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1. $1/n$
2^{100}
$\log\log n$
$\sqrt{\log n}$
$\log^2 n$
$n^{0.01}$
$\sqrt{n}, 3n^{0.5}$
$2^{\log n}, 5n$
$n\log_4 n, 6n\log n$
$2n\log^2 n$
$4n^{3/2}$
$4^{\log n}$
$n^2\log n$
n^3
2^n
4^n
2^{2^n}

2 $\because$ $d(n)$ is $O(f(n))$ and $e(n)$ is $O(g(n))$
$\therefore$ $d(n) = K\cdot f(n)$ for all $n \geq N$, $e(n) = L \cdot g(n)$ for all $n \geq M$.
$\therefore$ $d(n)e(n) = (K\cdot L) f(n)g(n)$ for all $n \geq n_0$
That is $d(n)e(n)$ is $O(f(n)g(n))$

3 $\because$ $\log_b f(n) = \log_2 f(n) / \log_2 b$
and $b > 1$ and b is a constant.
$\therefore$ $1/\log b$ is a constant.
$\therefore$ $\log_b f(n)$ is $O(\log_2 f(n))$ and $\log_2 f(n)$ is $O(\log_2 f(n))$.
That is if $b>1$ is a constant, $\log_b f(n)$ is $\Theta(\log_2 f(n))$

4. when $n=1$, $T(1)=1$

Assume when $n=k$, $T(k)=2^{k+1}-1$

When $n=k+1$,

$\therefore T(k+1)=T(k)+2^{k+1}=2^{k+1}-1+2^{k+1}=2^{k+2}-1=2^{(k+1)+1}-1$

Hence, $T(n)=2^{n+1}-1$

5. (a) $T_b(n)=4$

$T(n)=1+n\times(1+1+1)+1+1=3n+3$

(b) Let S_k : arrayFind returns the index of element or -1 after k^{th} iteration.

Base: show true for S_0

Case 1 : $X=A[0]$, returns $0 \rightarrow$ True.

Case 2 : $X\neq A[0]$, returns $-1 \rightarrow$ True.

I.H : Assume S_{i-1} is true.

Case 1 : $X=A[0]$ or $A[1]$ or $\cdots$ or $A[i-1]$, returns index $\rightarrow$ True.

Case 2 : $X\neq A[0]$ or $A[1]$ or $\cdots$ or $A[i-1]$, returns $-1 \rightarrow$ True.

I.S : Show S_i is true after i^{th} iteration.

Case 1 : $X=A[0]$ or $A[1]$ or $\cdots$ or $A[i]$, returns index, $\rightarrow$ True.

Case 2 : $X\neq A[0]$ or $A[1]$ or $\cdots$ or $A[i]$, returns -1, $\rightarrow$ True.

Therefore. arrayFind is correct.