ECE519B/496A Selected Topics

MIMO and UWB Communications

Part 3 Wireless Channel Modeling

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Representation of Bandpass Signals

- Wireless systems transmit and receive *real bandpass* signals with bandwidth $2B$ centered at carrier frequency f_c.

  e.g. QPSK signal over a symbol period T_s

$$s_i(t) = A\cos\left[\frac{2\pi(i-1)}{4}\right]g(t)\cos 2\pi f_c t - A\sin\left[\frac{2\pi(i-1)}{4}\right]g(t)\sin 2\pi f_c t$$

  where $g(t)$ is the pulse shaping function, assumed to be unit square wave of length T_s.

- To facilitate analysis, represent a real bandpass signal as the real part of a complex signal

$$s(t) = \mathrm{Re}\{u(t)e^{j2\pi f_c t}\} = s_I(t)\cos 2\pi f_c t - s_Q(t)\sin 2\pi f_c t$$

  where $u(t) = s_I(t)+js_Q(t)$ is baseband complex envelop, $s_I(t)$ is the in-phase component and $s_Q(t)$ is the quadrature component.

Note: $u(t)$ contains all the information in $s(t)$ except for f_c.

Effects of Multipath Wireless Channel

- Transmitted bandpass signal

$$s(t) = \mathrm{Re}\{u(t)e^{j2\pi f_c t}\}$$

- Received signal over multipath channel

$$r(t) = \mathrm{Re}\left\{\sum_{n=0}^{N(t)} \alpha_n(t)u(t-\tau_n(t))e^{j2\pi f_c(t-\tau_n(t))}\right\}$$

$$= \mathrm{Re}\left\{\left[\sum_{n=0}^{N(t)} \alpha_n(t)e^{-j\phi_n(t)}u(t-\tau_n(t))\right]e^{j2\pi f_c t}\right\}$$

where $\varphi_n(t) = 2\pi f_c \tau_n(t)$, $N(t)$ is the number of multipath, $\alpha_n(t)$ is the amplitude and $\tau_n(t)$ is the delay for the *n*th path.

- Consider complex baseband channel

$$\tilde{r}(t) = \sum_{n=0}^{N(t)} \alpha_n(t) e^{-j\phi_n(t)} u(t - \tau_n(t))$$

=> Channel impulse response

$$c(\tau, t) = \sum_{n=0}^{N(t)} \alpha_n(t) e^{-j\phi_n(t)} \delta(\tau - \tau_n(t))$$

Linear time-variant (LTV) systems

Doppler Effect

- Phase variation due to Doppler effect governs the rate of channel variation.
- Doppler frequency shift on the *n*th path caused by the relative movement.

$$f_{D_n}(t) = \frac{v(t)\cos\theta_n(t)}{\lambda}$$

where $v(t)$ is the moving speed, $\theta_n(t)$ is angle between incident wave and moving direction, $\lambda = c/f_c$ is the wavelength.

- Maximum Doppler shift is $f_D = v/ \lambda$.
- Considering all possible directions, Doppler effect expands signal bandwidth by $\pm f_D$.
- If signal bandwidth $B >> f_D$, Doppler effect negligible and channel variation considered slow => Slow fading channel.
- Knowledge of the channel can be explored to improve system performance and efficiency.

Channel Coherence Time

- Characterize time-domain variation.
- *Coherence time* T_c: time duration that c(τ, t) remains roughly constant.
- T_c approximately estimated as $0.4/f_D$.
- Fast/slow fading
  - If $T_c >> T_s$ (symbol period), fading is considered slow.
  - If $T_c \approx T_s$, we have fast fading.

- For slow fading, channel deemed not time varying for a short duration of interest

$$\tilde{r}(t) = \sum_{n=0}^{N} \alpha_n e^{-j\phi_n} u(t - \tau_n)$$

=> Channel impulse response

$$c(t) = \sum_{n=0}^{N} \alpha_n e^{-j\phi_n} \delta(t - \tau_n)$$

Linear time invariant (LTI) systems

Multipath Delay Spread

- Multipath signals experience different delays => time domain dispersion in received signal.
- When the dispersion is significant compared to signal symbol period T_s,
  - Inter-symbol interference (ISI) introduced.
  - Frequency selectiveness in frequency domain.
- Otherwise, ISI is negligible and channel frequency response considered flat.
- Needs proper characterization of delay spread.

- *Power delay profile* (PDP): plot of average path power α_n^2 versus path delay τ_n.
- Quantify time dispersion based on PDP.
- Maximum delay spread

$$T_m = \max_{i,j}\{|\tau_i - \tau_j|\}$$

fails to take into account power contribution of different paths.

Channel Coherence Bandwidth

- With the consideration of power distribution, *RMS delay spread* σ_T

$$\sigma_T = \sqrt{\frac{\sum_{n=0}^{N} \alpha_n^2 (\tau_n - \mu_T)^2}{\sum_{n=0}^{N} \alpha_n^2}}$$

where average delay spread μ_T is

$$\mu_T = \frac{\sum_{n=0}^{N} \alpha_n^2 \tau_n}{\sum_{n=0}^{N} \alpha_n^2}$$

- Coherence bandwidth B_c: bandwidth that frequency response remains roughly constant.
- B_c inverse proportional to σ_T and estimated as $B_c \approx 0.2/\sigma_T$.
- Frequency flat/selective fading
  - If $\sigma_T << T_s$ or equivalently $B_c >> B_s$, frequency flat fading.
  - Otherwise, frequency selective fading.

Frequency-flat Slow Fading Channel

- $T_s >> \sigma_T$ and ignore the common delay
$$c(\tau) = \left(\sum_{n=0}^{N} \alpha_n e^{-j\phi_n}\right)\delta(\tau)$$
where $\varphi_n = 2\pi f_c \tau_n$.
- Fading channel introduce a complex gain
$$z = \sum_{n=0}^{N} \alpha_n e^{-j\phi_n}$$
- Received complex envelope is
$$\tilde{r}(t) = zu(t) + n(t)$$

where $u(t)$ is the transmitted complex envelope and $n(t)$ is the Gaussian noise.

- $z = z_I + jz_Q$, where

$$z_I = \sum_{n=0}^{N} \alpha_n \cos\phi_n, \quad z_Q = \sum_{n=0}^{N} \alpha_n \sin\phi_n$$

- Apply central limit theorem (CLT), z_I and z_Q are modeled as independent Gaussian random variables.

Rayleigh Fading

- Received signal power is proportional to $|z|^2$.
- For NLOS, z_I and z_Q are Gaussian distributed with zero mean and variance σ^2 (isotropic scattering).
- Channel amplitude $|z| = \sqrt{z_I^2 + z_Q^2}$ has distribution function

$$p_{|z|}(x) = \frac{x}{\sigma^2}\exp\left[-\frac{x^2}{2\sigma^2}\right], \quad x \geq 0$$

=> $|z|$ is a *Rayleigh random variable*.

- Then $|z|^2$ is exponentially distributed, i.e.,

$$p_{|z|^2}(x) = \frac{1}{2\sigma^2}\exp\left[-\frac{x}{2\sigma^2}\right], \quad x \geq 0$$

where $2\sigma^2 = \overline{P}_r$ is the average received signal power, i.e. based on path loss/average shadowing.

- Channel phase $\theta = \arctan(z_Q/z_I)$ is uniformly distributed over $[0, 2\pi]$.

Well-known Rayleigh Fading Channel Model.

Slow Flat Fading with LOS

- When LOS exists, z_I and z_Q are Gaussian distributed with nonzero mean m_1 and m_2.
- Channel amplitude $|z| = \sqrt{z_I^2 + z_Q^2}$ has distribution function

$$p_{|z|}(x) = \frac{x}{\sigma^2} \exp\left[-\frac{x^2 + s^2}{2\sigma^2} \right] I_0\left(\frac{xs}{\sigma^2} \right)$$

where $s^2 = m_1^2 + m_2^2 = \alpha_0^2$ is received power from LOS, $2\sigma^2$ is the average power of NLOS components.

– $I_0()$ is the modified Bessel function of zeroth order.

=> $|z|$ is a *Rician random variable*.

- Average received power $\overline{P_r} = s^2 + 2\sigma^2$
- Rician fading parameter $K = s^2/2\sigma^2$.
- Alternative Nakagami fading model developed by fitting empirical measurements

$$p_{|z|}(x) = \frac{2m^m x^{2m-1}}{\Gamma(m)\overline{P_r}} \exp\left[-\frac{mx^2}{\overline{P_r}}\right]$$

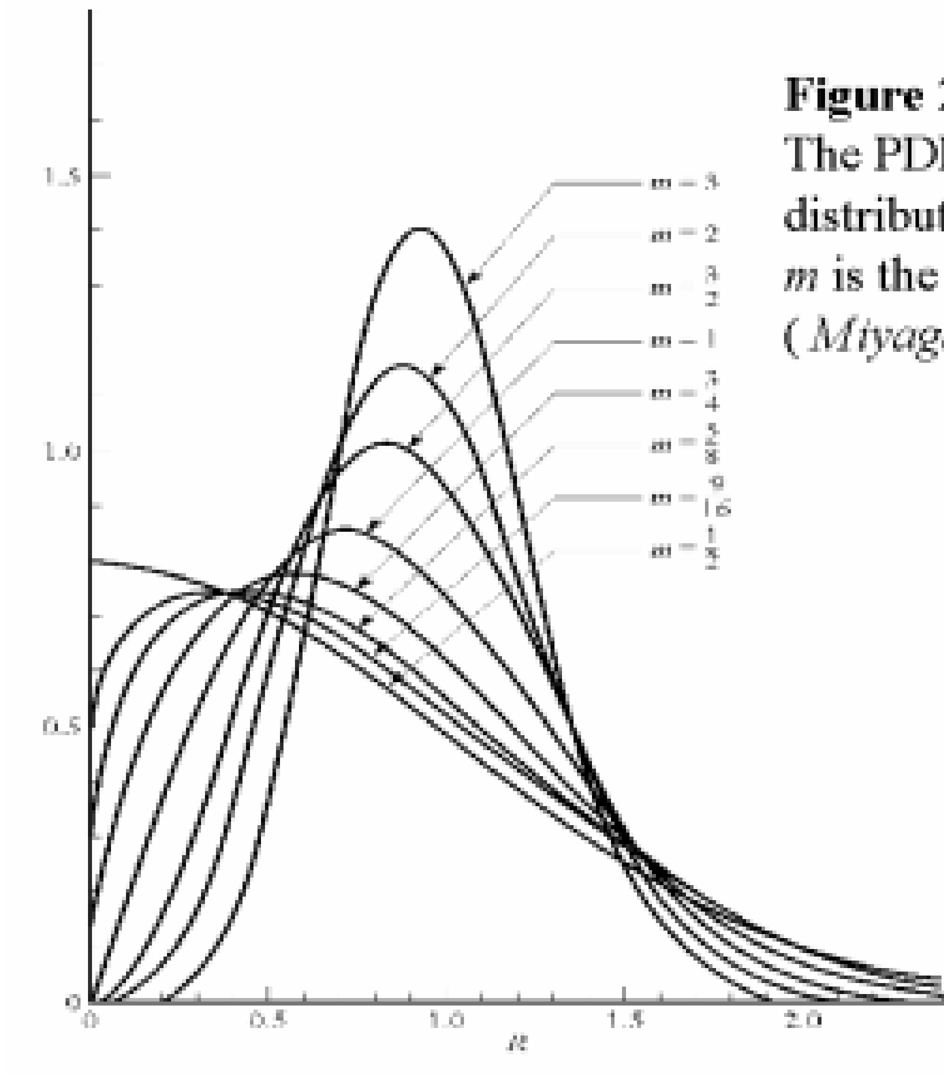
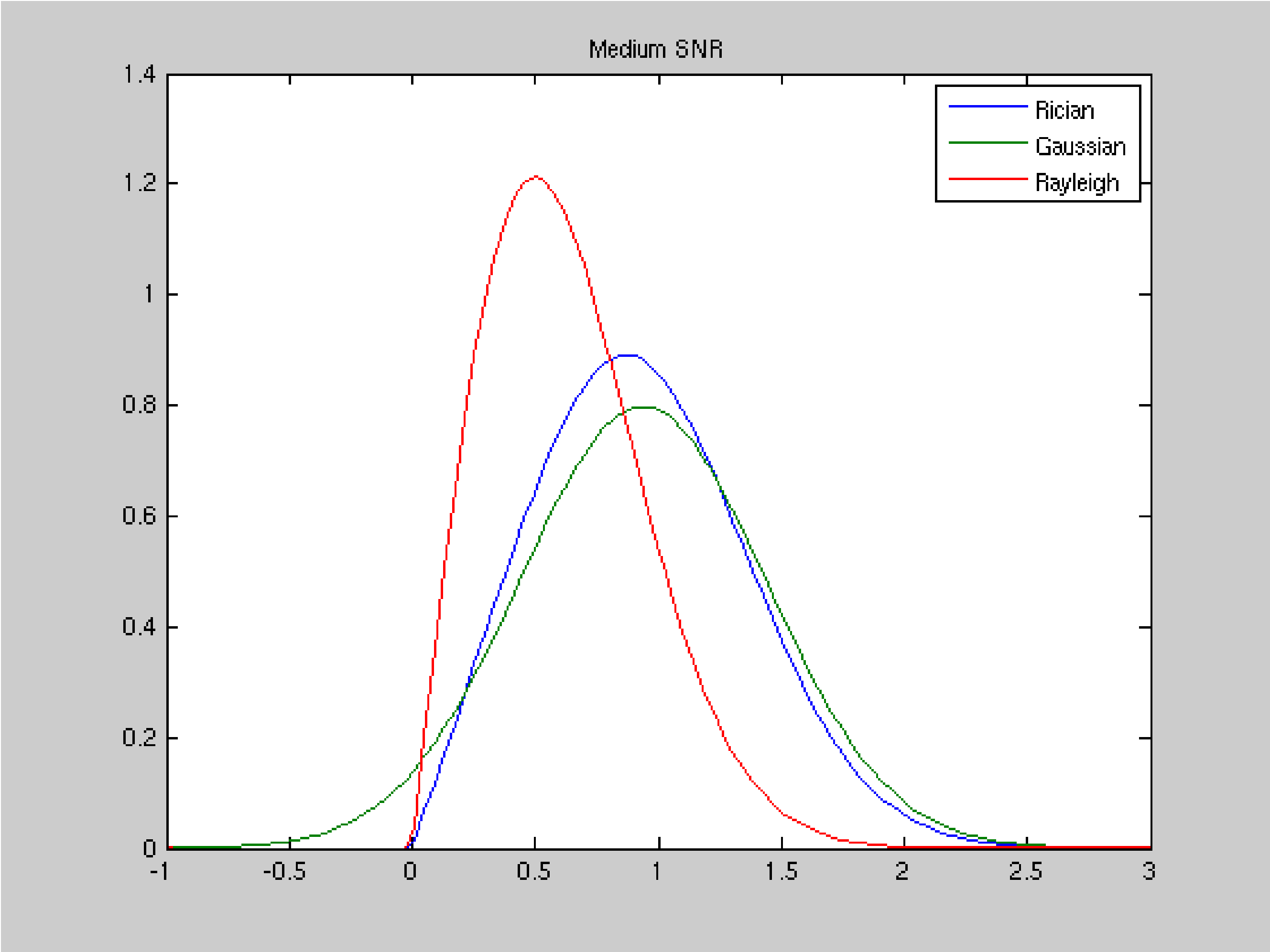


Figure 2.1-10
The PDF for the Nakagami-*m* distribution, shown with Ω = 1. *m* is the fading figure.
(*Miyagaki et al., 1978*)



Rice/Rician distribution *by [Ged Ridgway]*
from MATLAB Central

Frequency-flat Fast Fading Channel

- Channel impulse response

$$c(t,\tau) = \left(\sum_{n=0}^{N} \alpha_n e^{-j\phi_n(t)}\right)\delta(\tau)$$

where $\varphi_n(t) = 2\pi f_c \tau_n - 2\pi f_{Dn} t$.

- Fading channel introduce a complex gain

$$z(t) = \sum_{n=0}^{N} \alpha_n e^{-j\phi_n(t)} = z_I(t) + j z_Q(t)$$

- Received complex envelope is

$$\tilde{r}(t) = z(t)u(t) + n(t)$$

- Assuming isotropic scattering, the auto-correlation function

$$E[z_I(t)z_I(t+\tau)] = E[z_Q(t)z_Q(t+\tau)] = \overline{P}_r J_0(2\pi f_D \tau)$$

  where $J_0()$ is the Bessel function of the zeroth order. Decorrelates over roughly half a wavelength.

  Moreover, the cross-correlation

$$E[z_I(t)z_Q(t+\tau)] = E[z_Q(t)z_I(t+\tau)] = 0$$

  The in-phase and quadrature components of the fading are independent.

- $f_D T_s$ determines the fading rate.

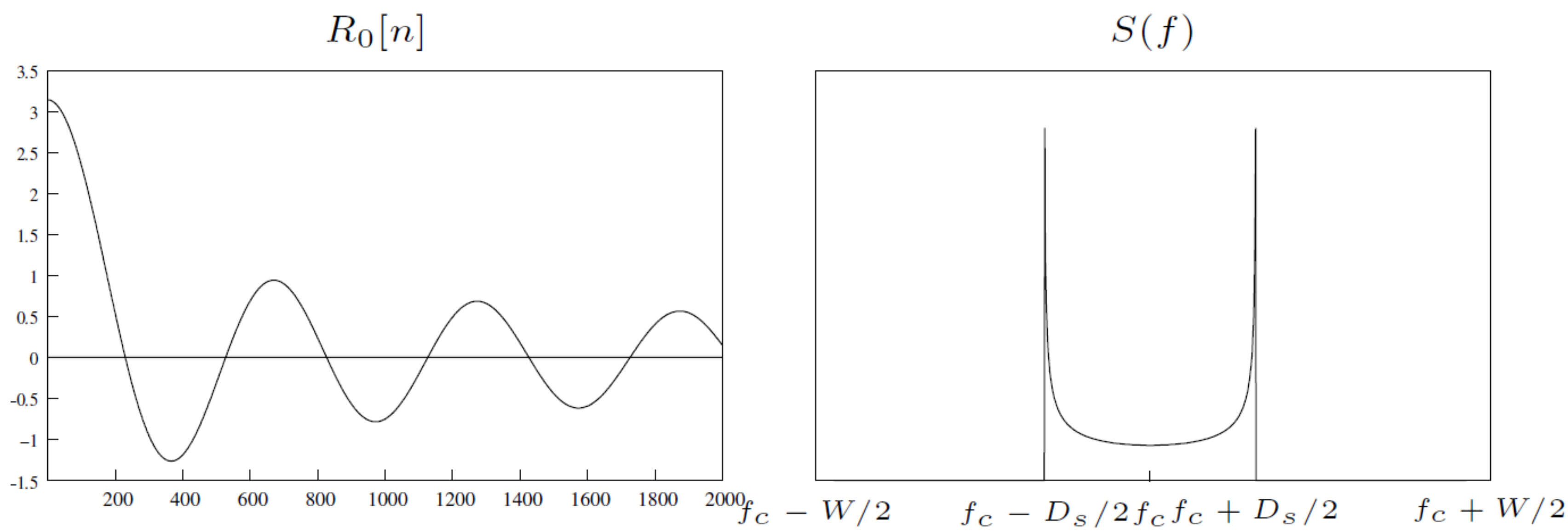


Figure 2.15: Plots of the autocorrelation function and Doppler spectrum in Clarke's model.

Frequency-selective Fading

- Applicable to wideband system $B_s >> B_c$.
- Receiver can resolve paths with delay difference greater than $1/ B_s$.
- Discrete-time model for selective fading
  - Divide the τ axis into slots of duration $\Delta\tau = 1/ B_s$,
  - Assume the same delay $i\Delta\tau$ for paths arrived in slot i, $i = 0, 1, \ldots, L - 1$,

– Model composite gain for paths in slot *i*

$$z_i = \sum_{n=0}^{N_i} \alpha_n e^{-j\phi_n}$$

– Arrive at the tapped delay line model

$$c(\tau) = \sum_{i=0}^{L-1} z_i \delta(\tau - i\Delta\tau)$$

- z_i follows frequency flat fading model, i.e. $|z_i|$ follows Rayleigh, Rician or Nakagami distribution.

Properties of Random Variables

- Captured in distribution functions of *X*.
  - Probability density function (pdf) $p_X(x)$: likelihood of *X* taking values around *x*.
  - Cumulative distribution function (cdf):

$$P_X(z) = \int_{-\infty}^{z} p_X(x)dx$$

i.e. the probability that the value of *X* is smaller than *z*.

- Mean or expected value of *X*:

$$\mu_X = \int_{-\infty}^{+\infty} x p_X(x) dx$$

- Variance of *X*:

$$\sigma_X^2 = \int_{-\infty}^{+\infty} (x - \mu_X)^2 p_X(x) dx$$

- Function of *X*, *Y* = g(*X*), has pdf

$$p_Y(y) = \left. \frac{p_X(x)}{dg(x)/dx} \right|_{x=g^{-1}(y)}$$

if g(·) is a one-to-one function.