Session 1 - 09/09/2025

- Revised material discussed in class Monday and Tuesday.
- Introduction of trigonometric functions: sine, cosine, and tangent.
- This included the presentation of SOH-CAH-TOA, and acute and obtuse angles.

Notes

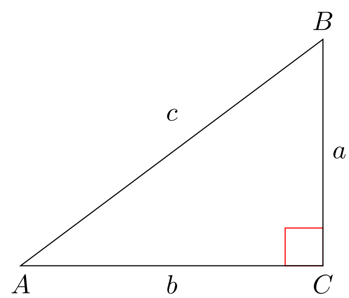
Definition. Acute Angle

An angle is acute if it's degrees lie (strictly) between 0 and 90 degrees.

Definition. Obtuse Angle

An angle is obtuse if it's degrees lie (strictly) between 90 and 180 degrees.

One of the first types of triangles we work with is right-angle triangles. These triangles are characterized by having one angle of 90 degrees. In the following diagram, we see a triangle given by vertices A, B, C (we may write ΔABC), where C is the 90 degree angle.



Remark. Sum of Angles in a Triangle

We recall that the sum of all three angles in a triangle is 180 degrees.

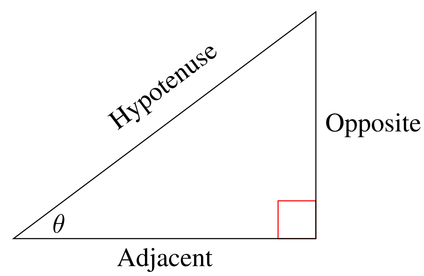
In a right-angle triangle, since one of the angles is necessarily 90 degrees (angle C) and that the sum of all angles in a triangle is 180 degrees, this implies that the angles A and B must sum to 90 degrees. By definition, this means that the angles A and B must be acute angles!

There are a few ways we can determine if a triangle is indeed a right-angle triangle. Perhaps the most evident characteristic would be to see if one of the angles measures 90 degrees. However, we may not always be told the measurements of the angles in a triangle, so we might require other methods. For one, we may use Pythagoras Theorem to determine if a given triangle is a right-angle triangle. We may use this theorem to determine if we have a right-angle triangle since it only holds for right-angle triangles.

> ***Remark.*** Right-Angle Triangle Test (Pythagoras)
>
> For a given triangle, if we know the length of its three sides, we may use Pythagoras Theorem to determine if we have a right-angle triangle.
>
> $$a^2 + b^2 = c^2 \implies \Delta ABC \text{ is a right-angle triangle}$$

In addition to the fact that Pythagoras Theorem only holds for right-angle triangles, there are also certain ratios maintained between the lengths of a right-angle triangles side and the other two acute angles. There are three principal ratios that we commonly care for, which we recall using the acronym SOH-CAH-TOA. This acronym is derived by combining the first letter of the relevant trigonometric function with the two sides that form the corresponding ratio.

$$\begin{aligned}
&\text{SOH} &&\implies \sin\theta = \tfrac{\text{Opposite}}{\text{Hypotenuse}}\\
&\text{CAH} &&\implies \cos\theta = \tfrac{\text{Adjacent}}{\text{Hypotenuse}}\\
&\text{TOA} &&\implies \tan\theta = \tfrac{\text{Opposite}}{\text{Adjacent}}
\end{aligned}$$



Here, we list the ratios using the two possible angles, A and B. Briefly, we remark that we may uses ratios in two particular cases. First, to find an angle using the ratio of two sides. Second, to find the length of a side using an angle and the length of one side.

$$\sin A = \tfrac{a}{c} \qquad \cos A = \tfrac{b}{c} \qquad \tan A = \tfrac{a}{b}$$

$$\sin B = \tfrac{b}{c} \qquad \cos B = \tfrac{a}{c} \qquad \tan B = \tfrac{b}{a}$$