Quadratic Equations

Q1. Solve the following quadratic equation: $x^2 + 4x - 8 = 0$

Give your answer correct to one decimal place.
(Use mathematical tables if necessary.) [2023]

Answer: x= 1.5, -5.5

Step-by-step explanation:

x2 + 4x – 8 = 0

comparing the above equation with $ax^2 + bx + c = 0$, we have,

$a = 1, b = 4, c = -8$

By Quadratic formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4 \times 1 \times (-8)}}{2 \times 1}$$

$$x = \frac{-4 \pm \sqrt{16 + 32}}{2}$$

$$x = \frac{-4 \pm \sqrt{48}}{2}$$

$$x = \frac{-4 \pm 6.928}{2}$$

$$x = \frac{-4 + 6.928}{2}, \frac{-4 - 6.928}{2}$$

$$x = \frac{2.928}{2}, \frac{-10.928}{2}$$

$$x = 1.464, -5.464$$

$$x = 1.5, -5.5$$

Q2. If 3 is a root of the quadratic equation $x^2 - px + 3 = 0$ then p is equal to:

(a) 4

(b) 3

(c) 5

(d) 2 [2023]

Answer: (a) 4

Step-by-step explanation:

$$x^2 - px + 3 = 0$$

Given, 3 is a root of the above equation.

Therefore,

$$(3)^2 - p \times 3 + 3 = 0$$
$$9 - 3p + 3 = 0$$
$$-3p + 12 = 0$$
$$-3p = -12$$
$$p = 4$$

Q3. One of the roots of the quadratic equation $x^2 - 8x + 5 = 0$ is 7.3166. The root of the equation correct to 4 significant figures is: [1]

(a) 7.3166

(b) 7.317

(c) 7.316

(d) 7.32 [2021 Semester-1]

Answer: (b) 7.317

Step-by-step explanation:

$$x^2 - 8x + 5 = 0$$

Given, 7.3166 is a root of the above equation.

Therefore,

The root of the equation correct to 4 significant figures is

7.317

Q4. Which of the following quadratic equations has 2 and 3 as its roots? [1]

(a) $x^2 - 5x + 6 = 0$

(b) $x^2 + 5x + 6 = 0$

(c) $x^2 - 5x - 6 = 0$

(d) $x^2 + 5x - 6 = 0$ [2021 Semester-1]

Answer: (a) $x^2-5x+6=0$

Step-by-step Explanation:

$$x^2 - 5x + 6 = 0$$

$$x^2 - 3x - 2x + 6 = 0$$

$$x(x - 3) - 2(x - 3) = 0$$

$$(x - 2)(x - 3) = 0$$

$$Either\ x - 3 = 0\ OR\ x - 2 = 0$$

$$Therefore,\ x = 3\ OR\ x = 2$$

Q5. Solve the following Quadratic Equation:

$x^2 - 7x + 3 = 0$

Give your answer correct to two decimal places. [2020]

Answer: x = 6.54 , 0.46

Step-by-step Explanation:

$$x^2 - 7x + 3 = 0$$

Comparing the given equation with $ax^2 + bx + c = 0$ *we have,*

$$a = 1,\ b = -7,\ c = 3$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4 \times 1 \times 3}}{2 \times 1}$$

$$x = \frac{7 \pm \sqrt{49 - 12}}{2}$$

$$x = \frac{7 \pm \sqrt{37}}{2}$$

$$x = \frac{7 \pm 6.083}{2}$$

$$x = \frac{7 + 6.083}{2},\ \frac{7 - 6.083}{2}$$

$$x = \frac{13.083}{2},\ \frac{0.917}{2}$$

$$x = 6.5416,\ 0.4585$$

$$x = 6.54,\ 0.46$$

Q6. Solve for x the quadratic equation $x^2 - 4x - 8 = 0$

Give your answer correct to three significant figures. [2019]

Answer: x= 5.46 , -1.46

Step-by-step Explanation:

$$x^2 - 4x - 8 = 0$$

Comparing the given equation with $ax^2 + bx + c = 0$, *we have,*

$$a = 1,\ b = -4,\ c = -8$$

By formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \times 1 \times (-8)}}{2 \times 1}$$

$$x = \frac{4 \pm \sqrt{16 + 32}}{2}$$

$$x = \frac{4 \pm \sqrt{48}}{2}$$

$$x = \frac{4 \pm 6.928}{2}$$

$$x = \frac{4 + 6.928}{2},\ \frac{4 - 6.928}{2}$$

$$x = \frac{10.928}{2},\ \frac{-2.928}{2}$$

$$x = 5.464,\ -1.464$$

$$x = 5.46,\ -1.46$$

Q7. Solve $x^2 + 7x = 7$ and give your answer correct to two decimal places. [4] [2018]

Answer: 0.89 , -7.89

Step-by-step Explanation:

$$x^2 + 7x = 7$$

$$x^2 + 7x - 7 = 0$$

Comparing the given equation with $ax^2 + bx + c = 0$, *we have,*

$$a = 1,\ b = 7,\ c = -7$$

By formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-7 \pm \sqrt{(7)^2 - 4 \times 1 \times (-7)}}{2 \times 1}$$

$$x = \frac{-7 \pm \sqrt{49 + 28}}{2}$$

$$x = \frac{-7 \pm \sqrt{77}}{2}$$

$$x = \frac{-7 \pm 8.775}{2}$$

$$x = \frac{-7 + 8.775}{2},\ \frac{-7 - 8.775}{2}$$

$$x = \frac{1.775}{2},\ \frac{-15.775}{2}$$

$$x = 0.8875,\ -7.8875$$

$$x = 0.89,\ -7.89$$

Q8. Find the value of k for which the following equation has equal roots. [3]

$x^2 + 4kx + (k^2 - k + 2) = 0$ **[2018]**

Answer: k= -1 or 2/3

Step-by-step Explanation:

$$x^2 + 4kx + (k^2 - k + 2) = 0$$

Comparing the equation with $ax^2 + bx + c = 0$ *we have,*

$$a = 1,\ b = 4k,\ c = (k^2 - k + 2)$$

Given, $b^2 - 4ac = 0$

Therefore, $(4k)^2 - 4 \times 1 \times (k^2 - k + 2) = 0$

$$16k^2 - 4k^2 + 4k - 8 = 0$$

$$12k^2 + 4k - 8 = 0$$

$$4(3k^2 + k - 2) = 0$$

$$3k^2 + k - 2 = 0$$

$$3k^2 + 3k - 2k - 2 = 0$$

$$3k(k + 1) - 2(k + 1) = 0$$

$$(k + 1)(3k - 2) = 0$$

Either $k + 1 = 0$ *or* $3k - 2 = 0$

Either $k = -1$ *or* $\frac{2}{3}$

Q9. Solve the equation $4x^2 - 5x - 3 = 0$ and give your answer correct to two decimal places. [4] [2017]

Answer: x = 1.69 , -0.44

Step-by-step Explanation:

$$4x^2 - 5x - 3 = 0$$

Comparing the equation with $ax^2 + bx + c = 0$ *we have,*

$$a = 4,\ b = -5,\ c = -3$$

By formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \times 4 \times (-3)}}{2 \times 4}$$

$$x = \frac{5 \pm \sqrt{25 + 48}}{8}$$

$$x = \frac{5 \pm \sqrt{73}}{8}$$

$$x = \frac{5 \pm 8.544}{8}$$

$$x = \frac{5 + 8.544}{8},\ \frac{5 - 8.544}{8}$$

$$x = \frac{13.544}{8},\ \frac{-3.544}{8}$$

$$x = 1.693,\ -0.443$$

$$x = 1.69,\ -0.44$$

Q10. Solve the quadratic equation $x^2 - 3(x + 3) = 0$; Give your answer correct to two significant figures. [3] [2016]

Answer: x = 5.9 , -0.85

Step-by-step Explanation:

$$x^2 - 3(x+3) = 0$$

$$x^2 - 3x - 9 = 0$$

Comparing the equation with $ax^2 + bx + c = 0$ *we have,*

$$a = 1,\ b = -3,\ c = -9$$

By formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \times 1 \times (-9)}}{2 \times 1}$$

$$x = \frac{3 \pm \sqrt{9 + 36}}{2}$$

$$x = \frac{3 \pm \sqrt{45}}{2}$$

$$x = \frac{3 \pm 6.708}{2}$$

$$x = \frac{3 + 6.708}{2},\ \frac{3 - 6.708}{2}$$

$$x = \frac{9.708}{2},\ \frac{-3.708}{2}$$

$$x = 4.854,\ -1.854$$

$$x = 4.9,\ -1.9$$

Q11. Find the value of 'K' for which x = 3 is a solution of the quadratic equation, (K + 2) x^2 – Kx + 6 = 0.Thus find the other root of the equation. [2015]

Answer: k= -4, other root = -1

Step-by-step Explanation:

$$(K + 2)x^2 - Kx + 6 = 0$$

$x = 3$ is a solution of the above equation.

Therefore,

$$(k+2)(3)^2 - k \times 3 + 6 = 0$$

$$9k + 18 - 3k + 6 = 0$$

$$6k + 24 = 0$$

$$6k = -24$$

$$k = -4$$

putting $k = -4$ in the given equation,

$$-2x^2 + 4x + 6 = 0$$

$$-2(x^2 - 2x - 3) = 0$$

$$x^2 - 2x - 3 = 0$$

$$x^2 - 3x + x - 3 = 0$$

$$x(x - 3) + 1(x - 3) = 0$$

$$(x - 3)(x + 1) = 0$$

Either $x + 1 = 0$ or $x - 3 = 0$

$$x = -1 \text{ or } 3$$

So, other root of the equation is -1.

Q12. Solve for x using the quadratic formula. Write your answer correct to two significant figures,

$(x - 1)^2 - 3x + 4 = 0$. [3] [2014]

Answer: 3.6 , 1.4

Step-by-step Explanation:

$$(x-1)^2 - 3x + 4 = 0$$

$$\Rightarrow x^2 - 2x + 1 - 3x + 4 = 0$$

$$\Rightarrow x^2 - 5x + 5 = 0$$

comparing the given equation with $ax^2 + bx + c = 0$, *we have,*

$$a = 1,\ b = -5,\ c = 5$$

By formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \times 1 \times 5}}{2 \times 1}$$

$$x = \frac{5 \pm \sqrt{25 - 20}}{2}$$

$$x = \frac{5 \pm \sqrt{5}}{2}$$

$$x = \frac{5 \pm 2.24}{2}$$

$$x = \frac{5 + 2.24}{2} \text{ or } \frac{5 - 2.24}{2}$$

$$x = \frac{7.24}{2} \text{ or } \frac{2.76}{2}$$

$$x = 3.62 \text{ or } 1.38$$

$$x = 3.6,\ 1.4$$

Q13. Solve the following equation and calculate the answer correct to two decimal places:

$x^2 - 5x - 10 = 0$ [3] [2013]

Answer: 6.53 , -1.53

Step-by-step Explanation:

$$x^2 - 5x - 10 = 0$$

comparing the given equation with $ax^2 + bx + c = 0$, *we have,*

$$a = 1,\ b = -5,\ c = -10$$

By formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \times 1 \times (-10)}}{2 \times 1}$$

$$x = \frac{5 \pm \sqrt{25 + 40}}{2}$$

$$x = \frac{5 \pm \sqrt{65}}{2}$$

$$x = \frac{5 \pm 8.062}{2}$$

$$x = \frac{5 + 8.062}{2} \text{ or } \frac{5 - 8.062}{2}$$

$$x = \frac{13.062}{2} \text{ or } \frac{-3.062}{2}$$

$$x = 6.531 \text{ or } -1.531$$

$$x = 6.53\ ,\ -1.53$$

Q14. Without solving the following quadratic equation, find the value of 'p' for which the given equation has real and equal roots: $x^2 + (p - 3)x + p = 0$ [2013]

Answer: p= 1 or 9

Step-by-step Explanation:

$$x^2 + (p-3)x + p = 0$$

comparing the given equation with $ax^2 + bx + c = 0$, *we have,*

$$a = 1,\ b = (p-3),\ c = p$$

$$\text{Given, } b^2 - 4ac = 0$$

$$\Rightarrow (p-3)^2 - 4 \times 1 \times p = 0$$

$$\Rightarrow p^2 - 6p + 9 - 4p = 0$$

$$\Rightarrow p^2 - 10p + 9 = 0$$

$$\Rightarrow p^2 - 9p - p + 9 = 0$$

$$\Rightarrow p(p-9) - 1(p-9) = 0$$

$$\Rightarrow (p-9)(p-1) = 0$$

$$\text{Either } p - 9 = 0 \text{ OR } p - 1 = 0$$

$$p = 9 \text{ or } 1$$

Q15. Without solving the following quadratic equation, find the value of 'm' for which the given equation has real and equal roots.
$x^2 + 2(m-1)x + (m+5) = 0$ [3] [2012]

Answer: m = -1 , 4

Step-by-step Explanation:

$$x^2 + 2(m-1)x + (m+5) = 0$$

comparing the given equation with $ax^2 + bx + c = 0$

$$a = 1,\ b = 2(m-1),\ c = (m+5)$$

$$\text{given, } b^2 - 4ac = 0$$

$$[2(m-1)]^2 - 4 \times 1 \times (m+5) = 0$$

$$4(m^2 - 2m + 1) - 4m - 20 = 0$$

$$4m^2 - 8m + 4 - 4m - 20 = 0$$

$$4m^2 - 12m - 16 = 0$$

$$4(m^2 - 3m - 4) = 0$$

$$m^2 - 3m - 4 = 0$$

$$m^2 - 4m + m - 4 = 0$$

$$m(m-4) + 1(m-4) = 0$$

$$(m+1)(m-4) = 0$$

$$\text{Either } (m+1) = 0 \text{ or } (m-4) = 0$$

$$m = -1 \text{ or } 4$$

Q16. Solve the following equation and give your answer correct to 3 significant figures:

$5x^2 - 3x - 4 = 0$ [3] [2012]

Answer: 1.24 , -0.643

Step-by-step Explanation:

$$5x^2 - 3x - 4 = 0$$

comparing the given equation with $ax^2 + bx + c = 0$

$$a = 5,\ b = -3\ ,\ c = -4$$

by quadratic formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \times 5 \times (-4)}}{2 \times 5}$$

$$x = \frac{3 \pm \sqrt{9 + 80}}{10}$$

$$x = \frac{3 \pm \sqrt{89}}{10}$$

$$x = \frac{3 \pm 9.434}{10}$$

$$x = \frac{3 + 9.434}{10}\ ,\ \frac{3 - 9.434}{10}$$

$$x = \frac{12.434}{10}\ ,\ \frac{-6.434}{10}$$

$$x = 1.2434\ ,\ -0.6434$$

$$x = 1.24\ ,\ -0.643$$

Q17. Solve the following equation:

$$x - \frac{18}{x} = 6$$

Give your answer correct to two significant figures. [3] [2011]

Answer: 8.2 , -2.2

Step-by-step Explanation:

$$x - \frac{18}{x} = 6$$

$$\Rightarrow \frac{x^2 - 18}{x} = 6$$

$$\Rightarrow x^2 - 18 = 6x$$

$$\Rightarrow x^2 - 6x - 18 = 0$$

comparing the given equation with $ax^2 + bx + c = 0$

$$a = 1,\ b = -6\ ,\ c = -18$$

by quadratic formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4 \times 1 \times (-18)}}{2 \times 1}$$

$$x = \frac{6 \pm \sqrt{36 + 72}}{2}$$

$$x = \frac{6 \pm \sqrt{108}}{2}$$

$$x = \frac{6 \pm 10.392}{2}$$

$$x = \frac{6 + 10.392}{2}\ ,\ \frac{6 - 10.392}{2}$$

$$x = \frac{16.392}{2}\ ,\ \frac{-4.392}{2}$$

$$x = 8.196\ ,\ -2.196$$

$$x = 8.2\ ,\ -2.2$$

Q18. Without solving the following quadratic equation, find the value of 'p' for which the roots are equal. $px^2 - 4x + 3 = 0$ [3] [2010]

Answer: p = 4/3

Step-by-step Explanation:

$$px^2 - 4x + 3 = 0$$

comparing the above equation with $ax^2 + bx + c = 0$, *we have,*

$$a = p\ ,\ b = -4\ ,\ c = 3$$

given, $b^2 - 4ac = 0$

$$(-4)^2 - 4 \times p \times 3 = 0$$

$$16 - 12p = 0$$

$$-12p = -16$$

$$p = \frac{16}{12}$$

$$p = \frac{4}{3}$$

Q19. A man covers a distance of 100 km, travelling with a uniform speed of x km/hr. Had the speed been 5 km/hr more it would have taken 1 hour less. Find x the original speed. [2023]

Answer: 20 km/h

Step-by-step Explanation:

Original speed = x km/hr

Distance = 100 km

Therefore, Time taken = 100/x hr

Now, if speed = (x+5) km/hr

Then, by the problem,

$$\frac{100}{x+5} = \frac{100}{x} - 1$$
$$\Rightarrow \frac{100}{x+5} - \frac{100}{x} = -1$$
$$\Rightarrow \frac{100x - 100(x+5)}{x(x+5)} = -1$$
$$\Rightarrow \frac{100x - 100x - 500}{x^2 + 5x} = -1$$
$$\Rightarrow -500 = -x^2 - 5x$$
$$\Rightarrow x^2 + 5x - 500 = 0$$
$$\Rightarrow x^2 + 25x - 20x - 500 = 0$$
$$\Rightarrow x(x+25) - 20(x+25) = 0$$
$$\Rightarrow (x+25)(x-20) = 0$$
$$either\ (x+25) = 0\ or\ (x-20) = 0$$
$$x = -25\ or\ 20$$

As speed cannot be negative, therefore original speed is 20 km/h

Q20. The difference of two natural numbers is 7 and their product is 450. Find the numbers. [2020]

Answer: 18 and 25

Step-by-step Explanation:

Let the two numbers be x and (x-7).

by the problem,

$x(x-7) = 450$

or, $x^2 - 7x - 450 = 0$

or, $x^2 - 25x + 18x - 450 = 0$

or, $x(x - 25) + 18(x - 25) = 0$

or, $(x - 25)(x + 18) = 0$

Either (x – 25) = 0 or (x + 18) = 0

x = 25 or -18

As natural numbers cannot be negative, therefore

One number is 25 and

the other number is (25-7) = 18.

Q21. The product of two consecutive natural numbers which are multiples of 3 is equal to 810. Find the two numbers. [3] [2019]

Answer: 27 and 30

Step-by-step Explanation:

Let the two numbers be 3x and 3(x+1).

by the problem,

$$3x \times 3(x+1) = 810$$
$$\Rightarrow 3x(3x+3) = 810$$
$$\Rightarrow 9x^2 + 9x = 810$$
$$\Rightarrow 9(x^2 + x) = 810$$
$$\Rightarrow x^2 + x = 90$$
$$\Rightarrow x^2 + x - 90 = 0$$
$$\Rightarrow x^2 + 10x - 9x - 90 = 0$$
$$\Rightarrow x(x+10) - 9(x+10) = 0$$
$$\Rightarrow (x+10)(x-9) = 0$$
$$either\ (x+10) = 0\ or\ (x-9) = 0$$
$$x = -10\ or\ 9$$
$$As\ natural\ numbers\ cannot\ be\ negative,\ therefore,$$
$$x = 9$$
$$Therefore\ the\ two\ numbers\ are\ 3x = 27\ and$$
$$3(x+1) = 30$$

Q22. ₹ 7500 were divided equally among a certain number of children. Had there been 20 less children, each would have received ₹ 100 more. Find the original number of children. [2018]

Answer: 50

Step-by-step Explanation:

Let the original number of children be x.

Each child gets Rs. $\frac{7500}{x}$.

By the problem,

$$\frac{7500}{x-20} - \frac{7500}{x} = 100$$

$$\Rightarrow \frac{7500x - 7500(x-20)}{x(x-20)} = 100$$

$$\Rightarrow \frac{7500x - 7500x + 150000}{x^2 - 20x} = 100$$

$$\Rightarrow 100x^2 - 2000x = 150000$$

$$\Rightarrow 100x^2 - 2000x - 150000 = 0$$

$$\Rightarrow 100(x^2 - 20x - 1500) = 0$$

$$\Rightarrow x^2 - 20x - 1500 = 0$$

$$\Rightarrow x^2 - 50x + 30x - 1500 = 0$$

$$\Rightarrow x(x-50) + 30(x-50) = 0$$

$$\Rightarrow (x+30)(x-50) = 0$$

either $(x+30) = 0$ *or* $(x-50) = 0$

$$x = -30 \text{ or } 50$$

As number of children cannot be negative, therefore,

$$x = 50.$$

Therefore, original number of children is 50.

Q23. Two cars X and Y use 1 litre of diesel to travel x km and (x + 3) km respectively. If both the cars covered a distance of 72 km, then:

i. The number of litres of diesel used by car X is: [1]

(a) $\frac{72}{x-3} litres$

(b) $\frac{72}{x+3}\ litres$

(c) $\frac{72}{x} litres$

(d) $\frac{12}{x} litres$

ii. The number of litres of diesel used by car Y is: [1]

(a) $\frac{72}{x-3} litres$

(b) $\frac{72}{x+3}\ litres$

(c) $\frac{72}{x} litres$

(d) $\frac{12}{x+3} litres$

iii. If car X used 4 litres of diesel more than car Y in the journey, then: [1]

(a) $\frac{72}{x-3} - \frac{12}{x} = 4$

(b) $\frac{72}{x+3} - \frac{72}{x} = 4$

(c) $\frac{72}{x} - \frac{72}{x+3} = 4$

(d) $\frac{72}{x-3} - \frac{72}{x+3} = 4$

iv. The amount of diesel used by the car X is: [1]

(a) 6 litres

(b) 12 litres

(c) 18 litres

(d) 24 litres [2021 Semester-1]

Answer: i. (c) , ii. (b) , iii. (c) , iv. (b)

Step-by-step Explanation:

$$\textit{By the problem,}$$

$$\frac{72}{x} - \frac{72}{x+3} = 4$$

$$\Rightarrow \frac{72(x+3) - 72x}{x(x+3)} = 4$$

$$\Rightarrow \frac{72x + 216 - 72x}{x^2 + 3x} = 4$$

$$\Rightarrow 4x^2 + 12x = 216$$

$$\Rightarrow 4x^2 + 12x - 216 = 0$$

$$\Rightarrow 4(x^2 + 3x - 54) = 0$$

$$\Rightarrow x^2 + 3x - 54 = 0$$

$$\Rightarrow x^2 + 9x - 6x - 54 = 0$$

$$\Rightarrow x(x+9) - 6(x+9) = 0$$

$$\Rightarrow (x+9)(x-6) = 0$$

either $(x + 9) = 0$ or $(x - 6) = 0$

$x = -9$ or 6

therefore,

$x = 6$ litres.

Therefore, The amount of diesel used by the car X is

$$\frac{72}{x} = \frac{72}{6} = 12 \textit{ litres.}$$

option (b).

Q24. The sum of the ages of Vivek and his younger brother Amit is 47 years. The product of their ages in years is 550. Find their ages. [4] [2017]

Answer: 25 years, 22 years

Step-by-step Explanation:

Let the age of Vivek be x years and that of his younger brother be (47-x) years.

By the problem,

$x(47 - x) = 550$

or, $47x - x^2 = 550$

or, $x^2 - 47x + 550 = 0$

or, $x^2 - 25x - 22x + 550 = 0$

or, $x(x - 25) - 22(x - 25) = 0$

or, (x – 25)(x – 22) = 0

either (x – 25) = 0 or (x – 22) = 0

x = 25 or 22

Therefore, Vivek's age is 25 years and his younger brother's age is 22 years.

Q25. A bus covers a distance of 240 km at a uniform speed. Due to heavy rain its speed gets reduced by 10 km/h and as such it takes two hours longer to cover the total distance. Assuming the uniform speed to be 'x' km/h, form an equation and solve it to evaluate 'x'. [3] [2016]

Answer: 40 km/h

Step-by-step Explanation:

$$Uniform\ speed = x\ km/h$$

$$distance = 240\ km$$

$$time = \frac{Distance}{speed} = \frac{240}{x}\ hours$$

$$Due\ to\ heavy\ rain\ speed = (x - 10)\ km/h$$

$$Now\ time = \frac{240}{x - 10} hours$$

By the problem,

$$\frac{240}{x-10} - \frac{240}{x} = 2$$

$$\Rightarrow \frac{240x - 240(x-10)}{x(x-10)} = 2$$

$$\Rightarrow \frac{240x - 240x + 2400}{x^2 - 10x} = 2$$

$$\Rightarrow 2x^2 - 20x = 2400$$

$$\Rightarrow 2x^2 - 20x - 2400 = 0$$

$$\Rightarrow 2(x^2 - 10x - 1200) = 0$$

$$\Rightarrow x^2 - 10x - 1200 = 0$$

$$\Rightarrow x^2 - 40x + 30x - 1200 = 0$$

$$\Rightarrow x(x-40) + 30(x-40) = 0$$

$$\Rightarrow (x-40)(x+30) = 0$$

$$either\ (x-40) = 0\ or\ (x+30) = 0$$

$$x = 40\ or\ -30$$

As speed cannot be negative, therefore,

$$x = 40$$

Therefore, speed of bus is 40 km/h.

Q26. Sum of two natural numbers is 8 and the difference of their reciprocal is 2 / 15. Find the numbers. [3] [2015]

Answer: 3 and 5

Step-by-step Explanation:

Let the two natural numbers be x and (8-x).

Now, by the problem,

$$\frac{1}{x} - \frac{1}{8-x} = \frac{2}{15}$$
$$\Rightarrow \frac{8-x-x}{x(8-x)} = \frac{2}{15}$$
$$\Rightarrow \frac{8-2x}{8x-x^2} = \frac{2}{15}$$
$$\Rightarrow 120 - 30x = 16x - 2x^2$$
$$\Rightarrow 2x^2 - 46x + 120 = 0$$
$$\Rightarrow 2(x^2 - 23x + 60) = 0$$
$$\Rightarrow x^2 - 23x + 60 = 0$$
$$\Rightarrow x^2 - 3x - 20x + 60 = 0$$
$$\Rightarrow x(x-3) - 20(x-3) = 0$$
$$\Rightarrow (x-3)(x-20) = 0$$
$$\textit{either } (x-3) = 0 \textit{ or } (x-20) = 0$$
$$x = 3, 20$$
$$20 \textit{ is not possible.}$$

Therefore, the two numbers are 3 *and* $(8-3) = 5$.

Q27. A two digit positive number is such that the product of its digits is 6. If 9 is added to the number, the digits interchange their places. Find the number. [4] [2014]

Answer: 23

Step-by-step Explanation:

Let the tens' digit be x *and unit's digit be* $\frac{6}{x}$.

Therefore the two digit number $= 10x + \frac{6}{x}$.

According to the problem,

$$10x + \frac{6}{x} + 9 = (10 \times \frac{6}{x}) + x$$

$$\Rightarrow 10x + \frac{6}{x} - \frac{60}{x} - x = -9$$

$$\Rightarrow \frac{10x^2 + 6 - 60 - x^2}{x} = -9$$

$$\Rightarrow 9x^2 - 54 = -9x$$

$$\Rightarrow 9x^2 + 9x - 54 = 0$$

$$\Rightarrow 9(x^2 + x - 6) = 0$$

$$\Rightarrow x^2 + x - 6 = 0$$

$$\Rightarrow x^2 + 3x - 2x - 6 = 0$$

$$\Rightarrow x(x + 3) - 2(x + 3) = 0$$

$$\Rightarrow (x + 3)(x - 2) = 0$$

$$Either\ (x + 3) = 0\ or\ (x - 2) = 0$$

$$x = -3, 2$$

Since number is positive, therefore

x=2

Therefore, the two – digit number is

$10x + \frac{6}{x}$

$20 + \frac{6}{2}$

23.

Q28. A shopkeeper purchases a certain number of books for Rs. 960. If the cost per book was 8 less, the number of books that could be purchased for Rs. 960 would be 4 more. Write an equation, taking the original cost of each book to be Rs. x, and solve it to find the original cost of the books.[4] [2013]

Answer: Rs. 48

Step-by-step Explanation:

$$\text{original cost of each book is Rs } x.$$

$$\text{Then, the no. of books } = \frac{960}{x}.$$

$$\text{Also, cost of each books } = \text{Rs } (x-8)$$

$$\text{Therefore, by the problem,}$$

$$\frac{960}{x-8} - \frac{960}{x} = 4$$

$$\Rightarrow \frac{960x - 960(x-8)}{x(x-8)} = 4$$

$$\Rightarrow \frac{960x - 960x + 7680}{x^2 - 8x} = 4$$

$$\Rightarrow 4x^2 - 32x = 7680$$

$$\Rightarrow 4x^2 - 32x - 7680 = 0$$

$$\Rightarrow x^2 - 8x - 1920 = 0$$

$$\Rightarrow x^2 + 40x - 48x - 1920 = 0$$

$$\Rightarrow (x-48)(x+40) = 0$$

$$\text{Either } x - 48 = 0 \text{ or } x + 40 = 0$$

$$x = 48 \text{ or } -40$$

$$\therefore \text{ Original cost of book } = \text{Rs } 48.$$

Q29. A car covers a distance of 400 km at a certain speed. Had the speed been 12 km/h more, the time taken for the journey would have been 1 hour 40 minutes less. Find the original speed of the car. [4] [2012]

Answer: 48 km/h

Step-by-step Explanation:

Let the original speed be x km/h.

$$\text{Then Time} = \frac{400}{x} \text{ hrs}$$

If speed $= x + 12$, *then*

$$\text{Time} = \frac{400}{x+12}$$

By the problem,

$$\frac{400}{x} - \frac{400}{x+12} = 1hr + \frac{40}{60}$$

$$\Rightarrow \frac{400(x+12) - 400x}{x(x+12)} = \frac{5}{3}$$

$$\Rightarrow \frac{400x + 4800 - 400x}{x^2 + 12x} = \frac{5}{3}$$

$$\Rightarrow 5x^2 + 60x = 14400$$

$$\Rightarrow 5x^2 + 60x - 14400 = 0$$

$$\Rightarrow x^2 + 12x - 2880 = 0$$

$$\Rightarrow x^2 + 60x - 48x - 2880 = 0$$

$$\Rightarrow x(x+60) - 48(x+60) = 0$$

$$\Rightarrow (x+60)(x-48) = 0$$

Either $x + 60 = 0$ *or* $x - 48 = 0$

$$x = -60 \text{ or } 48$$

Speed cannot be negative. therefore,

speed of the car is 48 *km/h.*

Q30. Rs.480 is divided equally among 'x' children. If the number of children was 20 more, then each would have got Rs. 12 less. Find 'x'. [3] [2011]

Answer: 20

Step-by-step Explanation:

$$Given,\ Number\ of\ children\ =\ x$$

$$Therefore,\ money\ received\ by\ each\ child\ =\ Rs.\ \frac{480}{x}$$

$$Now,\ if\ Number\ of\ children\ =\ x+20,\ then,$$

$$by\ the\ problem,$$

$$\frac{480}{x}-\frac{480}{x+20}=12$$

$$\Rightarrow \frac{480(x+20)-480x}{x(x+20)}=12$$

$$\Rightarrow \frac{480x+9600-480x}{x^2+20x}=12$$

$$\Rightarrow 12x^2+240x=9600$$

$$\Rightarrow x^2+20x-800=0$$

$$\Rightarrow x^2+40x-20x-800=0$$

$$\Rightarrow x(x+40)-20(x+40)=0$$

$$\Rightarrow (x+40)(x-20)=0$$

$$Either\ x+40=0\ or\ x-20=0$$

$$x=-40\ or\ 20.$$

$$Children\ cannot\ be\ in\ negative.$$

$$Therefore,\ number\ of\ children\ is\ 20.$$

Q31. A positive number is divided into two parts such that the sum of the squares of the two parts is 20. The square of the larger part is 8 times the smaller part. Taking x as the smaller part of the two parts, find the number. [4] [2010]

Answer: 6

Step-by-step Explanation:

$$\text{Let smaller part be } x.$$

$$\text{Then, } (\text{larger part})^2 = 8x \text{ Therefore, larger part} = \sqrt{8x}$$

$$\text{By the problem,}$$

$$x^2 + 8x = 20$$

$$\Rightarrow x^2 + 8x - 20 = 0$$

$$\Rightarrow x^2 + 10x - 2x - 20 = 0$$

$$\Rightarrow x(x + 10) - 2(x + 10) = 0$$

$$\Rightarrow (x + 10)(x - 2) = 0$$

$$\text{Either } (x + 10) = 0 \text{ or } (x - 2) = 0$$

$$x = -10 \text{ or } 2.$$

$$\text{As the number is positive, therefore,}$$

$$\text{smaller part is } 2 \text{ and}$$

$$\text{larger part is } \sqrt{8 \times 2} = 4$$

$$\text{Therefore the number is } 4 + 2 = 6.$$