Session 1 - 09/10/2025

- Reviewed sections 1.1 through 1.4.
- Completed some exercises (Q1 - Q4).

Review of Relevant Material

There are four principal variables we want to consider in the following work.

t This variable represents time, often in seconds (s).

d This variable represents distance, often in meters (m), as measured from the origin.

v This variable represents speed, often in meters per seconds $(\mathrm{m/s})$, as measured relative to the original position.

a This variable represents acceleration, often in meters per seconds squared $(\mathrm{m/s^2})$.

Further, we will also use subscripts to denote initial and final observations of these variables. Commonly, we will use a 2 subscript to represent the final observations and a 1 subscript to represent the initial observations.

Theorem. Equation of Speed

$$v = \frac{\Delta d}{\Delta t} = \frac{d_2 - d_1}{t_2 - t_1}$$

Theorem. Equation of Acceleration

$$a = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$$

Exercise

1. At 12:30, you leave the school for lunch. You start 1.3km north of the bridge, and find yourself 0.4km north of the bridge 20 minutes later.

   (a) Express your position in meters and your elapsed time in seconds.
   In physics, we often prefer to work with meters to measure distances and seconds to measure time. In this question, we see that we are provided with the measurements of distance in kilometres and of time in minutes. In particular, we may convert between these units using the following conversion.

$$1000\text{m} = 1\text{km} \quad 60\text{s} = 1\text{min}$$

   Hence, we may obtain our distances in metres by multiplying the distances in kilometres by 1000 and the times in seconds by multiplying the time in minutes by 60. Therefore, we have the following.

$$\begin{aligned} d_1 &= 1.3\text{km} & d_2 &= 0.4\text{km} & t_1 &= 0\text{min} & t_2 &= 20\text{min} \\ &= 1.3 \cdot 1000\text{m} & &= 0.4 \cdot 1000\text{m} & &= 0 \cdot \text{s} & &= 20 \cdot 60\text{s} \\ &= 1300\text{m} & &= 400\text{m} & &= 0\text{s} & &= 1200\text{s} \end{aligned}$$

   Consequently, our initial position is 1300m and our final position is 400m, whereas for time, our initial time is 0s and our final time is 1200s.

   (b) Determine your speed in m/s.
   Before we attempt to determine our speed, we shall first organize which variables we know and which variables we want to compute.

$$\begin{aligned} d_1 &= 1300\text{m} \\ d_2 &= 400\text{m} \\ t_1 &= 0\text{s} \\ t_2 &= 1200\text{s} \\ v &= ?\text{m/s} \end{aligned}$$

   In this question, we are provided with the total distance travelled and total time elapsed, and so, we may use our *Equation of Speed* to solve for the speed in m/s. Therefore, we have the following.

$$\begin{aligned} v &= \frac{d_2 - d_1}{t_2 - t_1} && \text{Equation of Speed.} \\ &= \frac{400\text{m} - 1300\text{m}}{1200\text{s} - 0\text{s}} && \text{Substituting Values.} \\ &= \frac{-900\text{m}}{1200\text{s}} && \text{Computing Differences.} \\ &= -\frac{3}{4}\text{m/s} && \text{Simplifying.} \\ &= -0.75\text{m/s} && \text{Numerical Value.} \end{aligned}$$

   Therefore, our speed in m/s is -0.75m/s

(c) If you were to use a frame of reference using the school as zero instead of the bridge, how would your answers to (a) and (b) change?
TBD.

2. You launch a bottle rocket into the air. It starts travelling at a speed of 15 m/s upwards, but comes to a stop 1.3 s later. What is the acceleration of the bottle rocket?
Before we attempt to determine our speed, we shall first organize which variables we know and which variables we want to compute.

$$\begin{aligned} v_1 &= 15\mathrm{m/s} \\ v_2 &= 0\mathrm{m/s} \\ t_1 &= 0\mathrm{s} \\ t_2 &= 1.3\mathrm{s} \\ a &= ?\mathrm{m/s^2} \end{aligned}$$

In this question, we are provided with the initial and final speed and total time elapsed, and so, we may use our *Equation of Acceleration* to solve for the acceleration. Therefore, we have the following.

$$\begin{aligned} a &= \frac{v_2 - v_1}{t_2 - t_1} && \text{Equation of Acceleration.} \\ &= \frac{0\mathrm{m/s} - 15\mathrm{m/s}}{1.3\mathrm{s} - 0\mathrm{s}} && \text{Substituting Values.} \\ &= \frac{-15\mathrm{m/s}}{1.3\mathrm{s}} && \text{Computting Differences.} \\ &= -\frac{150}{13}\mathrm{m/s^2} && \text{Simplifying.} \\ &\approx -11.54\mathrm{m/s^2} && \text{Numerical Value.} \end{aligned}$$

Therefore, we have determined that the acceleration of the rocket is -11.54$\mathrm{m/s^2}$, as needed.

3. How long will it take to make a 350 km trip if you are travelling at $20\mathrm{m/s}$?
Before we attempt to determine our total time elapsed, we shall first organize which variables we know and which variables we want to compute.

$$\begin{aligned} v &= 20\mathrm{m/s} \\ \Delta d &= 350\mathrm{km} = 350000\mathrm{m} \\ \Delta t &= ?\mathrm{s} \end{aligned}$$

In this question, we are provided with the speed and the total distance travelled. While it may not initially appear that there is a suitable formula for our computation, we remark that *Equation of Speed* contains all of our relevant variables: $v, \Delta d, \Delta t$. Further, we will show that we may rearrange this equation to solve for Δd instead. Therefore, we have the following.

$$\begin{aligned} v &= \frac{\Delta d}{\Delta t} && \text{Equation of Speed.} \\ v \cdot \Delta t &= \frac{\Delta d}{\Delta t} \cdot \Delta t && \text{Multiplying by } \Delta t. \end{aligned}$$

$$
\begin{aligned}
&= \Delta d && \text{Cancellation of } \Delta t. \\
\frac{v \cdot \Delta t}{v} &= \frac{\Delta d}{v} && \text{Dividing by } v. \\
\Delta t &= \frac{\Delta d}{v} && \text{Cancellation of } v. \\
&= \frac{350000\mathrm{m}}{20\mathrm{m/s}} && \text{Substituting Values.} \\
&= 17500\mathrm{s} && \text{Simplifying.}
\end{aligned}
$$

Therefore, to complete a $350\mathrm{km}$ trip travelling at a speed of $20\mathrm{m/s}$ it will require $17500\mathrm{s}$.

4. You are travelling at a speed of $60\mathrm{km/h}$. You then put your foot on the gas, accelerating at $3.00\mathrm{m/s}^2$ for 8.0s. What will your final speed be, in $\mathrm{kmh/h}$?
Before we attempt to determine our final speed, we shall first organize which variables we know and which variables we want to compute.

$$
\begin{aligned}
v_1 &= 60\mathrm{km/h} = 60 \cdot \frac{1000}{60^2}\mathrm{m/s} = \frac{100}{6}\mathrm{m/s} \\
a &= 3.00\mathrm{m/s}^2 \\
\Delta t &= 8.0\mathrm{s} \\
v_2 &= ?\mathrm{km/h}
\end{aligned}
$$

In this question, we are provided with the initial speed, acceleration, and total time elapsed. While it may not initially appera that there is a suitable formula for our computation, we remark that *Equation of Acceleration* contains all of our relevant variables: $v_1, a, \Delta t, v_2$. Further, we will show that we may rearrange this equation to solve for v_2 instead. Therefore, we have the following.

$$
\begin{aligned}
a &= \frac{v_2 - v_1}{\Delta t} && \text{Equation of Acceleration.} \\
a \cdot \Delta t &= \frac{v_2 - v_1}{\Delta t} \cdot \Delta t && \text{Multiplying by } \Delta t. \\
&= v_2 - v_1 && \text{Cancellation of } \Delta t. \\
a \cdot \Delta t + v_1 &= v_2 - v_1 + v_1 && \text{Adding } v_1. \\
&= v_2 && \text{Since } -v_1 + v_1 = 0.
\end{aligned}
$$

$$
\begin{aligned}
v_2 &= a \cdot \Delta t + v_1 && \text{Derived Expression.} \\
&= 3.00\mathrm{m/s}^2 \cdot 8.0\mathrm{s} + \frac{100}{6}\mathrm{m/s} && \text{Substituting Values.} \\
&= 24.00\mathrm{m/s} + \frac{100}{6}\mathrm{m/s} && \text{Computing Multiplication.} \\
&= \frac{144}{6}\mathrm{m/s} + \frac{100}{6}\mathrm{m/s} && \text{Equivalent Expression.} \\
&= \frac{244}{6}\mathrm{m/s} && \text{Adding the Fractions.} \\
&= \frac{244}{6} \cdot \frac{60^2}{1000}\mathrm{km/h} && \text{Converting to } \mathrm{km/h}.
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{732}{5}\text{km/h} && \text{Simplifying.} \\
&= 146.4\text{km/h} && \text{Numerical Value.}
\end{aligned}
$$

5. You make a trip down the 401 (speed limit: 100 km/h), travelling a distance of 200 km in 2 hours. However, you still end up with a speeding ticket. How is this possible?

   TBD.