1 Kompendiet:

1.1 Ultrafiolet katastrofe:

$$c = \lambda\nu \Leftrightarrow \lambda\frac{c}{\nu} \tag{1}$$

Radianse:

$$M_\nu(t) = \frac{2\pi}{c^2}\nu^2 \underbrace{\langle E\rangle_\nu}_{k_BT} \tag{2}$$

Total utstrålingenergi:

$$M(t) = \int_0^\infty M_\nu(t)d\nu = \int_0^\infty \frac{2\pi}{c^2}\nu^2 k_BTd\nu = \infty \tag{3}$$

Plank:

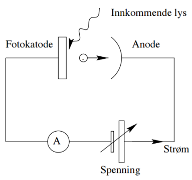
$$\vec{k} = \frac{2\pi}{\lambda}\vec{n} \tag{4}$$

$$E_k = n_k h\nu \tag{5}$$

$$\langle E\rangle = \frac{h\nu}{e^{h\nu/kT}-1} \propto \nu e^{h\nu/kT} \underbrace{\longrightarrow}_{\nu\to\infty} 0 \tag{6}$$

$$M(t) = \sigma T^4, \qquad \epsilon = 1W\cdot\frac{A_e}{A}\cdot t = 1W\cdot\frac{\pi r^2}{2\pi R^2}\cdot t \tag{7}$$

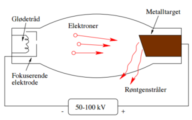
1.2 Fotoelektriske effekt:



- $K_{maks} = |eV_0|$ er uavhengig av intensiteten til lyset.
- Klassisk: fotoelektriske effekten skal skje for alle frekvenser. Dette er ikke tilfelle
- Klassisk: Tid mellom lyset treffer og elektronet løsrev seg. Ikke tilfellet

$$K_{maks} = h\nu - \omega_0 \tag{8}$$

1.3 Röntgen

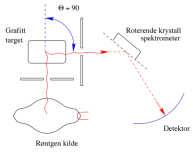


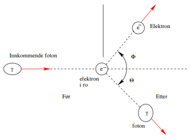
$$e^- \to e^- + \gamma \Rightarrow h\nu = \frac{hc}{\lambda} = K_e - K_e' \tag{9}$$

Antar elektronet står stille etter kollisjon(minste energi!):

$$h\nu = \frac{hc}{\lambda} = K_e \Rightarrow \lambda_{min} = \frac{hc}{eV_R} \tag{10}$$

1.4 Compton





$$E_\gamma + E_e = E'_\gamma + E'_e, \qquad \mathbf{p}_\gamma + 0 = \mathbf{p}'_\gamma + \mathbf{p}'_e \tag{11}$$

$$\Delta\lambda = \lambda_C(1 - \cos\theta) = \frac{h}{m_e c}(1 - \cos\theta) \tag{12}$$

1.5 Bohr

$$p = \frac{h}{r}, \qquad L = pr = n\hbar \Rightarrow 2\pi r = n\lambda \tag{13}$$

$$E = -\frac{ke^2}{r} + \frac{1}{2}mv^2 = -\frac{ke^2}{2r}, \qquad \Rightarrow r(n) = \frac{\hbar^2}{m_e k e^2} n^2 \tag{14}$$

$$\frac{1}{\lambda} = \frac{k^2 e^4 m_e}{2\hbar^2 hc}\left(\frac{1}{n_f} - \frac{1}{n_i}\right) = R_H\left(\frac{1}{n_f} - \frac{1}{n_i}\right) \tag{15}$$

1.6 Dobbelspalte, Brag diffraksjon, Davidsson-Gremer

$$n\lambda = d\sin\theta, \qquad 2d\sin\theta = n\lambda, \qquad d\sin\theta = m\lambda \tag{16}$$

2 Griffith:

2.1 Generelt:

$$\boxed{i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V\Psi} \tag{17}$$

$$\int_{-\infty}^{\infty} |\Psi(x,t)|^2 dx = \int_{-\infty}^{\infty} \Psi^*\Psi dx = 1 \tag{18}$$

$$\langle A\rangle = \langle\Psi|A|\Psi\rangle = \int_{-\infty}^{\infty} \Psi^* \hat{A}\Psi dx \tag{19}$$

$$\hat{p} = -i\hbar\frac{\partial}{\partial x}, \qquad \hat{x} = x, \qquad \hat{T} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} \tag{20}$$

$$\Psi(x,t) = \psi(x)\varphi(t), \qquad \Psi(x,0) = \sum_n c_n\psi_n(x) \tag{21}$$

$$\Psi_n(x,t) = \psi_n(x)e^{-iE_n t/\hbar}, \qquad \Psi(x,t) = \sum_n \psi_n(x)e^{-iE_n t/\hbar} \tag{22}$$

$$\int \psi_m^*\psi_n dx = \delta_{mn}, \qquad c_n = \int \psi_n(x)^*\Psi dx \tag{23}$$

$$\sum_n |c_n|^2 = \sum_n c_n^* c_n = 1, \qquad \langle H\rangle = \sum_n |c_n|^2 E_n \tag{24}$$

2.2 1D:

2.2.1 Uendelig brønn:

$$\frac{d^2\psi}{dx^2} = -k^2\psi, \qquad k = \frac{\sqrt{2mE}}{\hbar} \tag{25}$$

$$E_n = \frac{\hbar^2 k_n^2}{2m} = \frac{n^2\pi^2\hbar^2}{2ma^2}, \qquad \psi_n(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi}{a}x\right) \tag{26}$$

$$c_n = \sqrt{\frac{2}{a}}\int_0^a \sin\left(\frac{n\pi}{a}x\right)\Psi(x,0)dx \tag{27}$$

2.2.2 Harmonisk oscillator:

$$V = \frac{1}{2}m\omega^2 x^2 \tag{28}$$

Algebraisk:

$$a_\pm = \frac{1}{\sqrt{2\hbar m\omega}}(\mp ip + m\omega x), \qquad H = \hbar\omega(a_-a_+ - \frac{1}{2}), \qquad [a_-, a_+] = 1 \tag{29}$$

$$H = \hbar\omega(a_\pm a_\mp \pm \frac{1}{2})\psi = E\psi, \qquad H(a_\pm\psi) = (E \pm \hbar\omega)(a_\pm\psi) \tag{30}$$

$$a_-\psi_0 = 0 \Rightarrow \frac{1}{\sqrt{2\hbar m\omega}}\left(\hbar\frac{d}{dx} + m\omega x\right)\psi_0 = 0 \tag{31}$$

$$\psi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-\frac{m\omega}{2\hbar}x^2}, \qquad \psi_n = A_n(a_+)^n\psi_0 \tag{32}$$

$$E_0 = \frac{1}{2}\hbar\omega, \qquad E_n = \left(n + \frac{1}{2}\right)\hbar\omega \tag{33}$$

$$a_\pm\psi_n = c_n\psi_{n\pm1}, \qquad \int f^*(a_\pm g)dx = \int (a_\mp f)^* g dx \Leftrightarrow (a_\pm)^\dagger = a_\mp \tag{34}$$

$$a_+\psi_n = \sqrt{n+1}\psi_{n+1}, \qquad a_-\psi_n = \sqrt{n}\psi_{n-1} \tag{35}$$

$$\psi_n = \frac{1}{\sqrt{n!}}(a_+)^n\psi_0 \tag{36}$$

$$x = \sqrt{\frac{\hbar}{2m\omega}}(a_+ + a_-), \qquad p = i\sqrt{\frac{\hbar m\omega}{2}}(a_+ - a_-) \tag{37}$$

Analytisk:

$$\xi = \sqrt{\frac{m\omega}{\hbar}}x, \qquad K = \frac{2E}{\hbar\omega} = (!)2n+1, \qquad a_{j+2} = \frac{-2(n-j)}{(j+1)(j+2)}a_j \tag{38}$$

$$\psi_n(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4}\frac{1}{\sqrt{2^n n!}}H_n(\xi)e^{-\xi^2/2} \tag{39}$$

$$H_n(\xi) = (-1)^n e^{\xi^2}\left(\frac{d}{d\xi}\right)^n e^{-\xi^2} \text{ Rodrigues}, \qquad H_{n+1} = 2\xi H_n - 2nH_{n-1}, \qquad \frac{dH_n}{d\xi} = 2nH_{n-1} \tag{40}$$

2.2.3 Fri partikkel:

$$\frac{d^2\psi}{dx} = -k^2\psi, \qquad k = \frac{\sqrt{2mE}}{\hbar} \tag{41}$$

$$\Psi_k(x,t) = Ae^{i(kx - \frac{\hbar k^2}{2m}t)}, \qquad k = \pm\frac{\sqrt{2mE}}{\hbar} \text{ Right or left} \tag{42}$$

$$v_{quant} = \frac{\hbar|k|}{2m} = \sqrt{\frac{E}{2m}}, \qquad v_{class} = \sqrt{\frac{2E}{m}} = 2v_{qunat} \tag{43}$$

$$\int \Psi_k^* \Psi_k dx = |A|^2 \int dx = |A|^2 \cdot \infty \tag{44}$$

Ikke normaliserbar, og vi tar derfor en lineærkombo(Fourier):

$$\Psi(x,t) = \frac{1}{\sqrt{2\pi}} \int \phi(k) e^{i(kx - \frac{\hbar k^2}{2m}t)} dk \tag{45}$$

$$\Psi(x,0) = \frac{1}{\sqrt{2\pi}} \int \phi(k) e^{ikx} dk \Leftrightarrow \phi(k) = \frac{1}{\sqrt{2\pi}} \int \Psi(x,0) e^{-ikx} dx \tag{46}$$

$$v_g = \frac{d\omega}{dk}, \qquad v_f = \frac{\omega}{k}, \qquad v_{class} = v_g = 2v_f \tag{47}$$

2.2.4 Deltafuksjonspotensial:

$$V(x) = -\alpha\delta(x), \qquad \frac{d^2\psi}{dx^2} = -\frac{2mE}{\hbar^2}\psi = \kappa^2\psi \tag{48}$$

$$\psi(x) = \frac{\sqrt{m\alpha}}{\hbar} e^{-m\alpha|x|/\hbar^2}, \qquad E = -\frac{m\alpha^2}{2\hbar^2} \tag{49}$$

$$R = \frac{|B|^2}{|A|^2} = \frac{1}{1 + \frac{2\hbar^2 E}{m\alpha^2}}, \qquad T = \frac{|F|^2}{|A|^2} = \frac{1}{1 + \frac{m\alpha^2}{2\hbar^2 E}} \tag{50}$$

2.2.5 Endelig Brønn:

$$\frac{d^2\psi}{dx^2} = -\frac{2mE}{\hbar^2}\psi = \kappa^2\psi \text{ utenfor} \tag{51}$$

$$\frac{d^2\psi}{dx^2} = -l^2\psi, \qquad l = \frac{\sqrt{2m(E+V_0)}}{\hbar} \text{ innenfor brønnen} \tag{52}$$

$$\psi(x) = \begin{cases} Fe^{-\kappa x} & x > a \\ D\cos(lx) & 0 < x < a \\ \psi(-x) & x < a \end{cases} \tag{53}$$

$$\kappa = l\tan(la), \qquad z = la, \qquad z_0 = \frac{a}{\hbar}\sqrt{2mV_0}, \qquad \tan z = \sqrt{(z_0/z)^2 - 1} \tag{54}$$

Bare mulig å løse numerisk. Stor, dyp brønn:

$$z_n = \frac{n\pi}{2}, n \text{ odd}, \qquad E_n + V_0 \cong \frac{n^2\pi^2\hbar^2}{2m(2a)^2} \tag{55}$$

Tynn, grunn brønn:

$$E_n + V_0 = \frac{n^2\pi^2\hbar^2}{2m(2a)^2} \text{ for T = 1 (som i en uendelig brønn)} \tag{56}$$

$$T^{-1} = 1 + \frac{V_0^2}{4E(E+V_0)} \sin^2\left(\frac{2a}{\hbar}\sqrt{2m(E+V_0)}\right) \tag{57}$$

2.3 Formalisme:

$$\Psi = \sum_n c_n f_n(x), \qquad c_n = \langle f_n|\Psi\rangle = \int f_n(x)^* \Psi dx \tag{58}$$

$$\langle Q\rangle = \langle \Psi|Q|\rangle = \sum_n q_n |c_n|^2 \tag{59}$$

q_n er egenverdien til Q. Dette er de mulige observasjonene til Q, og $|c_n|^2$ er sannsynligheten for disse målingene. De tilhørende egenvektorene er tilstandene til Q.

2.3.1 Usikkerhetsprinsippet:

$$\sigma_A^2 \sigma_B^2 \geq \left(\frac{1}{2i}\langle[A,B]\rangle\right)^2 \tag{60}$$

$$[x,p] = i\hbar \Rightarrow \sigma_x \sigma_p \geq \frac{\hbar}{2} \tag{61}$$

To operatorer som ikke kommuterer er imkomplatible obervabler, og deler ingen egenfunksjoner(eller de deler ikke en komplett mengde egenfunksjoner!).

$$\Delta t \Delta E \geq \frac{\hbar}{2} \tag{62}$$

$$\frac{d}{dt}\langle Q\rangle = \frac{i}{\hbar}\langle[H,Q]\rangle + \langle\frac{\partial Q}{\partial t}\rangle \tag{63}$$

2.3.2 Dirac:

$$\Psi = \langle x|s(t)\rangle, \qquad \phi(p,t) = \langle p|s(t)\rangle \tag{64}$$

$$H|s\rangle = E|s\rangle \tag{65}$$

$s(t)$ er tilstanden til system. Denne er en lineærkombinasjon av egentilstanden til observabelen Q(egentilstandene danner en basis).

$$|s(t=0)\rangle = \sum a_n |s_n\rangle, \qquad a_n = \langle s_n|s(t=0)\rangle \tag{66}$$

for å få tidsavhengigheten må vi legge til $e^{-iE_n t/\hbar}$, hvor E_n er egenverdiene til H

For å finne Ψ, må tilstanden være laget fra Hamiltonianen.

Men generelt er de mulige målingen av Q den egenverdier, mens den tilsvarende tilstanden til denne målingen $|q_n\rangle$ er en egenvektor av Q. Tilstanden til system kan være i en lineærkominasjon av disse egenvektorene. Sannsynigheten for å måle $|q_n\rangle$ er

$$P(q_n) = |c_n|^2 = |\langle q_n|\Psi\rangle|^2 \tag{67}$$

og

$$\langle Q\rangle = \langle\Psi|Q|\Psi\rangle \tag{68}$$

2.4 3D:

2.4.1 Separation of variables

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\left(\frac{\partial^2}{\partial\phi 2}\right) \tag{69}$$

$$\psi(r,\theta,\phi) = R(r)Y(\theta,\phi) \tag{70}$$

2.4.2 Angular equation:

$$\sin\theta\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial Y}{\partial\theta}\right)+\frac{\partial^2 Y}{\partial\phi^2}=-l(l+1)\sin^2\theta Y \tag{71}$$

$$Y(\theta,\phi)=\Theta(\theta)\Phi(\phi) \tag{72}$$

$$\frac{1}{\Phi}\left[\sin\theta\frac{d}{d\theta}\left(\sin\theta\frac{d\Theta}{d\theta}\right)\right]+l(l+1)\sin^2\theta=m^2 \tag{73}$$

$$\frac{1}{\Phi}\frac{d^2\Phi}{d\phi^2}=-m^2 \tag{74}$$

$$\Phi(\phi)=e^{im\phi} \tag{75}$$

$$\Phi(\phi+2\pi)=\Phi(\phi)\Rightarrow m=0,\pm1,\pm2,... \tag{76}$$

$$\Theta(\theta)=AP_l^m(\cos\theta) \tag{77}$$

$$P_l^m(x)=(1-x^2)^{|m|/2}\left(\frac{d}{dx}\right)^{|m|}P_l(x) \tag{78}$$

$$P_l(x)=\frac{1}{2^l l!}\left(\frac{d}{dx}\right)^l(x^2-1)^l\ \text{Legendre/Rodrigues} \tag{79}$$

Normalisert får vi de sphæriske harmoniske:

$$Y_l^m(\theta,\phi)=\epsilon\sqrt{\frac{(2l+1)(l-|m|)!}{4\pi(l+|m|)!}}e^{im\phi}P_l^m(\cos\theta) \tag{80}$$

Hvor $\epsilon=(-1)^m$ for $m\geq 0$ og $\epsilon=1$ for $m\leq 0$. $l=0,1,2,...$ er 'azimuthal quantum number', og $m=-l,..,0,...,l$ er 'the magnetic quantum number'.

2.4.3 Radial equation:

$$-\frac{\hbar}{m}\frac{d^2u}{dr^2}+\left[V+\frac{\hbar^2}{2m}\frac{l(l+1)}{r^2}\right]u=Eu \tag{81}$$

$$V_{eff}=V+\frac{\hbar^2}{2m}\frac{l(l+1)}{r^2} \tag{82}$$

For en uendelig brønn:

$$\psi_{nlm}=A_{nl}j_l(\beta_{nl}\frac{r}{a})Y_l^m \tag{83}$$

$$j_l=(-x)^l\left(\frac{1}{x}\frac{d}{dx}\right)^l\frac{\sin x}{x} \tag{84}$$

$$E_{nl}=\frac{\hbar^2}{2ma^2}\beta_{nl}^2 \tag{85}$$

Er den sphæriske besselfunksjonen og β_{nl} er dens n'te nullpunkt.

2.4.4 Hydrogenatomet:

$$V(r) = -\frac{e^2}{4\pi\epsilon_0}\frac{1}{r} \tag{86}$$

$$E_n = -\left[\frac{m}{2\hbar^2}\left(\frac{e^2}{4\pi\epsilon_0}\right)^2\right]\frac{1}{n^2} \tag{87}$$

$$\psi_{nlm} = \sqrt{\left(\frac{2}{na}\right)^3\frac{(n-l-1)!}{2n[(n+l)!]^3}}e^{-r/na}\left(\frac{2r}{na}\right)^l\left[L_{n-l-1}^{2l+1}\frac{2r}{na}\right]Y_l^m \tag{88}$$

$$a = \frac{4\pi\epsilon_0\hbar^2}{me^2} \tag{89}$$

$$L_{q-p}^p = (-1)^p\left(\frac{d}{dx}\right)^p L_q(x), \qquad L_q = e^x\left(\frac{d}{dx}\right)^q(e^{-x}x^q) \text{ Lagguerre} \tag{90}$$

2.4.5 Angulærmoment:

$$\mathbf{L} = \mathbf{r}\times\mathbf{p}, \qquad L_x = yp_z - zp_y, p_x = -i\hbar\frac{\partial}{\partial x} \tag{91}$$

$$[L_x, L_y] = i\hbar L_z, \qquad [L^2, L_i] = 0 \tag{92}$$

$$L_\pm = L_x \pm iL_y \tag{93}$$

$$[L_z, L_\pm] = \pm\hbar L_\pm, \qquad [L^2, L_\pm] = 0 \tag{94}$$

$$L^2 f = \lambda f, \qquad L_z f = \mu f, \qquad L_z(L_\pm f) = (\mu\pm\hbar)(L_\pm f) \tag{95}$$

$$L^2 = L_\pm L_\mp + L_z^2 \mp \hbar L_z \tag{96}$$

$$L_+ f_{top} = 0 \tag{97}$$

$$L^2 f_l^m = \hbar l(l+1)f_l^m, \qquad L_z f_l^m = \hbar m f_l^m \tag{98}$$

$$l = 0, 1/2, 1, 3/2, ..., \qquad m = -l, -l+1, ...0, ...l \tag{99}$$

Egenfunksjonen:

$$L_z = -i\hbar\frac{\partial}{\partial\phi} \tag{100}$$

$$L^2 = -\hbar^2\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right] \tag{101}$$

Løsingene på L_z og L^2 er derfor de sfæriske harmoniske!

2.4.6 Spin

: Mange av de samme regnereglene gjelder også for spin.

$$S^2|sm\rangle = \hbar^2 s(s+1)|sm\rangle, \qquad S_z|sm\rangle = \hbar m|sm\rangle \tag{102}$$

$$S_\pm|sm\rangle = \hbar\sqrt{s(s+1)-m(m\pm1)}|s(m\pm1)\rangle \tag{103}$$

Spin 1/2:

$$S^2\chi_+ = \frac{3}{4}\hbar^2\chi_+, \qquad S^2\chi_- = \frac{3}{4}\hbar^2\chi_- \tag{104}$$

$$S^2 = \frac{3}{4}\hbar^2\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} \tag{105}$$

$$S_z\chi = \frac{\hbar}{2}\chi_+, \qquad S_z\chi = -\frac{\hbar}{2}\chi_- \tag{106}$$

$$S_z = \frac{\hbar}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} \tag{107}$$

$$S_+\chi_- = \hbar\chi_+, \qquad S_-\chi_+ = \hbar\chi_-, \qquad S_+\chi_+ = S_-\chi_- = 0 \tag{108}$$

$$S_+ = \hbar\begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}, \qquad S_- = \hbar\begin{pmatrix}0 & 0\\ 1 & 0\end{pmatrix} \tag{109}$$

$$S_\pm = S_x \pm iS_y, \qquad S_x = \frac{1}{2}(S_+ + S_-), \qquad S_y = \frac{1}{2i}(S_+ - S_-) \tag{110}$$

$$S_x = \frac{\hbar}{2}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}, \qquad S_y = \frac{\hbar}{2}\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix} \tag{111}$$

$$\mathbf{S} = \frac{\hbar}{2}\sigma \tag{112}$$

2.4.7 Electron in Magnetic field:

$$\vec{\mu} = \gamma\vec{S}, \qquad H = -\vec{\mu}\cdot\vec{B} = -\gamma\vec{B}\cdot\vec{S} \tag{113}$$

Larmor precession:

$$\vec{B} = B_0\hat{k}, \qquad a = \cos(\alpha/2) \qquad b = \sin(\alpha/2, \qquad \Rightarrow \omega = \gamma B_0 \tag{114}$$

Stern-Gerlach:

$$\vec{B} = -\alpha x\hat{i} + (B_0 + \alpha z)\hat{k}, \qquad \vec{F} = \gamma\alpha(-S_x\hat{i} + S_z\hat{k}) \tag{115}$$

Because of Larmor precession S_x oscillates and averages to zero.

$$H(t) = -\gamma(B_0 + \alpha z)S_z, \qquad 0 \le t \le T \tag{116}$$

$$\chi(t) = (ae^{i\gamma TB_0/2}\chi_+)e^{i(\alpha\gamma T/s)z} + (be^{-i\gamma TB_0/2}\chi_+)e^{-i(\alpha\gamma T/s)z} \tag{117}$$

$$E_\pm = \mp\gamma(B_0 + \alpha z)\frac{\hbar}{2}, \qquad p_z = \frac{\alpha\gamma T\hbar}{2} \tag{118}$$

2.4.8 Addition of Angular Momentum:

$$\vec{S} \equiv \vec{S}^{(1)} + \vec{S}^{(2)} \tag{119}$$

$$S_z\chi_1\chi_2 = (S_Z^{(1)} + S_z^{(2)})\chi_1\chi_2 = \hbar(m_1 + m_2)\chi_1\chi_2 \tag{120}$$

For 2 spin 1/2:

$$s = 1 \text{ (triplet) } \begin{cases} |11\rangle = \uparrow\uparrow \\ |10\rangle = \frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow) \\ |1-1\rangle = \downarrow\downarrow \end{cases} \tag{121}$$

$$s = 2 \text{ (singlet) } \left\{ |00\rangle = \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow) \right. \tag{122}$$

Kombinasjonen av spin s_1 og spin s_2 gir de mulige verdiene for spin:

$$s = (s_1 + s_2), (s_1 + s_2 - 1), ..., |s_1 - s_2| \tag{123}$$

$$|sm\rangle = \sum_{m_1+m_2=m} C^{s_1 s_2 s}_{m_1 m_2 m}|s_1 m_1\rangle|s_2 m_2\rangle \text{ Clebsch-Gordan} \tag{124}$$

2.5 Ideniske partikler

$$\Psi(\vec{r}_1, \vec{r}_2, t) \qquad H = -\frac{\hbar^2}{2m_1}\nabla_1^2 - \frac{\hbar^2}{2m_2}\nabla_2^2 + V(\vec{r}_1, \vec{r}_2, t) \tag{125}$$

2.5.1 Bosons and Fermions

$$\psi(r_1, r_2) = \psi_a(r_1)\psi_b(r_2) \tag{126}$$

indistinguishable in principle:

$$\psi_{\pm}(r_1, r_2) = A[\psi_a(r_1)\psi_b(r_2) \pm \psi_b(r_1)\psi_a(r_2)] \tag{127}$$

Bosons:'+', integer spin. Fermions: '-', half integer spin. Pauli exclusion principle,$\psi_a = \psi_b$:

$$\psi_{-}(r_1, r_2) = A[\psi_a(r_1)\psi_a(r_2) - \psi_a(r_1)\psi_a(r_2)] = 0 \tag{128}$$

Exchange operator:

$$Pf(r_1, r_2) = f(r_2, r_1) \tag{129}$$

$P^2 = 1$ and eigenvalues for P is ± 1 $[P, H] = 0$, $\quad \psi(r_1, r_2) = \pm\psi(r_2, r_1)$

2.5.2 Exchange force

$$\psi_{\pm}(r_1, r_2) = \frac{1}{\sqrt{2}}\psi_a(r_1)\psi_b(r_2) \pm \psi_b(r_1)\psi_a(r_2)] \tag{130}$$

For forskjellige:

$$\langle(x_1 - x_2)^2\rangle_d = \langle x_1^2\rangle + \langle x_2^2\rangle - 2\langle x_1 x_2\rangle = \langle x^2\rangle_a + \langle x^2\rangle_b - 2\langle x\rangle_a\langle x\rangle_b \tag{131}$$

For identiske:

$$\langle(\Delta x)^2\rangle_{\pm} = \langle(\Delta x)^2\rangle_d \mp 2|\langle x\rangle_{ab}|^2 \tag{132}$$

$$\langle x\rangle_{ab} = \int x\psi_a^*\psi_b dx \tag{133}$$

2.5.3 Atoms:

Neutral atom of number Z:

$$H = \sum_{j=1}^{Z}\left[-\frac{\hbar^2}{2m}\nabla_j^2 - \frac{1}{4\pi\epsilon_0}\frac{Ze^2}{r_j}\right] + \frac{1}{2}\frac{1}{4\pi\epsilon_0}\sum_{j\neq k}\frac{e^2}{|r_j - r_k|} \tag{134}$$

Because electrons are identical fermions not all solutions are acceptable: only those for witch the complete state $\psi(r_1, ..., r_Z)\chi(s_1, ..., s_Z)$ is antisymmetrical with respect to interchange of any two electrons. No electron can occupy the same state.
Helium: Z = 2:

$$\psi(r_1, r_2) = \psi_{nlm}(r_1)\psi_{n'l'm'}(r_2), \qquad E = 4(E_n + E_{n'} \tag{135}$$

$$\psi_0(r_1, r_2) = \psi_{100}(r_1)\psi_{100}(r_2) = \frac{8}{\pi a^2}e^{-2(r_1+r_2)/a}, \qquad E_0 = 8(-13.6 \text{ eV}) \tag{136}$$

One electron in ground state and one in excited state $\psi_{nlm}\psi_{100}$. We can use this to make a combination (127) in a symmetric way(with antisymmetric spin) and antisymmetric way(symmetric spin(triplet)), these are para- and orthohelium. Ground states are parahelium, exited states are both.

2.5.4 Periodic table:

Fill in when reading!!!!

2.5.5 Solids:

Free electron gas: Uendelig brønn formet som en boks med lengder l_x,l_y og l_z

$$\psi_{n_x n_y n_z} = \sqrt{\frac{8}{l_x l_y l_z}} \sin(\frac{n_x \pi}{l_x} x) \sin(\frac{n_y \pi}{l_y} y) \sin(\frac{n_z \pi}{l_z} z) \tag{137}$$

$$E_{n_x n_y n_z} = \frac{\hbar^2 \pi^2}{2m} \left(\frac{n_x^2}{l_x^2} + \frac{n_y^2}{l_y^2} + \frac{n_z^2}{l_z^2} \right) = \frac{\hbar^2 k^2}{2m} \tag{138}$$

$$E_F = \frac{\hbar^2}{2m} (3\rho\pi^2)^{2/3} \tag{139}$$

$$P_F = \frac{2}{3} E_{tot}/V = \frac{(3\pi^2)^{2/3}\hbar^2}{5m} \rho^{5/3} \tag{140}$$