Fys2160 Oblig 2

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1 Exercise 1)

1.1 a)

The partition function is given as

$$Z = \sum_s e^{-E(s)/kT} = \sum_s e^{-E(s)\beta} \tag{1}$$

We have the energies $\epsilon_1 = \epsilon$ and $\epsilon_2 = \epsilon_3 = \epsilon_4 = 2\epsilon$, giving us the partition function:

$$Z = e^{-\beta\epsilon} + 3e^{-2\beta\epsilon} \tag{2}$$

1.2 b)

From the partition function we are able to find the average energy from the following equation:

$$\langle E \rangle = -\frac{\partial \ln Z}{\partial \beta} \tag{3}$$

We can from this find the average energy for our system:

$$\langle E \rangle = -\frac{\partial}{\partial \beta} \ln\left(e^{-\beta\epsilon} + 3e^{-2\epsilon\beta}\right) \tag{4}$$

$$= -\frac{-\epsilon e^{-\beta\epsilon} - 6\epsilon e^{-2\beta\epsilon}}{e^{-\beta\epsilon} + 3e^{-2\epsilon\beta}} == \epsilon \frac{e^{\beta\epsilon} + 6}{e^{\beta\epsilon} + 3} \tag{5}$$