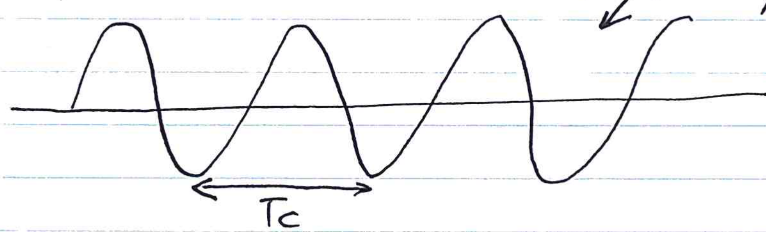
ECE519B/496A

Basic concepts:

- ~~frequency band~~ carrier frequency



$\cos 2\pi f_c t$ $(\cos \omega_c t)$

or

$\sin 2\pi f_c t$ $(\sin \omega_c t)$

$$f_c = \frac{1}{T_c} \qquad \omega_c = 2\pi f_c = \frac{2\pi}{T_c}$$

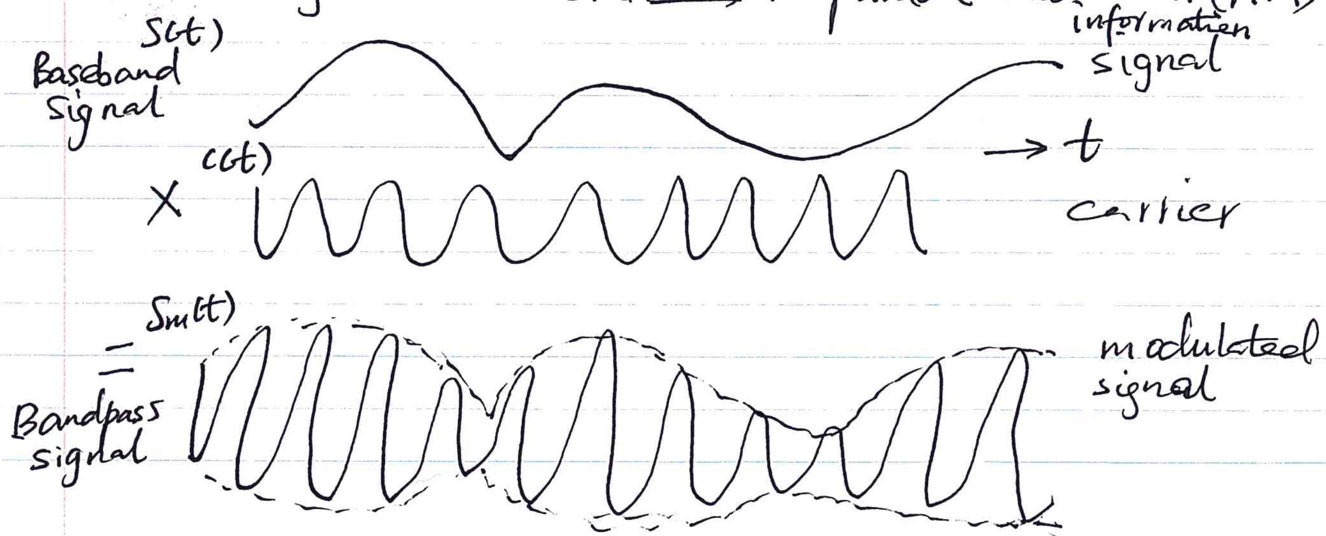
f_c: several hundred MHz 1G-5G

1.9, 2.4 GHz	2G, 3G, 4G
sub-6 GHz	5G term
28GHz and above	mmWave

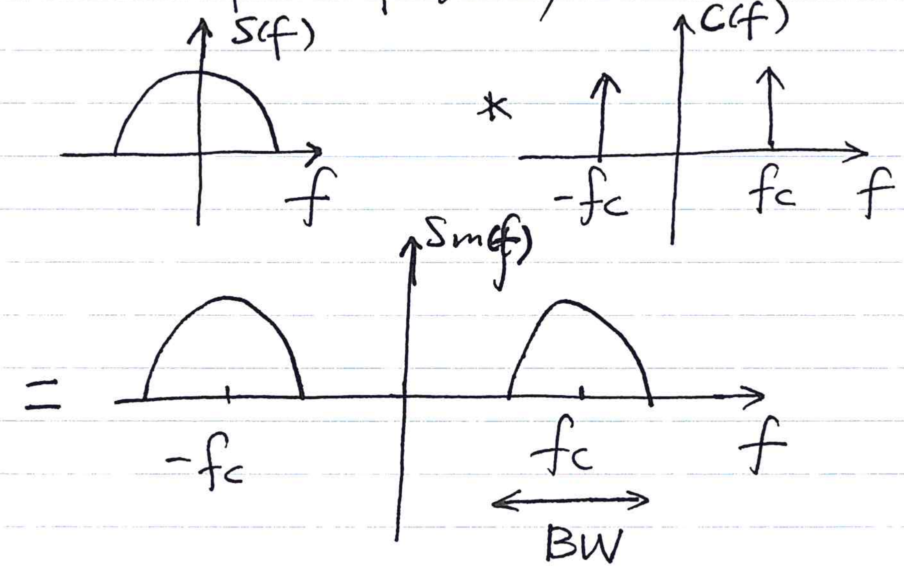
- Channel bandwidth

  * carrier is just a sinusoid signal. Ideally its BW is 0.
  * When the carrier signal is modulated to carry information data, the modulated signal has a bandwidth.

EX1. Analog modulation: ~~(AM)~~ Amplitude modulation (AM)



view from fequency domain



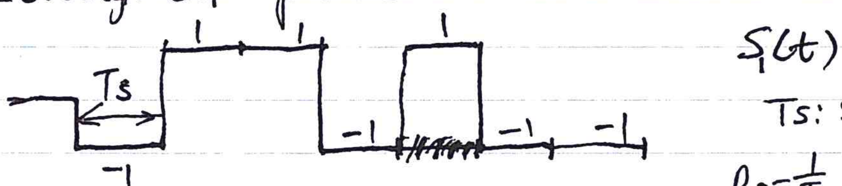
EX2. Digital modulation: pulse amplitude modulation (PAM)

information data is binary digits

0 1 1 0 1 0 0 $\xrightarrow{\text{mapping}}$ -1, 1, 1, -1, 1, -1, -1

(bit → symbol)

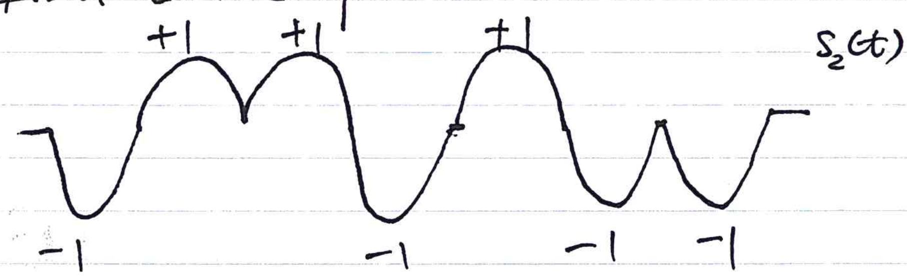
rectangular pulse



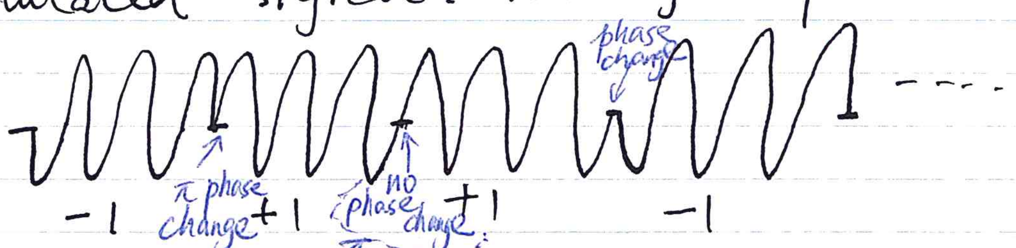
Ts: symbol duration

$R_s = \frac{1}{T_s}$: symbol rate or Baud rate

Half ~~Raised~~ Cosine pulse



Modulated signal: rectangular pulse



π phase change

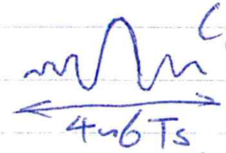
$-\cos x = \cos(x+\pi)$ ← phase change

Modulated signal: ~~half~~ half cosine pulse

rect. pulse

half cosine pulse

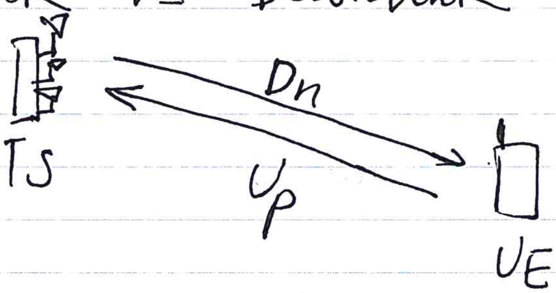
root raise cosine (RRC)



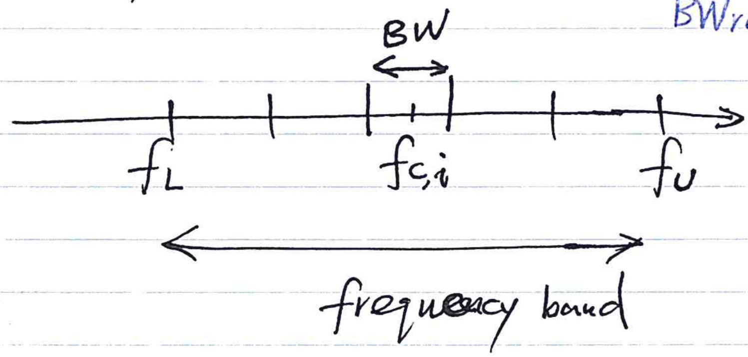
Digitally modulated signal's BW is largely determined by the BW of the shaping pulse.

$BW_{rect} > BW_{cosine} > BW_{RRC}$

- Up link vs Downlink



- frequency band



$$\#\text{channels} \simeq \frac{f_U - f_L}{BW}$$

- Tx power

Tx power determines the amplitude/magnitude of the modulated signal that is ~~radiated~~ fed ~~to~~ antenna for sent out.

$20\ \text{dBm} = 10^{\frac{20}{10}}\ \text{mW} = 10^2\ \text{mW} = 100\ \text{mW}$

$30\ \text{dBm} = 1\ \text{W}$

$-10\ \text{dBm} = 10^{\frac{-10}{10}}\ \text{mW} = 0.1\ \text{mW}$

$0\ \text{dBm} = 10^0\ \text{mW} = 1\ \text{mW}$

dB	linear
$X + 3\text{dB}$	$= 2X$
$X + 10\text{dB}$	$= 10X$

- Data rate   bps

  Data rate is directly related to BW and transmission power. Will be discussed more later.

- Link budget

  Link budget is related to the coverage distance that can be achieved with the received power above the minimum required for data detection.

Communication Principle Review

Take QPSK as example

Slides 30-31

At TX side.

$\underbrace{0\ 1}\ \underbrace{1\ 0}\ \cdots$

$\downarrow \qquad \downarrow$

$-d+jd \quad d-jd$

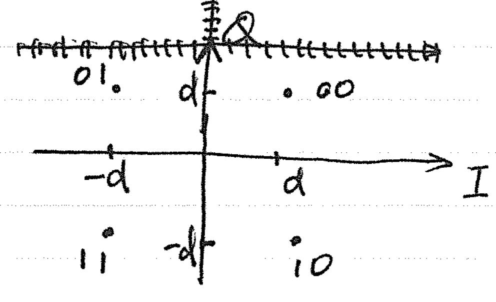
I: $-d$, d, ...

Q: d, $-d$, ...

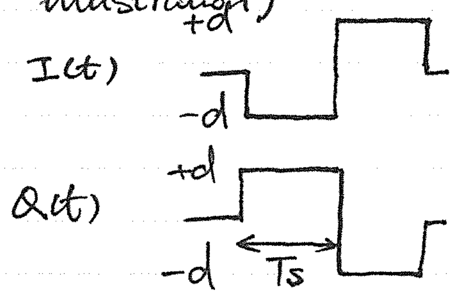
QPSK

· bit to symbol mapping rule

$$x_n = \begin{cases} d+jd & 00 \\ -d+jd & 01 \\ +d-jd & 10 \\ -d-jd & 11 \end{cases}$$



Let $p(t)$ be a rectangular pulse (for ease of illustration)



$$s(t) = I(t)A\cos\omega_c t - Q(t)A\sin\omega_c t$$

1st symbol $\quad -dA\cos\omega_c t - dA\sin\omega_c t$

$$= -dA(\cos\omega_c t + \sin\omega_c t) = -dA\sqrt{2}\cos(\omega_c t - \tfrac{\pi}{4})$$
$$= \sqrt{2}dA\cos(\omega_c t + \tfrac{3\pi}{4})$$

2nd symbol $\quad dA\cos\omega_c t - (-d)A\sin\omega_c t$

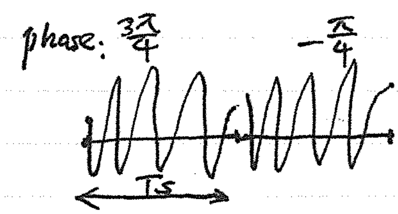
$$= dA(\cos\omega_c t + \sin\omega_c t)$$
$$= \sqrt{2}dA\cos(\omega_c t - \tfrac{\pi}{4})$$

transmit power (average)

$$P_{tx} = \frac{(\sqrt{2}dA)^2}{2} = d^2A^2$$

$$E_b = \frac{(\sqrt{2}dA)^2}{2}\cdot\frac{T}{2} = \frac{d^2A^2}{2}T$$

$s(t)$



One Ts sends 2 bits

$R_b = 2R_s = \frac{2}{T_s}$ bit rate

$T_b = \frac{T_s}{2}$ bit duration

At RX side, linear convolution

$$r(t) = s(t) * h(t) + n(t)$$

If the channel is only additive white Gaussian noise (AWGN)
$h(t) = \delta(t)$

$$r(t) = s(t) + n(t)$$

Assume lowpass filter (LPF) has coefficient of 1 for the passing band

1st symbol, for the I branch, ignoring noise

$$(r(t)A\cos\omega_c t)_{LPF} = (-dA^2\cos^2\omega_c t - dA^2\sin\omega_c t\cdot\cos\omega_c t)\Big|_{LPF}$$

$= \left(-dA^2 \cdot \frac{1+\cos 2w_c t}{2} - \frac{dA^2}{2}\sin 2w_c t\right)\Big|_{LPF}$

$= -\frac{dA^2}{2}$

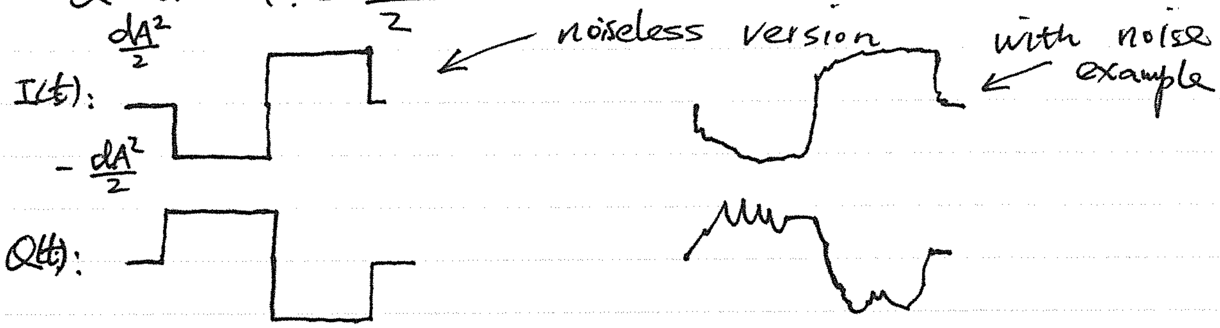
for the Q branch

$(r(t)\, A \sin w_c t)\big|_{LPF} = \frac{dA^2}{2} \leftarrow dA^2 \cdot \sin^2 w_c t\big|_{LPF} = \frac{dA^2}{2}(1-\cos 2w_c t)\Big|_{LPF}$

Similarly, 2nd symbol

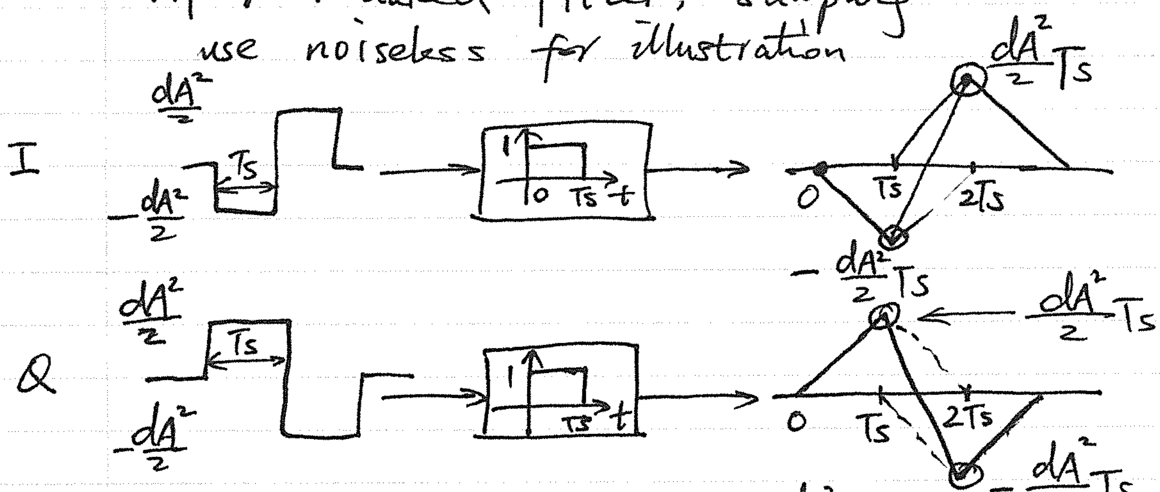
I branch : $\frac{dA^2}{2}$

Q branch : $-\frac{dA^2}{2}$



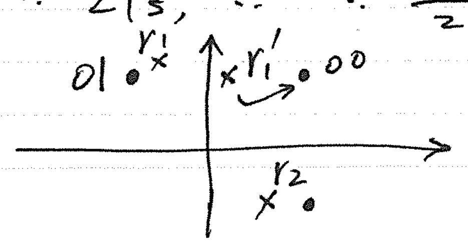
After matched filter, sampling

use noiseless for illustration



At time Ts, we get $-\frac{dA^2}{2}T_s$ for I and $\frac{dA^2}{2}T_s$ for Q

.. .. 2Ts, ... $\frac{dA^2}{2}T_s$ for I and $-\frac{dA^2}{2}T_s$ for Q



$r_1 = -\frac{dA^2}{2}T_s + j\frac{dA^2}{2}T_s \quad r_1' = r_1 + n_1$

$r_2 = \frac{dA^2}{2}T_s - j\frac{dA^2}{2}T_s \quad r_2' = r_2 + n_2$

- With noise, if we actually get sample points r_1 for 1st symbol at T_s and r_2 at $t = 2T_s$.

  r_1 is closest to signal point 01, so we determine 01 was sent. (this is called maximum likelihood rule) ML rule

  r_2 is closest to signal point 10, " " " 10 was sent.

- If we actually get sample points r_1', because of large noise, 00 will be decided as the tx data, and then an error occurs.

∴ It is important how large the noise is. In other words, the signal-to-noise ratio (SNR) is a key benchmark to error performance.

In digital communications, SNR is defined as

$SNR = \frac{E_b}{N_0}$ $\quad$ E_b : Average energy per bit

$\quad$ N_0 : Noise spectral density

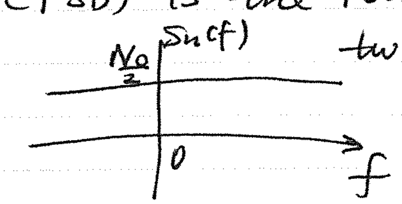
$n(t)$ is a random process with wide sense stationary property and zero mean, uncorrelated samples at two different time instants.

$E[n(t)] = 0$ $\quad$ white noise

$R_n(\tau) = E[n(t)n(t+\tau)] = \frac{N_0}{2}\delta(t)$ autocorrelation function

power spectral density (PSD) is the Fourier transform of $R_n(\tau)$.

$S_n(f) = \frac{N_0}{2}, \ -\infty < f < \infty$



two sided

$N_0 = kT$ k: Boltzmann's constant in Joules per kelvin
T: temperature in kelvins.

$k = 1.38 \times 10^{-23}$ J/K

$T = 288.4K$ (15°C)

$N_0 = 3.98 \times 10^{-21}$ J (W/HZ)

$= 3.98 \times 10^{-18}$ mW/HZ

$= -174$ dBm/HZ

noise power $P_n = N_0 B$ B: bandwidth $\propto \frac{1}{T_s}$

$$\frac{E_b}{N_0} = \frac{E_b \cdot B}{N_0 \cdot B} \Rightarrow \frac{E_b/T_s}{P_n} \propto \frac{P_t}{P_n}$$

To model the received signal (discrete) after MF and sampling,

$$y = x_i + n$$

where $E[|x_i|^2] = E_s$ $\boxed{E_s \text{ is energy per symbol}}$

$n = n_c + j n_s$; n_c and n_s are i.i.d. Gaussian

$n_c \sim N(0, \sigma_n^2)$, $n_s \sim N(0, \sigma_n^2)$

$\sigma_n^2 = \frac{N_0}{2}$

or $n \sim CN(0, N_0)$

Often SNR is defined as

$$SNR = \frac{\text{average received signal energy per symbol time}}{\text{noise energy per symbol time}}$$

So $E[|x_i|^2] = P$

$$SNR = \frac{E[|x_i|^2]}{\sigma_n^2} = \frac{P}{N_0}$$