Theorem 4 (Jensen's Inequality 1906) Let f be a convex function on the interval I. If $x_1, x_2, \ldots, a_n \in I$ and $t_1, t_2, \ldots, t_n$ are nonnegative real numbers such that $t_1 + t_2 + \ldots + t_n = 1$, then

$$f(\sum_{i=1}^{n} t_i x_i) \leq \sum_{i=1}^{n} t_i f(x_i).$$

Proof by induction: The case for $n = 2$ is true by the definition of convex. Assume the relation holds for n, then we have

$$\begin{aligned}
f\left(\sum_{i=1}^{n+1} t_i x_i\right) &= f\left(\sum_{i=1}^{n} t_i x_i + t_{n+1} x_{n+1}\right) = f\left(t_{n+1} x_{n+1} + (1 - t_{n+1}) \frac{1}{1 - t_{n+1}} \sum_{i=1}^{n} t_i x_i\right) \\
&\leq t_{n+1} f(x_{n+1}) + (1 - t_{n+1}) f\left(\frac{1}{1 - t_{n+1}} \sum_{i=1}^{n} t_i x_i\right) \\
&= t_{n+1} f(x_{n+1}) + (1 - t_{n+1}) f\left(\sum_{i=1}^{n} \frac{t_i}{1 - t_{n+1}} x_i\right) \\
&\leq t_{n+1} f(x_{n+1}) + (1 - t_{n+1}) \sum_{i=1}^{n} \frac{t_i}{1 - t_{n+1}} f(x_i) \\
&= \sum_{i=1}^{n} t_i f(x_i) + t_{n+1} f(x_{n+1}) \\
&= \sum_{i=1}^{n+1} t_i f(x_i).
\end{aligned}$$

Thus showing that the assumption implies that the relation holds for $n + 1$ and by the principle of Mathematical Induction holds for all natural numbers.