ON SOLVING RATIONALS

In general, when approaching solving rational functions, the aim should be to express the given rational function in a way that we can use previously seen techniques to solve for values of x. There is no formulaic approach, but again, we should maintain the general idea that we want to get to an expression which we can solve.

12. a) Explain why the inequalities $\frac{x+1}{x-1} < \frac{x+3}{x+2}$ and $\frac{x+5}{(x-1)(x+2)} < 0$ are equivalent.

We will show that the two expressions are equivalent by showing we can start with the first expression and obtain the second. Therefore, we have the following.

$$\begin{aligned}
\frac{x+3}{x+2} &> \frac{x+1}{x-1} && \text{First Expression.}\\
0 &> \frac{x+1}{x-1} - \frac{x+3}{x+2} && \text{Subtracting from both sides.}\\
0 &> \frac{x+2}{x+2}\cdot\frac{x+1}{x-1} - \frac{x-1}{x-1}\cdot\frac{x+3}{x+2} && \text{Cross Multiplying.}\\
0 &> \frac{x^2+3x+2}{(x-1)(x+2)} - \frac{x^2+2x-3}{(x-1)(x+2)} && \text{Simplifying.}\\
0 &> \frac{x^2+3x+2-(x^2+2x-3)}{(x-1)(x+2)} && \text{Adding the Fractions.}\\
0 &> \frac{x+5}{(x-1)(x+2)} && \text{Simplifying.}
\end{aligned}$$

Consequently, we see that indeed the two expressions are equal.

c) We will use a similar method to when we try to solve inequalities of polynomials. We note that zero can show up in our given function when $x = -5, 1, -2$. The first being from the numerator and the last two from the denominator. **NOTE:** we are not saying that the function evaluates to zero at these points (necessarily), just that we get some form of zero in the fraction (i.e., numerator OR denominator).

	$x < -5$	$-5 < x < -2$	$-2 < x < 1$	$x > 1$
$x+5$	$-$	$+$	$+$	$+$
NUM.	$-$	$+$	$+$	$+$
$x+2$	$-$	$-$	$+$	$+$
$x-1$	$-$	$-$	$-$	$+$
DEN.	$+$	$+$	$-$	$+$
FUNC.	$-$	$+$	$-$	$+$

Here, **NUM.** refers to the sign of the numerator, **DEN.** refers to the sign of the denominator, and **FUNC.** is the sign of the function on this interval, which we get by taking the product of **NUM.** and **DEN.** in a column.

So, the inequality is satisfied when $x < -5$ and $-2 < x < 1$, as needed.