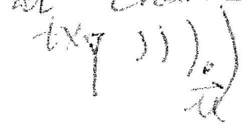
# Wireless Propagation Channels

- Path loss, shadowing and multipath
  See figure 2.1 in the slide

## I. Ray tracing method to look at channel

(Tse's book) – Free space, fixed tx antenna




$\boxed{\text{tx signal: } \cos 2\pi f t}$

Electric far field at time $t$, at point $\vec{u} = (r, \theta, \psi)$

$$E(f, t, (r, \theta, \psi)) = \frac{\alpha_s(\theta, \psi, f) \cos 2\pi f (t - \frac{r}{c})}{r} \quad V/m$$

where $\alpha_s(r, \theta, \psi)$: tx ant. radiation pattern at frequency $f$ in the direction of $(\theta, \psi)$

$c$ speed of light

– The delay at $\vec{u}$ is $\frac{r}{c}$, the phase change due to delay is $f\frac{r}{c}$

$E$ decreases with $r^{-1}$, the power per square meter decreases as $r^{-2}$.

Now place a rx antenna at $\vec{u}$. The received waveform (without noise) is

$$Er(f, t, \vec{u}) = \frac{\alpha(\theta, \psi, f) \cos 2\pi f (t - \frac{r}{c})}{r} \qquad h(t, f) = \frac{\alpha(\theta, \psi, f)}{r} \delta(t - \frac{r}{c})$$

where $\alpha(\theta, \psi, f)$: product of the antenna patterns of tx and rx antennas in the direction $(\theta, \psi)$ and frequency $f$.

Or tx signal $x(t) = e^{j2\pi f t}$

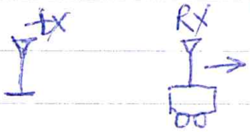
rx signal $y(t) = \frac{\alpha(\theta, \psi, f)}{r} e^{-j2\pi f \frac{r}{c}}$ $\quad x(t) = h(t) * x(t)$

Only for $e^{j2\pi ft}$ input

$$\therefore \; h(t, f) = \frac{\alpha(\theta, \psi, f)}{r} e^{-j2\pi f \frac{r}{c}} \quad \delta(t) \qquad x(t) * \delta(t) = x(t))$$

The channel is an linear time invariant (LTI) system

- Free space, moving antenna




Rx antenna moves with speed $v$ in the direction of increasing distance from the tx antenna.

$\vec{u}(t) = (r(t), \theta, \psi) \qquad r(t) = r_0 + vt$

$$E(f, t, (r_0+vt, \theta, \psi)) = \frac{\alpha_s(\theta,\psi,f)\cos 2\pi f\left(t - \frac{r_0}{c} - \frac{vt}{c}\right)}{r_0+vt}$$

$$= \frac{\alpha_s(\theta,\psi,f)\cos 2\pi\left[\left(f - \frac{vf}{c}\right)t - f\frac{r_0}{c}\right]}{r_0+vt}$$

The cosinewave has a new frequency $f - \frac{vf}{c} = f\left(1-\frac{v}{c}\right)$

$-\frac{vf}{c}$ is a Doppler shift due to the motion of the observation point.

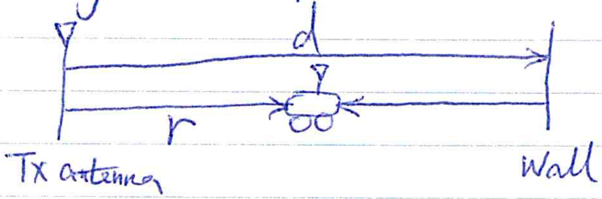
$\therefore$ The receive signal when a receive antenna is placed with moving speed $v$ away from tx antenna,

$$E_r(f, t, (r_0+vt, \theta, \psi)) = \frac{\alpha(\theta,\psi,f)\cos 2\pi\left[\left(f - \frac{vf}{c}\right)t - f\frac{r_0}{c}\right]}{r_0+vt}$$

where $\alpha = \alpha_s \cdot \alpha_r$

The system is not LTI. If $r_0+vt$ is considered as constant over the time of interest, the output can be considered as the input going through an LTI system and then translating in frequency by a Doppler shift.

- Reflecting wall, fixed antenna (two paths)




The 1st path length is $r$.

The 2nd path length is $d + (d-r) = 2d - r$

$$\therefore E_r(f,t) = \frac{\alpha\cos 2\pi f\left(t - \frac{r}{c}\right)}{r} - \frac{\alpha\cos 2\pi f\left(t - \frac{2d-r}{c}\right)}{2d-r}$$

reflection caused $\pi$ phase shift.

The two paths sinusoids add constructively if their phase difference is $2\pi$, destructively when the phase difference is $\pi$. (fading)

$$\Delta\Theta = 2\pi f \cdot \frac{2d-r}{c} + \pi - 2\pi f \frac{r}{c} = \frac{4\pi f}{c}(d-r) + \pi$$

The distance from a peak to a valley in $E_r$ is called coherence distance.

$$\frac{4\pi f}{c}(d-r_1) + \pi = 2\pi$$

$$\frac{4\pi f}{c}(d-r_2) + \pi = \pi$$

$$\Delta X_c = r_2 - r_1 = \frac{\lambda}{4} \qquad \lambda = \frac{c}{f}$$

Within much smaller distance than coherence distance, the received signal does not change significantly.

Similarly $\Delta f = \frac{1}{2}\Big(\underbrace{\frac{2d-r}{c} - \frac{r}{c}}_{Td}\Big)^{-1} = \frac{1}{2Td}$

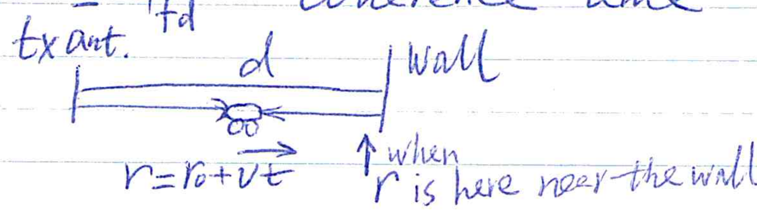
Td is delay spread.
Coherence bandwidth is defined as $\frac{1}{Td}$ related.

— Reflecting wall, moving antenna (two paths)
As the rx antenna is moving, it moves through the pattern of constructive and destructive interference of two waves, causing the received signal increases and decreases in strength. This is called multipath fading.

$$\Delta X_c = \frac{\lambda}{4} = \frac{c}{4f}$$

$$\Delta t = \frac{c}{4f \cdot v} \qquad f_d = \frac{vf}{c} = \frac{v}{\lambda} \text{ Doppler shift}$$

$$= \frac{1}{4f_d} \quad \text{coherence time}$$



Direct wave $\xrightarrow{\text{direct wave direction}}$

$\xrightarrow[v]{}$ rx antenna moving direction

Reflected wave $\longleftarrow$ reflected wave direction

$$E_r(f,t)=\frac{\alpha\cos 2\pi f[(1-\frac{v}{c})t-\frac{r_0}{c}]}{r_0+vt}-\frac{\alpha\cos 2\pi f[(1+\frac{v}{c})t\cancel{(r_0-2d)/c}-(2d-r_0)/c]}{2d-r_0-vt} \qquad (1)$$

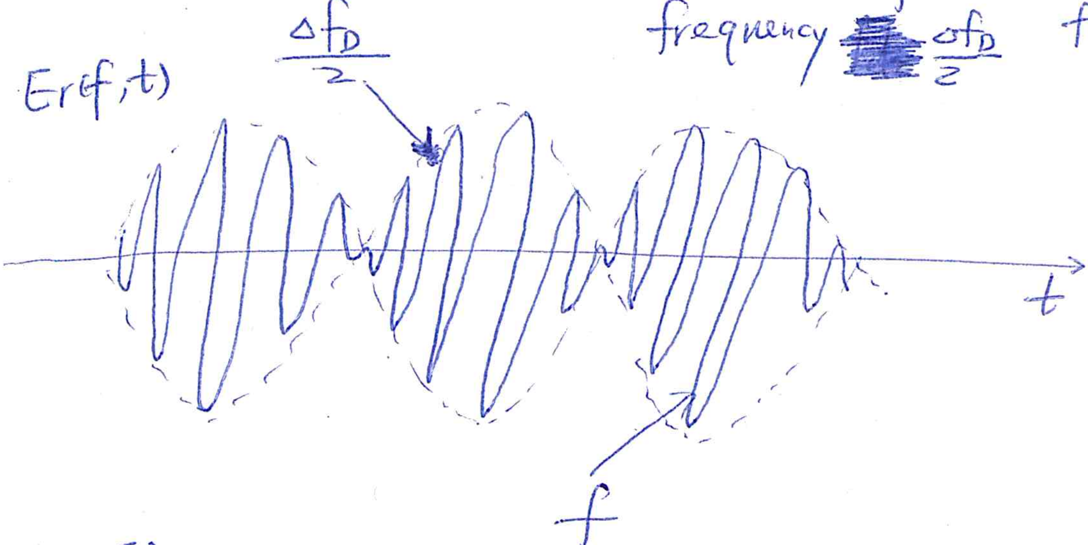
$$f_{d1}=-\frac{fv}{c} \qquad f_{d2}=\frac{fv}{c}$$

$$\Delta f_d=f_{d2}-f_{d1} = \frac{2fv}{c}$$ Doppler spread

For $r$ close to being at the wall, $(r_0+vt \simeq 2d-r_0 vt)$

$$E_r(f,t)\simeq\frac{2\alpha\sin 2\pi f[\frac{vt}{c}+(r_0-d)/c]\sin 2\pi f(t-\frac{d}{c})}{r_0+vt}$$

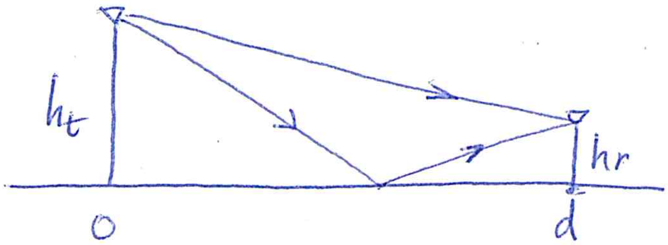
sine wave of frequency $\frac{\Delta f_D}{2}$ sine wave of frequency $f$




If $v=60km/h$, $f=900MHz$ $\Delta f_D=100Hz$

$\frac{\Delta f_D}{2}=50Hz$, the period of the envelope is $\frac{1}{50}s=20ms$.
The signal depicted above changes from peak amplitude to zero in 5ms.
The significant path length change on the denominator of (1) is related to $v$, on the level of seconds or minutes. Thus can be considered constant over ms.

– Groud reflection, 2-ray model




For $d \gg h_t, h_r$

the phase difference between 2 paths

$$\Delta\theta \simeq \frac{4\pi h_t h_r}{\lambda d} \propto d^{-1}$$

and the path length difference is $\propto d^{-1}$

The ground reflection path has reflection coefficient close to $-1$, so the reflected path is reversed in ~~phase~~ sign to the direct path, causing the two waves cancel each other. The received signal is attenuated as $d^{-2}$, and the received power is attenuated as $d^{-4}$.

See slide 11

– More complicated environment: extract major effects and parameters.

~~Power decay & shadowing~~

path length determines the delay and the phase.


Doppler shift

$f_d = \frac{v}{\lambda}\cos\theta$

# II. Modeling of Channels

Use mathematics modeling to describe the major characteristics of a wireless channel.

— Path loss & Shadowing

Only look at the large scale variation of the received signal power.

See slides for details

— Input/output model of the Wireless Channel

Previously sent $\cos 2\pi f t$, a sinusoid.

Now consider sending a signal with non-zero BW, $x(t)$

the received signal via multipath propagation

$$y(t) = \sum_i a_i(t)\, x(t - \tau_i(t))$$

$i$: path index

$a_i$: attenuation of i-th path

$\tau_i$: delay of i-th path

Note that $a_i(t)$ and $\tau_i(t)$ are written without $f$ in it. Strictly speaking, should be $a_i(f;t)$ and $\tau_i(f;t)$. Since $x(t)$ has BW usually much smaller than $f_c$, $a_i(t)$ and $\tau_i(t)$ can be approximated to drop $f$ in it.

Comments: 

- $a_i(t)$ include the distance related path loss, and shadowing.

  ~~multipath~~

  $\tau_i(t)$ ~~delay~~ include the path length related delay, and Doppler

$\frac{\phi}{2\pi f} = \tau$ any phase changes at the transmitter, reflector and receiver.

- From $x(t)$ and $y(t)$, we can get the time varying impulse response

$$h(\tau, t) = \sum_i a_i(t)\, \delta(\tau - \tau_i(t)) \xrightarrow[\tau \to f]{\tilde{F}} H(f, t) = \sum_i a_i(t) e^{-j2\pi f \tau_i(t)}$$

- When tx, rx, environment is stationary,

$$h(\tau) = \sum_i a_i\, \delta(\tau - \tau_i).$$

Typically ~~t~~ channel varies in time scale much larger than delay spread, and hence can be considered quasi-invariant.

- If the input signal $x(t)$ is a passband signal with center frequency $f_c$, the baseband equivalent input-output relationship is given by

$$y_b(t) = \sum_i a_i^b(t)\, x_b(t - \tau_i(t))$$

$$\therefore\ h_b(\tau, t) = \sum_i a_i^b(t)\, \delta(\tau - \tau_i(t))$$

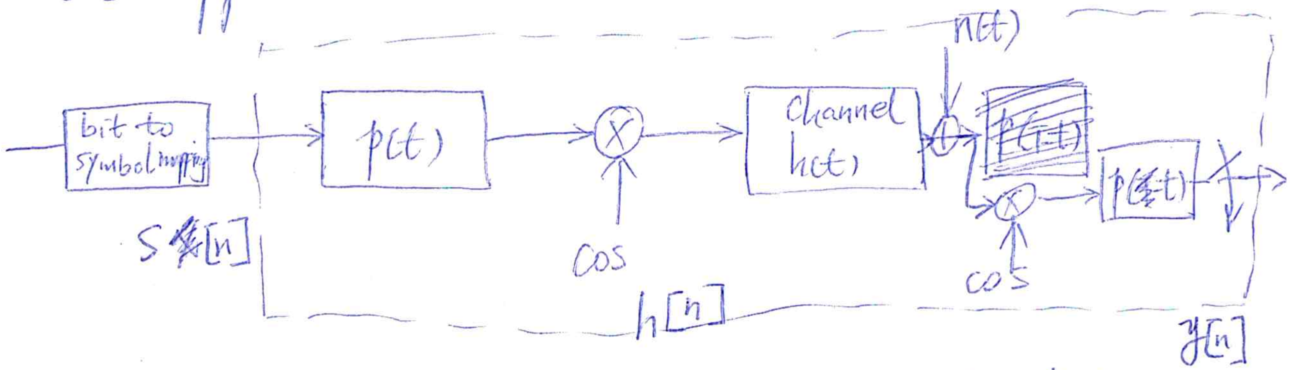
$$a_i^b(t) = a_i(t) e^{-j2\pi f_c \tau_i(t)} \quad \text{complex}$$

Delay caused phase change in included in $h(\tau, t)$.

We almost always use baseband equivalent representation in communication theory, and just use $h(\tau, t)$ to denote $h_b(\tau, t)$.

— Channel in discrete form

Recall the tx and rx, both ends of tx and rx are discrete symbols and samples. It is after pulse shaping at the tx, we have a baseband continous time signal. After upconversion, we have a bandpass signal. The opposite operations happen at the receiver.




$X(\tau)$ has BW W.

$T = \frac{1}{W}$

$$y[n] = h[n] * s[n] + v[n]$$

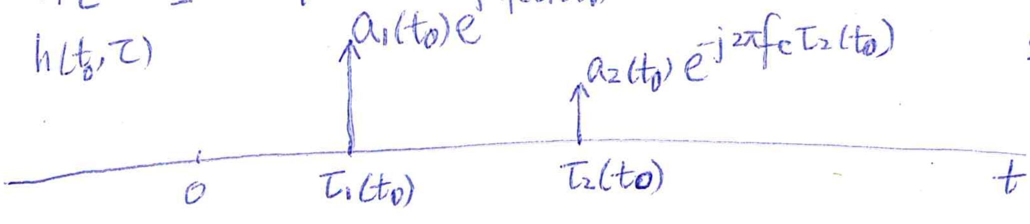
$$h[n,t] = h(t,\tau) * p(\tau) * p(-\tau) \Big|_{t=nT} = h(t,\tau) * x(\tau) \Big|_{\tau=nT_s}$$

Recall

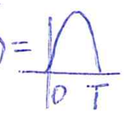
$$h(t,\tau) = \sum_i a_i(t) e^{-j2\pi f_c \tau_i(t)} \delta(\tau - \tau_i(t))$$

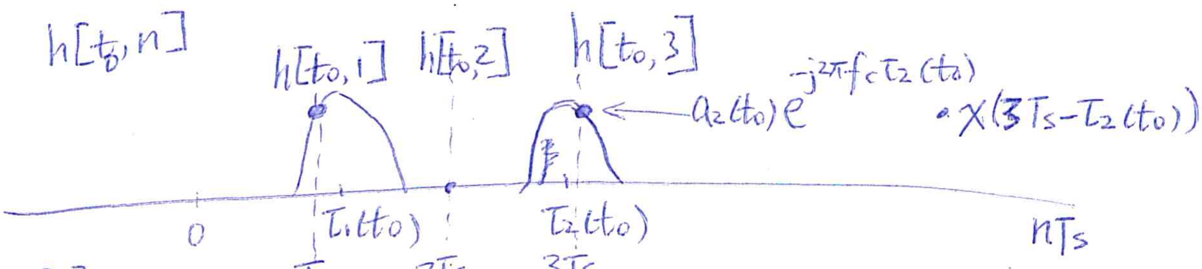
$$h[n,t] = \sum_i a_i(t) e^{-j2\pi f_c \tau_i(t)} x(\tau - \tau_i(t)) \Big|_{\tau=nT}$$

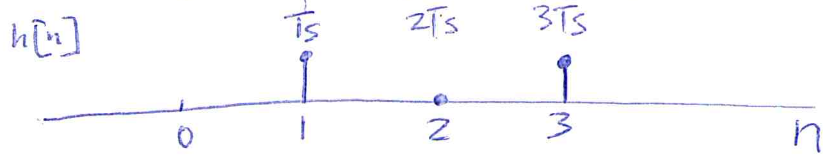
$t = t_0$




suppose $X(\tau) =$

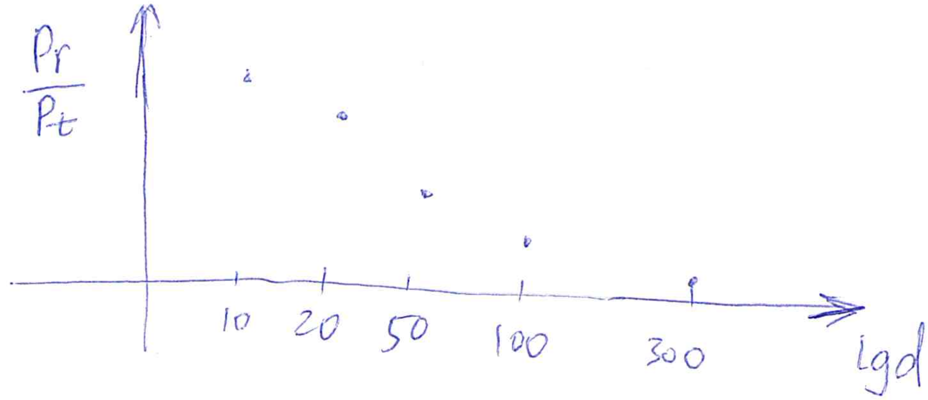


Ex 2.3

1a




$$\frac{P_t}{P_r} = PL\ (dB) = -K + 10\gamma \lg \frac{d}{d_0}$$

$d_0 = 1m$

$f_c = 900MHz$

$\lambda = 0.333\ldots$

$$-K = 20\lg\frac{4\pi}{\lambda} = 20\lg\frac{4\pi}{0.333} = 31.54dB$$

$$MSE = \sum_{i=1}^{5}\left[PL_i^m - (31.54 + 10\gamma \lg d_i)\right]^2$$

$$= (70 - 31.54 - 10\gamma\lg 10)^2 + (75 - 31.54 - 10\gamma\lg 20)^2$$

$$+ (90 - 31.54 - 10\gamma\lg 50)^2 + (110 - 31.54 - 10\gamma\lg 100)^2$$

$$+ (125 - 31.54 - 10\gamma\lg 300)^2$$

$$= 21676.3 - 11654.9\gamma + 1571.47\gamma^2$$

$$\frac{d\,MSE(\gamma)}{d\gamma} = -11654.9 + 3142.94\gamma = 0$$

$$\gamma = 3.71$$

Free space PL (dB) $= 20\lg \frac{4\pi d}{\lambda}$

Combined Path Loss and shadowing

$$\frac{P_r}{P_t} = 10\lg K - 10\gamma \lg \frac{d}{d_0} - \psi'_{dB} \qquad K = 20\lg \frac{4\pi d_0}{\lambda}$$

where $\psi'_{dB} \sim N(0, \sigma^2_{\psi_{dB}})$

In linear scale, $\qquad \psi'_{dB} = 10\lg \psi'$

$$P_r = P_t \cdot K \cdot \left(\frac{d}{d_0}\right)^{\gamma} \cdot \psi'$$

$t = t_1$

at time $t_1$, the channel is changed to 3 paths. The first two paths are closely spaced

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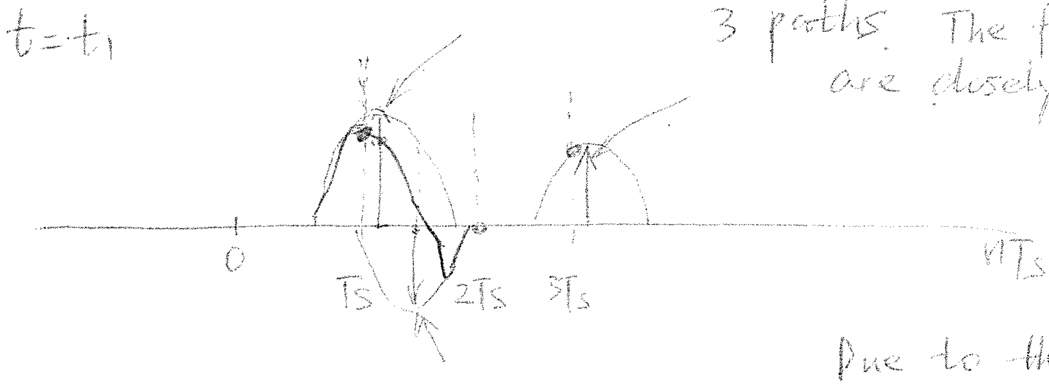


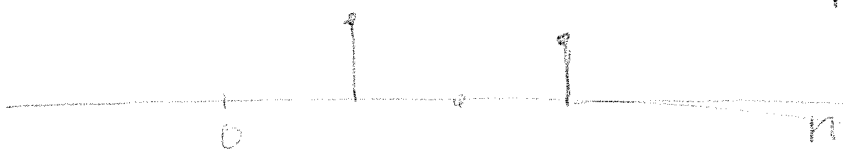


Due to the smooth out of $x(\tau)$, the equivalent discrete-time channel is two non-zero taps.

Observation

① Multipath propagation leads to each individual path with its own path length (delay), angle of depature and arrival, and Doppler shift. The aggregate effect is multiple channel taps, each with delay and fading effect, and the channel taps changes with time due to Doppler effect.

② A Propagation channel has its intrinsic multipath profile that is, each path has unique delay, AOA, AOD, Doppler and attenuation. When some paths are closely spaced in delay, and the spacing is less than the system pulse width ($\frac{1}{W}$), these multipaths are added constructively or destructively. For paths that are spaced farther apart than the system pulse, they are still resovlable as separate paths. So the effect of system bandwidth W is that when the transmitted pulse is very narrow, (i.e., large W), the multipath channel becomes more resolvable in closely spaced paths. It is like sounding the channel with higher resolution.

$$h[n,t] = \sum_{l=0}^{L-1} h_l^t \, \delta[n-l]$$

where $L$ is the # of channel taps, $T_D = l$
$h_l^t$ is the complex fading coefficient of the $l$-th tap at time $t$.

Special cases

- frequency flat fading, $L=1$

$$h[n,t] = h_0^t$$ the channel is just a random multiplicative factor.

- slow flat fading, $L=1$, time invariant

$$h[n,t] = h_0$$

* The ~~speed~~ rate of slow or fast fading is determined by the Doppler ~~spread~~ shift.

Cohence time $\propto \frac{1}{f_D}$ $\quad$ $f_D T$: normalized maximum Doppler ~~spread~~ shift.

- frequency selective fading

$L > 1$, multi-tap channel model.

Coherence bandwidth $\propto \frac{1}{T_D}$ $\quad$ $T_D$: normalized delay spread

$= \frac{\tau_{max}}{T}$

$= \tau_{max} \cdot W$

| Tse | Goldsmith |
| --- | --- |
| – coherence time | coherence time |
| $T_c = \frac{1}{4D_s}$ | $T_c = \frac{0.4}{f_D}$ |
| $D_s$ is the Doppler spread ($D_s = 2f_D$ if there are paths from both directions) | $f_D$ is the maximum Doppler shift |

A channel is fast or slow fading depending on the relationship between $T_c$ and $T_s$ (symbol interval)

| | |
| --- | --- |
| – Coherence bandwidth | coherence bandwidth |
| $W_c = \frac{1}{2T_d}$ | $B_c = \frac{0.2}{\sigma_T}$ |
| $T_d$ is the max delay spread | $\sigma_T$ is the rms delay spread |

A channel is frequency flat or selective depending on the relationship between $W_c/B_c$ and $W/B_s$ (system bandwidth)

See Fig. 2.13, Table 2.1, Table 2.2 of Tse's book.