AST5220 Milestone 2

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1 Introduction

In this exercise we are going to calculate the optical depth and visibility function. These quantities describe the how easy/difficult it is for a photon to travel. When later calculating the Boltzmann equations for the photons, we will have to use the optical depth .

Rewrite

And visibility function?

Due to the large amount of free electrons in the early Universe, photons could not travel far without being absorbed and scattered by the electrons. So to get a sense of the optical depth of the photons, we need to know the amount of electrons we have in our Universe. This will be done with the Saha and Peeble's equation. Having the density of electrons we can easily find the optical depth and visibility function.

2 Theory

We cannot find the electron density n_e directly. We need to go through the fractional electron density $X_e = n_e/n_H$, where n_H is the density of hydrogen in the Universe. To find X_e we are going to use two different equations. If $X_e \approx 1$ – meaning that there are way more free electrons than neutralhydrogen in the Universe – we can use the Saha equation

Does H mean neutral hydrogen

$$\frac{X_e^2}{1-X_e} = \frac{1}{n_b}\left(\frac{m_e T_b k_b}{2\pi\hbar^2}\right)^{3/2} e^{-\epsilon_0/T_b k_b}, \tag{1}$$

where n_b is the density of baryons, m_e the mass of an electron and T_b the temperature of the baryons. If we have $X_e < 0.99$ this equation becomes inaccurate and we need to solve Peebles' equation. This equation describes X_e for all values of X_e but is more difficult to solve, thus why be use the Saha equations for the earliest times. The Peebles' equation is given as

$$\frac{dX_e}{dx} = \frac{C_r}{H}\left[\beta(T_b)(1-X_e) - n_H\alpha^{(2)}(T_b)X_e^2\right], \tag{2}$$

where

$$C_r(T_b) = \frac{\Lambda_{2s\to1s} + \Lambda_\alpha}{\Lambda_{2s\to1s} + \Lambda_\alpha + \beta^{(2)}(T_b)}, \tag{3}$$

$$\Lambda_{2s\to1s} = 8.227 s^{-1}, \tag{4}$$

$$\Lambda_\alpha = \frac{H}{(\hbar c)^3}\frac{27\epsilon_0^3}{(8\pi)^2 n_{1s}}, \tag{5}$$

$$n_{1s} = (1-X_e)n_H, \tag{6}$$

$$\beta^{(2)}(T_b) = \beta(T_b)e^{3\epsilon_0/4T_b k_b}, \tag{7}$$

$$\beta(T_b) = \alpha^{(2)}(T_b)\left(\frac{m_e T_b k_b}{2\pi\hbar^2}\right)^{3/2} e^{-\epsilon_0/T_b k_b}, \tag{8}$$

$$\alpha^{(2)}(T_b) = \frac{64\pi}{\sqrt{27\pi}}\frac{\alpha^2}{m_e^2}\sqrt{\frac{\epsilon_0}{T_b k_b}}\frac{\hbar^2}{c}\phi_2(T_b), \tag{9}$$

$$\phi_2(T_b) = 0.448\ln(\epsilon_0/T_b k_b). \tag{10}$$

All of these equation use SI units.

From X_e we can find

$$n_e = X_e n_H, \tag{11}$$

where

$$n_H = n_b \approx \frac{\rho_b}{m_H} = \frac{\Omega_b \rho_c}{m_h a^3}. \tag{12}$$

We can use this because we assume that all baryonic matter in the Universe is hydrogen.

With this one could find the optical depth, defined as

$$\tau(\eta) = \int_{\eta}^{\eta_0} n_e \sigma_T a c d\eta \Leftrightarrow \tau(x) = \int_{x}^{x_0=0} -\frac{n_e \sigma_T a c}{\mathcal{H}} dx, \tag{13}$$

where σ_T is the Thompson cross section, c is the speed of light, a is the scale factor, x is the logarithm scale factor and $\mathcal{H}$ the scaled Hubble parameter aH. And finally from this we can get the visibility function

$$\tilde{g}(x) = -t' e^{-\tau} = -\frac{d\tau}{dx} e^{-\tau}. \tag{14}$$

3 Method