3

1. (a).

$$\begin{bmatrix} 2 & 1 & -1 \\ 5 & 2 & 2 \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -4 \\ 5 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 10 & 5 & -5 & 5 \\ -10 & -4 & -4 & 8 \\ 3 & 1 & 1 & 5 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 10 & 5 & -5 & 5 \\ 0 & 1 & -9 & 13 \\ 3 & 1 & 1 & 5 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 30 & 15 & -15 & 15 \\ 0 & 1 & -9 & 13 \\ -20 & -10 & 40 & -50 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 30 & 15 & -15 & 15 \\ 0 & 1 & -9 & 13 \\ 0 & 5 & -25 & -35 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 2 & 1 & -1 & 1 \\ 0 & -5 & 45 & -65 \\ 0 & 5 & -25 & -35 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 2 & 1 & -1 & 1 \\ 0 & -1 & 9 & -13 \\ 0 & 0 & -1 & 5 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 2 & 1 & -1 & 1 \\ 0 & -5 & 45 & -65 \\ 0 & 0 & 20 & -100 \end{array}\right]$$

Therefore $x_3 = -5$

$x_2 = (-13 + 45)/(-1) = -32$

$x_1 = (1 - 5 + 32)/2 = 14$

(b). $\begin{bmatrix} 2 & 1 & -1 & | & 1 \\ 5 & 2 & 2 & | & -4 \\ 3 & 1 & 1 & | & 5 \end{bmatrix}$

$\begin{bmatrix} 5 & 2 & 2 & | & -4 \\ 2 & 1 & -1 & | & 1 \\ 3 & 1 & 1 & | & 5 \end{bmatrix}$

$\begin{bmatrix} 5 & 2 & 2 & | & -4 \\ 0 & \frac{1}{5} & \frac{-9}{5} & | & \frac{13}{5} \\ 0 & -\frac{1}{5} & \frac{-1}{5} & | & \frac{37}{5} \end{bmatrix}$

$\begin{bmatrix} 5 & 2 & 2 & | & -4 \\ 0 & \frac{1}{5} & \frac{-9}{5} & | & \frac{13}{5} \\ 0 & 0 & -2 & | & 10 \end{bmatrix}$

$\therefore$ $X_3 = -5$, $X_2 = -32$, $X_1 = 14$.

(c). $det(A) = (-1)^3 (5)(\frac{1}{5})(-2) = 2$.

$det(A) = 2(-1)(-1) = 2$

Q2.

(a).

```
>> A = [1 -2 4 -8;1 0 0 0;1 2 4 8;1 4 16 64]

A =

   1   -2    4   -8
   1    0    0    0
   1    2    4    8
   1    4   16   64
```

(b).

```
>> X = [-2;0;2;4]

X =

   -2
    0
    2
    4

>> Y = sinh(cos(X))

Y =

  -0.4283
   1.1752
  -0.4283
  -0.7012
```

\>\> C = A\Y
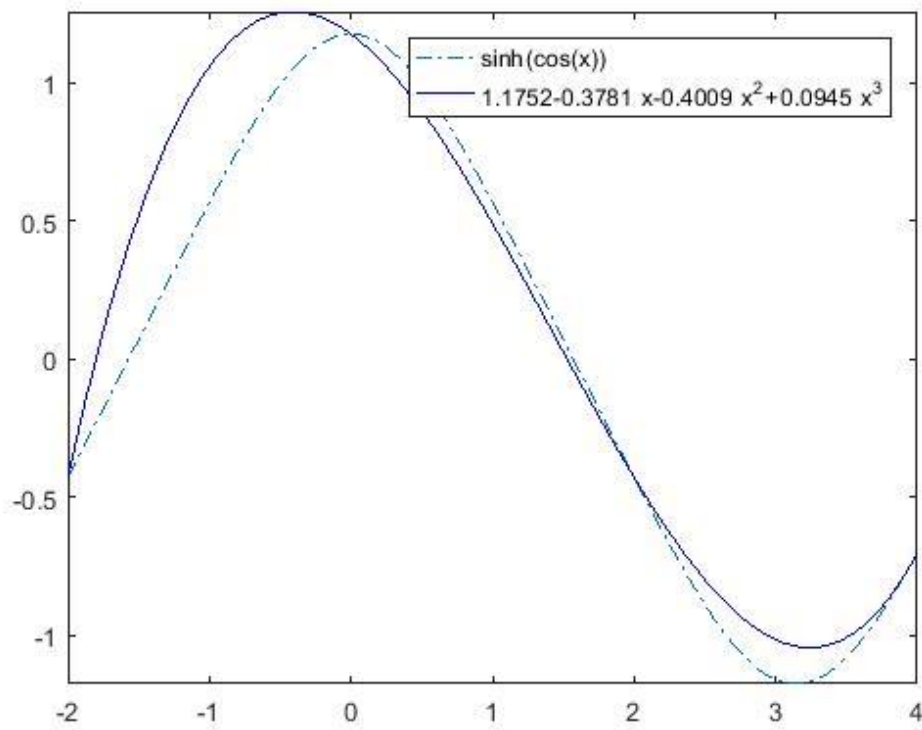
C =

   1.1752

  -0.3781

  -0.4009

   0.0945

Therefore, the coefficients are

$c_0 = 1.1752, c_1 = -0.3781, c_2 = -0.4009, c_3 = 0.0945$

(c).

Q3.

(a).

$$L_0(x) = \frac{(x - x_1)(x - x_2)}{(x_0 - x_1)(x_0 - x_2)} = \frac{(x - 2)(x - 6)}{(1 - 2)(1 - 6)} = \frac{x^2 - 8x + 12}{5}$$

$$L_1(x) = \frac{(x - x_0)(x - x_2)}{(x_1 - x_0)(x_1 - x_2)} = \frac{(x - 1)(x - 6)}{(2 - 1)(2 - 6)} = \frac{x^2 - 7x + 6}{-4}$$
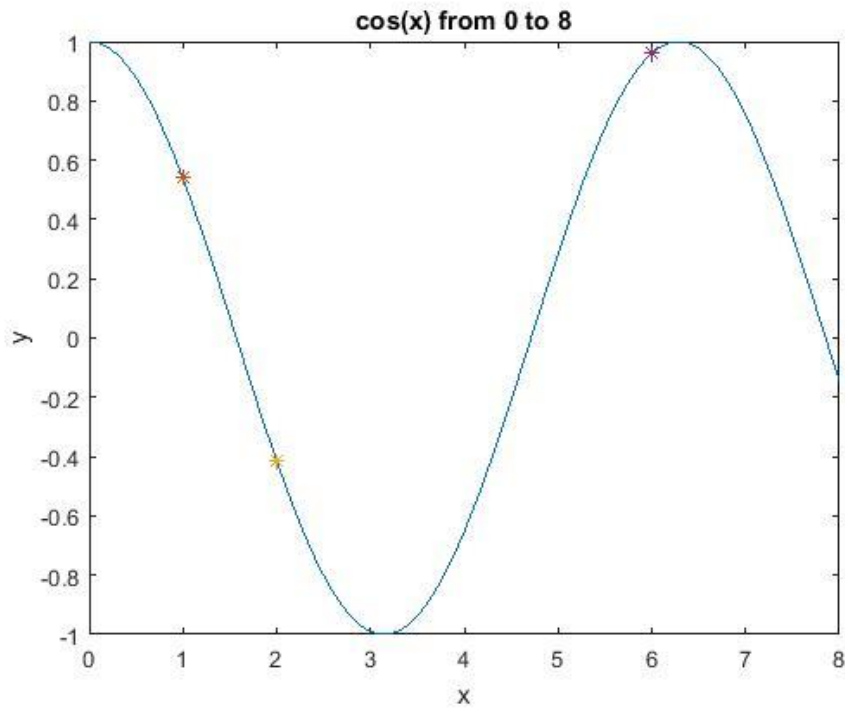
$$L_2(x) = \frac{(x - x_1)(x - x_0)}{(x_2 - x_1)(x_2 - x_0)} = \frac{(x - 2)(x - 1)}{(6 - 2)(6 - 1)} = \frac{x^2 - 3x + 2}{20}$$

Therefore,

$$p(x) = L_0(x)f(x_0) + L_1(x)f(x_1) + L_2(x)f(x_2)$$

$$= \frac{x^2 - 8x + 12}{5}\cos(1) - \frac{x^2 - 7x + 6}{4}\cos(2) + \frac{x^2 - 3x + 2}{20}\cos(6)$$

$$= 0.2601x^2 - 1.7367x + 2.0169$$

(b).

```
fplot(@cos,[0 8])
hold on
title('cos(x) from 0 to 8')
plot(1, 0.5403, '*')
plot(2, -0.4161, '*')
plot(6, 0.9602, '*')
xlabel('x');
ylabel('y');
>>A3Q2b
```

cos(x) from 0 to 8

(c).

```
x = 0:0.1:8;
fplot(@(x) ((x.^2)/5-(8*x)/5+12/5)*cos(1),[0 8],'b')
hold on
fplot(@(x) ((x.^2-7*x+6)/-4)*cos(2),[0 8],'r')
fplot(@(x) ((x.^2-3*x+2)/20)*cos(6),[0 8],'g')
xlabel('x');
ylabel('y');
hold off
>>A3Q3c
```
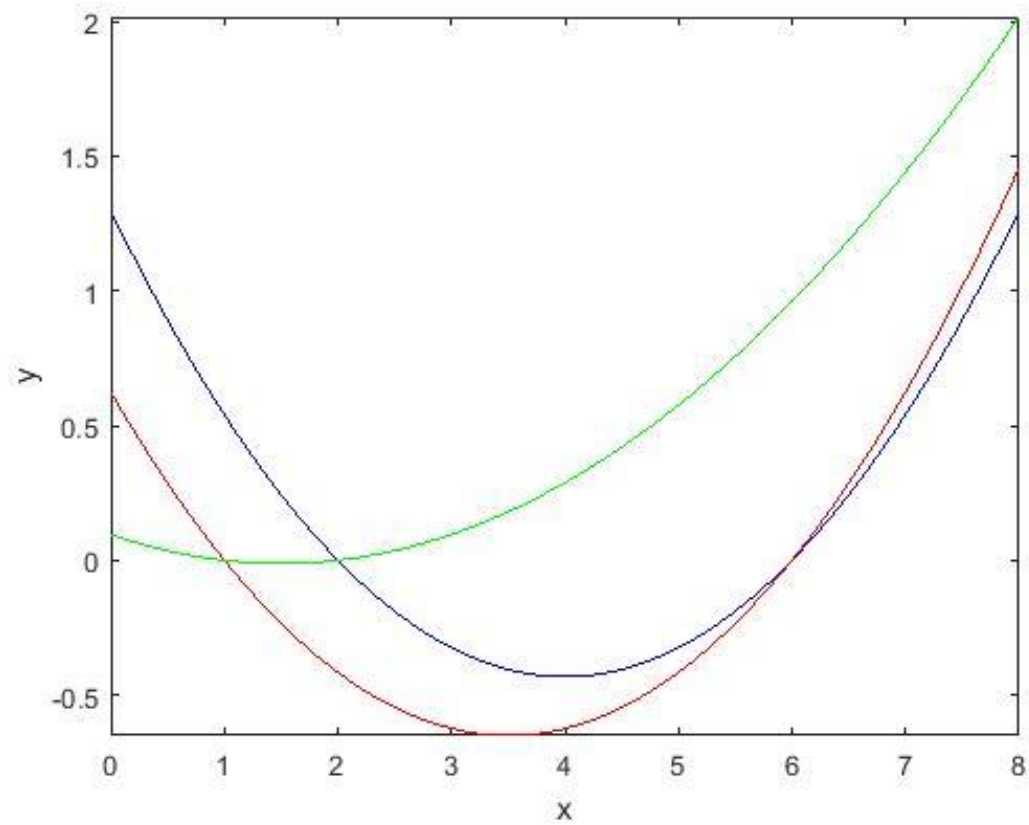
(d)

```
x = 0:0.1:8;

f = cos(x);

p = 0.2601*x.^2-1.7367*x+2.0169;

plot(x,f,'*',x,p,'*')

xlabel('x');

ylabel('y');

>>A3Q3d
```