University of Victoria
Department of Computer Science
CSC 355 Digital Logic and Computer Design
ASSIGNMENT 1 DUE Thursday September 24 AT BEGINNING OF CLASS

Neatness Counts! (5% of assignment grade allocated to following this format specification)
It is expected that answers to assignments are either typed or written *extremely* neatly. In all cases, the Karnaugh Maps formats below *must* be used, either copied and edited to add the required bits and circles or printed and written on. Circuits much be drawn using a circuit drawing program.

1. Determine the radix r given that $(2362)_r = (36D)_{16}$ (show all your work).

$(36D)_{16} = 3 * 16^2 + 6 * 16 + 13_{10} = 877_{10}$
Note: Because 877 < 2362 the radix r must be less than 10.
Using a trial and error method, guess 8:
$(36D)_{16} = (0011\ 0110\ 1101)_2 = (001\ 101\ 101\ 101)_2 = (1\ 5\ 5\ 5)_8$ $\therefore$ not base 8!
So, try 7:
$877_{10} / 7 = 125$ remainder 2
$125 / 7 = 17$ remainder 6
$17 / 7 = 2$ remainder 3
$2 / 7 = 0$ reminder 2 $\therefore 877_{10} = (2362)_7 = (36D)_{16}$, our radix r = 7

2. Show the bit configuration that represents the decimal number 251 in
   a. Binary: $251_{10} = 11111011_2$
   b. BCD $251_{10} = 0010\ 0101\ 0001_{BCD}$
   c. ASCII (7 bit) $251_{10} = 0110010\ 0110101\ 0110001_{ASCII(7\ bit)}$
   d. ASCII with even parity $251_{10} = 10110010\ 00110101\ 10110001_{ASCII(even)}$

3. Simplify the following expression, using Boolean algebra, into minimum sum of products form. Show each Boolean algebra rule used in the simplification.
   a.

$(D + \bar{C})(A + C)(\bar{D} + B)(C + D)$

$= (AD + A\bar{C} + CD + \bar{C}C)(BC + BD + C\bar{D} + D\bar{D})$,distributive

$= (AD + A\bar{C} + CD)(BC + BD + C\bar{D})$, complement, idempotent

$= (A\bar{C} + CD)(BD + C\bar{D})$, consensus, consensus

$= AB\bar{C}D + A\bar{C}C\bar{D} + BCDD + CCD\bar{D}$, distributive

$= AB\bar{C}D + BCD$, complement, idempotent

$= BC(A\bar{C} + C)$, distributive

$= BC(A + C)$, simplification

= ABC+BCD , distributive

b.

$$
\begin{aligned}
&(A+D+E)(\bar{A}+B+\bar{C})(\bar{A}+C+\bar{D})(A+\bar{C}) \\
&= (A+D+E)(A+\bar{C})(\bar{A}+B+\bar{C})(\bar{A}+C+\bar{D}), && \text{Commutative} \\
&= (A+(D+E)\bar{C})\big(\bar{A}+(B+\bar{C})(C+\bar{D})\big), && \text{Distributive} \\
&= (A+\bar{C}D+\bar{C}E)(\bar{A}+BC+\bar{C}\bar{D}+B\bar{D}), && \text{Distributive, Complement, ident.} \\
&= (A+\bar{C}D+\bar{C}E)(\bar{A}+BC+\bar{C}\bar{D}), && \text{Consensus} \\
&= ABC+A\bar{C}\bar{D}+\bar{A}\bar{C}D+\bar{A}\,\bar{C}E+\bar{C}\,\bar{D}E, && \text{Distributive, identity, complement} \\
&= ABC+A\bar{C}\bar{D}+\bar{A}\bar{C}D+\bar{A}\,\bar{C}E+\bar{C}\,\bar{D}E(A+\bar{A}), && \text{complement, ident.} \\
&= ABC+A\bar{C}\bar{D}+\bar{A}\bar{C}D+\bar{A}\,\bar{C}E && \text{, distributive, absorption}
\end{aligned}
$$

4. <u>Use Karnaugh maps</u> to simplify the following Boolean expressions, giving the result in <u>Sum of Products form</u>:

a. $\overline{A}\,\overline{B}\,C+\overline{B}C\,\overline{D}+B\bar{C}D+A\overline{B}\,\overline{D}+AC\overline{D}+BCD$

F = $\overline{A}\,\overline{B}\,C+\overline{B}C\,\overline{D}+B\bar{C}D+A\overline{B}\,\overline{D}+AC\overline{D}+BCD$

AB\CD	00	01	11	10
00			1	1
01		1	1	
11		1	1	1
10	1			1

∴ F= $BD+A\overline{B}\,\overline{D}+\overline{A}\,\overline{B}\,C+AC\overline{D}$

b. F(A,B,C) = $\left(A\,\overline{C}+\overline{A}\,BC+\overline{B}C+ABC\right)$

F(A,B,C) = $\left(A\,\overline{C}+\overline{A}\,BC+\overline{B}C+ABC\right)$

A\BC	00	01	11	10
0		1	1	
1	1	1	1	1

∴ F= $A+C$

c. F(W,X,Y,Z) = $\sum m(0,2,6,7,8,12)$

F(W,X,Y,Z) = $\sum m(0,2,6,7,8,12)$

WX\YZ

	00	01	11	10
00	1			1
01			1	1
11	1			
10	1			

$\therefore$ F= $\overline{W}\overline{X}\overline{Z} + \overline{W}XY + W\overline{Y}\overline{Z}$

d. F(w,x,y,z) = $\sum m(3, 4, 9, 13) + d(1, 5, 7, 14, 15)$

F(w,x,y,z) = $\sum m(3, 4, 9, 13) + d(1, 5, 7, 14, 15)$

wx\yz

	00	01	11	10
00		x	1	
01	1	x	x	
11		1	x	x
10		1		

$\therefore$ F= $\overline{Y}Z + \overline{W}Z + \overline{W}X\overline{Y}$

5. Use Karnaugh maps to simplify the following Boolean expressions, giving the result in Product of Sums form.

a. F(W,X,Y,Z) = $(X + Y + \overline{Z})(W + \overline{Y} + \overline{Z})(\overline{W} + Y + \overline{Z})(W + \overline{X} + \overline{Z})(\overline{W} + X + Y)$

F(W,X,Y,Z) = $(X + Y + \overline{Z})(W + \overline{Y} + \overline{Z})(\overline{W} + Y + \overline{Z})(W + \overline{X} + \overline{Z})(\overline{W} + X + Y)$

$\overline{F} = \overline{X}\overline{Y}Z + \overline{W}YZ + W\overline{Y}Z + \overline{W}XZ + W\overline{X}\overline{Y}$

WX\YZ

	00	01	11	10
00		0	0	
01		0	0	
11		0		
10	0	0		

$\therefore$ $\overline{F} = \overline{Y}Z + \overline{W}Z + W\overline{X}\overline{Y}$

$\therefore$ F = $(W + \overline{Z})(Y + \overline{Z})(\overline{W} + X + Y)$

b.

F(w,x,y,z) = $\sum m(0, 1, 2, 4, 6, 8, 10, 12, 13, 14)$

wx\yz

	00	01	11	10
00	1	1	0	0
01	1	0	0	1
11	1	1	0	1
10	1	0	0	1

$\therefore\ \ \overline{F} = yz + \overline{w}\,\overline{x}y + \overline{w}xz + w\overline{x}z$

$\therefore$ F = $(\overline{y} + \overline{z})(w + x + \overline{y})(w + \overline{x} + \overline{z})(\overline{w} + x + \overline{z})$

c. F(w,x,y,z) = $\sum m(0, 2, 7, 9, 10, 13, 14, 15)$

F(w,x,y,z) = $\sum m(0, 2, 7, 9, 10, 13, 14, 15)$

wx\yz

	00	01	11	10
00	1	0	0	1
01	0	0	1	0
11	0	1	1	1
10	0	1	0	1

$\therefore\ \ \overline{F} = \overline{w}\overline{y}z + \overline{x}yz + \overline{w}x\overline{z} + w\overline{y}\overline{z}$

$\therefore$ F = $(w + y + \overline{z})(x + \overline{y} + \overline{z})(w + \overline{x} + z)(\overline{w} + y + z)$

d. F(w,x,y,z) = $\sum m(1, 3, 7, 13) + d(5,6,8, 9, 12, 14)$

F(w,x,y,z) = $\sum m(1, 3, 7, 13) + d(5,6,8, 9, 12, 14)$

wx\yz

	00	01	11	10
00	0	1	1	0
01	0	x	1	x
11	x	1	0	x
10	x	x	0	0

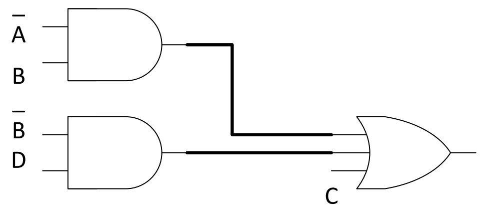
$\therefore\ \ \overline{F} = \overline{z} + wy$

$\therefore$ F = $z(\overline{w} + \overline{y})$

6. Design circuits as specified below. In each case assume that all variables are available in both true and inverted form (i.e., there is no need to put inverters on the main inputs). Gates may have any fan-in up to 4. More marks will be awarded for circuits that use fewer gates.

a. a two level circuit for: $b(c+\bar{a})+\bar{b}(c+d)+\bar{a}d$ using only AND, OR gates;

F = $b(c+\bar{a})+\bar{b}(c+d)+\bar{a}d = bc+\bar{a}b+\bar{b}c+\bar{b}d+\bar{a}d = \bar{a}b+\bar{b}d+c$



ab\cd

	00	01	11	10
00	0	1	1	1
01	1	1	1	1
11	0	0	0	1
10	0	1	1	1

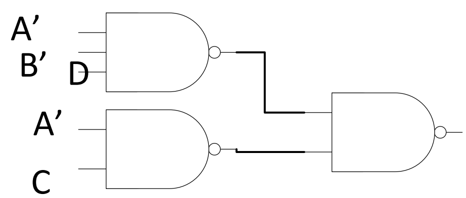
b. a circuit for $\bar{a}bc+\bar{a}\bar{b}d+\bar{a}c$ using NAND gates only

F = $\bar{a}bc+\bar{a}\bar{b}d+\bar{a}c$

F = $\bar{a}\bar{b}d+\bar{a}c$

AB\CD

	00	01	11	10
00		1	1	1
01			1	1
11				
10				



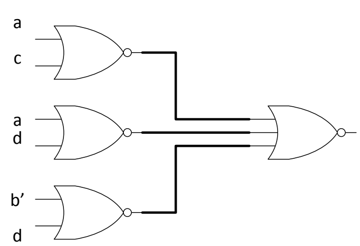
c. a circuit for $a\bar{b}\bar{c}+\bar{a}cd+a\bar{b}c+abd$ using NOR gates only.

F = $a\bar{b}\bar{c}+\bar{a}cd+a\bar{b}c+abd$

AB\CD

	00	01	11	10
00	0	0	1	0
01	0	0	1	0
11	0	1	1	0
10	1	1	1	1

$\bar{F}=\bar{a}\bar{c}+\bar{a}\bar{d}+b\bar{d}$ $\quad\therefore F=(a+c)(a+d)(\bar{b}+d)$



7. A switching network has 3 inputs (w, x and y) and 2 outputs (a and b). The output variables, a and b, represent the first and second bits, respectively of a binary number, N. N equals the total number of the inputs which are "0". For example, if w=1, x=0. y=1, then a=0, b=1, representing 1 "0" input. A second example, if w=1, x=0. y=0, then a=1, b=0, representing 2 "0" inputs.
   a. Find the minterm expansion for a and b. Express the answer in abbreviated notation (with Σ or π).
   b. Find the maxterm expansion for a and for b. Express the answer in abbreviated notation (with Σ or π).
   c. Provide a circuit, using AND and OR gates for this switching network.

w	x	y	a	b
0	0	0	1	1
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	0	0

a = $\bar{w}\,\bar{y} + \bar{w}\,\bar{x} + \bar{x}\,\bar{y} = \bar{w}\,\bar{x}\,\bar{y} + \bar{w}\,x\,\bar{y} + \bar{w}\,\bar{x}y + w\bar{x}\,\bar{y} = \sum m(0,1,2,4)$

w\xy	00	01	11	10
0	1	1	0	1
1	1	0	0	0

a

b = $\bar{w}\,\bar{x}\,\bar{y} + \bar{w}\,xy + w\bar{x}y + wx\bar{y} = \sum m(0,3,5,6)$

There is no simplification the SOM = SOP

w\xy	00	01	11	10
0	1	0	1	0
1	0	1	0	1

b

Note: It is correct to switch a and b. (I.e, a = $\sum m(0,3,5,6)$ and b = $\sum m(0,1,2,4)$

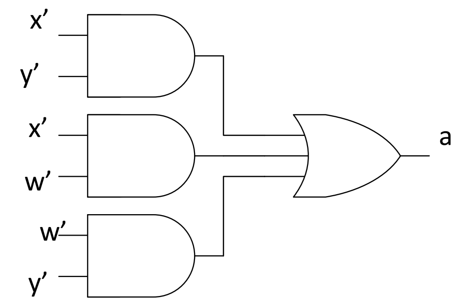
For max terms, first find $\bar{a}$ and $\bar{b}$ then invert:

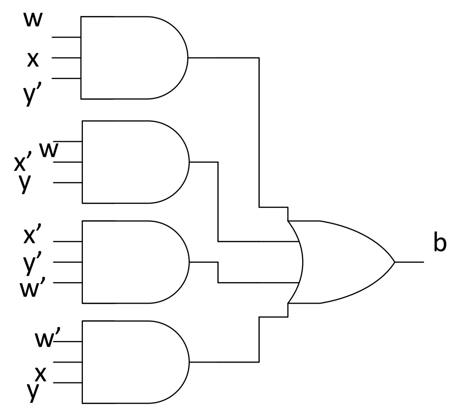
$\bar{a} = \bar{w}xy + w\bar{x}y + wxy + wx\bar{y}$, thus:

$a = (w + \bar{x} + \bar{y})(\bar{w} + x + \bar{y})(\bar{w} + \bar{x} + \bar{y})(\bar{w} + \bar{x} + y) = \prod M(3,5,6,7)$

$\bar{b} = \bar{w}\,\bar{x}y + w\bar{x}\bar{y} + wxy + \bar{w}x\bar{y}$, thus:

$b = (w + x + \bar{y})(\bar{w} + x + y)(\bar{w} + \bar{x} + \bar{y})(w + \bar{x} + y) = \prod M(1,2,4,7)$


x’
y’
x’
w’
w’
y’
a


w
x
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