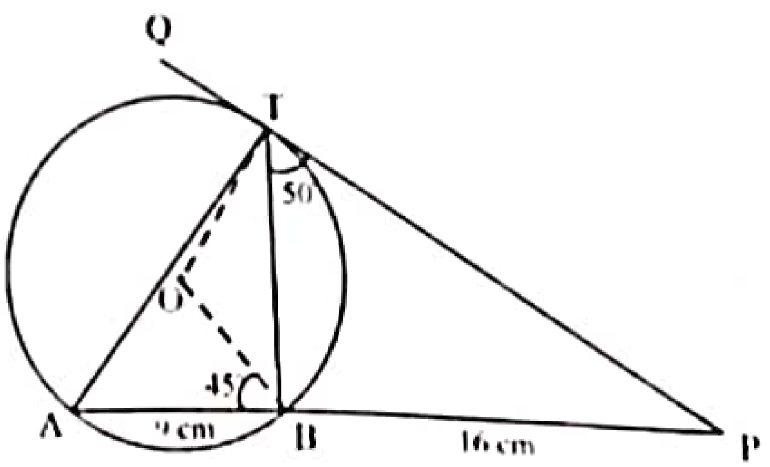
Circles

1. In the given figure, O is the centre of the circle. PQ is a tangent to the circle at T. Chord AB produced meets the tangent at P. AB = 9 cm, BP = 16 cm, ∠PTB = 50°, ∠OBA = 45°. Find:



(a) length of PT
(b) ∠BAT
(c) ∠BOT
(d) ∠ABT [2023]

Answer: (a) 20cm (b) 50° (c) 100° (d) 85°

Step-by-step Explanation:

(a) We know, $PT^2 = AP \times BP$ (When tangent and chord intersect externally, the product of the lengths of the segments of chord is equal to the square of the length of the tangent.)

$PT^2 = (16+9) \times 16$

$PT^2 = 25 \times 16$

$PT = \sqrt{25 \times 16}$

PT= 20 cm

(b) ∠BAT= ∠BTP = 50° (angle in the alternate segment)

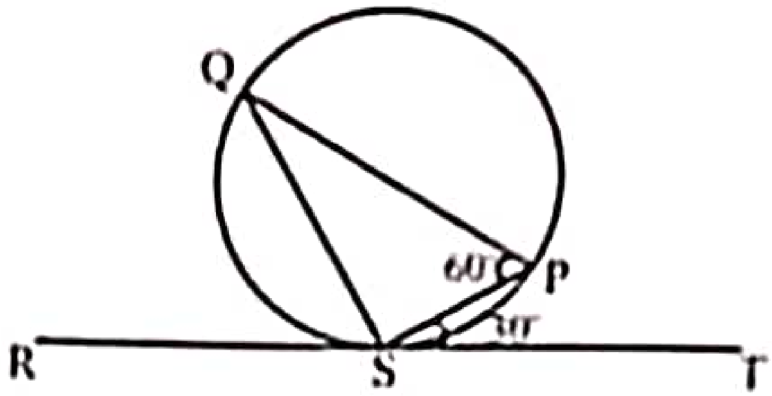
(c) ∠BOT =2∠BAT= 100° (Angle subtended by an arc at the center of a circle is double the angle subtended by it on remaining part of the circle.)

(d) In ΔBOT, OB = OT (radii of a circle)

∴∠OBT=∠BTO= 180°-100°/2= 40°

∴∠ABT= 45° + 40°= 85°

2. In the given diagram RT is a tangent touching the circle at S. If ∠PST= 50° and ∠SPQ = 60° then ∠PSQ is equal to:



(a) 40°
(b) 30°
(c) 60°
(d) 90° [2023]

Answer: (d)

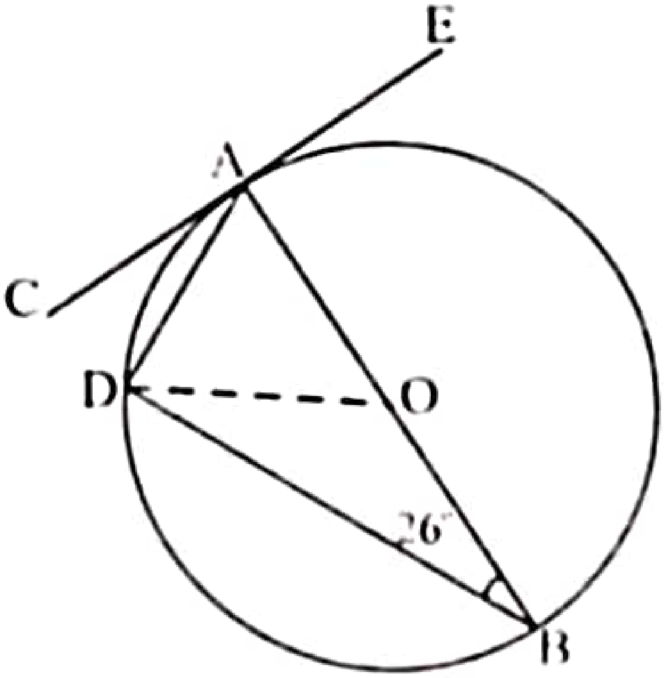
Step-by-step Explanation:

∠PQS=∠PST= 30°

In ΔPQS, ∠PQS +∠PSQ+∠QPS= 180°

∠PSQ= 180° – (60+30)°= 90°

3. In the given figure O, is the centre of the circle. CE is a tangent to the circle at A.
If ∠ABD=26°, then find



(a) ∠BDA
(b) ∠BAD
(c) ∠CAD
(d) ∠ODB [2023]

Answer: (a) 90° (b) 64° (c) 26° (d) 26°

Step-by-step Explanation:

(a) ∠BDA= 90° (angle in a semicircle is right angle.)

(b) ∠BAD= 180° – (90+26)° (sum of angles of a triangle is 180°

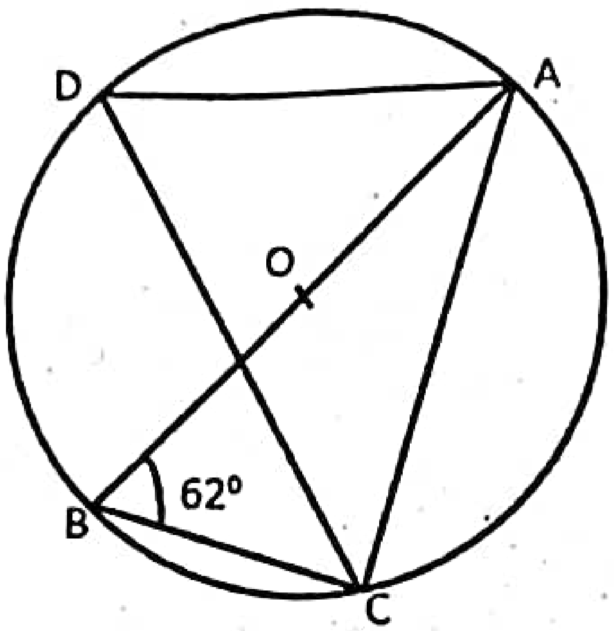
= 64°

(c) ∠CAD= 26° (angles in the alternate segments are equal.)

(d) ∠DOB= 2∠BAD= 2× 64= 128° (angle subtended by an arc at the center of a circle is double the angle subtended by it on any part on the remaining circle.)

∴ ∠ODB= 180° – (26+128)° =26° (sum of the angles of a triangle is 180°.)

4. In the given figure A, B, C and D are points on the circle with centre O. Given ∠ABC = 62°



Find:

(a) ∠ADC

(b) ∠CAB [2022 Semester-2]

Solution: (a) 62° (b) 28°

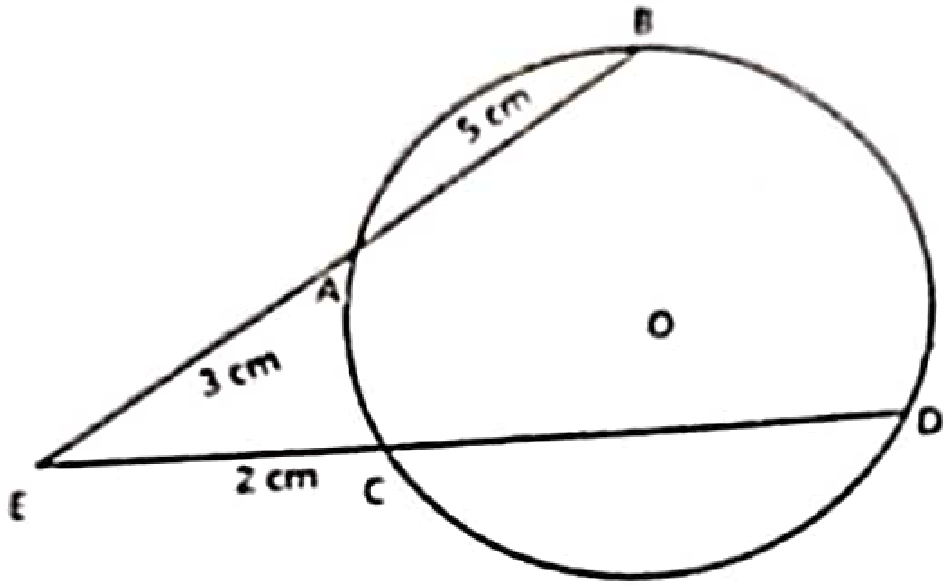
Step-by-step Explanation:

(a) ∠ADC= ∠ABC= 62° (Angles in the same segment are equal.)

(b) ∠ACB= 90° (angle in a semicircle is right angle.)

∴ ∠CAB= 180°– (62°+90°) =28° (sum of angles in a triangle is 180°.)

5. Two chords AB and CD of a circle intersect externally at E. If EC = 2 cm, EA = 3 cm and AB = 5 cm, Find the length of CD. [2022 Semester-2]



Answer: 10 cm

Step-by-step Explanation:

We know, AE × BE = CE × DE (when two chords intersect internally or externally, the products of the lengths of the segments of the chords are equal.)

$3 \times (3+5) = 2 \times (2+CD)$

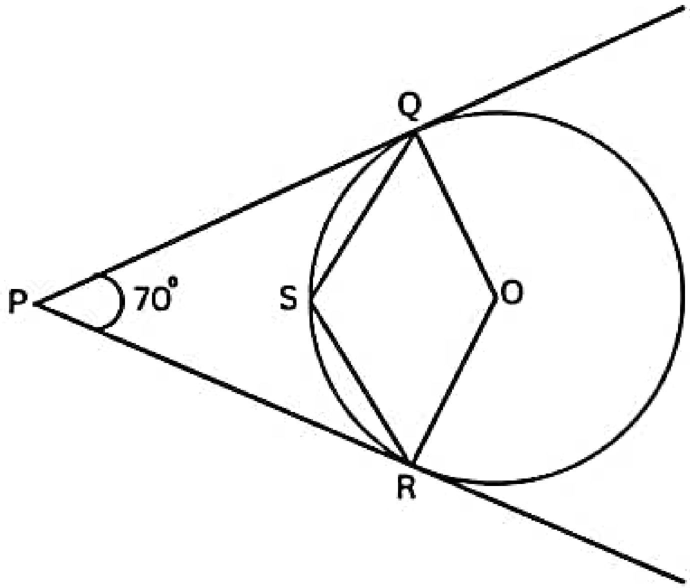
$24/2 = 2 + CD$

$10 = CD$

$CD = 10$ cm

6. In the given figure O is the centre of the circle. PQ and PR are tangents and $\angle QPR = 70°$. Calculate

(a) $\angle QOR$
(b) $\angle QSR$ [2022 Semester-2]



Answer: (a) 110° (b) 125°

Step-by-step Explanation:

(a) $\angle PQO = \angle PRO = 90°$ (tangent and the radius of a circle through the point of contact are perpendicular to each other.)

In Quadrilateral PQOR,

$\angle RPQ + \angle PQO + \angle QOR + \angle PRO = 360°$

$70° + 90° + \angle QOR + 90° = 360°$

$\angle QOR = 360° - 250° = 110°$

(b) reflex $\angle QOR = 360° - 110° = 250°$

$\angle QSR = 125°$ (angle subtended by an arc at the center of a circle is double the angle subtended by it on any part on the remaining circle.)

7. ABCD is a cyclic quadrilateral. If $\angle BAD = (2x + 5)°$ and $\angle BCD = (x + 10)°$ then x is equal to:
(a) 65° (b) 45° (c) 55° (d) 5° [2022 Semester-2]

Answer: (c)

Step-by-step Explanation:

We know, by theorem, opposite angles of a cyclic quadrilateral are supplementary.

$\therefore \angle BAD + \angle BCD = 180°$

$(2x + 5)° + (x + 10)° = 180°$

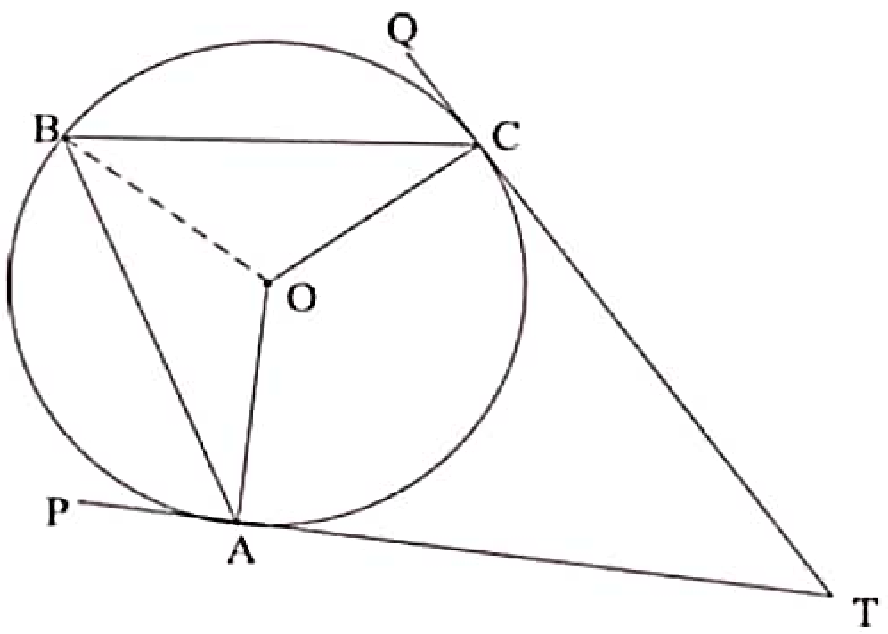
$3x + 15 = 180$

$3x = 165$

$x = 55°$

8. In the given figure TP and TQ are two tangents to the circle with centre O, touching at A and C, respectively. If $\angle BCQ = 55°$ and $\angle BAP = 60°$, find:

(i) $\angle OBA$ and $\angle OBC$
(ii) $\angle AOC$
(iii) $\angle ATC$ [2020]



Answer: (i) 30°, 35° (ii) 130° (iii) 50°

Step-by-step Explanation:

(i) PAT and QCT are tangents to the circle.

$\therefore \angle QCO = \angle PAO = 90°$ (tangent and the radius of a circle through the point of contact are perpendicular to each other.)

Now, $\angle BCQ = 55°$.

$\therefore \angle BCO = 90 - 55 = 35°$

In ΔBOC, $OB = OC$ (radii)

$\therefore \angle OBC = \angle OCB = 35°$

Similarly,

$\angle BAO = 90 - 60 = 30°$

In ΔOAB, $OA = OB$ (radii)

$\therefore \angle OBA = \angle BAO = 30°$

(ii) $\angle ABC = \angle OBA + \angle OBC = 30 + 35 = 65°$

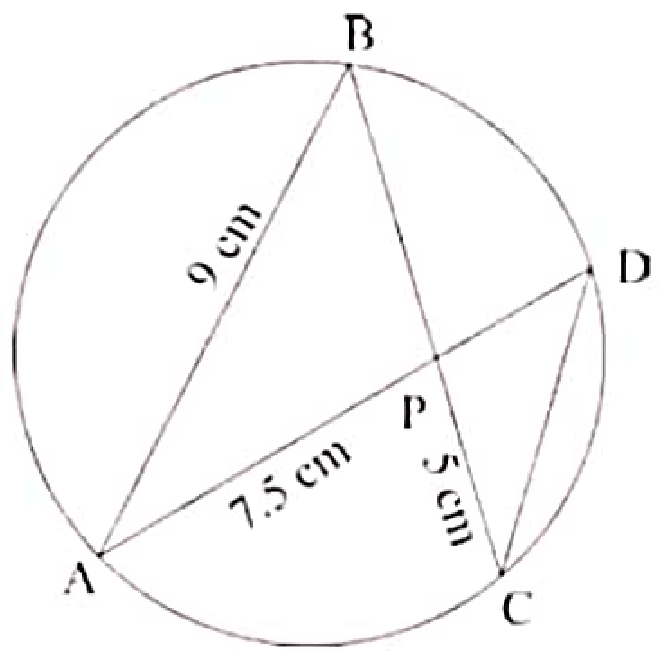
Hence, $\angle AOC = 2\angle ABC = 130°$ (angle subtended by an arc at the center is double the angle subtended by it on the remaining part on the circle.)

(iii) $\angle ATC = 360° - (\angle TAO + \angle AOC + \angle TCO)$ (sum of angles of a quadrilateral is 360°.)

$\therefore \angle ATC = 360° - (90+130+90)° = 360° - 310° = 50°$

9. In the given figure AB = 9 cm, PA = 7.5 cm and PC = 5 cm. Chords AD and BC intersect at P.

(i) Prove that $\Delta PAB \sim \Delta PCD$
(ii) Find the length of the CD.
(iii) Find area of ΔPAB : area of ΔPCD [2020]



Answer: (ii) 6 cm (iii) 9 : 4

Step-by-step Explanation:

(i) Chords AD and BC intersect internally. Therefore according to the theorem, the product of the lengths of their segments are equal.

$\therefore AP \times PD = BP \times PC$

or, $AP/PC = BP/PD$

Now, In ΔPAB and ΔPCD

$\angle APB = \angle CPD$ (vertically opposite angles)

$AP/PC = BP/PD$ (proved above)

$\therefore \Delta PAB \sim \Delta PCD$ (S-A-S condition of similarity)

(ii) As $\Delta PAB \sim \Delta PCD$

$\therefore AP/PC = BP/PD = AB/CD$

$AP/PC = AB/CD$

$7.5/5 = 9/CD$

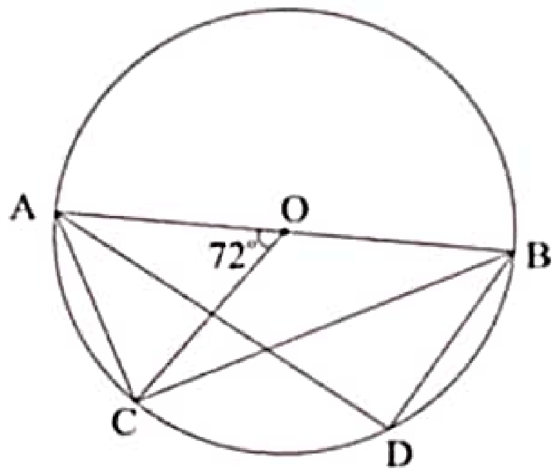
$CD = 9/1.5 = 6$ cm

(iii) Area of ΔPAB : Area of $\Delta PCD = (PA/PC)^2$ (ratio of areas of similar triangles is equal to the square of the ratio of their corresponding sides.)

Area of ΔPAB : Area of $\Delta PCD = (7.5/5)^2 = 9 : 4$

10. In the figure given below, O is the centre of the circle and AB is a diameter.

If AC = BD and $\angle AOC = 72°$. Find:
(i) $\angle ABC$
(ii) $\angle BAD$
(iii) $\angle ABD$ [2020]



Answer: (i) 36° (ii) 36° (iii) 54°

Step-by-step Explanation:

(i) $\angle ABC = 1/2\angle AOC = 36°$ (angle subtended by an arc at the center is double the angle subtended by it on the remaining part on the circle.)

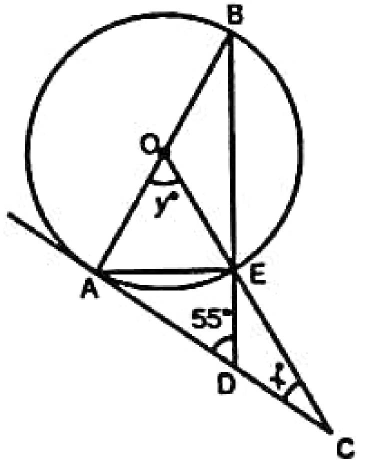
(ii) $\angle BAD = \angle ABC = 36°$ (equal chords subtend equal angles.)

(iii) $\angle ADB = 90°$ (angle in a semicircle is right angle.)

$\therefore \angle ABD = 180° - (\angle BAD + \angle ADB)$ (sum of angles of a triangle is 180°.)

or, $\angle ABD = 180° - 126° = 54°$

11. In the given figure, AC is a tangent to the circle with center 0. If $\angle ADB = 55°$, find x and y. Give reasons for your answers. [3] [2019]



Answer: x= 20° , y= 70°

Step-by-step Explanation:

∠AEB= 90° (angle in a semicircle is right angle.)

∴ ∠AED= 90° (linear pair)

∠DAE= 180°-(90°+55°)= 35°

∴∠ABE= 35° (angles in the alternate segments are equal.)

∴ ∠AOE= y°= 70° (angle subtended by an arc at the center is double the angle subtended by it on the remaining part on the circle.)

∠OEB=∠OBE= 35° (isosceles triangle property)

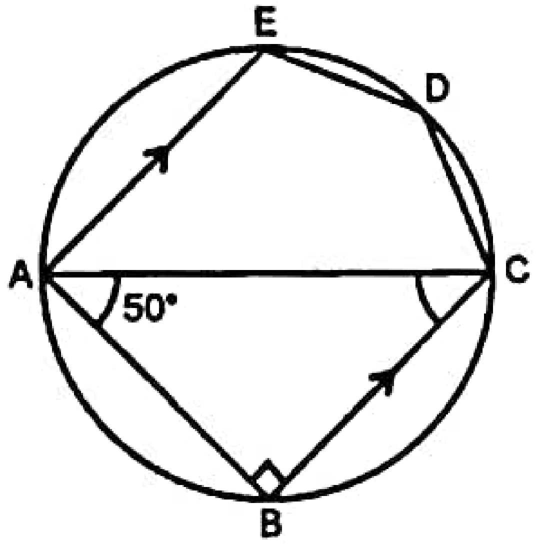
Hence, ∠DEC=∠OEB= 35°

∠EDC= 180 – 55=125° (linear pair)

Hence, x°= 180° – (125 + 35)° = 20°

12. In the given figure, ABCDE is a pentagon inscribed in a circle such that AC is a diameter and side BC || AE. If ΔBAC = 50°, find giving reasons : [4]

(i) ∠ACB
(ii) ∠EDC
(iii) ∠BEC [2019]



Answer: (i) 40° (ii) 140° (iii) 50°

Step-by-step Explanation:

(i) ∠ABC= 90° (angle in a semicircle is right angle.)

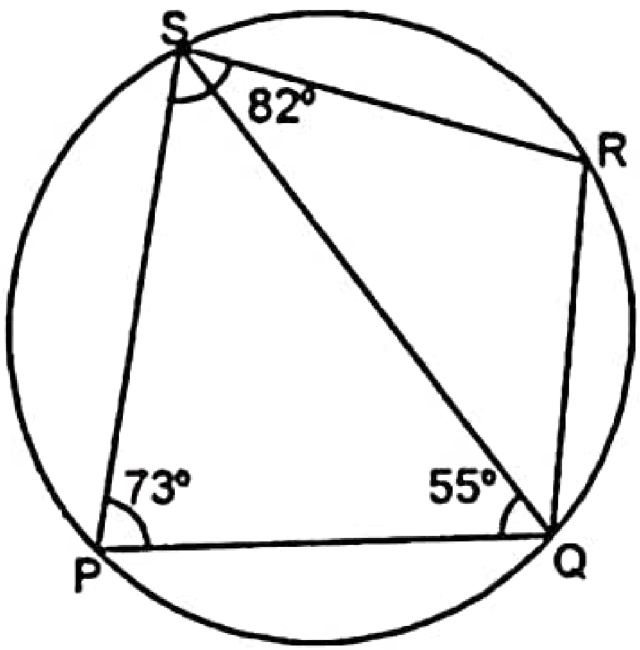
Hence, ∠ACB= 180° – (90+50)°= 40°

(ii) ∠CAE=∠ACB= 40°

Hence, ∠EDC= 180° - 40°= 140° (opposite angles of a cyclic quadrilateral are supplementary.)

(iii) ∠BEC= ∠BAC= 50° (angles in the same segment are equal.)

13. PQRS is a cyclic quadrilateral. Given $\angle QPS = 73°$, $\angle PQS = 55°$ and $\angle PSR = 82°$, calculate: [4]



(i) $\angle QRS$

(ii) $\angle RQS$

(iii) $\angle PRQ$ [2018]

Answer: (i) 107° (ii) 43° (iii) 52°

Step-by-step Explanation:

(i) $\angle QRS = 180° - 73° = 107°$ (opposite angles of a cyclic quadrilateral are supplementary.)

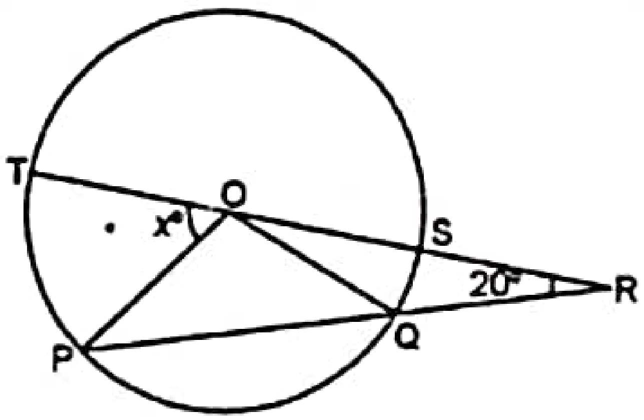
(ii) $\angle PSQ = 180° - (73 + 55)° = 52°$

$\therefore \angle RSQ = 82 - 52 = 30°$

Hence, $\angle RQS = 180° - (107 + 30)° = 43°$

(iii) $\angle PRQ = \angle PSQ = 52°$ (angles in the same segment are equal.)

14. In the figure given below 'O' is the center of the circle. If QR = OP and ∠ORP= 20°. Find the value of 'x ' giving reasons. [3] [2018]



Answer: 60°

Step-by-step Explanation:

OP=QR (given) and OP= OQ (radii)

Hence, OQ = QR

$\therefore \angle QOR = \angle ORQ = 20°$

$\therefore \angle OQR = 180° - 40° = 140°$ (angle sum property of triangle)

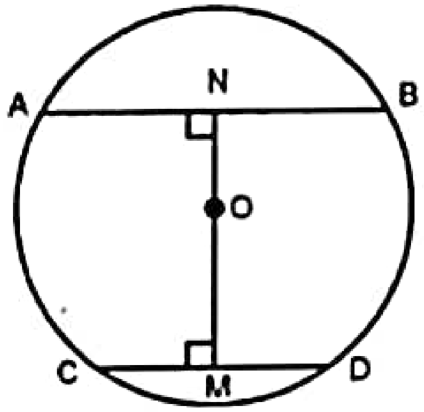
$\therefore \angle OQP = 180° - 140° = 40°$

$\therefore \angle OPQ = 40°$

$\therefore \angle POQ = 180° - 80° = 100°$

$\therefore x° = 180° - (100 + 20)° = 60°$ (angles in a straight line)

15. AB and CD are two parallel chords of a circle such that AB = 24 cm and CD = 10 cm. If the radius of the circle is 13 cm, find the distance between the two chords. [3] [2017]



Answer: 17 cm

Step-by-step Explanation:

Join OB and OD,

NB= 1/2 AB= 12 cm and MD= 1/2 CD= 5 cm (perpendicular drawn from the center of a circle to the chord bisects it.)

In ΔONB, By pythagoras theorem,

$ON = \sqrt{OB^2 - NB^2}$

$ON = \sqrt{169 - 144} = 5$ cm

In ΔOMD, By pythagoras theorem,

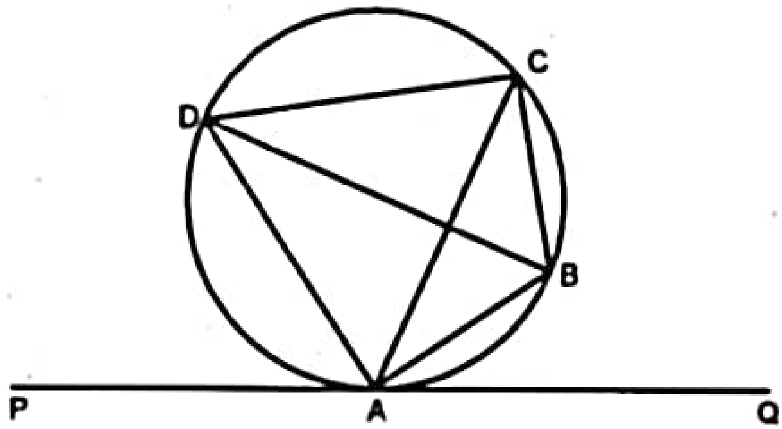
$OM = \sqrt{OD^2 - MD^2}$

$ON = \sqrt{169 - 25} = 12$ cm

$\therefore$ MN= 5 + 12= 17 cm

16. In the given figure PQ is a tangent to the circle at A. AB and AD are bisectors of $\angle CAQ$ and $\angle PAC$. If $\angle BAQ = 30°$ prove that :

(i) BD is a diameter of the circle.
(ii) ABC is an isosceles triangle. [2017]



Step-by-step Explanation:

(i) Given that AB and AD are bisectors of $\angle CAQ$ and $\angle PAC$.

Let $\angle CAB = \angle BAQ = x°$ and $\angle CAD = \angle DAP = y°$.

$\therefore \angle BAQ + \angle CAB + \angle CAD + \angle DAP = (2x + 2y)°$

$(2x + 2y)° = 180°$ (angles in a straight line.)

$2(x+y) = 180°$

$x + y = 90°$

or, $\angle BAD = 90°$

Hence, BD is the diameter of the circle. (angle in a semicircle is right angle.)