Mathematics

Formulas

GST (Goods and Services Tax)
1. $\text{Profit} = SP - CP$
2. $\text{Loss} = CP - SP$
3. $\text{Profit\%} = \frac{\text{Profit}}{\text{CP}} \times 100$
4. $\text{Loss\%} = \frac{\text{Loss}}{\text{CP}} \times 100$
5. $SP = MP - \text{Discount}$

BANKING
1. $\text{Interest} = P \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$
2. $\text{Maturity Value} = P \times n + I$

QUADRATIC EQUATIONS
1. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

SOLVING (SIMPLE) PROBLEMS
1. $\text{Time} = \frac{\text{Distance}}{\text{Speed}}$

ARITHMETIC PROGRESSION
1. $T_n = a + (n-1)d$
2. $d = t_2 - t_1$
3. $S_n = \frac{n}{2}[2a + (n-1)d]$
4. $S_n = \frac{n}{2}[a + l]$

SECTION AND MID-POINT FORMULA
1. $\text{Distance Formula} = \sqrt{(x2 - x1)^2 + (y2 - y1)^2}$
2. $\text{Section} = \left[\frac{m1x2 + m2x1}{m1 + m2}, \frac{m1y2 + m2y1}{m1 + m2}\right]$
3. $\text{Mid Point} = \left[\frac{x1 + x2}{2}, \frac{y1 + y2}{2}\right]$
4. $\text{Centroid} = \left[\frac{x1 + x2 + x3}{3}, \frac{y1 + y2 + y3}{3}\right]$

EQUATION OF A LINE
1. $m = \tan\theta$
2. $m = \frac{y2 - y1}{x2 - x1}$
3. $y = mx + c$
4. $y - y_1 = m(x - x_1)$

CYLINDER , CONE AND SPHERE

Cylinder
1. Area of Cross section(circle) $= \pi r^2$
2. Perimeter (circumference) of cross section $= 2\pi r$
3. Curved Surface area $= 2\pi rh$
4. Total Surface area $= 2\pi r(h + r)$
5. Volume $= \pi r^2 h$

Hollow Cylinder
1. Thickness $= R - r$
2. Area of cross section $= \pi(R^2 - r^2)$
3. External Curved surface area $= 2\pi Rh$
4. Internal Curved surface area $= 2\pi rh$
5. Total surface area $= 2\pi Rh + 2\pi rh + 2\pi(R^2 - r^2)$
6. Volume $= \pi(R^2 - r^2)h$

Cone
1. Volume $= \frac{1}{3}\pi r^2 h$
2. Curved (or) lateral surface area $= \pi rl$
3. Total surface area $= \pi r(l + r)$
4. $l^2 = h^2 + r^2$

Sphere
1. Volume $= \frac{4}{3}\pi r^3$
2. Surface area $= 4\pi r^2$

Spherical Shell
1. Volume $= \frac{4}{3}\pi(R^3 - r^3)$

Hemisphere
1. Volume $= \frac{2}{3}\pi r^3$
2. Total surface area $= 3\pi r^2$

Conversion of Solids
1. Number of solids $= \dfrac{\text{Volume of bigger solid}}{\text{Volume of smaller solid}}$

TRIGONOMETRICAL IDENTITIES

Trigonometrical Ratios :
1. Sine$(\sin\theta) = \dfrac{\text{Perpendicular}}{\text{Hypotenuse}}$
2. Cosine$(\cos\theta) = \dfrac{\text{Base}}{\text{Hypotenuse}}$
3. Tangent$(\tan\theta) = \dfrac{\text{Perpendicular}}{\text{Base}}$
4. Cotangent$(\cot\theta) = \dfrac{\text{Base}}{\text{Perpendicular}}$

5. $\text{Secant}(\sec \theta) = \dfrac{\text{Hypotenuse}}{\text{Base}}$

6. $\text{Cosecant}(\text{cosec } \theta) = \dfrac{\text{Hypotenuse}}{\text{Perpendicular}}$

Reciprocal Relations:

1. $\text{Sin A} = \dfrac{1}{\text{cosec A}}$
2. $\text{Cos A} = \dfrac{1}{\text{sec A}}$
3. $\text{Tan A} = \dfrac{1}{\text{cot A}}$
4. $\text{Cosec A} = \dfrac{1}{\text{sin A}}$
5. $\text{Sec A} = \dfrac{1}{\text{cos A}}$
6. $\text{Cot A} = \dfrac{1}{\text{tan A}}$

Quotient Relations:

1. $\tan A = \dfrac{\sin A}{\cos A}$
2. $\cot A = \dfrac{\cos A}{\sin A}$

Square relations:

1. $\sin^2 A + \cos^2 A = 1 \Rightarrow \sin^2 A = 1 - \cos^2 A \Rightarrow \cos^2 A = 1 - \sin^2 A$
2. $1 + \tan^2 A = \sec^2 A \Rightarrow \sec^2 A - \tan^2 A = 1 \Rightarrow \sec^2 A - 1 = \tan^2 A$
3. $1 + \cot^2 A = \text{cosec}^2 A \Rightarrow \text{cosec}^2 A - \cot^2 A = 1 \Rightarrow \text{cosec}^2 A - 1 = \cot^2 A$

Standard angles:

Angles	0°	30°	45°	60°	90°
sin	0	$\dfrac{1}{2}$	$\dfrac{1}{\sqrt{2}}$	$\dfrac{\sqrt{3}}{2}$	1
cos	1	$\dfrac{\sqrt{3}}{2}$	$\dfrac{1}{\sqrt{2}}$	$\dfrac{1}{2}$	0
tan	0	$\dfrac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞(not defined)
cot	∞(not defined)	$\sqrt{3}$	1	$\dfrac{1}{\sqrt{3}}$	0
sec	1	$\dfrac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞(not defined)
cosec	∞(not defined)	2	$\sqrt{2}$	$\dfrac{2}{\sqrt{3}}$	1

MEASURES OF CENTRAL TENDENCY
(MEAN , MEDIAN , QUARTILES AND MODE)

Mean:

1. Mean $= \dfrac{\text{Sum of all observations}}{\text{Total number of observations}}$
2. Direct method
    *   Mean $= \dfrac{\Sigma fx}{\Sigma f}$
    Shortcut method
    *   Mean $= A + \dfrac{\Sigma fd}{\Sigma f}$
    Step-deviation method
    *   Mean $= A + \dfrac{\Sigma ft}{\Sigma f} \times i$
3. $d = x - A$
4. $t = \dfrac{d}{i}$ (**or**) $\dfrac{x-A}{i}$

Median:

If n is odd

1. Median $= \left[\dfrac{n+1}{2}\right]^{\text{th}}$ term

If n is even

2. Median $= \left[\dfrac{n}{2}\right]^{\text{th}}$ term (**or**) $\dfrac{1}{2}\left[\left(\dfrac{n}{2}\right)^{\text{th}} \text{term} + \left(\dfrac{n}{2}+1\right)^{\text{th}} \text{term}\right]$

Quartiles:

1. If n is odd
    *   Lower quartile$(Q_1) = \left[\dfrac{n+1}{4}\right]^{\text{th}}$ term
    *   Upper quartile$(Q_3) = \left[\dfrac{3(n+1)}{4}\right]^{\text{th}}$ term
2. If n is even
    *   Lower quartile$(Q_1) = \left[\dfrac{n}{4}\right]^{\text{th}}$ term
    *   Upper quartile$(Q_3) = \left[\dfrac{3n}{4}\right]^{\text{th}}$ term
3. Inter quartile range $= Q_3 - Q_1$

PROBABILITY
1. Probability $= \dfrac{\text{Number of favourable outcomes}}{\text{Total number of possible outomes}}$

Important Points:

- **Discount** is always calculated on **MP(Marked Price)**
- **Tax** is calculated on **SP(Sale Price)** obtained after **deducting discount** or **adding the profit** , from the Marked Price.
- **Tax = Rate of tax × SP**
- **N** ⇒ Natural numbers
- **W** ⇒ Whole numbers
- **R** ⇒ Real numbers
- **I** or **Z** ⇒ Integer numbers
- The standard form of a quadratic equation is $ax^2 + bx + c = 0$
- $b^2 - 4ac = 0$ ⇒ The roots are **real & equal**
- $b^2 - 4ac > 0$ ⇒ The roots are **real & unequal** (positive)
- $b^2 - 4ac < 0$ ⇒ The roots are **imaginary [not real]** (negative)
- $a: b = c: d$ ⇒ **product of means = product of extremes**
- To find the order of matrix : **m×n × n×p = m×p**
- The general form of an AP is $a, a + d, a + 2d, a + 3d, \ldots \ldots \ldots \ldots \ldots$
- **The rth term from the end $= (n - r + 1)^{th}$ term from the beginning**
- The consecutive three terms of an AP is $(a - d), (a), (a + d)$
- The consecutive four terms of an AP is $(a - 3d), (a - d), (a + d), (a + 3d)$
- The consecutive five terms of an AP is $(a - 2d), (a - d), (a), (a + d), (a + 2d)$
- The consecutive six terms of an AP is $(a - 5d), (a - 3d), (a - d), (a + d),$ $(a + 3d), (a + 5d)$
- Centroid divides each median in the ratio **2:1**
- Trisection of a line segment means dividing the line into **3 equal parts**
- If three points A, B and C are collinear, then: **slope of AB = slope of BC**
- $m_1 = m_2$ **(parallel)**
- $m_1 \times m_2 = $ **-1 (perpendicular)**
- Each interior angle of the regular hexagon measures **120°**
- **Circumscribed** ⇒ Perpendicular bisector
- Inscribed ⇒ Angular bisector
- When the solid is **melted and recasted** into another solid, then the **volumes of both the solids remains the same**
- Volume is always represented or measured in **cubic units**
- Surface area is always represented in **square units**
- When the angle of elevation of the tower decreases, the shadow of the tower increases
- Probability of any event **can never be less than 0 or more than 1**
- **Or** ⇒ combined
- **And** ⇒ common

Theorems:

1. **A line drawn parallel to one side of a triangle divides the other two sides proportionally.**
2. **The angle at the centre is twice the angle at the remaining circumference of the circle.**
3. **Angles in the same segment of a circle are equal.**
4. **The angle in a semi-circle is a right angle (90°).**
5. **The opposite angles of a cyclic quadrilateral are supplementary.**
6. **The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.**
7. **The tangent at any point of a circle and the radius through this point are perpendicular (90°) to each other.**
8. **If two circles touch each other externally or internally, the point of contact lies on the straight line through the centres.**
9. **If two chords of a circle intersect internally or externally then the product of the lengths of their segments are equal. i.e.,$PA \times PB = PC \times PD$**
10. **The angle between a tangent and a chord through the point of contact is equal to an angle in the alternate segment.**
11. **If a chord and a tangent intersect externally, then the product of the lengths of the segments of the chord is equal to the square of the length of the tangent from the point of contact to the point of intersection. i.e.,$PT^2 = PA \times PB$**

Identities:

1. $(a + b)^2 = a^2 + b^2 + 2ab$
2. $(a - b)^2 = a^2 + b^2 - 2ab$
3. $a^2 - b^2 = (a + b)(a - b)$
4. $(a + b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$
5. $(a - b)^3 = a^3 - b^3 - 3a^2b + 3ab^2$

Properties of special kinds of Quadrilaterals

1. **Trapezium**:
   - One pair of opposite sides is parallel but the other pair of opposite sides is not-parallel.
   - If the non-parallel sides are equal, it is called an isosceles trapezium.
   - Sum of adjacent angles on non-parallel sides is 180°.

2. **Parallelogram**:
   - Opposite sides are parallel, equal & congruent.
   - Opposite angles are equal.
   - Consecutive angles are supplementary.
   - Diagonals bisect each other.

3. **Rectangle**:
   - Opposite sides are equal & parallel.
   - Diagonals are equal & congruent.
   - Diagonals bisect each other.
   - Each angles measures 90°.

4. **Rhombus**:
   - All sides are equal & parallel.
   - Opposite angles are equal.
   - Diagonals bisect each other at 90°.
   - Adjacent angles are supplementary.

5. **Square**:
   - All sides are equal.
   - Each angles measures 90°.
   - Diagonals are equal.
   - Diagonals bisect each other at 90°.

6. **Kite**:
   - Adjacent sides of kite are equal.
   - Diagonals bisect each other at 90°.
   - Longer diagonal is perpendicular bisector of the shorter diagonal.

Diagonal properties of different kinds of parallelogram

Properties	Parallelogram	Rectangle	Rhombus	Square
Diagonals bisect each other	✓	✓	✓	✓
Diagonals are equal	-	✓	-	✓
Diagonals bisect vertex angles	-	-	✓	✓
Diagonals are perpendicular to each other	-	-	✓	✓
Each diagonal forms 2 congruent triangles	✓	✓	✓	✓
Diagonals form 4 equal triangles (Equal triangles means, triangles equal in area)	✓	✓	✓	✓
Diagonals form 4 congruent triangles	-	-	✓	✓

Area and Perimeter

1. **Square**
   - Area $= (\text{side})^2$
   - $Perimeter = 4 \times side$
   - $Diagonal = \sqrt{2 \times area}$

2. **Rectangle**
   - $Area = length \times breadth$
   - $Perimeter = 2(length + breadth)$
   - $Diagonal = \sqrt{l^2 + b^2}$

3. **Parallelogram**
   - $Area = base \times height$
   - $Perimeter = 2(l + b)$

4. **Trapezium**
   - $Area = \frac{1}{2} \times (sum\ of\ parallel\ sides) \times height$
   - $Perimeter = 2(l + b)$

5. **Rhombus**
   - $Area = \frac{1}{2} \times (product\ of\ diagonals)$
   - $Perimeter = 4 \times side$

6. **Kite**
   - $Area = \frac{1}{2} \times (product\ of\ diagonals)$
   - $Perimeter = 2(a + b)$