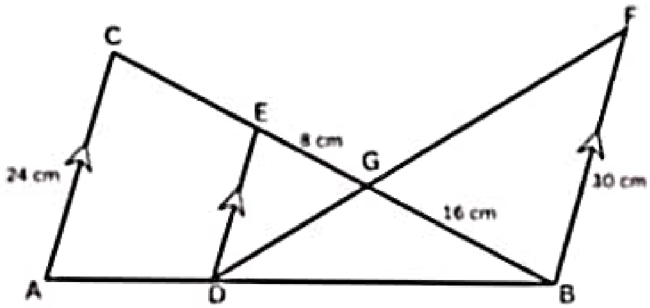
Similarity

1. In the given figure, AC ∥ DE ∥ BF. If AC = 24 cm, EG = 8 cm, GB = 16 cm, BF = 30 cm,



(a) prove ΔGED ~ ΔGBF

(b) find DE.

(c) DB : AB

Solution: (b) 15 cm (c) 5 : 8

Step-by-step Explanation:

(a) In ΔGED and ΔGBF,

∠EGD = ∠BGF (vertically opposite angles)

∠GED = ∠GBF (alternate interior angles)

∴ ΔGED ~ ΔGBF (by A-A axiom of similarity)

(b) as ΔGED ~ ΔGBF

∴ GE/BG = DE/BF = DG/GF

GE/BG = DE/BF

8/16 = DE/30

$DE = 8 \times 30/16$

$DE = 15$ cm

(c) In ΔDBE and ΔABC,

$\angle B = \angle B$ (common)

$\angle EDB = \angle CAB$ (corresponding angles)

$\therefore \Delta DBE \sim \Delta ABC$ (by A-A condition of similarity)

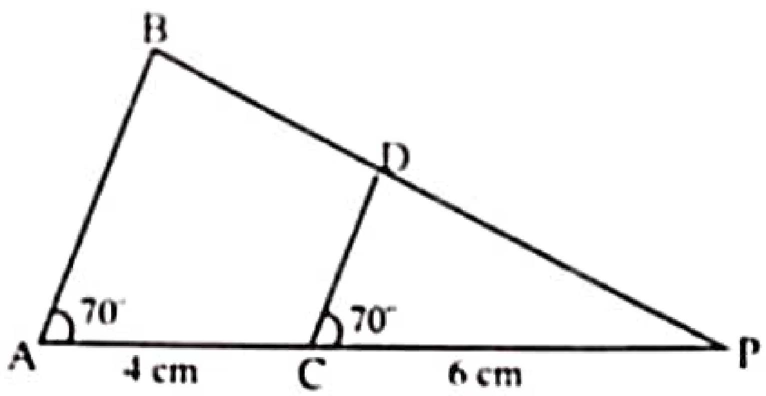
$\therefore DB/AB = BE/BC = DE/AC$

$DB/AB = DE/AC$

$DB/AB = 15/24 = 5/8$

$DB : AB = 5 : 8$

2. In the given figure, $\angle BAP = \angle DCP = 70°$, PC = 6 cm and CA = 4 cm, then PD : DB is



(a) 5 : 3

(b) 3 : 5

(c) 3 : 2

(d) 2 : 3 [2023]

Solution: (c) 3 : 2

Step-by-step Explanation:

In ΔPCD and ΔPAB,

$\angle PCD = \angle BAP = 70°$, (given)

$\angle DPC = \angle BPA$ (common)

$\therefore \Delta PCD \sim \Delta PAB$ (by A-A condition of similarity)

$\therefore PC/PA = PD/PB = CD/AB$

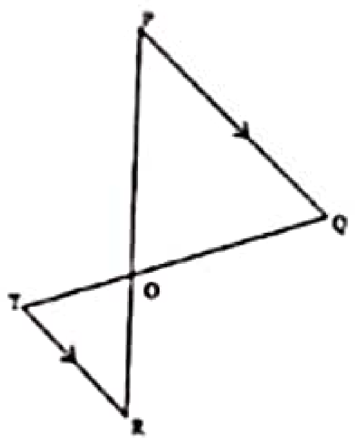
$PC/PA = PD/PB$

$6/6+4 = 6/10 = 3/5 = PD/PB$

As $PD : PB = 3 : 5$, therefore $PD : DB = 3 : (5\text{-}3) = 3 : 2$

option (c) is correct.

3. In the given figure, PQ is parallel to TR, then by using condition of similarity:



(a) PQ/RT = OP/OT = OQ/OR

(b) PQ/RT = OP/OR = OQ/OT

(c) PQ/RT = OR/OP = OQ/OT

(d) PQ/RT = OP/OR = OT/OQ

[2021 SEMESTER-1]

Solution: (b) $PQ/RT = OP/OR = OQ/OT$

Step-by-step Explanation:

In ΔPOQ and ΔROT,

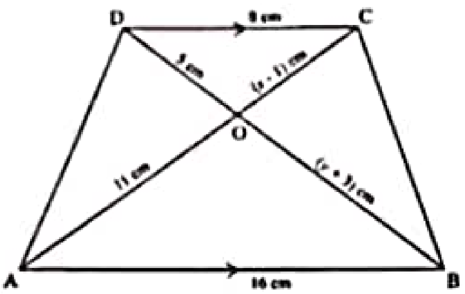
$\angle POQ = \angle ROT$ (vertically opposite angles)

$\angle OPQ = \angle ORT$ (alternate interior angles)

$\therefore \Delta POQ \sim \Delta ROT$ (by A-A condition of similarity)

so, $PQ/RT = OP/OR = OQ/OT$

4. In the given figure ABCD is a trapezium in which DC is parallel to AB. AB = 16 cm and DC = 8 cm. OD = 5 cm, OB = (y + 3) cm, OA = 11 cm and OC = (x – 1) cm. Using the given information answer the following questions. [2021 Semester-I]



(i.) From the given figure name the pair of similar triangles:
(a) ΔOAB, ΔOBC (b) ΔCOD, ΔAOB (c) ΔADB, ΔACB (d) ΔCOD, ΔCOB

(ii.) The corresponding proportional sides with respect to the pair of similar triangles obtained in (i):
(a) $CD/AB = OC/OA = OD/OB$ (b) $AD/BC = OC/OA = OD/OB$

(c) $AD/BC = BD/AC = AB/DC$ (d) $OD/OB = CD/CB = OC/OA$

(iii.) The ratio of the sides of the pair of similar triangles is:
(a) 1 : 3 (b) 1 : 2 (c) 2 : 3 (d) 3 : 1

(iv.) Using the ratio of sides of the pair of similar triangles values of x and y are respectively:
(a) $x = 4.6, y = 7$ (b) $x = 7, y = 7$ (c) $x = 6.5, y = 7$ (d) $x = 6.5, y = 2$

Solution: (i) (b) (ii) (a) (iii) (b) (iv) (c)

Step-by-step Explanation:

In ΔCOD and ΔAOB,

$\angle COD = \angle AOB$ (vertically opposite angles)

$\angle OCD = \angle OAB$ (alternate interior angles)

$\angle POQ = \angle ROT$ (vertically opposite angles)

$\therefore \Delta COD \sim \Delta AOB$ (by A-A condition of similarity)

Option (b) is correct.

(ii) Therefore, $CD/AB = OC/OA = OD/OB$

option (a) is correct.

(iii) $CD/AB = 8/16 = 1 : 2$

option (b) is correct.

(iv) We know, $CD/AB = OC/OA = OD/OB$

$CD/AB = OC/OA$

$1/2 = x\text{-}1/11$

$2(x-1) = 11$

$2x-2 = 11$

$2x = 11+2$

$x = 13/2 = 6.5$ cm

Now, CD/AB = OD/OB

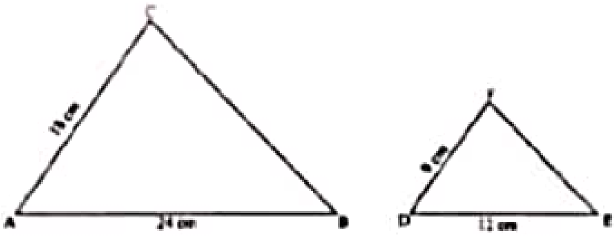
1/2 = 5/y+3

y+3 = 10

y = 7

option (c) is correct.

5. In the given figure, AB = 24 cm, AC = 18 cm, DE = 12cm, DF = 9 cm and ∠BAC = ∠EDF. Then ΔABC ~ ΔDEF by the condition:
(a) AAA
(b) SAS
(c) SSS
(d) AAS [2021 Semester-1]



Solution: (b)

Step-by-step Explanation:

In ΔABC and ΔDEF,

AC/DF = 18/9 =2/1

∠BAC = ∠EDF (given)

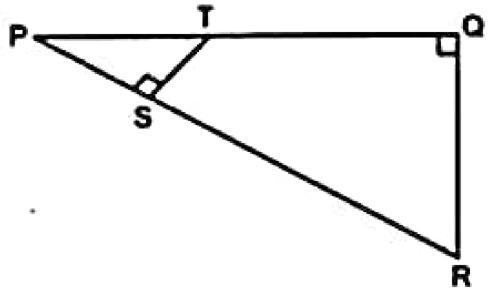
AB/DE = 24/12 = 2/1

∴ ΔABC ~ ΔDEF (by S-A-S condition of similarity)

option (b) is correct.

6. In the given figure, ∠PQR = ∠PST = 90°, PQ = 5 cm and PS = 2 cm.

(i) Prove that ΔPQR ~ ΔPST.
(ii) Find Area of ΔPQR: Area of quadrilateral SRQT. [2019]



Solution: (ii) 25 : 21

Step-by-step Explanation:

(i) In ΔPQR and ΔPST,

∠P is common.

∠PQR= ∠PST = 90° (given)

∴ ΔPQR ~ ΔPST (A-A condition of similarity)

(ii) ∴ PQ/PS = QR/ST = PR/PT

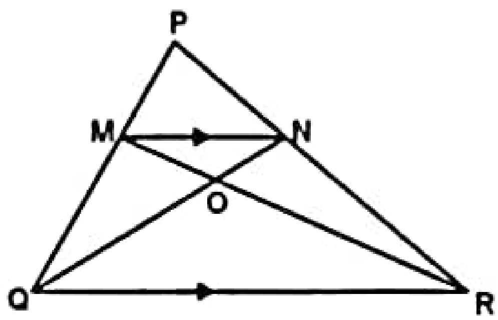
As ΔPQR ~ ΔPST, therefore

Area of ΔPQR / Area of ΔPST =$(PQ/PS)^2$ =$(5/2)^2$ =25 : 4

∴ Area of ΔPQR / Area of quadrilateral SRQT = Area of ΔPQR / Area of ΔPQR-area of ΔPST

=25/25-4 = 25/21 = 25 : 21

7. In Δ PQR, MN is parallel to QR and PM/MQ = 2/3 [3]
(i) Find MN / QR
(ii) Prove that ΔOMN and ΔORQ are similar.
(iii) Find, Area of ΔOMN: Area of ΔORQ [2018]



Solution: (i) 2/5 (iii) 4 : 25

Step-by-step Explanation:

(i) In ΔPMN and $\triangle PQR$,

$\angle P$ is common.

$\angle PMN = \angle PQR$ (corresponding angles)

$\therefore \Delta PMN \sim \Delta PQR$ (A-A condition of similarity)

We know, $PM/MQ = 2/3$

$\therefore PM/PQ = PM/PM + MQ = 2/2+3 = 2/5$

$\therefore PM/PQ = MN/QR = PN/PR = 2/5$

$\therefore MN/QR = 2/5$

(ii) In ΔOMN and $\triangle ORQ$,

$\angle MON = \angle ROQ$ (vertically opposite angles)

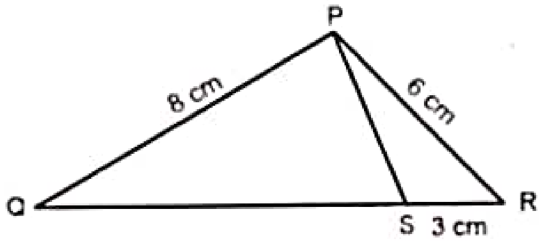
$\angle OMN = \angle ORQ$ (alternate interior angles)

$\therefore \Delta OMN \sim \Delta ORQ$ (A-A condition of similarity)

(iii) Area of ΔOMN / Area of $\Delta ORQ = (MN/QR)^2 = (2/5)^2 = 4 : 25$

8. PQR is a triangle. S is a point on the side QR of ΔPQR such that $\angle PSR = \angle QPR$. Given QP = 8 cm, PR = 6 cm and SR = 3 cm.

(i) Prove ΔPQR ~ ΔSPR
(ii) Find the length of QR and PS
(iii) area of ΔPQR/ area of ΔSPR
[2017]



Solution: (ii) OR=12cm, PS = 4cm (iii) 4 : 1

Step-by-step Explanation:

(i) In ΔPQR and ΔSPR,

∠R is common.

∠PSR = ∠QPR (given)

∴ ΔPQR ~ ΔSPR (A-A condition of similarity)

(ii) ∴ PQ/PS = QR/PR = PR/SR

PQ/PS = PR/SR

8/PS = 6/3

PS = 4 cm

QR/PR = PR/SR

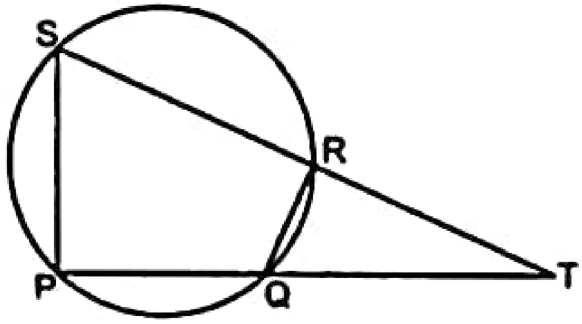
QR/6 = 6/3

QR = 12 cm

(iii) Area of ΔPQR / Area of ΔSPR $=(PR/SR)^2=(6/3)^2=4:1$

9. In the given figure PQRS is a cyclic quadrilateral PQ and SR produced meet at T.

(i) Prove Δ TPS ~ Δ TRQ.
(ii) Find SP if TP= 18cm, RQ= 4cm and TR= 6cm.
(iii) Find area of quadrilateral PQRS if area of Δ PTS = 27 cm^2 [2016]



Solution: (ii) 12 cm (iii) 24 cm^2

Step-by-step Explanation:

(i) In ΔTPS and ΔTRQ,

∠T is common.

∠TPS = ∠TRQ (exterior angle of a cyclic quadrilateral is equal to opposite interior angle)

∴ ΔTPS ~ ΔTRQ (A-A condition of similarity)

(ii) ∴ TP/TR = SP/RQ = TS/TQ

TP/TR = SP/RQ

18/6 = SP/4

SP = 12 cm

(iii) As ΔTPS ~ ΔTRQ, therefore

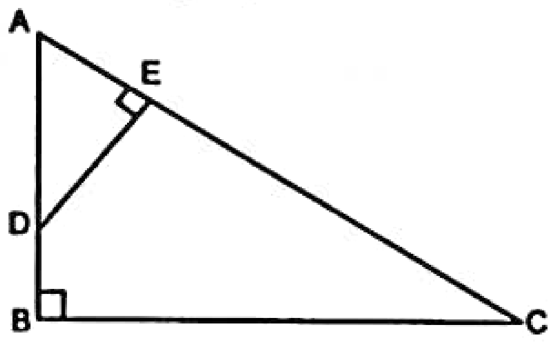
Area of ΔTPS / Area of ΔTRQ $=(TP/TR)^2 = (18/6)^2 = 9/1$

27/Area of ΔTRQ = 9/1

Area of ΔTRQ = 3 cm^2

∴ area of quadrilateral PQRS = 27 – 3 = 24 cm^2

10. ABC is a right angled triangle with $\angle ABC = 90°$. D is any point on AB and DE is perpendicular to AC.



Prove that:

(i) $\Delta ADE \sim \Delta ACB$.

(ii) If AC = 13 cm, BC = 5 cm and AE = 4 cm. Find DE and AD.

(iii) Find, area of Δ ADE: area of quadrilateral BCED. [2015]

Solution: (ii) DE = 1 2/3 cm, AD = 4 1/3 cm (iii) 1 : 8

Step-by-step Explanation:

(i) In ΔADE and △ACB,

$\angle A$ is common.

$\angle AED = \angle ABC = 90°$ (given)

$\therefore \Delta ADE \sim \Delta ACB$ (A-A condition of similarity)

(ii) $\therefore AD/AC = DE/BC = AE/AB$

In right-angled ΔACB, by pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$13^2 = AB^2 + 5^2$

$AB = \sqrt{169 - 25} = \sqrt{144} = 12$ cm

Now, $AD/AC = AE/AB$

$AD/13 = 4/12$

AD = 13/3 = 4 1/3 cm

DE/BC = AE/AB

DE/5 = 4/12

DE = 5/3 = 1 2/3 cm

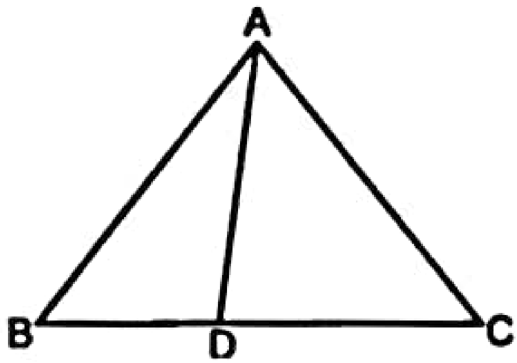
(iii) As ΔADE ~ ΔACB, therefore

Area of ΔADE / Area of ΔACB =$(AE/AB)^2$ =$(4/12)^2$ = 1 : 9

∴ Area of ΔADE / Area of quadrilateral BCED = Area of ΔADE / Area of ΔABC – area of ΔADE

= 1/9-1= 1/8= 1 : 8

11. In ΔABC, ∠ABC = ∠DAC, AB = 8 cm, AC = 4 cm, AD = 5 cm.



(i) Prove that ΔACD is similar to ΔBCA.
(ii) Find BC and CD
(iii) Find area of ΔACD : area of ΔABC.

Solution: (ii) BC = 6.4 cm, CD = 2.5 cm (iii) 25 : 64

Step-by-step Explanation:

(i) In ΔACD and △BCA,

∠C is common.

∠DAC = ∠ABC (given)

$\therefore \Delta ACD \sim \Delta BCA$ (A-A condition of similarity)

(ii) $\therefore AC/BC = CD/AC = AD/AB$

$CD/AC = AD/AB$

$CD/4 = 5/8$

$CD = 5/2 = 2.5$ cm

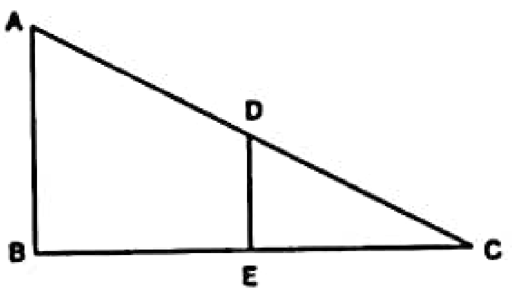
$AC/BC = AD/AB$

$4/BC = 5/8$

$BC = 32/5 = 6.4$ cm

(iii) Area of ΔACD / Area of $\Delta ABC = (AD/AB)^2 = (5/8)^2 = 25 : 64$

12. In the given figure, AB and DE are perpendiculars to BC.



(i) Prove that $\Delta ABC \sim \Delta DEC$

(ii) If AB = 6 cm, DE = 4 cm and AC = 15 cm. Calculate CD.

(iii) Find the ratio of the area of ΔABC : area of ΔDEC

Solution: (ii) CD = 10 cm (iii) 9 : 4

Step-by-step Explanation:

(i) In ΔABC and $\triangle DEC$,

$\angle C$ is common.

$\angle ABC = \angle DEC = 90°$ (given)

$\therefore \Delta ABC \sim \Delta DEC$ (A-A condition of similarity)

(ii) $\therefore AB/DE = BC/EC = AC/CD$

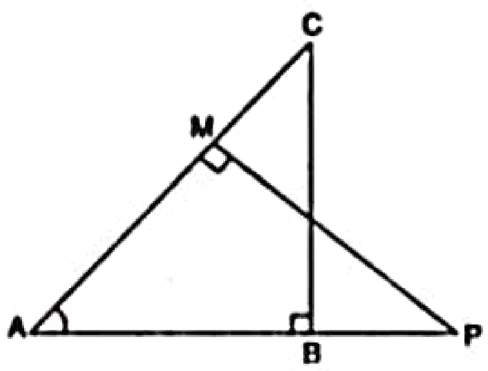
$AB/DE = AC/CD$

$6/4 = 15/CD$

$CD = 10$ cm

(iii) Area of ΔABC / Area of $\Delta DEC = (AB/DE)^2 = (6/4)^2 = 9 : 4$

13. In the given figure Δ ABC and Δ AMP are right angled at B and M respectively.
Given, AB = 10 cm, AP = 15 cm and PM = 12 cm



(i) Prove that Δ ABC ~ Δ AMP
(ii) Find AC and BC. [2012]

Solution: (ii) AC= 16 2/3 cm BC = 13 1/3 cm

Step-by-step Explanation:

(i) In ΔABC and $\triangle AMP$,

$\angle A$ is common.

$\angle ABC = \angle AMP = 90°$ (given)

$\therefore \Delta ABC \sim \Delta AMP$ (A-A condition of similarity)

(ii) In right-angled $\triangle AMP$, by pythagoras theorem,

$AP^2 = AM^2 + PM^2$

$15^2 = AM^2 + 12^2$

$AM = \sqrt{225 - 144} = \sqrt{81} = 9$ cm

now, as $\Delta ABC \sim \Delta AMP$

$\therefore AB/AM = BC/PM = AC/AP$

$AB/AM = BC/PM$

$10/9 = BC/12$

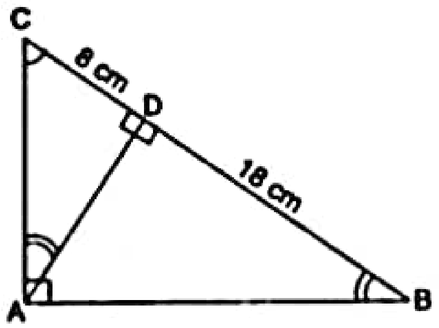
$BC = 40/3 = 13\ 1/3$ cm

$AB/AM = AC/AP$

$10/9 = AC/15$

$AC = 50/3 = 16\ 2/3$ cm

14. In the adjoining figure ABC is a right-angled triangle with $\angle BAC = 90°$,



(i) Prove $\Delta ADB \sim \Delta CDA$

(ii) If BD = 18 cm and CD = 8 cm, find AD.

(iii) Find the ratio of area of ΔADB is to area of ΔCDA. [2011]

Solution: (ii) AD = 12 cm (iii) 9 : 4

Step-by-step Explanation:

(i) In ΔADB, $\angle ABD = 180° - (\angle ADB + \angle DAB) = 90° - \angle DAB$

In ΔCDA, $\angle CAD = 90° - \angle DAB$

Hence, $\angle CAD = \angle ABD$

In ΔADB and $\triangle CDA$,

$\angle ADB = \angle CDA = 90°$ (given)

$\angle CAD = \angle ABD$ (proved above)

$\therefore \Delta ADB \sim \Delta CDA$ (A-A condition of similarity)

(ii) $\therefore AD/CD = BD/AD = AB/AC$

$AD/CD = BD/AD$

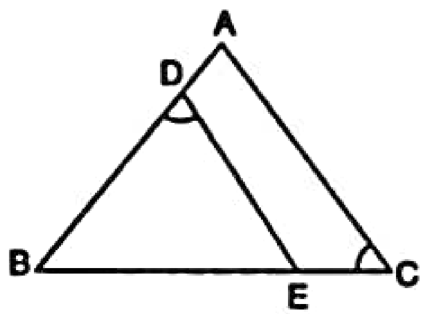
$AD/8 = 18/AD$

$AD^2 = 18 \times 8$

$AD = \sqrt{144} = 12$ cm

(iii) Area of ΔADB / Area of $\Delta CDA = (AD/CD)^2 = (12/8)^2 = 9 : 4$

15. In the figure ABC is a triangle with $\angle EDB = \angle ACB$.



Prove that $\Delta ABC \sim \Delta EBD$

If BE = 6 cm, EC = 4 cm BD = 5 cm and area of $\Delta BED = 9$ cm^2,

Calculate the (i) length of AB

(ii) area of ΔABC [2010]

Solution: (i) 12 cm (ii) 36 cm^2

Step-by-step Explanation:

In ΔABC and $\triangle EBD$,

$\angle B$ is common.

$\angle ACB = \angle BDE$ (given)

$\therefore \Delta ABC \sim \Delta EBD$ (A-A condition of similarity)

$\therefore AB/BE = BC/BD = AC/ED$

(i) so, $AB/BE = BC/BD$

$AB/6 = 6+4/5$

$AB/6 = 10/5$

$AB = 12$ cm

(iii) Area of ΔABC / Area of $\Delta EBD = (AB/BE)^2 = (12/6)^2$

Area of $\Delta ABC / 9 = 4/1$

Hence, Area of $\Delta ABC = 36$ cm^2