Q1

(a) $f'(x) = -\cos(x)$

```
>> -1.56*cos(1.56)/(1-sin(1.56))

ans =

  -288.9844
>> -0.51*cos(0.51)/(1-sin(0.51))

ans =

    -0.8696
```

Conclusion: Therefore $f(x)$ is ill-conditioned for values of x close to $\pi/2$ and well-conditioned for values of x close to 0.5

(b) In the case of x=$\frac{\pi}{2}$

$(x-\tilde{x})/\tilde{x}$= |1.5708−1.56|/1.56= 0.0069231

$(f(x)-f(\tilde{x}))/f(\tilde{x})$= |(1−sin(1.5708))−(1−sin(1.56))|/| 1−sin(1.56) | = 1.0

In the case of x=0.5

$(x-\tilde{x})/\tilde{x}$= |0.5−0.51|/0.51 = 0.019608

$(f(x)-f(\tilde{x}))/f(\tilde{x})$= |(1−sin(0.5))−(1−sin(0.51))|/|1−sin(0.51) | = 0.01709

Slight change in the input of x=$\frac{\pi}{2}$ cause a great difference of output, compared to the case of x=0.5 whereas relative error is smaller.

Q2.

(a) $\ln(1)+(x-1)-\frac{(x-1)^2}{2}+\frac{(x-1)^3}{3}-\frac{(x-1)^4}{4}+\frac{(x-1)^5}{5\xi^5}$

(b) ln(1)+0.5-0.25/2+0.125/3-0.0625/4=0.401042
ln(1.5)= 0.405465108108164
|Et|=ln(1.5)-0.401042=0.004423108108164

(c) Rn=$\frac{(x-1)^5}{5\xi^5}$, we choose x=1.5 and ξ=0.5 as we want Rn be the largest. Therefore Rn is

$\frac{0.5^5}{5(0.5)^5}$=0.200

Q3.

(a)

```
function [ root ] = Newton( x0, eps, imax, f, fp)
%UNTITLED Summary of this function goes here
```

```
%   Detailed explanation goes here
    i=1;
    fprintf ( ' iteration approximation \n')
    while i <= imax
        root = x0-feval(f,x0)/feval(fp,x0);
        fprintf ( ' %6.0f %18.8f \n', i, root)
        if abs(1-x0/root)<eps
            return;
        end
        i = i + 1;
        x0 = root;
    end
    fprintf ( ' failed to converge in %g 
iterations\n', imax )
end
```

(b)

```
function y=fQ3(x)
    y = 1/(x)^1/2 + 2.0*
log10(0.0000015/3.7/0.005+2.51/13743/(x)^1/2);
end

function y=fpQ3(x)
   y=(-1/2)x^(-3/2)-((2.51/13743)*x^(-
3/2))/((0.0000015/0.0185+(2.51/13743)*x^(-
1/2))*log(10));
end
```

(c)

```
>> Newton(0.008,1e-08,20,@fQ3,@fpQ3)
 iteration approximation
      1         0.01576881
      2         0.02415359
      3         0.02837022
      4         0.02895888
      5         0.02896782
      6         0.02896782
      7         0.02896782

ans =

    0.0290
```

(d)

```
>> Newton(0.08,1e-08,20,@fQ3,@fpQ3)
  iteration approximation
        1         -0.02184302
        2         -0.06807542
        3          0.03144000
        4          1.05877124
        5         -5.95292388
        6        216.42712658
        7     -18650.35472708
        8     549585.67351560
        9 194856091813.81741000
       10 -263745429741803260.00000000
       11 4317841837918353800000000000.00000000
       12 -1502578751186269700000000000000000000000000.00000000
       13
8297705467922336800000000000000000000000000000000000000000000000.00000000
       14                                                                   -
240047876797189910000000000000000000000000000000000000000000000000000000000
0000000000000000000000.00000000
       15
110142475090763960000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000.00
000000
       16                                                                   -
616725445174663930000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000.0000
0000
       17                  Inf
       18                  NaN
       19                  NaN
       20                  NaN
  failed to converge in 20 iterations

ans =

        NaN +       NaNi
```

(e)

The greater the x0 can be, the result will be further away from the actual correct value because in the Newton method, if $f'(x0)$ is relatively small, the approximated result will be far away from the actual result.