. \quad (1)$$

We need to integrate since we have Fourier transformed all our quantities. $a_{l,m}$ comes from the fact that we observe the temperature of the CMB projected on a sphere, meaning that we can write it as

$$T(\hat{n}) = \sum a_{l,m} Y_{l,m}(\hat{n}), \quad (2)$$

where $Y_{l,m}$ are spherical harmonics and $a_{l,m}$ the coefficients containing the actual physics of the temperature field. Instead of finding $|a_{l,m}|^2$ directly, we can instead define $|a_{l,m}|^2 = P(k)\Theta_l^2$, where $P(k)$ is the primordial power spectrum and Θ_l is the so called *transfer function* which contain all the information about how the power spectrum has evolved from inflation and until today. We can get the primordial power spectrum from the Harrison Zel'dovich spectrum

$$\frac{k^3}{2\pi^2} P(k) = \left(\frac{ck}{H_0} \right)^{n_s-1}, \quad (3)$$

where n_s is the spectral index. We can now write the power spectrum as

$$C_l = \int_0^\infty \left(\frac{ck}{H_0} \right)^{n_s-1} \Theta_l^2(k) \frac{dk}{k}. \quad (4)$$

We now need to find the transfer function Θ_l . As the choice of name implies, this is perturbation of the l^{th} momentum we found in milestone 3. The problem is the while we calculated these up to $l = 6$ the CMB spectrum is given for $l > 1200$. Calculating all these perturbations with the use of the method in milestone 3 is slow, so we need a faster way. Thanks to Zaldarriaga and Seljak we can calculate the momenta in minutes and not hours. They found the so called *line of sight integration* method. This means that instead of doing a multipole expansion of the temperature, then solve the differential equations, we instead solve the equations and then expand. This means that we can write, after some calculation,

$$\Theta_l(k, x=0) = \int_{-\infty}^0 \tilde{S}(k, x) j_l(k(\eta_0 - \eta)) dx, \quad (5)$$

where $j_l(k(\eta_0 - \eta))$ is the l^{th} spherical Bessel function evaluated at $k(\eta_0 - \eta)$, and \tilde{S} the source function. The source function is defined as

$$\tilde{S}(k, x) = \tilde{g} \left(\Theta_0 + \Psi + \frac{1}{4}\Pi \right) + e^{-\tau} (\Psi' - \Phi') - \frac{1}{ck} \frac{d}{dx} (\mathcal{H} \tilde{g} v_b) + \frac{3}{4c^2 k^2} \frac{d}{dx} \left[\mathcal{H} \frac{d}{dx} (\mathcal{H} \tilde{g} \Pi) \right], \quad (6)$$

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where (here) $\Pi = \Theta_2$. The source function can be thought of a function describing all the different things a photon encounters on our way to us that decreases or increases its energy. The first term describes the Sachs-Wolfe effect – the loss of energy as a function climbs out of a gravitational potential –, the second term describes the integrated Sachs-Wolfe effect – if the gravitational potential have change from when a photon enters to it leaves, this affects its energy –, and the third term is the Doppler term. Notice that the source function only needs temperature momenta up to Θ_2 , so we do not need to solve the differential equations for l higher than 2. In reality, since we use an approximation to find Θ'_{lmax} , there is a small error in the highest momentum. If we solve for l up to 6 this error is small enough for Θ_2 and below, that the error can be ignored.

Maybe write down all the equations used to calc S

3 Method

The method consists of two parts: Calculating the source function and integrating everything to get the final power spectrum.

3.1 Source Function

We want the source function from the start of recombination until today. From milestone 3 we have all the quantities we need to calculate \tilde{S} , but not with the precision we want. So with our quadratic k grid with 100 k s and our x grid with 500 points, we can find an initial grid of $\tilde{S}(k, x)$. We can then spline this 2D array. With new, finer k and x grids, both with 5000 points in the same intervals, we can use the spline to get a new \tilde{S} array with a finer grid. It is these grid we are going to use for the rest of the calculations.

3.2 Integrating Everything

To find Θ_l we see from eq. (5) that we need to find the spherical Bessel functions. Fortunately we are given functions to find these. But we need to save the Bessel function in a grid for each l and for 3500 x values in the same range as our x grid. For each l we will spline the Bessel function for use later.

We can now use eq. (5) to find the transfer functions Θ_l . This integration was done with a simple trapezoidal integration .

We then integrate the Θ_l s with the primordial power spectrum to get the C_l s, eq. (4). This again is done via an Euler Scheme . We start by finding C_l for 44 l s from 1 to 1200. We then spline and retrieve a C_l for each l in the same interval. We then normalize as $C_l \cdot l(l+1)/2\pi$, and we have our final result!

Check is this was used, and update if something else is used!

Change is something else is used!

Maybe write more?