INCLUSIVE COLLABORATION VIA CLASS PREPARATION

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Abstract *–This paper explores the implementation and effectiveness of pre-class preparation modules in a large-scale linear algebra course for engineering students, aimed at promoting inclusion, and active participation in an active learning environment. The study investigates the impact of tailored pre-class readings and post-class materials on student engagement, preparedness, and overall performance. Through survey data collected during the fall 2023 semester, we analyze students' preferences for pre-class content delivery methods, their perceived preparedness for class, and the correlation between class preparedness and final exam grades. Our findings suggest that high-quality pre-class content positively contributes to the students experience in an active learning classrooms, with potential implications for mitigating hierarchical structures and addressing disparities in student engagement and achievement.*

Keywords- *Equity and Inclusion, Pre-class preparedness, Flipped Classroom, Inquiry Based Teaching*

1. Introduction

There is a large body of educational research and literature in support of active learning, student-centered teaching, and learning practices. Specifically, in mathematical sciences there is a strong community of support for active inquiry-based learning [1], [2], [3]. At the heart of any active lecture is a series of collaborative student-centered learning activities such as group work with peers and whole-class discussions facilitated by the instructor. Student participation lays the foundation for these activities. As shown in a variety of studies, both in university and K-12 setting, participation is significantly related to students learning [4], [5]. Hence, providing an equitable and inclusive environment in which all students are given a chance to participate, and feel comfortable and supported to do so, is crucial for success in an active classroom.

Although there seems to be a consensus on the effectiveness of inquiry and student-centered models when implemented well, there are concerns about the equity and inclusion in an active learning class, which affects participation, and hence learning. For instance, [6] found the beneficial effects of active learning are driven by white males and higher-achieving students, while certain minorities with lower baseline academic performance did not experience these gains. Moreover, as discussed in [7] inequality in inquiry-oriented learning (in favor of men), may be attributed to lower participation rates of women in the classroom. They showed that classrooms that achieved higher participation rates of women resulted in more equitable outcomes.

To gain additional perspective on the above-mentioned concerns, we can view active learning as a social process. Within social contexts, there are social dynamics, which lead to social hierarchies. These hierarchies, if not actively and intentionally interrupted, affect who gets to participate in classroom activities, and how they get to participate. There is evidence [8] for a link between hierarchies, student participation, and student learning.

Let's give a concrete example of such social dynamics making their way into the classroom. In some cultures, mathematical sciences are associated with intelligence, and economic prosperity. As a signifier of intelligence, performance in mathematics classes can play a unique role in the development of a student's identity. This is even more profound among engineering student who directly connect their performance in mathematics to their future success in their program. This association puts students under intensive pressure to perform and creates anxiety. The fear of "being wrong" in front of peers plays into the narrative that some students are good at math while others are not. Therefore, students who have some preconceived notion of being weak in mathematics are reluctant to participate in classroom activities, which in turn leads to less learning.

As educators, our goal is to promote more learning for more students. The social dynamics, such as the example discussed above, can interfere with how our students interact with our in-class activities and their intended learning objectives. Although the root cause of these social dynamics may be diverse, and often beyond the control of the course instructors and designers, the methods to interrupt them can be effective regardless of the source. For instance, authors of this paper observed a link between students' content-specific preparedness for class, their self-confidence, and the level of their participation in active classes. Therefore, if we can improve student preparedness for class, we may see an increase in engagement and willingness to participate in class discussions. In this paper we explore this idea of redesigning pre-class activities with the aim of boosting students' confidence, improving their in-class experience, and encouraging them to be an active agent in their learning. Our goal is to show that if we can effectively prepare students for an active class, that we can disrupt the social dynamics, or social hierarchies, which lead to inequality and a lack of diverse perspectives in our classrooms. In the remainder of this paper, we will explain how we designed our pre-class preparation modules and how our students interacted with them. Furthermore, we will discuss how preparation affected students' perception of preparedness in lectures and the effect it had on their overall performance in the course.

2. Motivation and Background

Active classes have become increasingly common. A popular framework for an active class is to implement a *flipped class* format. As defined in [9], a flipped classroom is a "set of pedagogical approaches that (1) move most information-transmission teaching out of class; (2) use class time for learning activities that are active and social; and (3) require students to complete pre- and/or post-class activities to fully benefit from in-class work". Thus, through part (1), the flipped classroom format aims to help prepare students for the upcoming active lectures, with part (3) solidifying the knowledge gained in-class or adding some accountability for sufficient engagement in the pre-class work. Our teaching method does fit in the *flipped class* framework described above. A meta study done across 198 institutions [3] found that the flipped classroom format was beneficial regardless of discipline, with some institutions showing more dramatic results than others.

Authors of [9] suggest that the self-paced nature of pre-class activities, as well as facilitating tailored instruction, may reduce cognitive load and thus improve learning outcomes. There is also some evidence in [10] showing that pre-class learning can influence post-class performance. Motivated by the result in [9] we worked on tailoring our pre-class preparation modules to target specific learning objectives.

"*To what extent does the form of pre-class engagement affect student performance*?" is among the research questions asked in [3]. Their results indicate that even a basic approach to flipping, which emphasizes pre-class and in-class engagement without pre-class testing, can lead to a moderate improvement in student performance, suggesting that educators' efforts in clarifying tasks may be a crucial factor in flipped classroom effectiveness. They also found that pre-class tests did not seem to provide significant differences in outcomes.

As discussed in [11], the limited research that is available regarding the optimal delivery methods for pre-class content points towards videos being advantageous versus textbook readings due to the simultaneous auditory and visual sensory stimulation [12]. We explored the same question in our set up, by giving students the flexibility to choose from a variety of preparation methods including reading the textbook, watching videos, or reading our tailored preparation modules, followed by a pre-class test, regardless of how they chose to prepare. We collected feedback on students' preference for preparation and then we looked at how their choice affected their in-class experience and performance in the course.

Finally, in our literature review we looked for classroom practices in which pre-class preparation was specifically used as a tool to interrupt social dynamics and the hierarchical feedback loop. We did not find any study that views pre-class preparation from the equity and inclusion lens. Hence, we believe that our perspective on promoting equity and improving the in-class experience for students through tailored and approachable pre-class content is novel.

3. Context

Two of the authors of this paper have co-taught a large (+1200) first year linear algebra course for engineering students at the University of Toronto for three consecutive years. One author coordinated this course for three years, and the others has been a member of the teaching team for the past two years. The course comprises of 8 lecture sections taught by a mix of faculty, post docs and graduate student instructors. The course structure, assessments and the teaching method are coordinated among all sections.

3.1. Students

Our students have passed a competitive admission process and have mostly been at the very top of their high school classes. They are fairly new to the content of the course and almost entirely new to our active learning format. In an entrance survey of our most recent offering of the course, 60% of coming students say that they have never seen linear algebra before. Even among those who have seen some matrix theory in high school, this class is their first robust exposure to Linear Algebra and their first non-lecture-based learning experience. The level of exposure to mathematical language, and comfort with reading mathematics also varies. In the same survey, although 55% said they were confident reading mathematics only 12% felt fully confident in reading mathematical notation and language.

3.2. Pre Class-Reading Design

All sections are taught actively, following closely the inquiry-based learning (IBL) approaches, as described in [13]. In our IBL lectures, the instructors guide students through a series of activities which promote exploration of concepts and peer discussion. This teaching method works best if students have already familiarized themselves with the main ideas being explored in each lecture before attending the class. We will refer to any method of preparing for lectures as *pre-class preparation*.

A priori, pre-class preparation can be any combination of watching a video, reading sections of the textbook, or reading a document specifically tailored for pre-class preparation of a specific lecture, or series of lectures. If the preparation includes reading, we call it *pre-class reading*. Our pre-class preparation was paired with an accountability ticket in the form of a check-in quiz which often contained 1 to 3 questions on the core ideas students needed to have thought about before attending the upcoming lecture. The check-ins were automatically graded and lightly counted towards the final course grades. We refer to the combination of the pre-class reading and the check-in quizzes as *Pre-Class Essentials (PCEs)*.

Over the past three offerings of the course, we investigated various methods of designing and incorporating PCEs into the course, observed the student interaction with them and how it affected equity, inclusion, and learning within the classroom and in the course.

In our first implementation, pre-class essentials consisted of readings directly from the course textbook together with a check-in quiz. The quiz was designed to clearly define the minimum expectation of preparedness by directing them to specific parts of the reading (a definition, theorem, or an example) and then asking a targeted question that allowed students to review what they needed to know before coming to the lectures. Although the textbook exposes students to high-quality mathematical writing and communication, we noticed that most of students have little experience in reading mathematical textbooks, and doing so, given the workload of the first-year engineering program, was cumbersome.

For the second offering we decided to write documents specifically tailored to our lectures. The goals were to design a document that 1) expose students to the correct use of mathematical language 2) are easy to read and digest before lectures and 3) equip students with the knowledge they need to feel confident to **participate** in lecture activities 4) serves as a reference for later use during the semester. We wrote 12 pre class reading. Each pre-class reading addressed the material covered in one week over three lectures. Students were instructed to read the pre-class reading and do the check-in quiz before their first lecture of the week. We also advised students to revisit the document to refresh their memory before each lecture and utilize it as reference throughout the semester. The readings were accompanied with a short video that gave an overview of that week's reading. These documents were popular among students. They specifically appreciated the depth and breadth of the writings and their brevity. Our observation was that reading a week's worth of material in one shot before the first lecture was challenging to some students in terms of workload and cognitive load. Although the instruction encouraged students to refresh their memory before each lecture, in practice most students relied on their first reading at the top of the week, which at times was not enough to be able to fully engage during the lectures.

In the most recent offering in Fall 2023, which is the focus of this paper, we further tailored the pre-class readings. We introduce two major interventions to our pre-class preparation process: 1) we split each pre-class reading into three sections: Introduce, Solidify and Expand. Each section was designed to hold essential ideas of a single lecture in the week. The pre-class check-in quizzes were also tailored according to these sections and hence there was a check-in quiz before each lecture, making a total of three pre-class check-ins per week. 2) each pre-class reading was accompanied by a *post-class reading*.

Introduce aimed to give a gentle opening to the week's topic. We avoided heavy notations and mathematical language, and instead, we focused on setting the stage through presenting examples. In Solidify, we aimed to deepen their understanding by examining theorems and their implications, while still staying away from formal proofs, theorems, and the punch lines to that week's topics. Expand was the most robust portion of the reading, containing a summary of the pre-class readings, and if we saw a pedagogical reason for including proofs or heavy notations, they were included in Expand. Students were instructed to read **at least** Introduce before the first lecture of the week, Solidify before the second, and Expand before the third.

Each pre-class reading was accompanied by a *post-class reading.* In the post-class reading, we included proofs, justification, and more rigor in mathematical notations, which we had pedagogical reasons to omit from the pre-class reading. The difference between the post-class and pre-class were colour coded so that students who has already read the pre-class version can quickly identify the additions. We suggested that students who chose to prepare through our documents read the post-class reading before their tutorial session, which were scheduled one week behind the lecture content.

3.2. Design Philosophy

The decision to divide the content into three stages was guided by a multifaceted philosophy. Firstly, we aimed to enhance readability by gradually, through a three-stage process, introducing rigor and mathematical notations. Secondly, we sought to alleviate workload pressure by clearly indicating the essential reading for each lecture while allowing students the option to delve deeper into the material or use a different mode of preparation if desired. We took a flexible approach of making the entire pre-class reading (as well as the *post-class reading*), and the corresponding textbook sections available, and allowing students to choose the method that help them prepare best. Lastly, these changes were implemented with the overarching goal of maintaining rigor while also serving as exemplars of effective mathematical writing and communication. We also changed the frequency of the check-in quizzes to support the stages in the pre-class reading.

While designing the pre-class readings, we realized that every week there is material that, while not essential for lecture preparedness, we need our student to *eventually* read. We decided to create two versions of readings: pre-class and post-class readings. It is fair to think of the post-class readings sitting between the textbook and pre-class readings. They were still more concise and more targeted to our specific purposes compared to the textbook, and more thorough and rigorous compared to pre-class readings.

4. Methodology

In the fall of 2023, in our most recent offering of Linear Algebra for Engineering students, we set out to obtain feedback from students on their experience with our pre and post class readings, discussed above. In particular, we were interested in how the pre-class reading prepared them for lecture, if they preferred our course readings over other options, such as watching videos or reading the textbook, and how these experiences correlated with their final exam grade. We obtained this feedback from students via course surveys at various times throughout the semester. The bulk of the results which we present in this paper are from three main sources: 1) An entrance survey given at the start of the semester 2) An exit survey given at the end of the semester, and 3) Weekly surveys given regularly before or after class, with some questions appearing intermittently throughout the semester.

As discussed above, we believe class preparedness plays an important role in a student's confidence and willingness to engage with their peers in group and class discussions. Furthermore, we believe that class preparedness could act as a catalyst for increasing equity and inclusion in the classroom through increased engagement. Therefore, many of our survey questions are related to class preparedness, such as, how long they spent preparing for certain lectures, how much content they covered in preparation for a particular class, and how well they thought their pre-class work prepared them for the class they just attended. Several of our survey questions are also related to a student's preferred method of preparation. As discussed in [11], the limited research that is available regarding the optimal delivery methods for pre-class content points towards video being advantageous versus textbook reading due to the simultaneous auditory and visual sensory stimulation [12]. Thus, we are interested in collecting feedback to determine if students still prefer the video format when offered a reading alternative that targets specific learning objectives for each class in an approachable and concise manner. Survey data on preferred content for class preparation is collected on the weekly and exit surveys.

The final objective of our study is to investigate if class preparedness correlates with overall performance in the course measured by their final exam grade. The table below lists the survey questions given to students, categorized based on the survey they appeared in: Entrance, Weekly or Exit survey. Note that in the table, and the rest of the paper, whenever we say PCE, we mean the document we prepared for pre class reading. Full PCE means introduction, Solidify and Expand. Essential or minimum PCE means the section corresponding to a single lecture.

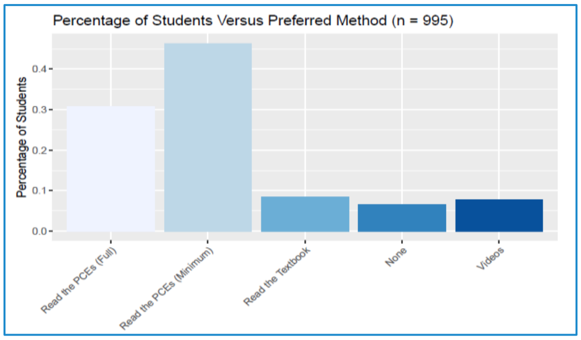
Asked in	Question	*Possible responses*
Entrance Survey	(1) In the past, which of the following learning methods works best for you?	*Attending lectures, Reading textbooks, Solving problems on my own, Solving problems with my peers, Watching online videos*
	(2) I learned mathematics by reading a textbook.	*Strongly disagree, Disagree, Neutral, Agree, Strongly Agree*
Weekly Surveys	(1) Choose the option that best describes how you prepared for this lecture.	*Read the full PCEs, Read the minimum PCE, Read the textbook, Watched Videos, Nothing*
	(2) How long did it take you to prepare for this lecture?	*0 hrs., under 30 mins, between 30 and 60 mins, between 1hr and 2 hrs., more than 2 hrs*
	(3) How long did your pre-class reading take?	*0 hrs., under 30 mins, between 30 and 60 mins, between 1hr and 2 hrs., more than 2 hrs*
	(4) On a scale from 0 to 5, how well did your pre-class reading prepare you for this lecture?	0 to 5
Exit Survey	(1) Throughout the semester, what did you do to prepare for lectures before coming to class?	*Read the full PCEs, Read the minimum PCE, Read the textbook, Watched Videos, Nothing*
	(2) When I invested the time to prepare for class, it significantly improved my lecture experience.	*Strongly disagree, Disagree, Neutral, Agree, Strongly Agree*
	(3) The post-class readings were a helpful reference material throughout the semester.	*Strongly disagree, Disagree, Neutral, Agree, Strongly Agree*

5. Results and Discussion

We now present the results from the course surveys presented in the table above. We begin by looking at data related to how students chose to prepare for class. Specifically, the Weekly Surveys and Exit (1). Next, we present the results on students' perception of the readings, how much they read to prepare for class, and how this correlates with class preparedness. In particular, we look into Entrance (1), Entrance (2), Weekly (1), Exit (1) and Exit (3). Finally, we conclude with an analysis of how class preparation, and the form of class preparation, particularly Weekly (1) and Exit (1), correlates with final course grade.

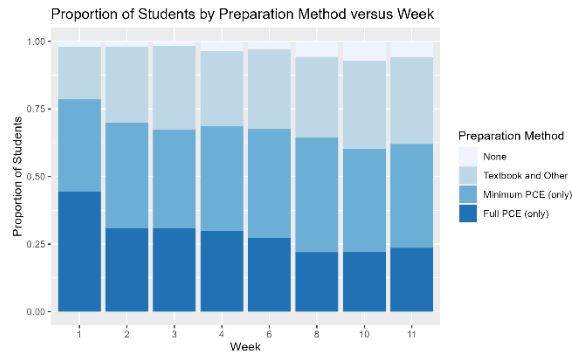
In the end of semester survey, we asked students to look back over the semester and conclude what their primary method for pre-class preparation was (Exit (1), n=995).

We see most students chose to prepare for class via the PCEs, by reading the minimum required for a given lecture (46%) or reading all sections before a given class (31%). This was followed by textbook reading (8.5%), watching videos (8%) and no preparation (6.5%).



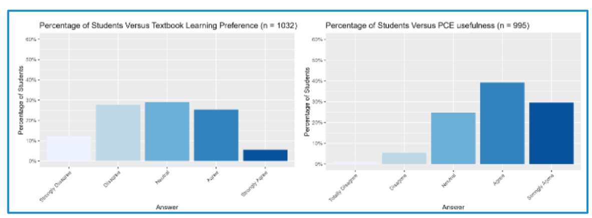
Firstly, this result shows that creating documents specifically for each lecture is preferred by students over the textbook. This could be attributed to many things, for instance, textbook sections can be long and often cover material in greater depth than what is needed for a pre-class primer. This places the task of filtering what is important and what is not onto the student, which requires judgement and some maturity. We also see that the PCE documents were preferred by students over learning via online videos. This result is interesting given other studies found evidence in favour of videos [12]. We note that we did not provide students with course tailored videos to watch or provide guidance on where to find relevant and quality content. Although there is a plethora of content easily accessible via a simple Google or YouTube one can argue that part of the popularity of PCEs can be attributed to the fact that students associated them with the course material specifically made by their teaching team.

Similarly, throughout the semester we asked students to choose what best describes how they prepared for this lecture (Weekly (1)). Unlike the Exit (1) where students could only choose one option, here students could choose any combination of textbook reading, PCE reading and/or watching videos. Our interest here is if students thought the PCE reading was sufficient, or if they felt the need to incorporate other materials to aid in their class preparation. Thus, if students read the textbook or watched videos in addition to their PCE readings, then we added them to the Textbook and Other category.



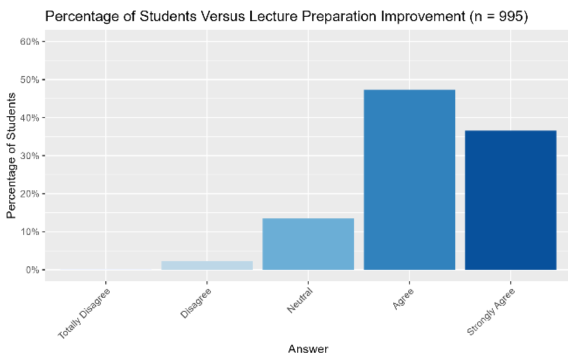
In this plot, we see the Textbook and Other options gradually increasing as the Full PCE group decreases, yet we still see most students rely exclusively on the PCEs for preparation throughout the semester.

Next, we look at how students' perception of reading mathematics evolved from the beginning to the end of the course by looking at Entrance (2) compared with Exit (3). Although these questions are not worded identically, both questions inquire into students' perceived value of reading mathematics reference materials. We see a dramatic shift in perception here from textbooks (asked upon entry into the course) to PCE readings (asked at the end of the course), from a mean of 2.838 to 3.908, where strongly disagree counts for 1 and strongly agree counts for 5.



Next, we asked if students found that coming to class prepared significantly improved their lecture experience (Exit (2)). This question is closely related to our thesis statement that class preparation improves students' class experience.

As seen in the histogram above, the students perceived a positive correlation between class preparedness and lecture experience, with the average student providing an average score of 4.18 out 5 on the strongly disagree to strongly agree scale. Since most students used the PCEs for their preparation, this is, in part, positive feedback about the value of investing time into reading the PCEs. More broadly, this along the lines of results in [3], [9] is positive feedback for the flipped classroom in general, regardless of a students' preferred method of preparation that class preparation leads to an improved lecture experience.

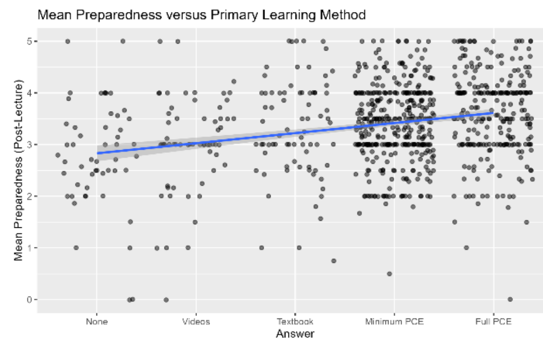


We now compare how students, on average, chose to prepare for class, versus how well they thought their pre-class work prepared them for lecture (Exit (1) vs Weekly (4)). We note that Weekly (4) was asked after the first class each week and the results in the table below are the combined results from throughout the semester. The separation between mean scores of Weekly 4 responses is statistically significant, with all p-values being under 0.006.

Type of preparation	Mean Score	Std. Dev.
Read Full PCE	3.616	0.05065
Read Minimum PCE	3.4	0.06539
Read Textbook	3.32	0.10681
Watched Videos	3.017	0.11578
Did no preparation	2.7951	0.12446

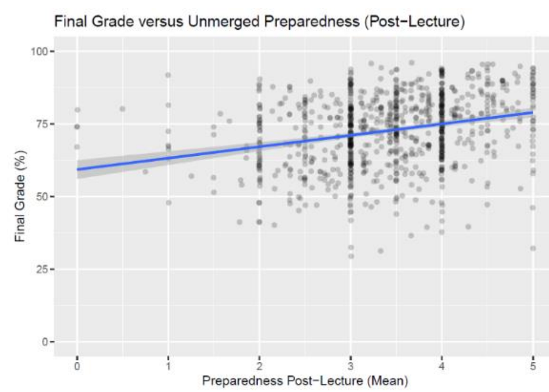
These results indicate, on average, that students felt more prepared for class if they read the PCEs, and particularly, they felt the most prepared if they read full PCE before the first class of the week. Surprisingly, textbook reading scored higher than videos, and unsurprisingly, any form of preparation had better results than doing no preparation.

The line of best fit shown below indicates a clear trend upwards from doing no preparation, to doing thorough preparation via reading the full PCE.



We conclude the results section by looking at how certain survey data correlates with final course grades. We present two findings, the first comparing preferred form of preparation throughout the term versus final course grade, and the second compares average perceived preparedness by students throughout the semester versus final course grade.

First, we present the results for class preparedness (Weekly (4)) versus final course grade. The included figure shows a clear correlation between average class preparedness and positive outcomes in the course, with



the linear regression suggesting, with p-value <10^-16, that a line of best fit has slope 3.9 with a y-intercept of 59.3.

This result clearly indicates that, on average, being adequately prepared for lecture and able to leverage the benefits of an active classroom will yield a better grade in the course. Furthermore, if students are not engaging in the pre-class content and coming to class underprepared, they are likely to perform wors in the course.

Finally, we investigate the relationship between preferred material for pre-class preparation (Exit (1)) and final course grade. The results are shown in the table below, with the separation between all groups reaching statistical significance except for the first two, meaning there wasn't a statistically significant difference in course grade between reading the full PCE or the minimum amount, with a p-value of 0.114.

Type of preparation	Mean Grade	Std. Dev.
Read Full PCE	75.0517	0.7373
Read Minimum PCE	73.5465	0.9518
Read Textbook	69.3552	1.5549
Watched Videos	68.9907	1.6854
Did no preparation	67.3271	1.8118

Thus, the students who, on average, chose to read the PCEs, rather in full or the minimum amount, had improved outcomes in the course, with the difference being statistically significant.

There were a few sets of survey questions which did not yield positive correlations or show significant improvement throughout the semester. For example, we did not see a correlation between the amount of time spent preparing for class and students feeling like their pre-class work prepared them well for class (Weekly (2) vs Weekly (4)). That is students who spent more time studying were not necessarily more prepared than students who spent less time preparing. Our results seemed to suggest that students should aim to feel adequately prepared after spending between 30 minutes to 1 hour preparing for class. We also asked students in the entrance and exit surveys on a scale of strongly disagree to strongly agree, how confident they are with reading mathematical language and notation. We did not see a significant improvement in their feedback between the start and end of the semester. We believe this lack of improvement is due to the significant increase in the amount of mathematical language and notation that they were exposed to during their first semester linear algebra class. We do think that their reading and comprehension skills increased, but relative to the literature they were exposed to, students did not perceive a significant improvement.

6. Conclusion

We believe that a pre-class preparation that reduces the gap in students background knowledge can improve inclusion and equity in the classroom. Over the past three years, we have iterated on our approach to pre-class content with the goal of improving student readiness for in-class participation and engagement. We have assessed the efficacy of this intervention through a quantitative analysis of survey data, collected during our third offering of the course in the fall of 2023. The results of this investigation clearly indicate that our pre-class readings were preferred by students over any other form of preparation, and students who consistently read the pre-class readings on average felt more prepared for class and achieved a higher final grade in the course when compared with students who chose other forms of preparation. Furthermore, students recognized the benefits of coming to class prepared, with most students agreeing that their in-class experience was improved when they had done sufficient preparation. Additionally, students that consistently felt prepared for active classes throughout the term also achieved a higher final grade in the class. We believe these results strongly support the argument that high-quality pre-class content is a vital component of the flipped classroom format, and that pre-class content using videos is not the only option for effective delivery of this material. Furthermore, the clear connection between student preparedness, the PCE readings being their preferred method of preparation and its positive influence on in-class experience suggests that pre-class preparation plays a role in the efficacy of the active learning classroom and student engagement.

Our data doesn't directly address whether the intervention had uniform effectiveness across genders, cultural backgrounds, or other minority groups. By minimizing the complex mathematical concepts from pre-class readings to make them more approachable, we speculate that positive outcomes weren't driven solely by high-achieving students. This can be verified in future studies.

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References

[1] S. Freeman *et al.*, "Active learning increases student performance in science, engineering, and mathematics," *Proceedings of the National Academy of Sciences*, vol. 111, no. 23, pp. 8410–8415, Jun. 2014, doi: 10.1073/pnas.1319030111.

[2] Z. Zainuddin and S. H. Halili, "Flipped Classroom Research and Trends from Different Fields of Study," *The International Review of Research in Open and Distributed Learning*, vol. 17, no. 3, May 2016, doi: 10.19173/irrodl.v17i3.2274.

[3] P. Strelan, A. Osborn, and E. Palmer, "The flipped classroom: A meta-analysis of effects on student performance across disciplines and education levels," *Educ Res Rev*, vol. 30, p. 100314, Jun. 2020, doi: 10.1016/j.edurev.2020.100314.

[4] L. C. Banes, R. M. Restani, R. C. Ambrose, H. A. Martin, and R. Bayley, "Relating Performance on Written Assessments to Features of Mathematics Discussion," *Int J Sci Math Educ*, vol. 18, no. 7, pp. 1375–1398, Oct. 2020, doi: 10.1007/s10763-019-10029-w.

[5] N. M. Webb, M. L. Franke, N. C. Johnson, M. Ing, and J. Zimmerman, "Learning through explaining and engaging with others' mathematical ideas," *Math Think Learn*, vol. 25, no. 4, pp. 438–464, Oct. 2023, doi: 10.1080/10986065.2021.1990744.

[6] E. Setren, K. Greenberg, O. Moore, and M. Yankovich, "Effects of Flipped Classroom Instruction: Evidence from a Randomized Trial," *Educ Finance Policy*, vol. 16, no. 3, pp. 363–387, Jul. 2021, doi: 10.1162/edfp_a_00314.

[7] D. Reinholz *et al.*, "When Active Learning Is Inequitable: Women's Participation Predicts Gender Inequities in Mathematical Performance," *J Res Math Educ*, vol. 53, no. 3, pp. 204–226, May 2022, doi: 10.5951/jresematheduc-2020-0143.

[8] R. A. Lotan, "Equitable Classrooms," in *Unequals*, Oxford University Press, 2022, pp. 178–199. doi: 10.1093/oso/9780197600009.003.0009.

[9] L. Abeysekera and P. Dawson, "Motivation and cognitive load in the flipped classroom: definition, rationale and a call for research," *Higher Education Research & Development*, vol. 34, no. 1, pp. 1–14, Jan. 2015, doi: 10.1080/07294360.2014.934336.

[10] M. Moravec, A. Williams, N. Aguilar-Roca, and D. K. O'Dowd, "Learn before Lecture: A Strategy That Improves Learning Outcomes in a Large Introductory Biology Class," *CBE—Life Sciences Education*, vol. 9, no. 4, pp. 473–481, Dec. 2010, doi: 10.1187/cbe.10-04-0063.

[11] J. L. Jensen, E. A. Holt, J. B. Sowards, T. Heath Ogden, and R. E. West, "Investigating Strategies for Pre-Class Content Learning in a Flipped Classroom," *J Sci Educ Technol*, vol. 27, no. 6, pp. 523–535, Dec. 2018, doi: 10.1007/s10956-018-9740-6.

[12] A. Yadav, M. M. Phillips, M. A. Lundeberg, M. J. Koehler, K. Hilden, and K. H. Dirkin, "If a picture is worth a thousand words is video worth a million? Differences in affective and cognitive processing of video and text cases," *J Comput High Educ*, vol. 23, no. 1, pp. 15–37, Apr. 2011, doi: 10.1007/s12528-011-9042-y.

[13] S. L. Laursen and C. Rasmussen, "I on the Prize: Inquiry Approaches in Undergraduate Mathematics," *International Journal of Research in Undergraduate Mathematics Education*, vol. 5, no. 1, pp. 129–146, Apr. 2019, doi: 10.1007/s40753-019-00085-6.