Quiz 5 (Sections 2.3)

You will have 30 minutes to complete the quiz.

Name:	
Student Number:	

Q1 (a) What are the two essential properties that make a transformation linear? (1 Point)

(b) Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation. Show that for any vectors $\vec{u}, \vec{v} \in \mathbb{R}^n$ and scalars $r, s \in \mathbb{R}$, we have $T(r\vec{u} + s\vec{v}) = rT(\vec{u}) + sT(\vec{v})$. (2 Points)

Q2 Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be a linear transformation such that the folowing holds.

$$T(\vec{e}_1) = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad T(\vec{e}_2) = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \quad T\left(2\vec{e}_1 + 3\vec{e}_2 + \vec{e}_3\right) = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

Find the standard matrix representation of the transformation T. (3 Points)
HINT: Consider the domain and codomain to ensure you have the correct matrix dimensions.

Q3 Determine whether the following statements are true or false. You do not need to justify your work. (2 Points)

(a) If $S, T : \mathbb{R}^n \to \mathbb{R}^n$ are linear transformations, then $T \circ S(\vec{x}) = S \circ T(\vec{x})$. (0.5 Points)

(b) There exists a linear transformation T such that $\mathrm{Im}(T)$ cannot be written as a span of vectors. (0.5 Points)

(c) A linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$ is invertible if and only if $n = m$. (0.5 Points)

(d) For any subspace $S \subseteq \mathbb{R}^n$ and linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$, the set $T(S) = \{T(\vec{x}) \in \mathbb{R}^m | \vec{x} \in S\}$ is a subspace of $\mathbb{R}^m$. (0.5 Points)

Q1

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Q2

Q3

(a) **TRUE** **FALSE**

(b) **TRUE** **FALSE**

(c) **TRUE** **FALSE**

(d) **TRUE** **FALSE**