BYJU'S The Learning App

Question 1: If R = {(x, y) : x, y $\in$ Z, $x^2 + 3y^2 \leq 8$} is a relation on the set of integers Z, then the domain of R^{-1} is

(a) {-2, -1, 1, 2}

(b) {0, 1}

(c) {-2, -1, 0, 1, 2}

(d) {-1, 0, 1}

Solution:

Given R = {(x, y) : x, y $\in$ Z, $x^2 + 3y^2 \leq 8$}

when x = 0, $3y^2 \leq 8$

Domain of R^{-1} = value of y

= {-1, 0, 1}

Hence option d is the answer.

Question 2: Let the function f: [0, 1] → R be defined by $f(x) = 4^x/(4^x + 2)$. Then the value of f(1/40) + f(2/40) + f(3/40) + … + f(39/40) - f(1/2)

Solution:

Given $f(x) = 4^x/(4^x + 2)$

$f(1-x) = 4^{1-x}/(4^{1-x} + 2)$

$f(x) + f(1-x) = 4^x/(4^x + 2) + 4^{1-x}/(4^{1-x} + 2)$

$= 4^x/(4^x + 2) + (4/4^x)/((4/4^x) + 2)$

$= 4^x/(4^x + 2) + 2/(2 + 4^x)$

$= (4^x + 2)/(4^x + 2)$

= 1

f(1/40) + f(2/40) + f(3/40) + … + f(39/40) - f(1/2) = (f(1/40) + f(39/40)) + f(2/40) + f(38/40) + …+ f(20/40) - f(½)

= (1 + 1 + ...19 times) + f(½) - f(½)

= 19

Question 3: The function f: [0, 3] → [1, 29], defined by $f(x) = 2x^3 - 15x^2 + 36x + 1$ is

(a) one one and onto

(b) onto but not one one

(c) one one but not onto

(d) neither one one nor onto

Solution:

Given $f(x) = 2x^3 - 15x^2 + 36x + 1$

$f'(x) = 6x^2 - 30x + 36$

$= 6(x - 2)(x - 3)$

$f'(x) > 0$

$x \in (-\infty, 2) \cup (3, \infty)$

f (x) is increasing [0, 2) and is decreasing in [2, 3]

So f(x) is not one-one.

It is many one.

f(0) = 1, f (2) = 29, f (3) = 28

Range = [1, 29]

So f(x) is onto.

Hence option b is the answer.

Question 4: Let the function, f:[-7, 0] →R be continuous on [-7, 0] and differentiable on (-7, 0). If f(-7)= -3 and $f'(x) \leq 2$, for all $x \in (-7, 0)$, then for all such functions f, f(-1) + f(0) lies in the interval:

(a) [-6, 20]

(b) $(-\infty, 20]$

(c) $(-\infty, 11]$

(d) [-3, 11]

Solution:

Given $f(-7) = -3$ and $f'(x) \leq 2$

Applying LMVT in [-7,0], we get

$(f(-7) - f(0))/-7 = f'(c) \leq 2$

$(-3-f(0))/-7 \leq 2$

$f(0) + 3 \leq 14$

$f(0) \leq 11$

Applying LMVT in [-7, -1], we get

$(f(-7) - f(-1))/(-7 +1) = f'(c) \leq 2$

$-3 - f(-1))/-6 = f'(c) \leq 2$

$f(-1) + 3 = \leq 12$

$f(-1) \leq 9$

Therefore $f(-1)+ f(0) \leq 20$

Hence option b is the answer.

Question 5: Let X be a set with exactly 5 elements and Y be a set with exactly 7 elements. If α is the number of one-one functions from X to Y and β is the number of onto functions from Y to X, then the value of (1/5!)(β - α) is

Solution:

Given $n(X) = 5$

$n(Y) = 7$

α = number of one-one functions from X to Y = ${}^7C_5 \times 5! = 2520$

β = number of onto functions from Y to X = $({}^7C_3 + 3.\ {}^7C_3)5!$

$= 4\times {}^7C_3 \times 5!$

$= 16800$

$(\beta - \alpha)/5! = (16800 - 2520)/5!$

$= 119$

Question 6: Let [t] denote the greatest integer ≤ t. Then the equation in x, $[x]^2 + 2[x+2] - 7 = 0$ has

(a) exactly four integral solutions.

(b) infinitely many solutions.

(c) no integral solution.

(d) exactly two solutions.

Solution:

Given $[x]^2+2[x+2]-7 = 0$

$[x]^2 +2[x]-3 = 0$

let $[x] = y$

$y^2+3y-y -3 = 0$

$(y -1) (y+3) = 0$

$[x] = 1$, $[x] = -3$

$\Rightarrow x \in [-3, -2) \cup [1, 2)$

Hence option b is the answer.

Question 7: Let f(x) be a quadratic polynomial such that f(-1) + f(2) = 0. If one of the roots of f(x) = 0 is 3, then its other root lies in

(a) (-3, -1)

(b) (1, 3)

(c) (-1, 0)

(d) (0, 1)

Solution:

Let $f(x) = ax^2 + bx + c$

$f(-1) = a - b + c$

$f(2) = 4a + 2b + c$

Given $f(-1) + f(2) = 0$

$\Rightarrow a - b + c + 4a + 2b + c = 0$

$\Rightarrow 5a + b + 2c = 0$ ..(i)

Given that one root is 3.

$f(3) = 0 \Rightarrow 9a + 3b + c = 0$ ..(ii)

Multiply (i) by 3 and subtract (ii) from it, we get $a/-5 = c/6$

$\Rightarrow c/a = -6/5$

Product of roots, $\alpha\beta = c/a = -6/5$

$\alpha = 3$

So $\beta = -2/5 \in (-1,0)$

Hence option c is the answer.

Question 8: If $f(x + y) = f(x) f(y)$ and $\sum_{x=1}^{\infty} f(x) = 2$. $x, y \in N$, where N is the set of all natural number, then the value of $f(4)/f(2)$ is:

(a) 2/3

(b) 1/9

(c) 1/3

(d) 4/9

Solution:

$f(x + y) = f(x) f(y)$

Put $x = 1, y = 1$

$f(2) = (f(1))^2$

Put $x = 2, y = 1$

$f(3) = f(2).\ f(1) = (f(1))^3$

Put $x = 2$, $y = 2$

$f(4) = (f(2))^2 = (f(1))^4$

So $f(n) = (f(1))^n$

$\sum_{x=1}^{\infty} f(x) = f(1) + f(2) + f(3) + \ldots = 2$

$f(1)+(f(1))^2+(f(1))^3+\ldots = 2$

$f(1)/(1-f(1)) = 2$

$f(1) = 2/3$

$f(2) = (2/3)^2$

$f(4) = (2/3)^4$

$f(4)/f(2) = (2/3)^4/(2/3)^2 = 4/9$

Hence option d is the answer.

Question 9: If $f(x) = \log_e((1 - x)/(1+x))$, $|x| < 1$, then $f(2x/(1+x^2))$ is equal to:

(a) $2f(x)$

(b) $2f(x^2)$

(c) $(f(x))^2$

(d) $-2f(x)$

Solution:

Given $f(x) = \log_e((1 - x)/(1+x))$

$f(2x/(1+x^2)) = \log_e((1 - 2x/(1+x^2))/(1+2x/(1+x^2)))$

$= \log_e((1+x^2 - 2x)/(1+x^2+2x))$

$= \log_e((1-x)^2/(1+x)^2)$

$= \log_e ((1-x)/(1+x))^2$

$= 2 \log ((1-x)/(1+x)$

= 2 f(x)

Hence option a is the answer.

Question 10: Let f(n) = [⅓ + 3n/100]n, where [n] denotes the greatest integer less than or equal to n. Then $\sum_{n=1}^{56}$f(n) is equal to

(a) 56

(b) 689

(c) 1287

(d) 1399

Solution:

f(x) = [(⅓) + 3x/100]x

f(1) to f(22) = 0

f(23) = [(⅓) + (69/100)]23

f(24) = [(⅓) + (72/100)]24

f(23) + f(24) + ...f(56) = 23 + 24 + …+55 + 2×56

= 33×39 + 112

= 1399

Hence option d is the answer.