Arithmetic Progression

1. Which term of the Arithmetic Progression (A.P.) 15, 30, 45, 60... is 300? Hence find the sum of all the terms of the Arithmetic Progression. [2023]

Solution: 20th term , 3150

Step-by-step Explanation:

Let 300 be nth term.

a = 15, d = 30 – 15 = 15

We know, T_n =a + (n – 1) d

Therefore, 300 = 15 + (n – 1) 15

300 = 15 + 15n – 15

300 = 15n

15n = 300

n = 20

Hence, 300 is the 20th term.

$$S_n = \frac{n}{2}(a + l)$$

$$S_{20} = \frac{20}{2}(15 + 300)$$

$$S_{20} = 10 \times 315$$

$$S_{20} = 3150$$

2. The 5th term and the 9th term of an Arithmetic Progression are 4 and -12 respectively.

Find:
(a) the first term
(b) common difference
(c) sum of 16 terms of the AP. [2023]

Solution: (a) 20 (b) -4 (c) -160

Step-by-step Explanation:

$$Given,\ T_5 = 4\ and\ T_9 = -12$$
$$So,\ a + 4d = 4 \ldots\ldots (1)$$
$$a + 8d = -12 \ldots\ldots (2)$$
$$subtracting\ equation\ (2)\ from\ (1),\ we\ have$$
$$4d - 8d = 4 + 12$$
$$-4d = 16$$
$$d = -4$$
$$putiing\ the\ value\ of\ d\ in\ (1)$$
$$a + 4(-4) = 4$$
$$a = 4 + 16$$
$$a = 20$$
$$(a)\ first\ term = 20$$
$$(b)\ common\ difference = -4$$
$$(c)\ S_{16} = \frac{n}{2}[2a + (n-1)d]$$
$$\Rightarrow \frac{16}{2}[2 \times 20 + 15 \times (-4)]$$
$$\Rightarrow 8(40 - 60)$$
$$\Rightarrow -160$$

3. The nth term of an Arithmetic Progression (A.P.) is 2n + 5. The 10th term is:

(a) 7
(b) 15
(c) 25
(d) 45 [2023]

Solution: (c)

Step-by-step Explanation:

$$\begin{aligned} &Given,\ nth\ term\ of\ an\ A.P. = 2n + 5 \\ &T_n = 2n + 5 \\ &T_{10} = 2 \times 10 + 5 \\ &T_{10} = 25 \end{aligned}$$

4. The sum of the first 20 terms of the Arithmetic Progression 2, 4, 6, 8,....is :

(a) 400
(b) 840
(c) 420
(d) 800 [2021 Semester-1]

Solution: (c)

Step-by-step Explanation:

$$\begin{aligned} S_n &= \frac{n}{2}[2a + (n-1)d] \\ \therefore S_{20} &= \frac{20}{2}[2 \times 2 + (20-1)2] \\ S_{20} &= 10\ [4 + 38] \\ S_{20} &= 420 \end{aligned}$$

5. The first four terms of an Arithmetic Progression (A. P.), whose first term is 4 and common difference is -6, are:
(a) 4, -10, -16, -22
(b) 4, 10, 16, 22
(c) 4, -2, -8, -14
(d) 4, 2, 8, 14 [2021 Semester-1]

Solution: (c)

Step-by-step Explanation:

$$\begin{aligned} &Given, \\ &First\ term\ a\ =\ 4 \\ &common\ difference\ d\ =\ -6 \\ &\therefore T_2 = 4 - 6 = -2 \\ &T_3 = -2 - 6 = -8 \\ &T_4 = -8 - 6 = -14 \end{aligned}$$

6. In an Arithmetic Progression (A.P.) if, first term is 5, common difference is – 3 and then nth term is – 7, then n is equal to:

(a) 5
(b) 17
(c) – 13
(d) 7 [2021 Semester-1]

Solution: (a)

Step-by-step Explanation:

$$
\begin{aligned}
&Given,\\
&first\ term\ a = 5,\\
&common\ difference\ d = -3\\
&nth\ term = -7,\\
&\therefore T_n = a + (n-1)d\\
&-7 = 5 + (n-1)(-3)\\
&-7 = 5 + -3n + 3\\
&3n = 15\\
&n = 5
\end{aligned}
$$

7. If 70, 75, 80, 85 are the first four terms of an Arithmetic Progression. Then the 10th term is:

(a) 35
(b) 25
(c) 115
(d) 105 [2021 Semester-1]

Solution: (c)

Step-by-step Explanation:

$$
\begin{aligned}
&first\ term\ a = 70,\\
&common\ difference\ d = a_2 - a_1 = 75 - 70 = 5\\
&T_n = a + (n-1)d\\
&\therefore T_{10} = 70 + (10-1)5\\
&= 70 + 45\\
&= 115
\end{aligned}
$$

8. If the 6th term of an A.P. is equal to four times its first term and the sum of first six terms is 75, find the first term and the common difference. [2020]

Solution: a = 5 , d = 3

Step-by-step Explanation:

$$
\begin{aligned}
&Given,\\
&T_6 = 4 \times a\\
\Rightarrow\ &a + 5d = 4a\\
\Rightarrow\ &5d = 3a\\
\Rightarrow\ &d = \frac{3a}{5}\\
&S_6 = 75\\
\Rightarrow\ &\frac{n}{2}[2a + (n-1)d] = 75\\
\Rightarrow\ &\frac{6}{2}[2a + 5 \times \frac{3a}{5}] = 75\\
\Rightarrow\ &3[2a + 3a] = 75\\
\Rightarrow\ &15a = 75\\
\Rightarrow\ &a = 5\\
&Hence,\ d = \frac{3 \times 5}{5} = 3
\end{aligned}
$$

9. In an Arithmetic Progression (A.P.) the fourth and sixth terms are 8 and 14 respectively. Find
(i) first term
(ii) common difference
(iii) sum of the first 20 terms. [2019]

Solution: (i) -1, (ii) 3 , (iii) 550

Step-by-step Explanation:

$$Given,$$
$$T_4 = 8$$
$$T_6 = 14$$
$$so,\ a + 3d = 8\ \ldots\ldots(1)$$
$$a + 5d = 14\ \ldots\ldots\ (2)$$
$$subtracting\ (2)\ from\ (1)\ we\ have,$$
$$-2d = -6$$
$$d = 3$$
$$putting\ the\ value\ of\ d\ in\ (1)$$
$$a + 3 \times 3 = 8$$
$$a = -1$$
$$Therefore,$$
$$(i)\ a = -1$$
$$(ii)\ d = 3$$
$$(iii)\ S_{20} = \frac{20}{2}[2 \times (-1) + (20 - 1)(3)]$$
$$\Rightarrow S_{20} = 10\ [-2 + 57]$$
$$\Rightarrow S_{20} = 550$$

10. The sum of the first three terms of an Arithmetic Progression (A:P.) is 42 and the product of the first and third term is 52. Find the first term and the common difference. [3] [2019]

Solution: first term= 14, common difference= 12

Step-by-step Explanation:

Let the first three terms of an A.P. be (a − d), a, (a + d)

$$Given,$$
$$a - d + a + a + d = 42$$
$$\Rightarrow 3a = 42$$
$$\Rightarrow a = 14$$
$$and\ (a - d)(a + d) = 52$$
$$\Rightarrow a^2 - d^2 = 52$$
$$\Rightarrow 14^2 - d^2 = 52$$
$$\Rightarrow 196 - 52 = d^2$$
$$\Rightarrow 144 = d^2$$
$$\Rightarrow d = 12$$

11. If (k – 3), (2k + 1) and (4k + 3) are three consecutive terms of an A.P., find the value of k. [3] [2018]

Solution: k=2

Step-by-step Explanation:

(k – 3), (2k + 1) and (4k+3) are three consecutive terms of an A.P.

Therefore,

a3 − a2 = a2 − a1

$$\Rightarrow (4k + 3) - (2k + 1) = (2k + 1) - (k - 3)$$
$$\Rightarrow 4k + 3 - 2k - 1 = 2k + 1 - k + 3$$
$$\Rightarrow 2k + 2 = k + 4$$
$$\Rightarrow k = 2$$

12. The 4th term of an A. P. is 22 and 15th term is 66. Find the first term and the common difference. Hence find the sum of the series to 8 terms. [2018]

Solution: a = 10, d = 4, S_8 =192

Step-by-step Explanation:

$$\textit{Given},$$
$$T_4 = 22$$
$$T_{15} = 66$$
$$a + 3d = 22 \ \ldots\ldots (1)$$
$$a + 14d = 66 \ \ldots\ldots (2)$$
$$\textit{subtracting } (1) \textit{ from } (2) \textit{ we have},$$
$$11d = 44$$
$$d = 4$$
$$\textit{putting the value of } d \textit{ in } (1)$$
$$a + 3 \times 4 = 22$$
$$a = 10$$
$$S_8 = \frac{8}{2}[2 \times 10 + (8 - 1)(4)]$$
$$\Rightarrow S_8 = 4[20 + 28]$$
$$\Rightarrow S_8 = 192$$