Quiz 4 (Sections 1.6, 2.1, 2.2)

You will have 30 minutes to complete the quiz.

Name:
Student Number:

Q1 For each of the following concepts, write their definitions (8 Points).

(a) Subspace.

(b) Basis.

(c) Rank.

(d) Nullity.

Q2 For each of the following statements, provide a sufficient condition to prove and disprove the statement (10 Points).

(a) There exists a subspace $S \subseteq \mathbb{R}^n$ such that $\vec{0} \notin S$.

(b) Every subspace $S \subseteq \mathbb{R}^n$ is unbounded.

(c) Every subset of $\mathbb{R}^n$ containing the zero vector is dependent.

(d) A $n \times n$ matrix A is invertible if and only if $\text{rank}(A) = n$

(e) There may be a non-zero matrix A such that $\text{rank}(A) = 0$.

Q1

(a) A subspace $S \subseteq \mathbb{R}^n$ is a set of vectors which satisfies the following.

- $\vec{0} \in S$.
- For any $\vec{x}, \vec{y} \in S$ it must be that $\vec{x} + \vec{y} \in S$.
- For any $\vec{x} \in S$ and $k \in \mathbb{R}$ it must be that $k\vec{x} \in S$.

(b) Let $S \subseteq \mathbb{R}^n$ be a subspace. A set $W = \{\vec{w}_1, ..., \vec{w}_n\}$ is a basis for the subspace S if it satisfies the following.

- The set W is linearly independent.
- The set W spans S.

(c) The rank is defined for any matrix and is the number of pivot columns in the REF of said matrix.

(d) The nullity is defined for any matrix and is the number of non-pivot columns in the REF of said matrix.

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Q2

(a) There exists a subspace $S \subseteq \mathbb{R}^n$ such that $\vec{0} \notin S$.

Prove To prove the claim, it is sufficient to produce an example. That is, we want to show that there exists a subspace of $\mathbb{R}^n$ that does not contain the zero vector.

Disprove To disprove the claim, it is sufficient to show that every subspace contains the zero vector. This can be done by using the definition of subsapce.

(b) Every subspace $S \subseteq \mathbb{R}^n$ is unbounded.

Prove To prove this claim, it is sufficient to show that every subspace is unbounded.

Disprove To disprove this claim, it is sufficient to produce a counterexample of a bounded subspace.

(c) Every subset of $\mathbb{R}^n$ containing the zero vector is dependent.

Prove To prove this claim, it is sufficient to show that any subset containing the zero vector must be dependent.

Disprove To disprove the claim, it is sufficient to show that there exists a linearly independent subset containing the zero vector.

(d) A $n \times n$ matrix A is invertible if and only if $\operatorname{rank}(A) = n$.

Prove To prove this claim, we would need to show both implications (as we have an if and only if statement). Firstly, that if a $n \times n$ matrix is invertible, then it must have full rank. Secondly, that if a $n \times n$ matrix has full rank, then it must be invertible.

Disprove To disprove this claim, there are two implications we must consider. Firstly, we may produce a counterexample of a matrix which is invertible, but does not have full rank. Secondly, we may produce a counterexample of a matrix with full rank, which is not invertible. It may be that only of the two conditions fails.

(e) There may be a non-zero matrix A such that $\operatorname{rank}(A) = 0$.

Prove To prove this claim, it is sufficient to produce an example.

Disprove To disprove this claim, we may show that a matrix has rank zero if and only if it is the zero matrix (or more simply, we would need to show that for any non-zero matrix the rank is greater than 0).