Quiz 2 (Sections 1.2, 1.3)

You will have 30 minutes to complete the quiz.

Name:
Student Number:

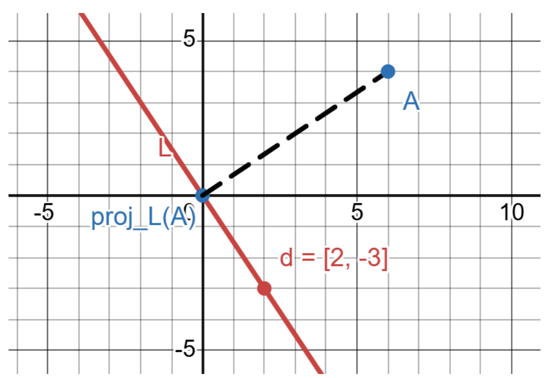
Q1 Consider the line given by $\mathcal{L} = \left\{ t \begin{bmatrix} 2 \\ -3 \end{bmatrix} \in \mathbb{R}^2 \, : \, t \in \mathbb{R} \right\}$.

a. Sketch the line $\mathcal{L}$. Be sure to include labels. (1 Point)
b. Determine the projection, either by computation or graphically, of the point $A = (6, 4)$ onto the line $\mathcal{L}$. Include these point in your sketch. (2 Points)

Q2 Find the equation of the line through the point $(1, 3)$ and parallel to the vector $[2, -1]$. Is your solution unique? (2 Points)

Q3 Let A be a 1×2 matrix and $t \in \mathbb{R}$. Show that if $\vec{x} \in \mathbb{R}^2$ is a solution to $A\vec{x} = \vec{0}$, then $t\vec{x}$ is also a solution. (2 Points)

Q1



Visualization available here.

Next, we want to compute the projection of A onto the line $\mathcal{L}$. We can view the point A as the vector $\vec{a} = \begin{bmatrix} 6 \\ 4 \end{bmatrix}$ and fix the direction vector of the line $\mathcal{L}$ as $\vec{d} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$. Therefore, we compute the following.

$$\text{proj}_{\mathcal{L}}(\vec{a}) = \frac{\vec{a} \cdot \vec{d}}{\vec{d} \cdot \vec{d}} \vec{d} = \frac{(6)(2) + (4)(-3)}{(2)(2) + (-3)(-3)} \begin{bmatrix} 2 \\ -3 \end{bmatrix} = \frac{0}{13} \begin{bmatrix} 2 \\ -3 \end{bmatrix} = \vec{0}$$

Hence, we see that the projection of A onto the line $\mathcal{L}$ is the zero vector.

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Q2

Recall that a line $\mathcal{L} \subseteq \mathbb{R}^n$ can be described using a point $\vec{p} \in \mathbb{R}^n$ and a direction $\vec{d} \in \mathbb{R}^n$. Therefore, to find the necessary equation we must find a suitable $\vec{p}$ and $\vec{d}$.

Since we know that the line must pass through the point $(1, 3)$, we may fix $\vec{p} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$.

Since we know that the line is parallel to the vector $[2, -1]$, we may fix $\vec{d} = k \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ for any nonzero $k \in \mathbb{R}$. For simplicity, we consider the case where $k = 1$. Hence, the line can be described by $t\vec{d} + \vec{p}$ where $t \in \mathbb{R}$.

$$\mathcal{L} : t\vec{d} + \vec{p}, t \in \mathbb{R} \Longrightarrow \mathcal{L} : t \begin{bmatrix} 2 \\ -1 \end{bmatrix} + \begin{bmatrix} 1 \\ 3 \end{bmatrix}, t \in \mathbb{R}$$

Furthermore, this solution (i.e., the equation) is not unique. Notably, we could have selected $\vec{p}$ to be any point on the line $\mathcal{L}$, or taken a different scalar multiple of $\vec{d}$.

Q3

We want to show that if $x \in \mathbb{R}^2$ is a solution to $A\vec{x} = 0$, then $t\vec{x}$ is also a solution.

Let $A = \begin{bmatrix} a_1 & a_2 \end{bmatrix}$ be a 1×2 matrix and $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \in \mathbb{R}^2$. Fix $t \in \mathbb{R}$. Assume that $A\vec{x} = 0$. This implies that $0 = A\vec{x} = \begin{bmatrix} a_1 & a_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = a_1x_1 + a_2x_2$. To show that $t\vec{x}$ is also a solution, we must show that $A(t\vec{x}) = 0$. Notice that we have the following.

$$\begin{aligned}
A(t\vec{x}) &= \begin{bmatrix} a_1 & a_2 \end{bmatrix} \left(t \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right) && \text{By Definition.} \\
&= a_1(tx_1) + a_2(tx_2) && \text{Matrix Multiplication.} \\
&= t(a_1x_1 + a_2x_2) && \text{Factoring.} \\
&= t(A\vec{x}) && \text{Matrix Multiplication.} \\
&= t(0) && \text{By Assumption.} \\
&= 0 && \text{Zero multiplication.}
\end{aligned}$$

Hence, we see that if $\vec{x}$ is a solution to $A\vec{x} = 0$, then it must also be the case that $t\vec{x}$ is a solution, as needed.

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