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324CC

Tema 1 - AA

1) a)

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even_until_neg(int A[][], int n)                    | Operatie
    for i = 0 ... n-1                               | 1
        for j = 0 ... n-1                           | 2
            if A[i][j] < 0                          | 3
                return                              | 4
            if A[i][j] % 2 == 0                     | 5
                print A[i][j] + '␣'                 | 6
```

b)

Operație	Cost	Repetiții
1	C1	t ($1 \le t \le n$; există cel puțin un nr. neg.)
2	C2	$\sum_{1}^{t-1}(n+1) + 1$ ($1 \le 1 \le n$)
3	C3	$\sum_{1}^{t-1}(n) + 1$
4	C4	1
5	C5	$\sum_{1}^{t-1}(n) + 1 - 1$
6	C6	u ($0 \le u \le \sum_{1}^{t-1}(n) + 1 - 1$)

Explicații:

C1: for-ul nu se va executa o dată în plus, deoarece este garantată de cerință prezența unui număr negativ ce va opri funcția

C2, C3, C5: operațiile se vor executa de un număr fix de ori pentru liniile din matrice ce nu conțin numere negative ($\sum_{1}^{t-1}$(n sau n+1)) și se vor executa de un număr variabil de ori pentru linia unde se află primul număr negativ (adică s = poziția pe linie a primului număr negativ din matrice)

C6: u = câte numere pare sunt până la primul număr negativ

Toate operațiile:

$$T(n) = t \cdot C_1 + \left[\sum_{1}^{t-1}(n+1) + 1\right] \cdot C_2 + \left[\sum_{1}^{t-1}(n) + s\right] \cdot C_3 + C_4 + \left[\sum_{1}^{t-1}(n) + s - 1\right] \cdot C_5 + u \cdot C_6$$

Cazul cel mai favorabil:

$$\left.\begin{array}{l} t = 1 = \text{nr. neg. se află pe prima linie} \\ s = 1 = \text{nr. neg. se află pe prima coloană a primei linii} \end{array}\right\} \Rightarrow$$

$\Rightarrow u = 0$

$$T(n) = C_1 + C_2 + C_3 + C_4 + 0 \cdot C_5 + 0 \cdot C_6 =$$

$$= (C_1 + C_2 + C_3 + C_4) \cdot 1 = \boxed{\Theta(1)}$$

Cazul cel mai puțin favorabil:

$t = n$ = primul nr. neg e pe ultima linie

$s = n$ = primul nr. neg e pe ultima coloană

$u = n^2 - 1$ = toate numerele sunt pare înafară de ultimul

$$T(n) = n \cdot C_1 + [(n-1)(n+1) + n] \cdot C_2 +$$
$$[(n-1) \cdot n + n] \cdot C_3 + C_4 + [(n-1) \cdot n + n - 1] \cdot C_5 +$$
$$(n^2 - 1) \cdot C_6$$

$$T(n) = n \cdot C_1 + (n^2 + n - 1) \cdot C_2 + n^2 \cdot C_3 + C_4 +$$
$$+ (n^2 - 1) \cdot C_5 + (n^2 - 1) \cdot C_6$$

$$T(n) = n^2 (C_2 + C_3 + C_5 + C_6) + n(C_1 + C_2) -$$
$$-(C_2 - C_4 + C_5 + C_6) = \boxed{\Theta(n^2)}$$

Cazul mediu:

$$\left.\begin{aligned} t &= \frac{n+1}{2} \\ s &= \frac{n+1}{2} \end{aligned}\right\} \Rightarrow$$ primul element din matrice negative se află în mijloc

$u = \cancel{0 + \frac{n^2-1}{2} \ldots \frac{n^2}{2}} = \frac{n^2}{4}$ = jumătate din elementele parcurse sunt pare

$$T(n) = \frac{n+1}{2} \cdot C_1 + \left[\sum_{1}^{\frac{n-1}{2}} (n+1) + \frac{n+1}{2} \right] \cdot C_2 +$$
$$+ \left[\sum_{1}^{\frac{n-1}{2}} (n) + \frac{n+1}{2} \right] \cdot C_3 + C_4 + \left[\sum_{1}^{\frac{n-1}{2}} (n) + \frac{n+1-1}{2} \right] \cdot C_5 +$$
$$+ C_6 \cdot \frac{n^2}{4}$$

$$T(n) = \frac{n+1}{2} \cdot C_1 + \left[\frac{n^2-1}{2} + \frac{n+1}{2}\right] \cdot C_2 +$$

$$+ \left[\frac{n^2-n}{2} + \frac{n+1}{2}\right] \cdot C_3 + C_4 + \left(\frac{n^2-n}{2} + \frac{n+1}{2} - 1\right) \cdot C_5 +$$

$$+ C_6 \cdot \frac{n^2}{4}$$

$$T(n) = n^2\left(\frac{C_2}{2} + \frac{C_3}{2} + \frac{C_5}{2} + \frac{C_6}{4}\right) + n\left(\frac{C_1}{2} + \frac{C_2}{2}\right) +$$

$$+ \left(\frac{C_1}{2} + \frac{C_3}{2} + C_4 - C_5\right) = \Theta(n^2)$$

<u>Operații critice</u>:

<u>Cazul cel mai favorabil</u>:

$$\left.\begin{aligned} t &= 1 \\ s &= 1 \\ u &= 0 \end{aligned}\right\} \Rightarrow$$ Operațiile critice sunt C_3 și C_4

$$T(n) = 1 \cdot C_3 + 1 \cdot C_4$$

$$T(n) = 1 \cdot (C_3 + C_4) = \boxed{\Theta(1)}$$

<u>Cazul cel mai puțin favorabil</u>:

$$\left.\begin{aligned} t &= n \\ s &= n \\ u &= n^2 - 1 \end{aligned}\right\} \Rightarrow$$ Operațiile critice sunt C_3

$$T(n) = (n^2 - n + n) \cdot C_3 = n^2 \cdot C_3 = \boxed{\Theta(n^2)}$$

Cazul mediu:

$$\left.\begin{aligned} t &= \frac{n+1}{2} \\ s &= \frac{n+1}{2} \\ u &= \frac{n^2}{4} \end{aligned}\right\} \Rightarrow \text{Operațiile critice sunt: } C_3$$

$$T(n) = \left(\frac{n^2-n}{2} + \frac{n+1}{2}\right) \cdot C_3$$

$$T(n) = \left(\frac{n^2+1}{2}\right) \cdot C_3$$

$$T(n) = \left(\frac{C_3}{2}\right) \cdot n^2 + \left(\frac{C_3}{2}\right) \cdot 1 = \boxed{\Theta(n^2)}$$

Caz / Metodă	Fave	Mediu	Nefave.
Toate op.	$\Theta(1)$	$\Theta(n^2)$	$\Theta(n^2)$
Op. critice	$\Theta(1)$	$\Theta(n^2)$	$\Theta(n^2)$

2) Dem.: $\Theta(2n^3+n^2)=\Theta(n^3)$

<u>Verificăm dacă $2n^3+n^2=\Theta(n^3)$</u>

Această relație ar implica că $\exists\, c_1, c_2 \in \mathbb{R}_+^*$ și $\exists\, n_0 \in \mathbb{N}^*$ a. î. $c_1 \cdot n^3 \le 2n^3+n^2 \le c_2 \cdot n^3 \ \forall n \ge n_0$

Fie $c_1=1$ și $c_2=3$ (ambele aparțin R_+^*) $\Rightarrow$

$\Rightarrow n^3 \le 2n^3+n^2 \le 3n^3 \ \forall n \ge n_0 \Rightarrow \begin{cases} n^3 \le 2n^3+n^2 \ \forall n \ge n_0 \\ 2n^3+n^2 \le 3n^3 \ \forall n \ge n_0 \end{cases} \Rightarrow$

$\Rightarrow \begin{cases} 0 \le n^3+n^2 \ \forall n \ge n_0 \\ n^2 \le n^3 \ \forall n \ge n_0 \end{cases} \Rightarrow n_0 = 1 \in \mathbb{N}^*$

Deci $\exists\, c_1=1,\ c_2=3 \in \mathbb{R}_+^*$ și $\exists\, n_0 = 1 \in \mathbb{N}^*$ a. î.

$c1 \cdot n^3 \le 2n^3+n^2 \le c_2 \cdot n^3 \ \forall n \ge n_0 \Rightarrow$

$\Rightarrow 2n^3+n^2 = \Theta(n^3) \xRightarrow{\text{Simetrie}} n^3 = \Theta(2n^3+n^2)$

<u>Demonstrăm că $\Theta(2n^3+n^2) \subseteq \Theta(n^3)$</u>

$\left.\begin{matrix} \text{Fie } f(n) = \Theta(2n^3+n^2) \\ 2n^3+n^2 = \Theta(n^3) \end{matrix}\right\} \xRightarrow{\text{Tranzitivitate}} f(n) = \Theta(n^3) \Rightarrow$

$\Rightarrow \Theta(2n^3+n^2) \subseteq \Theta(n^3)$ (1)

<u>Demonstrăm că $\Theta(n^3) \subseteq \Theta(2n^3+n^2)$</u>

$\left.\begin{matrix} \text{Fie } f(n) = \Theta(n^3) \\ n^3 = \Theta(2n^3+n^2) \end{matrix}\right\} \xRightarrow{\text{Trans.}} f(n) = \Theta(2n^3+n^2) \Rightarrow$

$\Rightarrow \Theta(n^3) \subseteq \Theta(2n^3+n^2)$ (2)

(1), (2) $\Rightarrow \Theta(2n^3+n^2) = \Theta(n^3)$

3) a)

$$T(n) = \begin{cases} k_1, & n = 1 \\ 4 \cdot T(\lfloor n/4 \rfloor) + k_2 \cdot n, & n > 1 \end{cases}$$

Metoda iterativă:

$T(n) = 4T(n/4) + \theta(n)$; $T(1) = \theta(1)$

Unde:
$n = 4^{k+1}$
$k+1 = \log_4 n$
$k = \log_4(n) - 1$

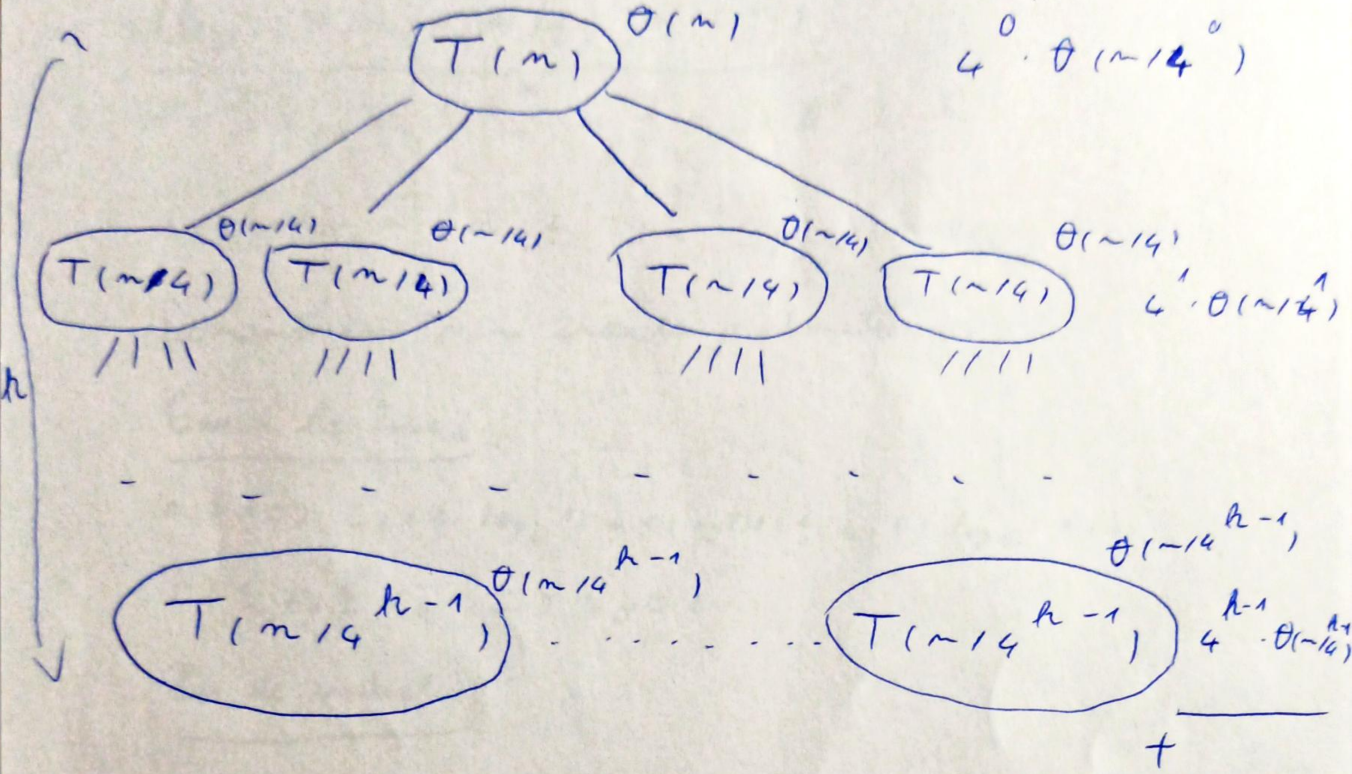
$$T(n) = \cancel{4T(n/4)} + \theta(n)$$
$$\cancel{T(n/4)} = 4\cancel{T(n/4^2)} + \theta(n/4) \mid \cdot 4$$
$$\cancel{T(n/4^2)} = 4\cancel{T(n/4^3)} + \theta(n/4^2) \mid \cdot 4^2$$
$$\cdots$$
$$\cancel{T(n/4^k)} = 4T(n/4^{k+1}) + \theta(n/4^k) \mid \cdot 4^k$$

$+$

$$T(n) = 4^{k+1} T(n/4^{k+1}) + \sum_{i=0}^{k} \theta(n/4^i) \cdot 4^i$$
$$T(n) = 4^{k+1} \cdot \theta(1) + \sum_{i=0}^{k} \theta(n)$$
$$T(n) = n \cdot \theta(1) + (k+1) \cdot \theta(n)$$
$$T(n) = \theta(n) + \log_4 n \cdot \theta(n)$$
$$T(n) = \theta(n \cdot \log_4 n + n)$$
$$\boxed{T(n) = \theta(n \cdot \log_4 n) = \theta(\tfrac{1}{2} n \lg n) = \theta(n \lg n)}$$

Metoda arborelui de recurență:

$T(n) = 4T(n/4) + \Theta(n)$; $T(1) = \Theta(1)$



$T(n) = \sum_{i=0}^{h-1} 4^i \cdot \Theta(n/4^i)$ unde

$$\boxed{\begin{aligned} 4^{h-1} &= n \\ h-1 &= \log_4 n \\ h &= \log_4(n) - 1 \end{aligned}}$$

$T(n) = \sum_{i=0}^{h-1} \Theta(n)$

$T(n) = h \cdot \Theta(n)$

$T(n) = (\log_4(n) - 1) \cdot \Theta(n)$

$T(n) = \Theta(n \log_4(n) - n)$

$$\boxed{T(n) = \Theta(n \log_4(n)) = \Theta(\tfrac{1}{2} n \lg n) = \Theta(n \lg n)}$$

Metoda substituției:

$T(n) = 4T(n/4) + k_2 n$; $T(1) = k_1$

Alegem $T(n) = \Theta(n \log_4 n + n)$

$\Rightarrow \exists\, c_1, c_2 \in \mathbb{R}_+^*$ și $\exists\, n_0 \in \mathbb{N}^*$ a. î.

$c_1(n \log_4 n + n) \leq T(n) \leq c_2(n \log_4 n + n)$

Demonstrăm prin inducție matematică:

Cazul de bază:

$n = 1 \Rightarrow c_1(1 \cdot \log_4 1 + 1) \leq T(1) \leq c_2(1 \cdot \log_4 1 + 1)$

$c_1 \leq k_1 \leq c_2 \Rightarrow n_0 = 1$

Pas de inducție:

$n/4 \rightarrow n$

Ipoteza de inducție:

$c_1\left(\frac{n}{4} \cdot \log_4 \frac{n}{4} + \frac{n}{4}\right) \leq T(n/4) \leq c_2\left(\frac{n}{4} \cdot \log_4 \frac{n}{4} + \frac{n}{4}\right)$

Arătăm că:

$c_1(n \cdot \log_4 n + n) \leq T(n) \leq c_2(n \log_4 n + n)$

Plecând de la ipoteză:

$c_1\left(\frac{n}{4} \cdot \log_4 \frac{n}{4} + \frac{n}{4}\right) \leq T(n/4) \leq c_2\left(\frac{n}{4} \log_4 \frac{n}{4} + \frac{n}{4}\right) \Big| \cdot 4 \Big| + k_2 n$

$k_2 n + 4c_1\left(\frac{n}{4} \log_4 \frac{n}{4} + \frac{n}{4}\right) \leq 4T(n/4) + k_2 n \leq 4c_2 \cdot$

$\left(\frac{n}{4} \log_4 \frac{n}{4} + \frac{n}{4}\right) + k_2 n$

$$c_1(n \cdot \log_4 \frac{n}{4} + n) + k_2 n \leq T(n) \leq c_2 \cdot (n \log_4 \frac{n}{4} + n) + k_2 n$$

$$c_1 n \log_4 n + c_1 \cdot n - c_1 \cdot n + k_2 n \leq T(n) \leq$$

$$c_2 n \log_4 n + c_2 \cdot n - c_2 \cdot n + k_2 n$$

$$c_1 n \log_4 n + c_1 n + n(k_2 - c_1) \leq T(n) \leq$$

$$c_2 n \log_4 n + c_2 n + n(k_2 - c_2)$$

Dar:

$$\begin{cases} c_1 n \log_4 n + c_1 n + n(k_2 - c_1) \geq c_1 n \log_4 n + c_1 n \\ \quad \text{dacă } k_2 - c_1 \geq 0 \\ c_2 n \log_4 n + c_2 n + n(k_2 - c_2) \leq c_2 n \log_4 n + c_2 n \\ \quad \text{dacă } k_2 - c_2 \leq 0 \end{cases}$$

$$\Rightarrow \begin{cases} k_2 - c_1 \geq 0 \\ k_2 - c_2 \leq 0 \end{cases} \Rightarrow \begin{cases} k_2 \geq c_1 \\ k_2 \leq c_2 \end{cases} \Rightarrow c_1 \leq k_2 \leq c_2 \Rightarrow$$

$\Rightarrow \exists\ c_1 = \min(k_1, k_2)$ și $c_2 = \max(k_1, k_2) \in \mathbb{R}_+^*$

și $\exists\ n_0 = 1 \in \mathbb{N}^*$ a.î. $c_1(n \log_4 n + n) \leq T(n) \leq$
$c_2(n \log_4 n + n)\ \forall n \geq n_0 \Rightarrow$

$$\Rightarrow T(n) = \Theta(n \log_4 n + n) = \boxed{\Theta(n \log_4 n)} =$$

$$= \Theta(\tfrac{1}{2} n \lg n) = \Theta(n \lg n)$$

Metoda Master:

$T(n) = 4T(n/4) + \Theta(n)$; $\Theta(n) \to k_2 n$

$\left.\begin{array}{l} a = 4 \\ b = 4 \end{array}\right\} \Rightarrow n^{\log_b a} = n^{\log_4 4} = n$

$f(n) = \Theta(n) = k_2 n$

Caz II:

$f(n) = \Theta(n) = \Theta(n^1) = \Theta(n^{\log_4 4}) = \Theta(n^{\log_b a}) \Rightarrow$

$\overset{\text{MASTER II}}{\Longrightarrow} T(n) = \Theta(n^{\log_4 4} \cdot \lg n) = \boxed{\Theta(n \lg n)}$

le)

$$T(n) = \begin{cases} k_1, & n = 1 \\ 4T(\lceil n/2 \rceil) + T(\lfloor n/2 \rfloor) + k_2 n^2, & n > 1 \end{cases}$$

Metoda iteratireă:

$$T(n) = 5T(n/2) + \theta(n^2); \quad T(1) = \theta(1)$$

Unde:
$2^{k+1} = n$
$k+1 = \lg n$
$k = \lg n - 1$

$$T(n) = 5\cancel{T(n/2)} + \theta(n^2)$$
$$\cancel{T(n/2)} = 5\cancel{T(n/2^2)} + \theta\left(\left(\frac{n}{2}\right)^2\right) \,|\cdot 5$$
$$\cancel{T(n/2^2)} = \cancel{5T(n/2^3)} + \theta\left(\left(\frac{n}{2^2}\right)^2\right) \,|\cdot 5^2$$
$$\ldots\ldots\ldots\ldots$$
$$\cancel{T(n/2^k)} = 5T(n/2^{k+1}) + \theta\left(\left(\frac{n}{2^k}\right)^2\right) \,|\cdot 5^k$$
$$+ \overline{}$$
$$T(n) = 5^{k+1} T(n/2^{k+1}) + \sum_{i=0}^{k} \theta\left(\left(\frac{n}{2^i}\right)^2\right) \cdot 5^i$$
$$T(n) = 5^{\lg n} \cdot T(1) + \theta\left(\sum_{i=0}^{k} \frac{5^i}{4^i} \cdot n^2\right)$$
$$T(n) = 5^{\lg n} \cdot \theta(1) + \theta\left(n^2 \sum_{i=0}^{k} \frac{5^i}{4^i}\right)$$
$$T(n) = 5^{\lg n} \cdot \theta(1) + \theta\left(n^2 \frac{(5/4)^{k+1} - 1}{5/4 - 1}\right)$$
$$T(n) = \theta(5^{\lg n}) + \theta\left(n^2 \frac{(5/4)^{\lg n} - 1}{1/4}\right)$$
$$T(n) = \theta\left(5^{\lg n} + 4n^2\left(\frac{5^{\lg n}}{4^{\lg n}} - 1\right)\right)$$
$$T(n) = \theta\left(n^{\lg 5} + 4n^2 \frac{n^{\lg 5} - n^{\lg 4}}{n^{\lg 4}}\right)$$

$$T(n) = \theta\left(n^{\lg 5} + 4 n^2 \frac{n^{\lg 5} - n^2}{n^2}\right)$$

$$T(n) = \theta\left(n^{\lg 5} + 4 \frac{n^{\lg 5} - n^2}{1}\right)$$

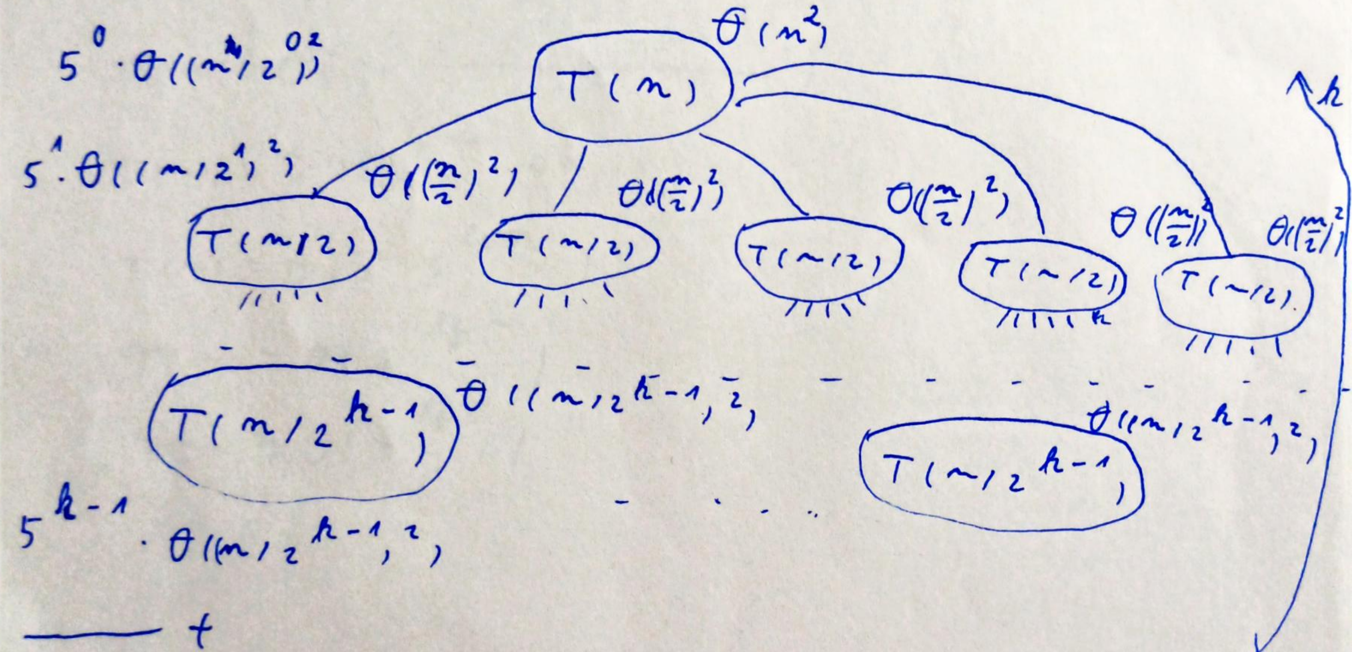
$$T(n) = \theta(5 n^{\lg 5} - 4 n^2)$$

$$T(n) = \theta(5 n^{\lg 5})$$

$$\boxed{T(n) = \theta(n^{\lg 5})}$$

Metoda arborelui de recurență:

$$T(n) = 5\,T(n/2) + \theta(n^2)\ ;\ T(1) = \theta(1)$$



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$$T(n) = \sum_{i=0}^{k-1} \theta((n/2^i)^2 \cdot 5^i$$

$$n = 2^{k-1}$$

$$k - 1 = \lg n$$

$$k = \lg n + 1$$

$$T(n) = \Theta\left(n^2 \sum_{i=0}^{k-1} \frac{5^i}{4^i}\right)$$

$$T(n) = \Theta\left(n^2 \frac{\left(\frac{5}{4}\right)^{\lg n + 1} - 1}{\frac{5}{4} - 1}\right)$$

$$T(n) = \Theta\left(n^2 \frac{\left(\frac{5}{4}\right)^{\lg n + 1} - 1}{\frac{1}{4}}\right)$$

$$T(n) = \Theta\left(4 n^2 \frac{5^{\lg n + 1} - 4^{\lg n + 1}}{4^{\lg n} \cdot 4}\right)$$

$$T(n) = \Theta\left(n^2 \frac{5 \cdot 5^{\lg n} - 4 n^2}{n^2}\right)$$

$$T(n) = \Theta\left(n^2 \frac{5 \cdot 5^{\lg n} - 4 n^2}{n^2}\right)$$

$$T(n) = \Theta(5 \cdot 5^{\lg n} - 4 n^2)$$

$$T(n) = \Theta(5 \cdot 5^{\lg n})$$

$$T(n) = \Theta(5^{\lg n})$$

$$T(n) = \Theta(n^{\lg 5})$$

Metoda substituției:

$$T(n) = 5T(n/2) + k_2 n^2;\quad T(1) = \theta(1) = k_1$$

Alegem $T(n) = \theta(3n^{\lg 5} - 2n^2)$

$\Rightarrow \exists\ c_1, c_2 \in \mathbb{R}_+^*$ și $\exists\ n_0 \in \mathbb{N}^*$ a.î.

$$c_1(3n^{\lg 5} - 2n^2) \leq T(n) \leq c_2(3n^{\lg 5} - 2n^2)$$

Demonstrăm prin inducție matematică:

Cazul de bază:

$T(1) = k_1$

$$n = 1 \Rightarrow c_1(3 \cdot 1^{\lg 5} - 2) \leq T(1) \leq c_2(3 \cdot 1^{\lg 5} - 2)$$

$$c_1 \leq k_1 \leq c_2 \Rightarrow n_0 = 1$$

Pas de inducție:

$n/2 \rightarrow n$

Ipoteza de inducție:

$$c_1\left[3\left(\frac{n}{2}\right)^{\lg 5} - 2\cdot\left(\frac{n}{2}\right)^2\right] \leq T(n/2) \leq c_2\left[3\left(\frac{n}{2}\right)^{\lg 5} - 2\cdot\left(\frac{n}{2}\right)^2\right]$$

Arătăm că:

$$c_1(3n^{\lg 5} - 2n^2) \leq T(n) \leq c_2(3n^{\lg 5} - 2n^2)$$

$$c_1\left[3\left(\frac{n}{2}\right)^{\lg 5}-2\left(\frac{n}{2}\right)^2\right]\le T(n/2)\le c_2\left[3\left(\frac{n}{2}\right)^{\lg 5}-2\left(\frac{n}{2}\right)^2\right]$$

$$|\cdot 5| + k_2 n^2$$

$$k_2n^2+5c_1\left(\frac{3n^{\lg 5}}{5}-\frac{n^2}{2}\right)\le 5T(n/2)+k_2n^2\le 5c_2\left(\frac{3n^{\lg 5}}{5}-\frac{n^2}{2}\right)+k_2n^2$$

$$c_1\left(3n^{\lg 5}-\frac{5n^2}{2}\right)+k_2n^2\le T(n)\le c_2\left(3n^{\lg 5}-\frac{5n^2}{2}\right)+k_2n^2$$

$$c_1\left(3n^{\lg 5}-\frac{4n^2}{2}\right)+k_2n^2-\frac{c_1}{2}n^2\le T(n)\le$$

$$c_2\left(3n^{\lg 5}-\frac{4n^2}{2}\right)+k_2n^2-\frac{c_2}{2}n^2$$

$$c_1(3n^{\lg 5}-2n^2)+n^2\left(k_2-\frac{c_1}{2}\right)\le T(n)\le$$

$$c_2(3n^{\lg 5}-2n^2)+n^2\left(k_2-\frac{c_2}{2}\right)$$

Dar:

$$\begin{cases} c_1(3n^{\lg 5}-2n^2)+n^2\left(k_2-\frac{c_1}{2}\right)\ge c_1(3n^{\lg 5}-2n^2) \\ \quad \text{dacă } k_2-\frac{c_1}{2}\ge 0 \\ c_2(3n^{\lg 5}-2n^2)+n^2\left(k_2-\frac{c_2}{2}\right)\le c_2(3n^{\lg 5}-2n^2) \\ \quad \text{dacă } k_2-\frac{c_2}{2}\le 0 \end{cases}$$

$$\Rightarrow \begin{cases} k_2-\frac{c_1}{2}\ge 0 \\ k_2-\frac{c_2}{2}\le 0 \end{cases} \Rightarrow \begin{cases} k_2\ge\frac{c_1}{2} \\ k_2\le\frac{c_2}{2} \end{cases} \Rightarrow \begin{cases} k_2\in\left[\frac{c_1}{2},\frac{c_2}{2}\right] \\ \frac{c_1}{2}\le k_2\le\frac{c_2}{2} \end{cases}$$

$\Rightarrow \exists\, c_1=\min(k_1,k_2),\ c_2=\max(2k_1,2k_2)\in\mathbb{R}_+^*$ și

$\exists\, n_0=1\in\mathbb{N}_0^*$ a.î. $c_1(3n^{\lg 5}-2n^2)\le T(n)\le c_2(3n^{\lg 5}-2n^2)$

$\Rightarrow T(n)=\Theta(3n^{\lg 5}-2n^2)=\Theta(3n^{\lg 5})=\Theta(n^{\lg 5})$

Metoda Master:

$T(n) = 5 \cdot T(n/2) + \theta(n^2)$

$\theta(n^2) \rightarrow k_2 n^2$

$\left.\begin{matrix} a = 5 \\ b = 2 \end{matrix}\right\} \Rightarrow n^{\log_2 5}$

$f(n) = k_2 n^2 = \theta(n^2)$

Caz II:

$f(n) = k_2 n^2$

$k_2 n^2 = \theta(n^{\log_2 5}) \Rightarrow \cancel{\text{Fo}} \quad 2 < \log_2 5$

Caz I:

$f(n) = \theta(n^2) = \theta(n^{\log_2 5 - (\log_2 5 - 2)})$

$\log_2 5 - 2 > 0 \Rightarrow \varepsilon = \log_2 5 - 2$

$f(n) = \theta(\cancel{\ } n^{\log_2 5 - (\log_2 5 - 2)}) \Rightarrow$

$\overset{\text{relatie}}{\Rightarrow} O(n^{\log_2 5 - (\log_2 5 - 2)}) \Rightarrow$

$\overset{\text{Master I}}{\Rightarrow} T(n) = \theta(n^{\log_2 5}) = \theta(n^{\lg 5})$