Quiz 8 (Sections 5.3, 9.4)

You will have 30 minutes to complete the quiz.

Name:
Student Number:

Q1 Consider the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ induced by the matrix $A = \begin{bmatrix} 3 & -1 \\ 0 & 2 \end{bmatrix}$.

(a) Sketch the unit square and the unit square under the transformation T. (1 Point)

(b) Using your sketch, determine two linearly independent eigenvectors of A.
INCLUDE THESE VECTORS IN YOUR SKETCH. (2 Point)

(c) Knowing that $\lambda_1 = 2, \lambda_2 = 3$ are the eigenvalues of A, verify computationally that the eigenvectors from (b) are indeed eigenvectors for the transformation T. (2 Points)

Q2 For the following, determine whether the matrix B is diagonalizable, not diagonalizable, or we cannot tell.

(a) The eigenvalues λ_i of B are given by $\lambda_k = 2\lambda_{k-1}$, where $\lambda_1 < 0$. (2 Points)

(b) There is some set of eigenvectors $\{\vec{v}_1, ..., \vec{v}_n\}$ that forms a basis for $\mathbb{R}^n$. (2 Points)

Q3 Let the sequence $c_0, c_1, c_2, ...$ be given by $c_k = \frac{1}{2}(c_{k-1} + c_{k-2})$ for $k \geq 2$. Here, we assume c_0, c_1 are some real numbers.

(a) Determine a matrix A that can be used for the following linear system: $A \begin{bmatrix} c_{k-1} \\ c_{k-2} \end{bmatrix} = \begin{bmatrix} c_k \\ c_{k-1} \end{bmatrix}$. (1 Point)

(b) Knowing that $A \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, A \begin{bmatrix} -1 \\ 2 \end{bmatrix} = -\frac{1}{2} \begin{bmatrix} -1 \\ 2 \end{bmatrix}$, determine c_{100}. (3 Points)

Q1

Q2

Q3