Quiz 1 (Sections 1.1, 1.2)

You will have 30 minutes to complete the quiz.

Name:
Student Number:

Q1 For the following, find all scalars $c \in \mathbb{R}$, if any exists, such that the statement is true.

a. The vectors $\begin{bmatrix} -3 \\ c \end{bmatrix}$ and $\begin{bmatrix} 5 \\ 2 \end{bmatrix}$ are parallel. (1 point)

b. The vectors $\begin{bmatrix} 2 \\ c \end{bmatrix}$ and $\begin{bmatrix} -2 \\ c \end{bmatrix}$ are perpendicular. (1 point)

c. The norm of the vector $\begin{bmatrix} 3 \\ c \end{bmatrix}$ is zero. (1 point)

Q2 Here, we will show that for any vector $\vec{v} \in \mathbb{R}^n, ||v||^2 = v \cdot v$.

a. Write the equation for the norm of a vector in $\mathbb{R}^n$. (1 point)

b. Write the equation for the dot product of a vector in $\mathbb{R}^n$ with itself. (1 point)

c. Prove that for any vector $\vec{v} \in \mathbb{R}^n, ||\vec{v}||^2 = \vec{v} \cdot \vec{v}$. (1 point)

Q3 Using vectors, show that the midpoints of the four sides of a quadrilateral are the vertices of a parallelogram. **Hint.** It may be useful to make a sketch and label vertices/intersections. (2 points)

Q1

a.

$$\begin{bmatrix} -3 \\ c \end{bmatrix} \text{ and } \begin{bmatrix} 5 \\ 2 \end{bmatrix} \text{ are parallel.} \implies k \begin{bmatrix} -3 \\ c \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}, k \in \mathbb{R} \setminus \{0\} \implies k = -\frac{5}{3} \implies c = \frac{2}{k} = 2 \cdot \left(-\frac{3}{5}\right) = -\frac{6}{5}$$

b.

$$\begin{bmatrix} 2 \\ c \end{bmatrix} \text{ and } \begin{bmatrix} -2 \\ c \end{bmatrix} \text{ are perpendicular} \implies 0 = \begin{bmatrix} 2 \\ c \end{bmatrix} \cdot \begin{bmatrix} -2 \\ c \end{bmatrix} = (2)(-2) + (c)(c) = -4 + c^2 \implies c^2 = 4 \implies c = \pm 2$$

c.

$$0 = \left\| \begin{bmatrix} 3 \\ c \end{bmatrix} \right\| = \sqrt{3^2 + c^2} \implies c^2 = -9 \implies c = \pm\sqrt{-9} \notin \mathbb{R} \implies c \text{ D.N.E.}$$

Next Page

Q2

Let $\vec{v} \in \mathbb{R}^n$ be defined by $\vec{v} = \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix}$.

a. The norm (or magnitude) of a vector is given by the following.

$$||\vec{v}|| = \sqrt{v_1^2 + \cdots + v_n^2}$$

b. The equation for the dot product of a vector with itself is given by the following.

$$\vec{v} \cdot \vec{v} = (v_1)(v_1) + \cdots + (v_n)(v_n) = v_1^2 + \cdots + v_n^2$$

c. We want to show that for any vector $\vec{v} \in \mathbb{R}^n$, $||\vec{v}||^2 = \vec{v} \cdot \vec{v}$.

Consider that we have the following.

$$\begin{aligned} ||\vec{v}||^2 &= (\sqrt{v_1^2 + \cdots + v_n^2})^2 && \text{Definition of norm.} \\ &= v_1^2 + \cdots + v_n^2 && \text{Simplifying.} \\ &= \vec{v} \cdot \vec{v} && \text{From b.} \end{aligned}$$

Hence, it must be that $||\vec{v}||^2 = \vec{v} \cdot \vec{v}$.

Therefore, we have shown that for any vector $\vec{v} \in \mathbb{R}^n$, $||\vec{v}||^2 = \vec{v} \cdot \vec{v}$, as needed.

Q3

See solution in the *Assignment Solution* module, file `s1.pdf`.