Problem 18: Mandelbrot Set

Points: 45

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Problem Background

The Mandelbrot set is drawn by considering the recursive function $_{+1} =$ ^{2} $+$, where is a complex number of the form + (in mathematics, is an imaginary number with the value of $\sqrt{-1}$; thus, $^2 = -1$). By iterating repeatedly, using each value of to calculate the next value, we find that for some input values of , grows without bound. For others, remains bound.

To draw the Mandelbrot set, we use the "complex plane", where the horizontal x-axis represents the value of , and the vertical y-axis represents the value of . Each point is colored based on the number of iterations ( ) we can perform before the absolute value of (| |) becomes greater than a specified value. When this happens, it is said that the function "diverges". In the image below, black indicates that | | remained below a prescribed value for all values of . Blue pixels represent points at which it took many iterations to get | | above that value; red pixels required fewer iterations.

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Let’s consider the function using a value of = 1.1 + 2 .

Regardless of the value of , the value of $_0$ always equals 0. We can use this to determine the value of $_1$:

$$_1 = _0{}^2 +$$
$$_1 = 0^2 + 1.1 + 2$$
$$_1 = 1.1 + 2$$

From this, we can see that for any value of , $_1 =$. Now we need to determine if the function has diverged. For the purposes of this problem, we’ll consider the function to have diverged if $| \, | \geq 100$. Since is an imaginary number, we use this formula to determine the absolute value of numbers of the form + :

$$|_1| = \sqrt{_1{}^2 + _1{}^2}$$
$$|_1| = \sqrt{1.1^2 + 2^2}$$
$$|_1| = \sqrt{1.21 + 4}$$
$$|_1| \approx 2.2825$$

2.2825 is less than 100, so the function hasn’t diverged yet. We need to do more iterations to determine when it diverges, if ever:

$$_2 = {}_1 \qquad {}^2 +$$
$$_2 = ({}_1 + {}_1)_2 + {}_0 + {}_0$$
$$_2 = (1.1 + 2\ )_2 + 1.1 + 2$$
$$_2 = 1.1_2 + 1.1(2\ ) + 1.1(2\ ) + (2\ )_2 + 1.1 + 2$$
$$_2 = 1.21 + 4.4 - 4 + 1.1 + 2$$
$$_2 = -1.69 + 6.4$$
$$_2 = -1.69$$
$$_2 = 6.4$$
$$|\ _2| = \sqrt{-1.69_2 + 6.4_2}$$
$$|\ _2| \approx \sqrt{2.8561 + 40.96}$$
$$|\ _2| \approx 6.6194$$

(Remember that $_2 = -1$, so above, $(2\ )_2 = 2_2 * {}_2 = 4 * -1 = -4$.)

$|\ _2|$ is still less than 100, so it hasn't diverged yet. How many iterations do we need to do to reach that point?

n	***Z***	***a***	***b***	**|*Z*|**
1	1.1 + 2*i*	1.1	2	2.2825
2	-1.69 + 6.4*i*	-1.69	6.4	6.6194
3	-37.0039 - 19.632*i*	-37.0039	-19.632	41.8892
4	984.9732 + 1454.9211*i*	984.9732	1454.9211	1756.9769

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So at = 4, we see that the value of $|\ |$ > 100. This means that for this value of , the function has diverged at 4. We color the point at x = 1.1, y = 2 an appropriate color for that value, and move on to the next value of to be checked.

Problem Description

Your program must identify the color to use in a rendering of the Mandelbrot set for a given value of . Use the following table and the explanation above to determine what colors should be used:

Value of when function diverges	Color
≤ 10	RED
11-20	ORANGE

21-30	YELLOW
31-40	GREEN
41-50	BLUE
≥ 51	BLACK

For the example calculation above, the function diverged at = 4, so the color for that value of should be red.

Sample Input

The first line of your program's input, **received from the standard input channel**, will contain a positive integer representing the number of test cases. Each test case will include a single line of input with two decimal numbers separated by spaces. These numbers represent the values for and , respectively. Remember that = + .

4
1.1 2.0
-0.7 0.2
-0.5 0.65
-0.5 0.608

Sample Output

For each test case, your program must output the value of , followed by a space, followed by the color used to render that value of according to the table above. The color should be printed in uppercase letters. Decimal values should be printed as they were received from the input.

1.1+2.0i RED
-0.7+0.2i BLACK
-0.5+0.65i ORANGE
-0.5+0.608i BLUE