1(a) $f = 10$

$P = (3,2,1)$

Let point coordinate at image be (y_1, y_2)

projection equations are :-

$y_1 = -f \frac{x_1}{x_3}$ $y_2 = -f \frac{x_2}{x_3}$

$y_1 = \frac{-(3)(10)}{1}$ $y_2 = \frac{-(2)(10)}{1}$

$y_1 = -30$ $y_2 = -20$

$(y_1, y_2) = (-30, -20)$

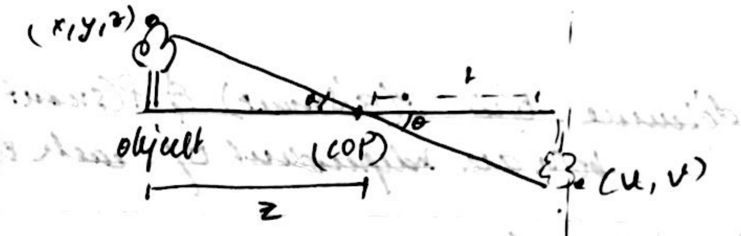
1 (b) Model, where image plane is behind centre of projection yield inverted image,

whereas in model where both image plane ~~and object are~~ is in front of the center of projection the image formed is upright.

The model where image plane is behind centre of projection corresponds better to a physical pinhole camera model.

model with image plane behind centre of projection:-

(model I)



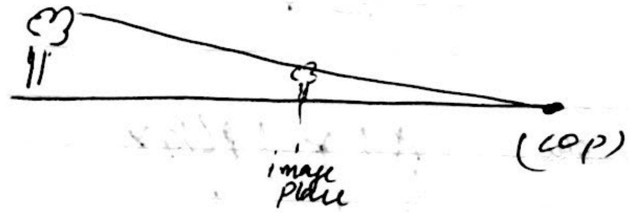
here projection equation is:-

$$\frac{v}{f} = -\frac{y}{z} \quad [\because \text{ both } \Delta\text{s are similar}]$$

$$\boxed{v = -\frac{y}{z} f \quad \text{similarly } u = -\frac{x f}{z}} \quad \text{— (I)}$$

for model with image plane is in front of centre of projection

(model II)



here projection equation is

$$\frac{y}{z} = \frac{v}{f}$$

$$\boxed{v = \left(\frac{y}{z}\right) f \quad \text{similarly } u = \frac{x f}{z}} \quad \text{— (II)}$$

Thus both (I) and (II) are similar with just a negative in (I) which denotes inverted image. Thus both are similar & 2nd model [represented by (II)] is justified

1 (c) $x_i = f\frac{x_0}{z_0}$, $y_i = f\frac{y_0}{z_0}$

here (x_i, y_i) → coordinates of image

(x_0, y_0, z_0) → coordinate of object in real world.

by projection equations, $x_i = f\frac{x_0}{z_0}$.

— If focal length is increased the size of image is increased. projection is bigger. [scaled up image] as $\boxed{x_i, y_i \propto f}$,

— If distance of object is increased the size of image is decreased projection is smaller [scaled down image] $\boxed{x_i, y_i \propto \frac{1}{z_0}}$

1 (d) Point → (1,1) → 2D.

corresponding 2DH is $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ → 1 is added

so, to find another 2DH point which corresponds to same 2D point, we can take all multiple of 2DH.
[multiply by any random number] [Let multiply by 2]

so → other 2DH point will be

$= [2, 2, 2]$

Q(e) 2DH → (1, 1 2)

The corresponding 2D is :-

$$\begin{bmatrix} U \\ V \\ W \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$$u = \frac{U}{W} = \frac{1}{2}$$

so point is $\left(\frac{1}{2}, 1/2\right)$

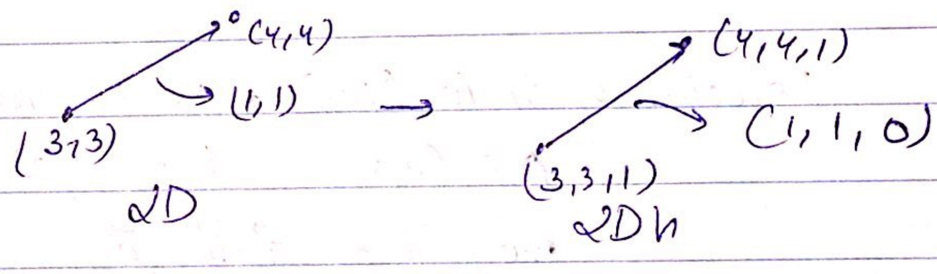
$$v = \frac{V}{W} = 1/2$$

Q(f) Meaning of 2DH (1,1,0) is that we cannot homogeneous it, as while converting to 2D, divide by zero won't make sense at it will be then '∞'.

~~So these points in 2D will be at '∞'~~

They represents directions & given by vector (1,1).

example → suppose vector points to (4,4) & (3,3) so vector is (1,1)



Thus they represent direction.

1 (g) Accordingly, projection equation is.

$$2D \rightarrow \begin{bmatrix} u \\ v \end{bmatrix} = \frac{1}{z} \begin{bmatrix} f & 0 \\ 0 & f \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \quad \text{—(I)}$$

→ which is non-linear in nature.

this equation in 2DH is:-

$$2DH = \begin{bmatrix} U \\ V \\ W \end{bmatrix} = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \text{—(II)}$$

→ which is linear in nature.

Here final points $u = \frac{U}{W}$, $v = \frac{V}{W}$

then 'u' in (I) & U in (II) are different.

also $\boxed{u = \frac{U}{W} = \frac{fx}{z}}$, $\boxed{v = \frac{V}{W} = \frac{fy}{z}}$

Here in (II) now we are dividing by 'z' in the later step than what we were doing in (I), thus making it linear in (II). This division by 'z' is called perspective division, which is postponed in (II).

1(h) $M = K[I|0]$

Dimension of $M = 3\times 4$

Dimension of $K = 3\times 3$

Dimension of $I = 3\times 3$

Dimension of $0 = 3\times 1$

$$M = \underbrace{\begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{K} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

I 0

1(b)

$$M \rightarrow \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 1 & 2 & 1 & 2 \end{bmatrix}$$

- 3D point $P \rightarrow [1, 2, 3]$

To find 2D coordinate (u, v)

$$\begin{bmatrix} U \\ V \\ W \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 1 & 2 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix}$$

$$U = 1 + 4 + 9 + 4 = 18$$

$$V = 5 + 12 + 21 + 8 = 46$$

$$W = 1 + 4 + 3 + 2 = 10$$

2D point $u = \frac{U}{W} = \frac{18}{10}$

$$v = \frac{V}{W} = \frac{46}{10}$$

$$= \left(\frac{18}{10}, \frac{46}{10}\right)$$

2 (a) Point = (1,1)

Translate by (2,3).

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & x \\ 0 & 1 & y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$x' = 1+2 = 3$

$y' = 1+3 = 4$ so new coordinate → [3,4].

2 (b) Point = (1, 1)

Scale by (2, 2).

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$x' = 2$

$y' = 2$ so new coordinate → [2, 2].

2(C) Point = (1,1)

Rotate by 45^0

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos 45 & -\sin 45 & 0 \\ \sin 45 & \cos 45 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$x' = \cos 45 - \sin 45$

$= 0$

$y' = \sin 45 + \cos 45$

$= 2/\sqrt{2} = \sqrt{2}$

2(d) Point → (1,1)

Rotate by 45° about point (2,2)

$$P' = T_{(1,1)} R_{(\theta - 45)} T_{(-1,-1)}$$

step 1 translate it to point (2,2)

$$\begin{bmatrix} x' \\ y' \\ 1' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$x' = 2$

$y' = 2$

step 2 rotate with 45°

$$\begin{bmatrix} x'' \\ y'' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos 45 & -\sin 45 & 0 \\ \sin 45 & \cos 45 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x'' \\ y'' \\ 1 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$$

$x'' = 0$, $y'' = 4/\sqrt{2} = \underline{\underline{2\sqrt{2}}}$

step 3 ~~retrans~~translate to (1,1)

$$\begin{bmatrix} x''' \\ y''' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 2\sqrt{2} \\ 1 \end{bmatrix}$$

$x''' = -1$, $y''' = 2\sqrt{2} - 1$

so point is $(-1, 2\sqrt{2} - 1)$

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2 (c) Operations done

(i) rotate object using matrix R.

(ii) translate object using matrix T.

so

combined matrix used to be applied to the object for transformation are:-

$$(P') = \underbrace{R \cdot T}_{\text{matrix multiplication of matrix R \& T}} (P)$$

$$= \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & Tx \\ 0 & 1 & Ty \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta & -\sin\theta & Tx \\ \sin\theta & \cos\theta & Ty \\ 0 & 0 & 1 \end{bmatrix}$$

2(f) $M \rightarrow$ transformation matrix.

$$M = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

This is similar to $\begin{bmatrix} Sx & 0 & 0 \\ 0 & Sy & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow$ Scaling transformation

So Here M represents scaling matrix.

2(g) $M \rightarrow$ transformation matrix

$$M = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

This is similar to $\begin{bmatrix} 1 & 0 & Tx \\ 0 & 1 & Ty \\ 0 & 0 & 1 \end{bmatrix} \rightarrow$ Translating transformation

so here M represents translating matrix

Q (h) $M \rightarrow$ transformation matrix

$$M = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

since it is similar to $\begin{bmatrix} Sx & 0 & 0 \\ 0 & Sy & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow$ scaling matrix

thus to reverse scaling effect we do M^{-1} where

$$M^{-1} = \begin{bmatrix} 1/Sx & 0 & 0 \\ 0 & 1/Sy & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

so reversed matrix is $\begin{bmatrix} 1/3 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ //

Q(i) $M = [R(45)\, T(1,2)]$

so inverse of this transformation matrix is M^{-1}

$$M^{-1} = \left[R(45)\ \ T(1,2)\right]^{-1}$$

$$= T^{-1}(1,2)\ R^{-1}(45^\circ)$$

$$= T^{-1}(1,2)\ \ R^{T}(45) \qquad [\because R^{-1} = R^{T}]$$

$$= T(-1,-2)\ \ R^{T}(45)$$

$$= \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(45) & \sin(45) & 0 \\ -\sin(45) & \cos(45) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos 45 & \sin 45 & -1 \\ -\sin 45 & \cos 45 & -2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & -1 \\ -1/\sqrt{2} & 1/\sqrt{2} & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

2(j)

Vector perpendicular to (1, 3)

so rotate ~~the~~ this vector by 90° so the resultant vector is perpendicular to given (1, 3)

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos 90 & -\sin 90 & 0 \\ \sin 90 & \cos 90 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$$

$x' = -3$, $y' = 1$ so vector $\perp$ to (1, 3) is $[-3, 1]$

Q2(K) Projection of vector (1,3) onto direction defined by (2,5)

$a = (2,5) \qquad b = (1,3)$

$$\text{proj}_{\vec{a}}\vec{b} = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|}\frac{\vec{a}}{|a|} = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|^2}\vec{a}$$

$$= \frac{(2,5)\cdot(1,3)}{(\sqrt{4+25})^2}(2\hat{i}+5\hat{j})$$

$$= \frac{2+15}{(\sqrt{29})^2}(2\hat{i}+5\hat{j})$$

$$= \frac{17}{(\sqrt{29})^2}[2,5] = \boxed{\frac{34}{\cancel{(\sqrt{29})}}, \cancel{\frac{85}{(\sqrt{29})}}}$$

$$= \left(\frac{34}{29}, \frac{85}{29}\right)$$

Q 1(a). For General Projection matrix, in different coordinate system.

$$p^{(i)} = M_{c \leftarrow i} K [I|0] M_{c \leftarrow w} p^{(w)}$$

in image

in world.

To convert ~~from~~ to image from camera

$$p^{(i)} = \underbrace{M_{c \leftarrow i}}_{\text{Transformation matrix}} \underbrace{K [I|0]}_{\text{Projection matrix}} p^{(c)}$$

Projection matrix =

$$\begin{bmatrix} U \\ V \\ W \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

This is used to convert 3DH to 2DH as:-

$$\begin{bmatrix} U \\ V \\ W \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \underbrace{\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}}_{3DH}$$

2D points $\leftarrow$ 2DH

$$u = \frac{U}{W}, \quad v = \frac{V}{W}$$

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3(b) Given that camera is rotated by 'R' and translated by T to convert world to camera coordinates.

a) cancel translation (T^{-1})

b) cancel rotation (R^{-1})

so

~~Let~~ transformation matrix is:

$$= R^{-1} \; T^{-1}$$

$$= \left[\begin{array}{c|c} R^{-1} & \begin{matrix}0\\0\\0\end{matrix} \\ \hline 0\;0\;0 & 1 \end{array}\right] \left[\begin{array}{c|c} I & (-T) \\ \hline 0\;0\;0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{c|c} R^{T} & -R^{T}T \\ \hline 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{c|c} R^{*} & T^{*} \\ \hline 0 & 1 \end{array}\right]$$

where

R^{*} = rotation of world wrt camera

T^{*} = translation of world wrt camera.

3(c). unit vectors $\rightarrow \hat{x}, \hat{y}, \hat{z}$
for camera.

so rotation of camera wrt world is given by a orthonormal matrix

$$\begin{bmatrix} P_x^c \\ P_y^c \\ P_z^c \end{bmatrix} = \begin{bmatrix} \hat{x}^T \\ \hat{y}^T \\ \hat{z}^T \end{bmatrix} (P^w - T^w)$$

where $P_x^c, P_y^c, P_z^c \rightarrow$ camera coordinates.

& $\hat{x}^T, \hat{y}^T, \hat{z}^T$ comprises of transpose of the defined unit vector in camera; which together acts as a orthogonal columns making up a orthonormal matrix which is also denoted as ROTATION MATRIX

3 (d) Given : $M = \begin{bmatrix} R^* & T^* \\ 0 & 1 \end{bmatrix}$

So

$R^* = R^T$ → means rotation of world wrt camera

$T^* = R^T(-T)$ → means translation of world wrt camera

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(c) $u_0, v_0 = (512, 512)$

$$M_{i \leftarrow c} = \left[\begin{array}{c|c} I & \begin{matrix} u_0 \\ v_0 \end{matrix} \\ \hline 0\ 0 & 1 \end{array}\right] \begin{bmatrix} K_u & 0 & 0 \\ 0 & K_v & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} K_u & 0 & u_0 \\ 0 & K_v & v_0 \\ 0 & 0 & 1 \end{bmatrix}$$

Transformation matrix $M_{i \leftarrow c}$ $= \begin{bmatrix} K_u & 0 & 512 \\ 0 & K_v & 512 \\ 0 & 0 & 1 \end{bmatrix}$

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(f) $M = K^* [R^* | T^*]$.

K^* = Intrinsic Parameters.
$[R^* | T^*]$ = External parameters.

where

$$K = \begin{bmatrix} fK_u & 0 & u_0 \\ 0 & fK_v & v_0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow$$ involves K_u & focal length 'f'

External parameters K^* → f → focal length (mm)
K_u, K_v → scale (pixels / mm)
u_0, v_0 → translation of optical centre [pixels]

internal parameter $[R^* | T^*]$ → : Rotation + translation of world wrt camera.

(g) To make intrinsic camera parameter more accurate, we assume that image is distorted (skewed), so we introduce skew parameter to take into account even smaller details giving more accurate results.

3 (h) To make camera model more accurate we take into account radial line distortion, this is caused because of wide angled lens, which results in distortion of image.

Due to this consideration, now $p^{(i)}$ will become:-

$$p^{(i)} = \begin{bmatrix} 1/\lambda & & \\ & 1/\lambda & \\ & & 1 \end{bmatrix} K^{*} \cdot [R^{*} | T^{*}] \, p^{(\omega)}$$

where $\lambda = 1 + k_1 d^2 + k_2 d^4$

$k_1 \rightarrow$ linear distortion coefficient

$k_2 \rightarrow$ quadratic distortion coefficient

$d \rightarrow$ distance from centre

as a result of which the equation is no-more linear and is non-linear in nature now.

3(i) Weak perspective Camera → (M_∞)

In weak perspective camera angles are better preserved. Thus, the resulting projection of parallel lines are (almost) parallel, ~~becau~~ as when depth variation is small, compared to distance of object.

$$e = |M_\infty P - MP| \cong \frac{\overset{\text{depth variation}}{\Delta}}{d_0} (M\underset{\text{world}}{P} - \underbrace{P_0}_{\text{distance from centre of image.}})$$

d_0 → distance from camera

so error is small when:- → depth variation is small
→ or distance from centre of image is small.

Affine Camera →

It is a special case of projective camera where in matrix T, $a_{31} = a_{32} = a_{33} = 0$, $a_{34} = 1$

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix}$$

image & Object mapping takes as

$$x = MX + t$$

M → 2x3 matrix = $M_{ij} = \dfrac{a_{ij}}{a_{34}}$

t → 2D-vector represents image centre

This is even worse than weak perspective & also preserves parallism.

(4) (a)

Surface Radiance	Image irradiance
i) It is defined as power of light per surface area reflected from surface of object.	i) It is defined as power of light per surface area received at each pixel of the image.

4 (b) Radiosity equations relating surface radiance & image irradiance is:

$$E(p) = L(p) \frac{\pi}{4} \left(\frac{d}{f}\right) (\cos \alpha)^4$$

where,

$E(p)$ = image irradiance

$L(p)$ = surface radiance

d = diameter of the lens

f = focal length of the lens.

'α' = angle between optical axis and surface normal.

4(c) Albedo of a measure of how much light is reflected back after being hit on a surface without being absorbed.

A good reflecting surface will have high albedo and vice-versa.

4(d) Since, human eyes can only have Red, Green & blue sensors, and also camera are designed like that, we tend to model using RGB color model to represent color.

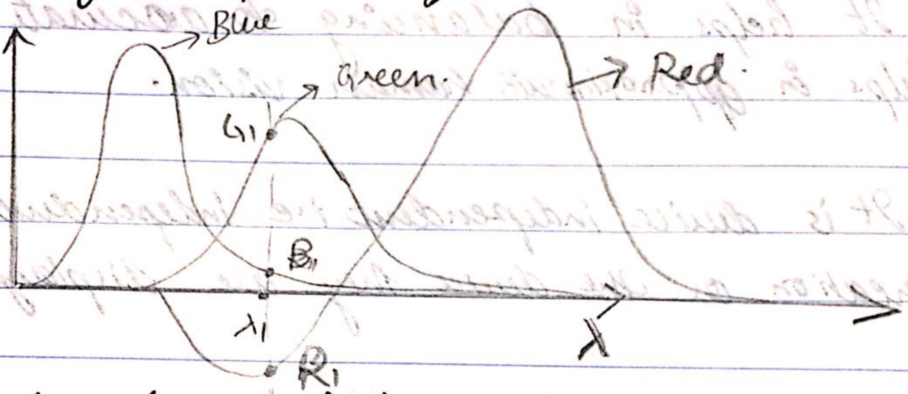
Also Red, Blue & Green being primary colours cannot be created using other colours.

4(e)

4(e) Since (0,0,0) → represents black } on a rgb
(1,1,1) → represents white } color cube.

Thus 'grey' is the color that we will get along line joining (0,0,0) (1,1,1)

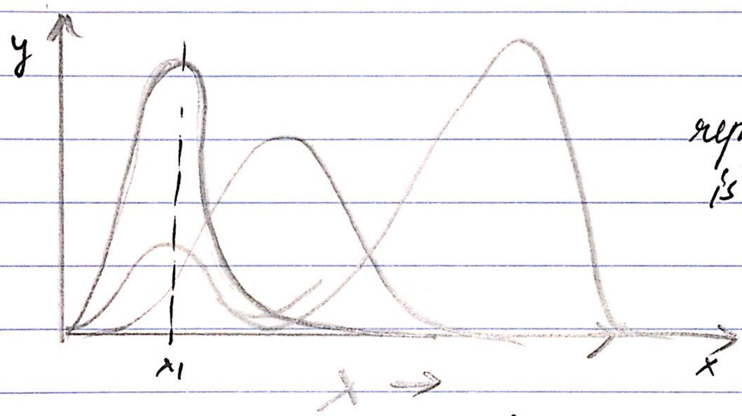
4 (f) RGB colors are mapped to real-world colors using graph plot as follows:-



For a color having 'λ_1' wavelength in (nm) in real-world the vertical line is stretched, wherever the line intersects the graph we define points value for R, G and B. which is R_1, G_1 and B_1.

4(g) The conversion of RGB to XYZ is given by:-

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \frac{1}{0.17697} \begin{bmatrix} 0.49 & 0.31 & 0.20 \\ 0.176 & 0.812 & 0.01 \\ 0.00 & 0.01 & 0.99 \end{bmatrix} \begin{bmatrix} r \\ g \\ b \end{bmatrix}$$



Here the 'y' component represents luminance → which is related to brightness.
So, for λ_1 as one color 'y' component will show much brighter a color (R, G, B) has been taken into account, to produce desired color.

4(h) Advantages of LAB color space are:-

→ It helps in balancing to accurate color, thus helps in approximate human vision.

→ It is device independent i.e Independent of their nature of creation or the device they are displayed on.