Quiz 6 (Sections 2.3)

You will have 30 minutes to complete the quiz.

Name:
Student Number:

Q1 Consider the linear transformation given by $\mathbb{R}^2 \to \mathbb{R}^2$ that reflects vectors along the line $y = x$. (4 Points)

(a) Determine the standard matrix representation of T. (1 Point)
HINT. It may be useful to draw a graph containing $\vec{e}_1, \vec{e}_2, T(\vec{e}_1), T(\vec{e}_2)$.

(b) Any reflection along a line through the origin is an invertible transformation. Determine the standard matrix representation of T^{-1}. (2 Points)

(c) Based on your previous computation, what can we say about the standard matrix representation of a reflection and its inverse? (1 Point)

Q2 For the following linear transformations $T : \mathbb{R}^n \to \mathbb{R}^n$, justify whether they are injective, not injective, or we cannot tell. (6 Points)

(a) For some nonzero $\vec{v} \in \mathbb{R}^n$, $T(\vec{v}) = \vec{0}$.

(b) T is a surjective transformation.

(c) There exists some linear combination $T(\alpha_1\vec{v}_1 + \cdots + \alpha_n\vec{v}_n) = \vec{0}$, where $\alpha_1, ..., \alpha_n \in \mathbb{R}; \vec{v}_1, ..., \vec{v}_n \in \mathbb{R}^n$.

Q3 (Bonus) Let $S : \mathbb{R}^n \to \mathbb{R}^m, T : \mathbb{R}^m \to \mathbb{R}^n$ be linear transformations. Assume S is surjective and T is injective (6 Points).

(a) Determine whether $S \circ T$ can be injective, surjective, bijective, or none.

(b) Determine whether $T \circ S$ can be injective, surjective, bijective, or none.

Q1

Q2

Q3