Generate B_z Fields Induced by AP/RL/SI Gradient Coil from Spherical Harmonic (SPH) Coefficients (R3.0)

(Yuxi Pang, Dariya Malyarenko and Thomas Chenevert 02/22/2018)

1. Given the spherical harmonic (SPH) coefficients (C_{nm} and S_{nm}) for each of three gradient coils (AP, RL and SI) and the system characteristic sphere radius (r_0), the corresponding induced magnetic field (Z component) within this sphere could be characterized as

$$B_z(r,\theta,\phi) = \sum_n \sum_m \left(\frac{r}{r_0}\right)^n \left(C_{nm}P_{nm}(\cos\theta)\cos(m\varphi) + S_{nm}P_{nm}(\cos\theta)\sin(m\varphi)\right)$$

where $r(\le r_0), \theta[0,\pi], \varphi[0,2\pi]$ are distance from the origin, polar angle and azimuth angle in Spherical coordinates, which are in turn related to Cartesian coordinates (x, y, z) as shown below,

$$x = r\sin\theta\cos\varphi$$
$$y = r\sin\theta\sin\varphi$$
$$z = r\cos\theta$$

$P_{nm}(\cos\theta)$ is the associated Legendre polynomials. The spherical harmonics are defined as shown in *Numerical Recipes in C: The Art of Scientific Computing, 2nd Edition* (Eq. 6.8.2 in p252).

$$Y_{nm}(\theta,\varphi) = \sqrt{\frac{(2n+1)(n-m)!}{4\pi(n+m)!}} P_{nm}(\cos\theta)e^{im\varphi}$$

Note, SPH coefficients (C_{nm} and S_{nm}) from different vendors may not include this normalization term ($\sqrt{\frac{(2n+1)(n-m)!}{4\pi(n+m)!}}$) depending on the conventions used.

2. We have created a matlab (compiled) script (**gbzmap_combo.p**) to output magnetic field maps based on provided SPH coefficients and the characteristic radius. Two SPH scaling options are provided to accommodate potential different SPH definitions. The vendor-provided gradient coil system information should be contained in a plain text file following the format as seen below …

SPH_REF_RADIUS_IN_MM = **Rsph** **OR**

SPH_REF_RADIUS_IN_MM_**[AP|RL|SI]** = **R_[AP|RL|SI]sph (for different r_0)**

G**[AP|RL|SI]**_FIELD_[**C|S**]_COEFFS_**N_M** = **NUM**

where **Rsph** is the system characteristic radius numerical value (in mm), **[AP|RL|SI]** represents either AP or RL or SI, and [**C|S**] refers to either cosine- or sine-term coefficient, SPH function's *degree* and *order* are indicated by **N** and **M,** and **NUM** is the SPH coefficient numerical value (in unit chosen by the vendor). ***Please note that the unit in the calculated***

field maps is the same as that used in SPH. Each line should contain only one coefficient, with the entered item self-explanatory. No space should be left in front of the line, and the order of the entry is not relevant. The comment lines start with “#” and will be ignored (see two example files). For an instance, if 2nd degree, 1st order SPH cosine-term coefficient for SI Gradient coil is 999.9, the entry should be typed in the text file as

GSI_FIELD_C_COEFFS_2_1 = 999.9

3. The calculated ***output gradient fields (3D MAT-structures) are represented in the left-hand “Magnet Coordinate System”: RL=dim1(rows); AP=dim2(cols); SI=dim3(slices).*** When you stand in front of magnet, the X direction is from RL, the Y direction is AP and Z direction moves superior (IS) from you. Note that the ***output XYZ convention maybe different from input coefficient convention*** (some vendors assign RL to “Y”), so the input coefficients are labeled explicitly as AP, RL or SI. ***Also note, that the (saved) MHD uses RAI convention, while DWI DICOM uses LPS (not equivalent to MAT “magnet” frame).***

4. To use the provided script, run the following command in matlab ...

BzXYZ = gbzmap_combo(np[int], sph_opt[int], save_mhd_opt[str]);

where the input/output arguments are

np - number of points in dim1 (dim2 or dim3)

sph_opt - yes(1) or no(0) SPH coeffs scaled by NORM factor

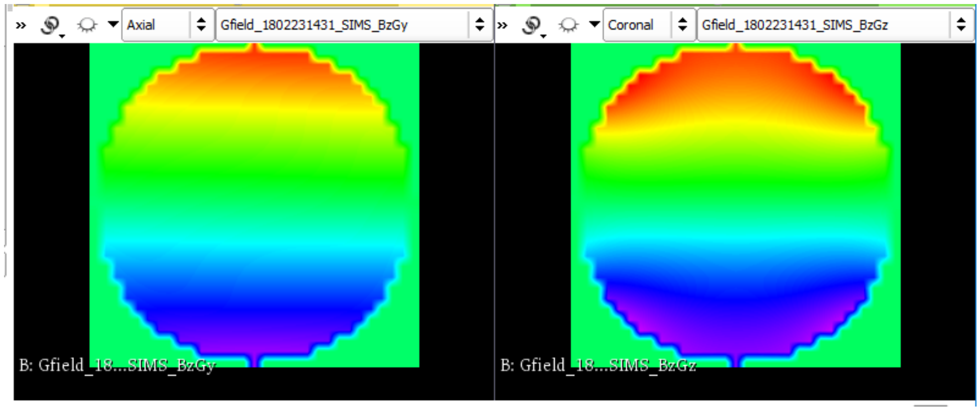
save_mhd_opt - ‘y’ or ‘n’ to save Bx/By/Bz field maps

BzXYZ.{zx,zy,zz} - 3D volumetric (np*np*np) magnetic field map induced by RL/AP/SI gradients.

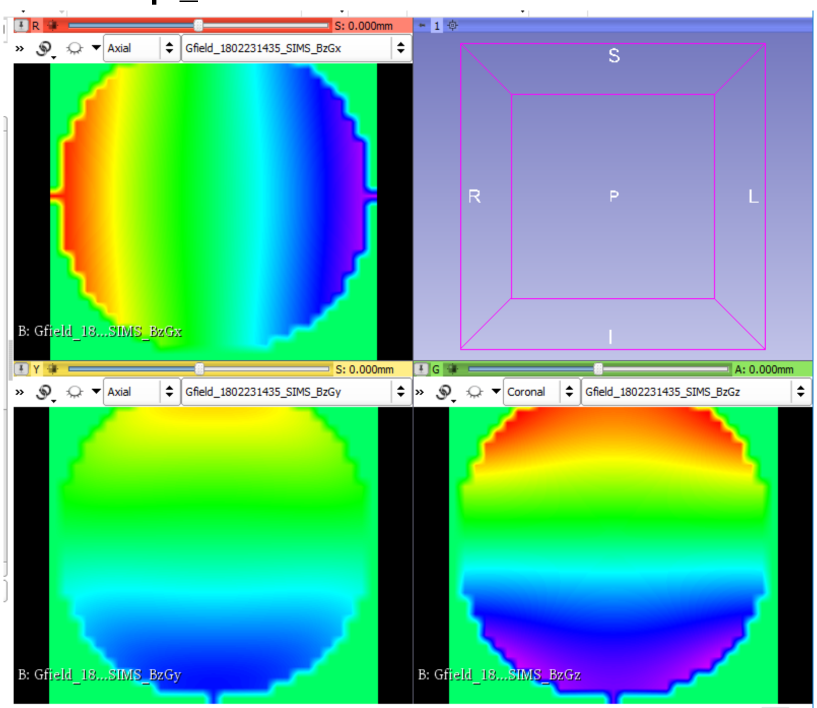
The generated final field maps will be automatically saved in MATLAB format (*.mat). ***Please note, if different Rsph are input for different channels, “np” will be adjusted to keep uniform voxel size across the output maps, and the FOV=2*minimum(Rx,Ry,Rz).***

Here are two examples using the included demo SPH files:

(1). Using **DEMO_ONE_SPH_COEFFS.txt**, ***BzXYZ =gbzmap_combo(31, 1, ‘n’),*** this will generate 3 volumetric (31*31*31) field maps from provided SPH coeffs that **DID NOT** include SPH NORM factor. Note, RL Gradient field map was empty due to missing SPH coefficients. These field maps were NOT saved in MHD format. The below two screenshots were taken from 3D Slicer software after loading AP Gradient field map (left, in axis mid plane) and SI Gradient field map (right, in coronal mid plane). The input files were grouped under the directory “**DEMO_ONE_GBZMAP\sfopt_yes**”.



(2). Using **DEMO_TWO_SPH_COEFFS.txt**, ***BzXYZ =gbzmap_combo(11, 0, 'y'),*** this will generate 3 volumetric (31*31*31) field maps from provided SPH coeffs that already included SPH NORM factor. The field maps were also saved in both MHD format and MATLAB format. The below screenshot was taken from 3D Slicer software after loading AP (upper left, in axis mid plane), RL (lower left, in axis mid plane) and SI Gradient field maps (lower right, in coronal mid plane). The input files were grouped under the directory "**DEMO_TWO_GBZMAP\sfopt_no**".



If you have any questions, please do not hesitate to contact us at: [yuxipang@umich.edu; dariya@umich.edu; tlchenev@umich.edu](mailto:yuxipang@umich.edu).