Consider a time-invariant indoor wireless channel with LOS component at delay 23 nsec with mean power 0.7, a multipath component at delay 48 nsec with mean power 0.2, and another multipath component at delay 67 nsec with mean power 0.1. Find the mean delay spread, rms delay spread and maximum delay spread, assuming the demodulator synchronizes to the LOS component.

Consider a high-speed data signal with bandwidth .5 MHz and a data rate of .5 Mbps. The signal is transmitted over a wireless channel with a delay spread of 10 μsec. If multicarrier modulation with nonoverlapping subchannels is used to mitigate the effects of ISI, approximately how many subcarriers are needed? What is the data rate and symbol time on each subcarrier? (We do not need to eliminate the ISI completely. So Ts = Tm is enough)

3. $\mu_T = \frac{\sum_n \alpha_n^2 \tau_n}{\sum_n \alpha_n^2} = \frac{0.7\times 23 + 0.2\times 48 + 0.1\times 67}{0.7+0.2+0.1} = 32.4\,ns$

$\sigma_T = \sqrt{\frac{\sum_n \alpha_n^2 (\tau_n-\mu_T)^2}{\sum_n \alpha_n^2}} = \sqrt{0.7\times(23-32.4)^2 + 0.2\times(48-32.4)^2 + 0.1\times(67-32.4)^2}$

$= 15.17\ ns$

$\tau_{max} = 67\,ns$

4. $T_s = \frac{1}{0.5}\mu s = 2\mu s \qquad B = \frac{1}{T} = 0.5\,MHz$

$T_m = 10\mu s = 5T_s \qquad \Delta f = \frac{1}{T_m} = \frac{1}{5T} = \frac{1}{10}MHz = 0.1\,MHz$

$\#$ of subcarriers $= \frac{B}{\Delta f} = 5$

Under a free space path loss model, find the transmit power required to obtain a received power of 1 dBm for a wireless system with isotropic antennas ($G_l = 1$) and a carrier frequency $f = 5$ GHz, assuming a distance $d = 10m$. Repeat for $d = 100m$.

Q2-1

$P_r = P_t\left[\frac{\sqrt{G_l}\lambda}{4\pi d}\right]^2 \qquad \lambda = \frac{c}{f} = \frac{3\times10^8}{5\times10^9} = 0.06\,m$

$G_l = 1,\ P_r = 1\,dBm = P_t(dBm) - PL$

$- d = 10m, \quad = 1.259mW$

$PL = 20\lg\frac{4\pi d}{\sqrt{G_l}\lambda} = 20\lg\frac{4\pi\cdot 10}{0.06} = 66.4\ dB$

$P_t = P_r + PL = 1\ dBm + 66.4 = 67.4dBm = 5.495\,kW$

$- d = 100m$

$PL = 20\lg\frac{4\pi\times100}{0.06} = 86.4\ dB$

$P_t = 1 + 86.4 = 87.4dBm = 549.5\,kW$

Consider a receiver with noise power -160 dBm within the signal bandwidth of interest. Assume a simplified path loss model with $d_0 = 1$ m, K obtained from the free space path loss formula with omnidirectional antennas and $f_c = 1$ GHz, and $\gamma = 4$. For a transmit power of $P_t = 10$ mW, find the maximum distance between the transmitter and receiver such that the received signal-to-noise power ratio is 20 dB.

$K = -20\lg\frac{\lambda}{4\pi d_0} = -20\lg\frac{c/f_c}{4\pi\cdot1} = -20\lg\frac{3\times10^8}{10^9\times4\pi} = -20\lg\frac{3}{40\pi}$

$= 32.44\ dB$

SNR in dB form

$P_r - P_{noise} = 20\ dB = P_r + 160\ dBm \quad \therefore P_r = -140\,dBm$

$P_r = P_t - K - PL' \quad \therefore$ the room left for PL is

$P_t = 10mW = 10\lg10 = 10\,dBm \qquad PL' = P_t - K - P_r = 10 - 32.44 - (-140)$

$= 150 - 32.44 = 117.56\ dB$

(2)

$PL' = 10\gamma\lg\frac{d}{d_0} = 40\lg\frac{d}{1} = 117.56$

$\therefore d = 10^{\frac{117.56}{40}} = 869\ m$

Find the coverage area for a microcellular system where path loss follows the simplied model with $\gamma = 3$, $d_0 = 1$, and $K = 0$ dB and there is also log normal shadowing with $\sigma = 4$ dB. Assume a cell radius of 100 m, a transmit power of 80 mW, and a minimum received power requirement of $P_{min} = -100$ dBm.

Assume a Rayleigh fading channel with the average signal power $2\sigma^2 = -80$ dBm. What is the power outage probability of this channel relative to the threshold $P_0 = -95$ dBm? How about $P_0 = -90$ dBm?

4. Q2-23

$C = Q(a) + e^{\frac{2-2ab}{b^2}} Q\left(\frac{2-ab}{b}\right) \qquad (2)$

where $a = \frac{P_{min} - P_t + PL(d_0) + 10\gamma\lg\frac{R}{d_0}}{\sigma_{dB}}$

$b = \frac{10\gamma\lg(e)}{\sigma_{dB}}$

$\gamma = 3,\ d_0 = 1\ \ K = 0\ dB\ \ R = 100\ \ P_{min} = -100\,dBm$

$P_t = 80mW = 19\ dBm \qquad \sigma_{dB} = 4\ dB$

$a = \frac{-100 - 19 + 0 + 10\cdot3\lg100}{4} = -14.75$

$b = \frac{30\lg e}{4} = 3.2572 \qquad \therefore C = 1$ using (2)

What is the energy inside the above rectangular pulse?

Parseval's theorem

$E_p = \int_{-\infty}^{\infty} p^2(t)\,dt = \int_0^T A^2 dt = A^2T = \int_{-\infty}^{\infty} |P(f)|^2 df$

5. Q3-b

$|z|^2$ is exponentially distributed, when $|z|$ is a Rayleigh r.v.

$P_{out} = Pr(|z|^2 < P_0) \qquad 2\sigma^2 = -80dBm = 10^{-8}\,mW$

$P_0 = -95dBm = 10^{-9.5}\,mW$

$= \int_0^{P_0} \frac{1}{2\sigma^2} e^{-\frac{x}{2\sigma^2}} dx$

$= -e^{-\frac{x}{2\sigma^2}}\Big|_0^{P_0} = 1 - e^{-\frac{P_0}{2\sigma^2}}$

$P_0 = 10^{-9.5}mW \qquad P_{out} = 1 - e^{-10^{-1.5}} = 0.0311$

$P_0 = -90dBm = 10^{-9}mW \qquad P_{out} = 1 - e^{-0.1} = 0.0952$

The following table lists a set of empirical path loss measurements.

Distance from Transmitter	P_r/P_t
5 m	-60 dB
25 m	-80 dB
65 m	-105 dB
110 m	-115 dB
400 m	-135 dB
1000 m	-150 dB

Find the parameters of a simplified path loss model plus log normal shadowing that best fit this data. Find the path loss at 2 Km based on this model.Find the outage probability at a distance d assuming the received power at d due to path loss alone is 10 dB above the required power for nonoutage.

(a) $\underset{(dB)}{P_r - P_t} = -K - 10\gamma\lg\frac{d}{d_0}$ (1), where $K = 20\lg\frac{4\pi d_0}{\lambda}$

Since this question did not give λ nor d_0, and d_0 is needed in (1), so we assume $d_0 = 1m$ here

$P_r - P_t = -K - 10\gamma\lg d = a + b\cdot\lg d$ Find a and b that lead to minimum MSE

$6\cdot MSE = (a+b\lg5+60)^2 + (a+b\lg25+80)^2 + (a+b\lg65+105)^2 + (a+b\lg110+115)^2 + (a+b\lg400+135)^2 + (a+b\lg1000+150)^2$

$\frac{dMSE}{da} = 0 \Rightarrow 6a + 11.5533b + 645 = 0$

$\frac{dMSE}{db} = 0 \Rightarrow 11.5533a + 25.6675b + 1380.2 = 0$

$\therefore a = -29.702 \qquad -K = a \ \therefore K = 29.702\ dB$

$b = -40.403 \qquad -10\gamma = b \ \therefore \gamma = 4$

(b) PL at 2km $= +K + 10\gamma\lg2000 = 29.702 + 40\lg2000$

$= 161.7\ dB$

(path loss is a positive number in dB. Path gain is a negative number in dB)

(c) First find out the standard deviation of the log-normal shadowing.

$\sigma_{\psi dB}^2 = MSE = 55.476516 \qquad \psi_{dB} \sim \mathcal{N}(0, \sigma_{\psi dB})$

$\sigma_{\psi dB} = \sqrt{MSE} = 3.0407$

$P_{out} = P(P_r(d) \le P_{min}) \qquad P_r = \underbrace{P_t - PL}_{=10+P_{min}} - \psi$

$= P(P_t - PL - \psi < P_{min})$

$= P(10 + P_{min} - \psi < P_{min})$

$= P(\psi > 10) = Q\left(\frac{10}{\sigma_{\psi dB}}\right) = 5.0323\times10^{-4}$

Q function in Matlab is qfunc.m

What is the spectrum of the above rectangular pulse $p(t)$? Explain the relationship of the zero crossing bandwidth and the pulse duration T.

A 16 quadrature amplitude modulation has symbols $\pm d \pm jd, \pm d \pm j3d, \pm 3d \pm jd, \pm 3d \pm j3d$. Plot its constellation and the corresponding minimum Euclidean distance decision boundaries for receiver detection. Gray mapping is the type of bit to symbol mapping where the nearest neighbor symbols are encoded with bit patterns that differ by one bit only. Try to label the symbols with bit patterns that follow the Gray mapping rule.

2. The spectrum of $p(t)$ is the fourier transform of $p(t)$.

$q(t) \xrightarrow{\mathcal{F}} AT\,\text{sinc}(fT)$

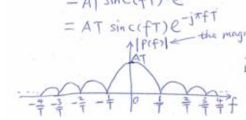
$\text{sinc}\,x = \frac{\sin\pi x}{\pi x}$

$P(f) = \mathcal{F}\{p(t)\} = \mathcal{F}\{q(t-\frac{T}{2})\}$

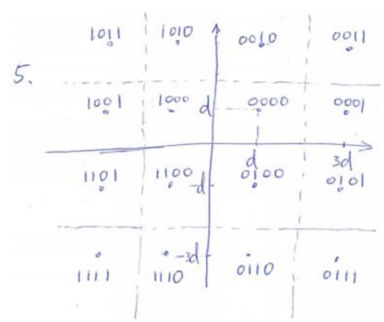
$= AT\,\text{sinc}(fT)\cdot e^{-j2\pi f\frac{T}{2}}$

$= AT\,\text{sinc}(fT)e^{-j\pi fT}$

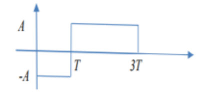
sinc$(fT) = 0$ when $fT = k$, $k \in \mathbb{Z}$ integer



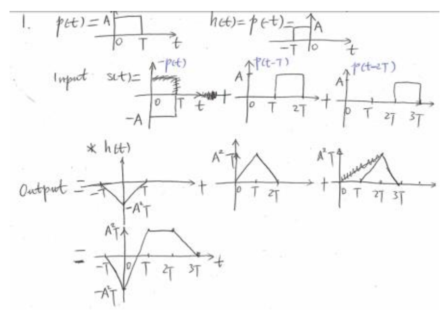
5.



Show the output of a matched filter with impulse response $h(t) = p(-t)$, where $p(t)$ is a rectangular pulse with amplitude A and duration T, when the input is the following binary phase shift keying signal of 3 bit long.



1.



Propagation mechanisms
- Direct line of sight (LOS)
- Reflection, by wall or terrain
- Diffraction, by edges of objects
- Scattering, through smaller spaces

Popular models to predict path loss
- Free space model: too simple.
- Ray-tracing model (two-ray here): requires site-specific information.
- Empirical models: not always generalizable.
- Log-distance (simplified) model: good for high-level analysis.