ICSE Selina Concise Solutions for Grade 10

Mathematics

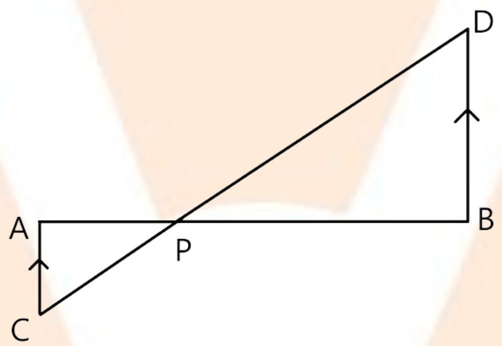
Chapter 15 - Similarity (With Applications to Maps and Models)

Exercise 15(A)

1. In the figure, given below, straight lines AB and CD intersect at P; and AC || BD. Prove that:

(i) ΔAPC and ΔBPD are similar.

(ii) If BD = 2.4 cm AC = 3.6 cm, PD = 4.0 cm and PB = 3.2 cm; find the lengths of PA and PC.



Ans: At point P, two line segments AB and CD intersect each other. AC || BD and we have to prove that

(i) $\Delta APC \sim \Delta BPD$

(ii) If BD = 2.4cm, AC = 3.6cm, PD = 4.0 cm and PB = 3.2, find length of PA and PC

Proves

(i) In ΔAPC and ΔAPD

$\angle APC = \angle BPD$ (Vertically opp. angles)

$\angle PAC = \angle PBD$ (Alternate angles)

$\Delta APC \sim \Delta BPD$ (AA axiom)

(ii) From the figure we can say that the corresponding parts of the similar triangle are equal, then we have

$$\frac{PA}{PB}=\frac{PC}{PD}=\frac{AC}{BD}$$

In the questions its given that BD = 2.4 cm, AC = 3.6 cm, PD = 4.0 cm and PB = 3.2 cm

On substituting the values, we have

$$\frac{PA}{3.2}=\frac{PC}{4}=\frac{3.6}{2.4}$$

$$\frac{PA}{3.2}=\frac{3.6}{2.4} \text{ and } \frac{\text{Area of } \Delta BPQ}{\text{Area of } \Delta CPD}=\frac{(BP^2)}{(CP^2)}=\frac{1}{4}$$

Thus,

$$\frac{PA}{3.2}=\frac{3.6}{2.4} \text{ and } \frac{PC}{4}=\frac{3.6}{2.4}$$

Furthermore we can say that

PA = (3.6 x 3.2)/ 2.4 = 4.8 cm and

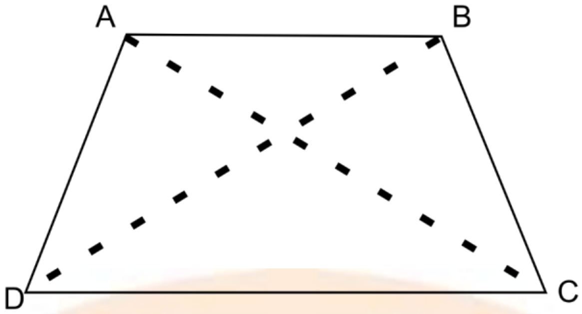
PC = (3.6 x 4)/ 2.4 = 6 cm

2. In a trapezium ABCD, side AB is parallel to side DC; and the diagonals AC and BD intersect each other at point P. Prove that:

(i) ΔAPB is similar to ΔCPD.

(ii) PA × PD = PB × PC.

Ans:



To prove that,

(i) ΔAPB is similar to ΔCPD.

In ΔAPB and ΔCPD, we have

∠APB = ∠CPD as they are vertically opposite angles

∠ABP = ∠CDP, Alternate angles as, AB||DC

Thus, ΔAPB ~ ΔCPD is as per the AA similarity criterion.

(ii) To prove that, PA × PD = PB × PC.

As we know that As ΔAPB ~ ΔCPD

And since the corresponding sides of similar triangles are proportional, we have

$$\frac{PA}{PC} = \frac{PB}{PD}$$

Thus after cross multiplying, we have,

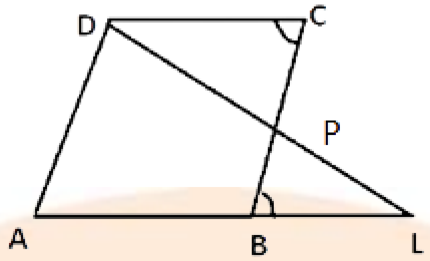
PA × PD = PB × PC

3. P is a point on side BC of a parallelogram ABCD. If DP produced meets AB produced at point L, prove that:

(i) DP : PL = DC : BL.

(ii) DL : DP = AL : DC.

Ans:



To prove that

(i) DP: PL = DC: BL.

As AD||BC, we can also have AD|| BP also

So, from BPL

$$\frac{DP}{PL} = \frac{AB}{BL}$$

And, since ABCD is a parallelogram, AB = DC

Hence,

$$\frac{DP}{PL} = \frac{DC}{BL}$$

And since it's known that ABCD is a parallelogram, AB = DC

Therefore, $\frac{DP}{PL} = \frac{DC}{BL}$

Which is nothing but DP: PL = DC: BL.

(ii) DL: DP = AL: DC

As it has been mentioned that AD||BC, we also can say that AD|| BP

From BPL, we have

$$\frac{DL}{DP} = \frac{AL}{AB}$$

$$\frac{DL}{DP}=\frac{AL}{AB}$$

As ABCD is a parallelogram, we can say that AB = DC

Therefore, $\frac{DL}{DP}=\frac{AL}{AB}$

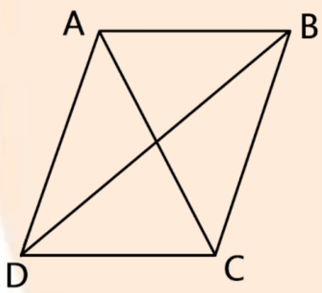
Which is nothing but DL: DP = AL: DC

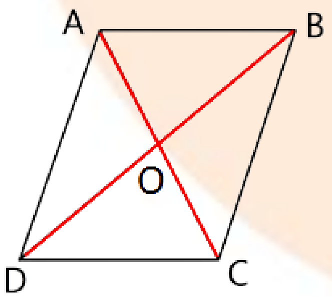
4. In quadrilateral ABCD, the diagonals AC and BD intersect each other at point O. If AO = 2CO and BO = 2DO; show that:

(i) ΔAOB is similar to ΔCOD.

(ii) OA x OD = OB x OC.

Ans:





To show that

(i) ΔAOB is similar to ΔCOD

Given in the question that

AO = 2CO and BO = 2DO,

$$\frac{AO}{CO} = \frac{BO}{DO} = \frac{2}{1}$$

From the vertically opposite angles property, we can say that ∠AOB = ∠DOC

Hence by SAS criterion for similarity, we can say that ΔAOB is similar to ΔCOD

(ii) OA × OD = OB × OC.

As we know that $\frac{AO}{CO} = \frac{BO}{DO} = \frac{2}{1}$

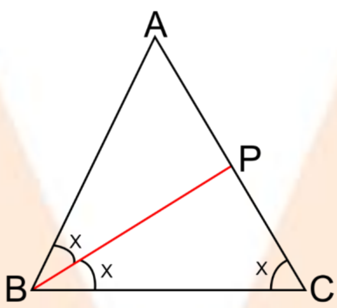
Thus from the given information, we can say that OA × OD = OB × OC

5. In ΔABC, angle ABC is equal to twice the angle ACB, and the bisector of angle ABC meets the opposite side at point P. Show that:

(i) CB: BA = CP: PA

(ii) AB x BC = BP x CA

Ans:



(i) As given,

In Δ ABC, we have

∠ABC = 2 ∠ACB

Now let us consider that ∠ACB = x

So, from this, we can also say that ∠ABC = 2x

Also, it is given that, BP is the bisector of ∠ABC

Hence, ∠ABP = ∠PBC = x

As we know from the angle bisector theorem, that the bisector of an angle divides the side opposite to it in the ratio of the other two sides.

Therefore, CB: BA = CP: PA.

(ii)To show that AB x BC = BP x CA

In Δ ABC and Δ APB,

From exterior angle property, we can say that ∠ABC = ∠APB

∠BCP = ∠ABP

Thus by AA criterion for similarity, ΔABC ~ ΔAPB

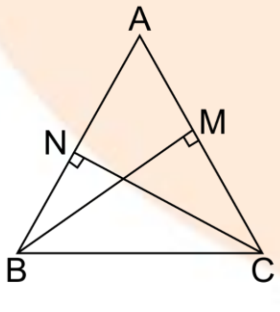
In similar triangles, the corresponding sides are proportional ad from that we have

$$\frac{CA}{AB} = \frac{BC}{BP}$$

Therefore, AB × BC = BP × CA

6. In ΔABC; BM ⊥ AC and CN ⊥ AB; show that:

$$\frac{AB}{AC} = \frac{BM}{CN} = \frac{AM}{AN}$$



Ans:

Let us consider two triangles, Δ ABM and Δ ACN.

∠AMB = ∠ANC as BM ⊥ AC and CN ⊥ AB(given)

∠CAN=∠BAM(Common angle)

Hence by AA criterion for similarity, we can say that $\Delta ABM \sim \Delta ACN$

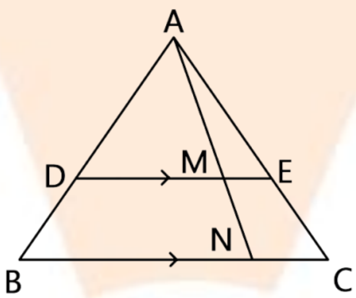
Furthermore by corresponding sides of similar triangles we have

$$\frac{AB}{AC}=\frac{BM}{CN}=\frac{AM}{AN}$$

7. In the given figure, DE // BC, AE = 15 cm, EC = 9 cm, NC = 6 cm and BN = 24 cm.

(i) Write all possible pairs of similar triangles.

(ii) Find lengths of ME and DM.



Ans: (i) In Δ AME and Δ ANC,

Since DE || BC so, ME || NC, we can say that $\angle AME = \angle ANC$

$\angle MAE = \angle NAC$(common angle)

Hence, by AA criterion for similarity, we can say that $\Delta AME \sim \Delta ANC$.

Let us consider the Δ ADM and Δ ABN,

Since DE || BC so, BC || BN, we can say that $\angle ADM = \angle ABN$

$\angle DAM = \angle BAN$(common angle)

Hence, by the AA criterion for similarity, we can say that $\Delta ADM \sim \Delta ABN$

For Δ ADE and Δ ABC,

Since DE || BC so, ME || NC, we can say that, $\angle ADE = \angle ABC$

$\angle AED = \angle ACB$ as DE || BC

Hence, by the AA criterion for similarity, we can say that ΔADE ~ ΔABC

(ii) To find lengths of ME and DM.

Since we have proven that ΔAME ~ ΔANC

So, the corresponding sides of the similar triangle are proportional, we can say that

$$\frac{AE}{AC} = \frac{ME}{NC}$$

15/24 = ME/6

ME = 3.75 cm

As we have proved above that ΔADE ~ ΔABC

As the corresponding sides of a similar triangle are proportional, we have

$$\frac{AE}{AC} = \frac{AD}{AB} = \frac{15}{24} \quad \ldots\ldots\ldots\ldots (1)$$

Also, ΔADM ~ ΔABN (As proven above)

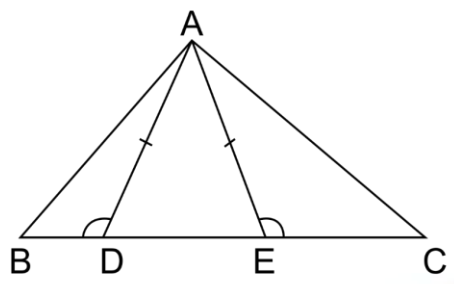
As the corresponding sides of a similar triangle are proportional, we have

$$\frac{AD}{AB} = \frac{DM}{BN} = \frac{15}{24} \text{ from equation(1)}$$

$$\frac{DM}{24} = \frac{15}{24}$$

DM = 15 cm

8. In the given figure, AD = AE and AD² = BD x EC Prove that: triangles ABD and CAE are similar.



Ans: To prove that Δ ABD and Δ CAE are similar.

∠ADE = ∠AED (Angles opposite to equal sides are equal)

As ∠ADB + ∠ADE = 180° and ∠AEC + ∠AED = 180°, so we can say that ∠ADB = ∠AEC

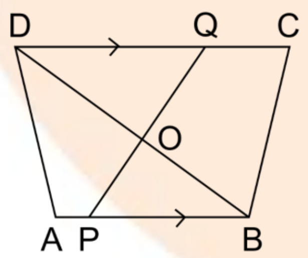
And as its given that AD2 = BD × EC

$$\frac{AD}{BD} = \frac{EC}{AD}$$

$$\frac{AD}{BD} = \frac{EC}{AE}$$

By SAS criterion for similarity, we can say that ΔABD ~ ΔCAE.

9. In the given figure, AB // DC, BO = 6 cm and DQ = 8 cm; find: BP × DO



Ans: Let us consider Δ DOQ and Δ BOP,

∠QDO = ∠PBO (As it given that AB || DC, it also means that PB || DQ)

So ∠DOQ = ∠BOP (As they are vertically opposite angles)

By AA criterion for similarity, we can say that ΔDOQ ~ ΔBOP.

As the corresponding sides of similar triangles are proportional we have

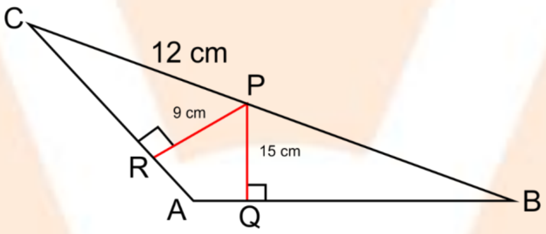
$$\frac{DO}{BO} = \frac{DQ}{BP}$$

$$\frac{DO}{6} = \frac{DQ}{8}$$

BP × DO = 48 cm²

10. Angle BAC of triangle ABC is obtuse and AB = AC. P is a point in BC such that PC = 12 cm. PQ and PR are perpendiculars to sides AB and AC respectively. If PQ = 15 cm and PR = 9 cm; find the length of PB.

Ans:



Let us consider Δ ABC,

As it's given that AC = AB

So, ∠ABC = ∠ACB(As angles opposite to equal sides are equal)

In Δ PRC and Δ PQB,

∠ABC = ∠ACB

∠PRC = ∠PQB(As both are right angles)

Hence by the AA criterion for similarity, we can say that ΔPRC ~ ΔPQB

Since corresponding sides of similar triangles are proportional we have

$$\frac{PR}{PQ} = \frac{RC}{QB} = \frac{PC}{PB}$$

$$\frac{PR}{PQ} = \frac{RC}{PB}$$

$$\frac{9}{15} = \frac{12}{PB}$$

Hence PB = 20 cm

11. State, true or false:

(i) Two similar polygons are necessarily congruent.

(ii) Two congruent polygons are necessarily similar.

(iii) All equiangular triangles are similar.

(iv) All isosceles triangles are similar.

(v) Two isosceles-right triangles are similar.

(vi) Two isosceles triangles are similar, if an angle of one is congruent to the corresponding angle of the other.

(vii) The diagonals of a trapezium, divide each other into proportional segments.

Ans: (i) False

(ii) True

(iii) True

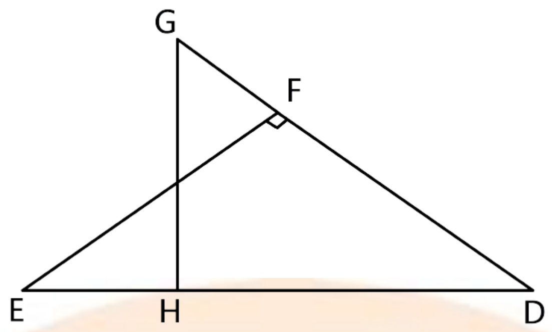
(iv) False

(v) True

(vi) True

(vii) True

12. Given: $\angle GHE = \angle DFE = 90°$, DH = 8, DF = 12, DG = 3x – 1 and DE = 4x + 2.Find the lengths of segments DG and DE.



Ans: Let us consider the Δ DHG and Δ DFE,

∠DFE=∠GHD= 90°

∠D = ∠D(Common)

Hence by the AA criterion for similarity, we can say that ΔDHG ~ ΔDFE

Thus, we have

$$\frac{DH}{DF} = \frac{DG}{DE}$$

$$\frac{8}{12} = \frac{(3x-1)}{(4x+2)}$$

32x + 16 = 36x – 12

28 = 4x

x = 7

Now as its given in the question that

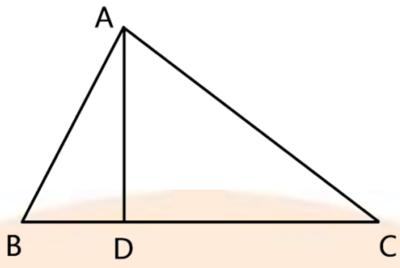
DG = 3x – 1 and DE = 4x + 2.

DG = 3 × 7 – 1 = 20

DE = 4 × 7 + 2 = 30

13. D is a point on the side BC of triangle ABC such that angle ADC is equal to angle BAC. Prove that: CA2 = CB × CD.

Ans:



Let us consider Δ ADC and Δ BAC,

From the given information we can say that $\angle ADC = \angle BAC$

$\angle ACD = \angle ACB$(common angles)

Hence by the AA criterion for similarity, we can say that $\Delta ADC \sim \Delta BAC$

Thus, from thus we have

$\frac{CA}{CB} = \frac{CD}{CA}$ which is nothing but,

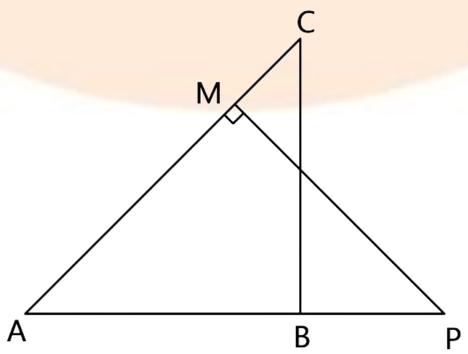
$CA^2 = CB \times CD.$

14. In the given figure, Δ ABC and Δ AMP are right-angled at B and M respectively. Given AC = 10 cm, AP = 15 cm and PM = 12 cm.

(i) Δ ABC ~ Δ AMP.

(ii) Find AB and BC.

Ans:



(i) From the figure let us consider Δ ABC and Δ AMP, from the triangles we have

∠BAC = ∠PAM (Common angles)

∠ABC = ∠PMA (Each angle is equal to 90°)

Hence by the AA criterion for similarity, we can say that ΔABC ~ ΔAMP

(ii) To find AB and BC.

Let us consider the right triangle AMP

By applying Pythagoras theorem, we will have

$$AM = \sqrt{(AP^2 + PM^2)} = \sqrt{(15^2 + 12^2)} = 9$$

As earlier we have proven that ΔABC ~ ΔAMP, we can say that

$$\frac{AB}{AM} = \frac{BC}{PM} = \frac{AC}{AP}$$

$$\frac{AB}{9} = \frac{BC}{12} = \frac{10}{15}$$

$$\frac{AB}{9} = \frac{10}{15}$$

Hence, AB = 6 cm

$$\frac{BC}{12} = \frac{10}{15}$$

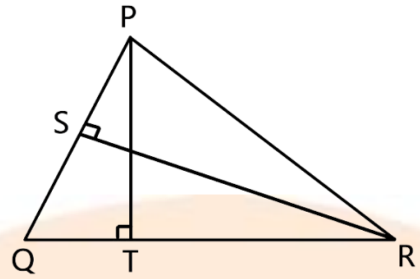
BC = 8 cm

15.Given: RS and PT are altitudes of ΔPQR prove that:

(i) ΔPQT ~ ΔQRS,

(ii) PQ x QS = RQ x QT.

Ans:



Proof:

Given, RS and PT are altitudes of ΔPQR

From the figure let us consider ΔPQT and ΔQRS,

$\angle PTQ = \angle RSQ$ as they are equal to $90°$

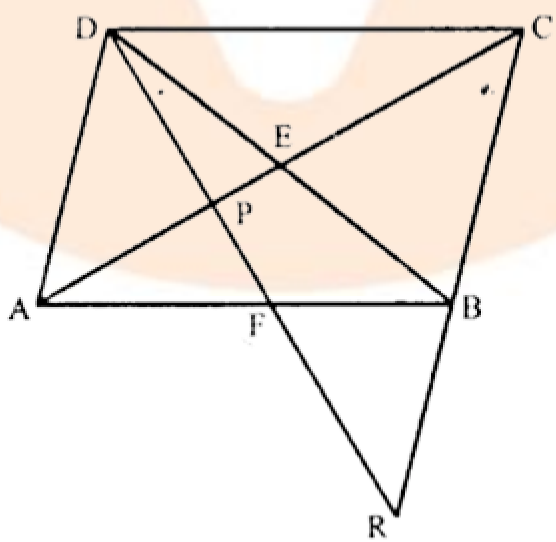
$\angle Q = \angle Q$ (Common angles)

Hence by the AA criterion for similarity, we can say that $\Delta PQT \sim \Delta QRS$

$$\frac{PQ}{RQ} = \frac{QT}{QS}$$

$PQ \times QS = RQ \times QT$, hence proved.

16.Given: ABCD is a rhombus, DPR and CBR are straight lines.



Prove that: DP × CR = DC × PR.

Ans: Let us consider the triangles ΔAPD and ΔPRC

∠CPR=∠DPA as they are vertically opposite angles

∠PAD = ∠PCR as they are alternate angles

Hence by the AA criterion for similarity, we can say that ΔAPD ~ ΔPRC

Thus, $\frac{DP}{PR} = \frac{AD}{CR}$

$\Rightarrow \frac{DP}{PR} = \frac{DC}{CR}$

Hence, AD=DC

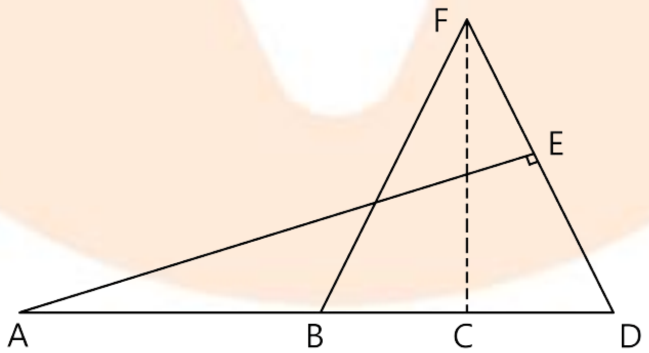
$\Rightarrow$ DP ×CR = DC ×PR(sides of the rhombus)

Hence proved

17. Given: FB = FD, AE ⊥ FD and FC ⊥ AD.

Prove: $\frac{FB}{AD} = \frac{BC}{ED}$

Ans:



Let us consider the Δ FBC and ΔADE

∠FCB=∠AED(As each angle is equal to 90°)

$\angle FBC = \angle ADE$(As FB=FD)

Hence by the AA criterion for similarity, we can say that Δ FBC ~ ΔADE

Therefore, $\frac{FB}{AD} = \frac{BC}{ED}$

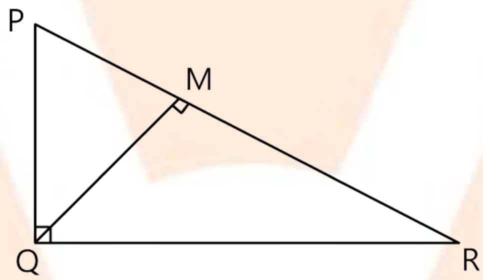
18. In ΔPQR, $\angle Q = 90°$ and QM is perpendicular to PR, Prove that:

(i) $PQ^2 = PM \times PR$

(ii) $QR^2 = PR \times MR$

(iii) $PQ^2 + QR^2 = PR^2$

Ans:



It is given that in ΔPQR, $\angle Q = 90°$ and $QM \perp PR$.

Let us consider that triangles ΔPQM and ΔPQR

$\angle QMP = \angle PQR$(As each angle is equal to 90°)

$\angle P = \angle P$

Hence by the AA postulate, we can say that $\Delta PQM \sim \Delta PQR$

As $\Delta PQM \sim \Delta PQR$, now we can say that

$$\frac{PQ}{PR} = \frac{PM}{PQ}$$

$PQ^2 = PM \times PR$ let us consider this to be (i)

Now consider the triangles Δ QRM and Δ PQR,

∠QMR= ∠Q (As each angle is equal to 90°)

∠R=∠R

Therefore, we can say that Δ QRM ~Δ PQR(by the AA postulate)

$$\frac{QR}{PR}=\frac{MR}{QR}$$

$QR^2 = PR \times MR$ let us consider this to be (ii)

On adding (i) and (ii) we get

$PQ^2 + QR^2 = PR^2$

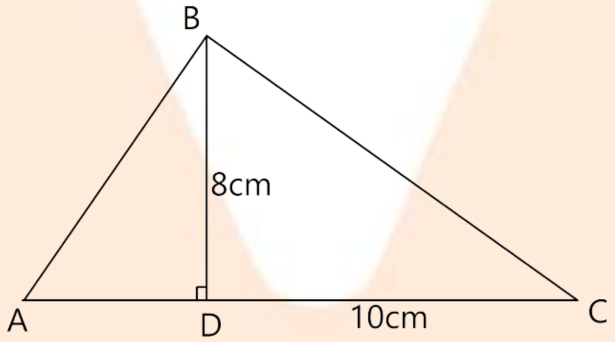
19. In ΔABC, ∠B = 90° and BD x AC.

(i) If CD = 10 cm and BD = 8 cm; find AD.

(ii) If AC = 18 cm and AD = 6 cm; find BD.

(iii) If AC = 9 cm, AB = 7 cm; find AD

Ans:



Given that in ΔABC, ∠B = 90°

∠A + ∠C = 90° let this be …….(i)

In ΔBDC, ∠D = 90°

∠CBD + ∠C = 90° let this be ….(ii)

By equating (i) and (ii)

$\angle A + \angle C = \angle CBD + \angle C$

$\angle A = \angle CBD$

Similarly, $\angle C = \angle ABD$

Now let us consider ΔABD and ΔCBD,

$\angle A = \angle CBD$ and $\angle ABD = \angle C$

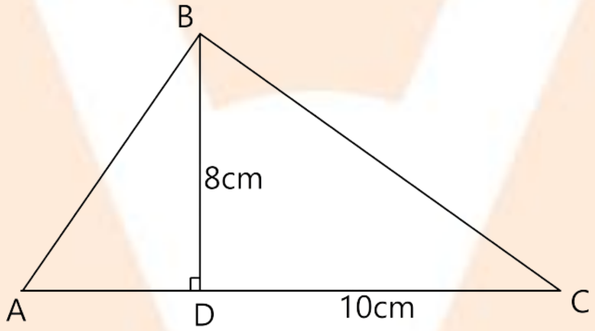
By AA Postulate we can say that $\Delta ABD \sim \Delta CBD$

$\therefore \dfrac{BD}{CD} = \dfrac{AD}{BD} = \dfrac{AB}{AC}$ .... (i)

$\Rightarrow BD^2 = AD \times CD$

In the question its given that CD = 10 cm and BD = 8 cm, so

AD= 6.4 cm



(ii) Let us consider $BD^2 = AD \times CD$, it is also given that

AC = 18 cm and AD = 6 cm

Substitute the values of AD and CD in $BD^2 = AD \times CD$, we have

$BD^2 = 6 \times 12 = 72$

BD = 8.5 cm

(iii) In ΔABD and ΔABC

$\angle ADB = \angle ABC$(As its equal to 90°)

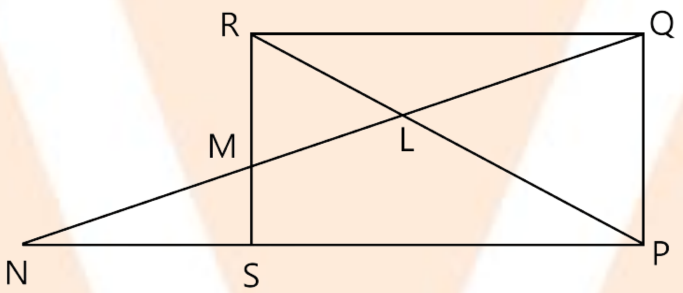
$\angle A = \angle A$

By AA postulate we can say that ΔABD ~ Δ ABC

$$\therefore \frac{AB}{AC} = \frac{AD}{AB} = \frac{AB^2}{AC} = AD$$

$$\Rightarrow AD = \frac{49}{9}$$

20.In the figure, PQRS is a parallelogram with PQ = 16 cm and QR = 10 cm. L is a point on PR such that RL : LP = 2 : 3. QL produced meets RS at M and PS produced at N.



Find the lengths of PN and RM.

Ans: Let us consider ΔLNP and ΔRLQ

∠LNP = ∠LQR(Alternate angles)

∠NLP = ∠QLR (Vertically opposite angles)

By AA Postulate we can say that ΔLNP ~ ΔRLQ

$$\therefore \frac{PN}{QR} = \frac{LP}{RL}$$

$$\frac{PN}{10} = \frac{3}{2}$$

PN = 15 cm.

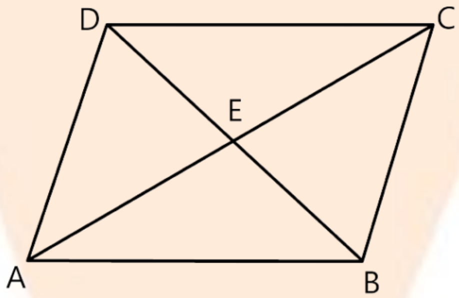
Similarly, we can prove that Δ LPQ and Δ LMR.

$$\frac{RM}{QP} = \frac{RL}{LP}$$

$RM = \frac{3}{2}$ cm

21. In quadrilateral ABCD, diagonals AC and BD intersect at point E. Such that AE : EC = BE : ED. Show that ABCD is a parallelogram.

Ans:



As it is given in quadrilateral ABCD, diagonal AC and BD intersect each other at E and

In AE : EC = BE : ED

$$\frac{EA}{EC} = \frac{BE}{ED}$$

$$\frac{EA}{BE} = \frac{EC}{ED}$$

In Δ AEB and Δ CED

$$\frac{AE}{BE} = \frac{EC}{ED}$$

$\angle AEB = \angle CED$(As they are vertically opposite angles)

$\Delta AEB \sim \Delta CED$(by SAS axiom)

$\angle EAB = \angle ECB$

$\angle EBA = \angle CDE$, as these are pairs of alternate angles

AB || CD …. (i)

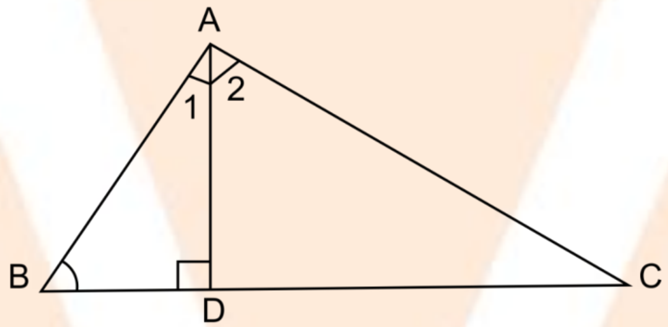
Similarly, we can prove that

AD || BC …. (ii)

from (i) and (ii)

ABCD is a parallelogram.

22. In ΔABC, AD is perpendicular to side BC and $AD^2 = BD \times DC$. Show that angle BAC = 90°



Ans: It is given that ΔABC, AD × BC and $AD^2 = BD \times DC$

To Prove: ∠BAC = 90°

$AD^2 = BD \times DC$

$$\frac{AD}{DC} = \frac{BD}{AD}$$

Now in Δ ABD and ΔACD,

$$\frac{AD}{DE} = \frac{BD}{AD}$$ (Given information)

∠ADB = ∠ADC (As they are equal to 90°)

∴ By SAS Postulate we can say that ΔADB ~ Δ ACD

∴∠B = ∠DAC……...(i)

And ∠BAD = ∠C……...(ii)

Add (i) and (ii)

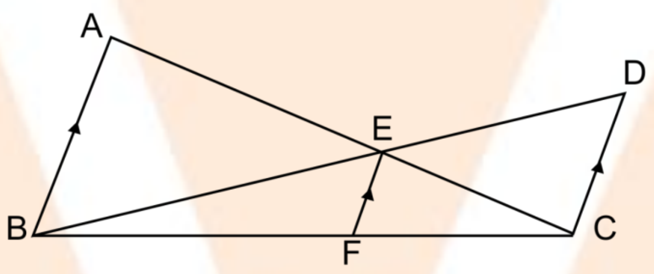
∠B + ∠C= ∠DAC+ ∠BAD= ∠BAC= ∠A

But ∠A + ∠B + ∠C =180° (Angles of the triangle)

∠A+∠A=180°

∠A = 90° or we could say ∠BAC = 90°

23. In the given figure AB || EF || DC; AB = 67.5 cm. DC = 40.5 cm and AE = 52.5 cm.



(i) Name the three pairs of similar triangles.

(ii) Find the lengths of EC and EF

Ans: (i) In the figure AB || EF || DC

There are three pairs of similar triangles.

(i) ΔAEB ~ ΔDEC

(ii) ΔABC ~ ΔEEC

(iii) ΔBCD ~ ΔEBF

(ii) ΔAEB ~ ΔDEC

$$\therefore \frac{EA}{EC} = \frac{BE}{EB} = \frac{AB}{DC}$$

It is given that AB = 67.5 cm. DC = 40.5 cm and AE = 52.5 cm.

$\therefore \frac{52.5}{EC}=\frac{7.5}{40.5}$

EC = 31.5 cm

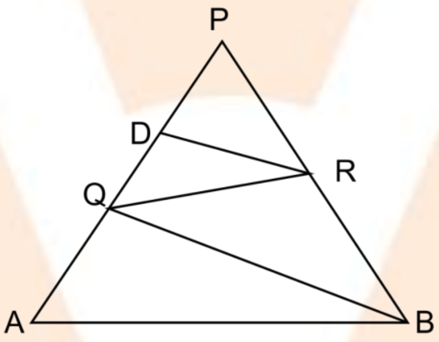
In ΔABC, EF $\parallel$AB

$\therefore \frac{AC}{EC}=\frac{AB}{EF}$

$\frac{84}{31.5}=\frac{67.5}{EF}$

EF=$\frac{405}{16}$

24. In the given figure, QR is parallel to AB and DR is parallel to QB.



Prove that $PQ^2 = PD \times PA$.

Ans: Given that in the figure QR is parallel to AB and DR is parallel to QB.

To prove that $PQ^2 = PD \times PA$.

Let us consider ΔPQB

DR || QB

$\therefore$ PD/PQ = PR/PB……....(i)

In ΔPAB

QR || AB

$$\frac{PQ}{PA} = \frac{PR}{PB} \ldots\ldots\ldots\ldots\text{(ii)}$$

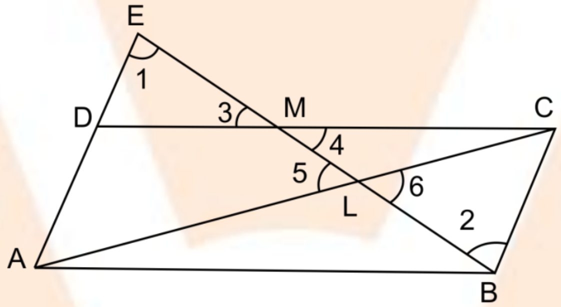
From (i) and (ii)

$$\frac{PD}{PQ} = \frac{PQ}{PA}$$

$PQ^2 = PD \times PA.$

25. Through the midpoint M of the side CD of a parallelogram ABCD, the line BM is drawn intersecting diagonal AC in L and AD produced in E.

Prove that: EL = 2 BL.



Ans: In parallelogram ABCD, M is the mid-point A of CD.

AC is the diagonal.

BM is joined and produced meeting AD produced in E and, intersecting AC in L.

To Prove: EL = 2 BL.

Proof: In ΔEDM, and ΔMBC,

DM = MC (M is midpoint of DC)

$\angle$EMD = $\angle$CMD (vertically opposite angles)

$\angle$EDM = $\angle$MCB (Alternate angles)

ΔEDM = ΔMBC (ASA postulate of congruence)

ED = CB = AD

EA = 2 AD = 2 BC

AB = BC(Opposite sides of the parallelogram)

∠DEM = ∠MBC

Now in ΔELA and ΔBLC,

∠ELA = ∠BLC (vertically opposite angles)

∠DEM or ∠AEL = ∠LBC (proved)

ΔELA ~ ΔBLC (AA postulate)

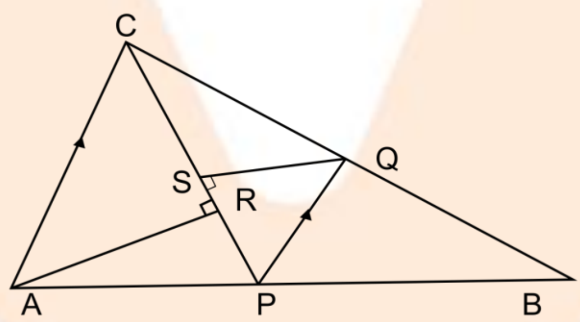
$$\therefore \frac{EA}{BC} = \frac{EL}{LB}$$

$$\frac{2BC}{BC} = \frac{EL}{LB}$$

EL=2LB

∴ EL=2BL

26. In the figure given below P is a point on AB such that AP: PB = 4 : 3. PQ is parallel to AC.



(i) Calculate the ratio PQ: AC, giving the reason for your answer.

(ii) In triangle ARC, ∠ARC = 90° and in triangle PQS, ∠PSQ = 90°.

Given QS = 6 cm, calculate the length of AR.

Ans: It is been given that, In ΔABC, P is a point on AB such that AP: PB = 4 : 3

and PQ || AC is drawn meeting BC in Q. It is also given that CP is joined and QS ⊥ CP and AR ⊥ CP

(i) In ΔABC, PQ || AC

$$= \frac{PQ}{AC} = \frac{BP}{AB} = \frac{BP}{BP + AP} = \frac{3}{3+4} = \frac{3}{7}$$

PQ: AC =3:7

(ii) Now let us consider ΔARC and ΔPSQ

∠ARC= ∠PSQ (As each is equal to 90°)

∠ACR= ∠QPS (Alternate angles)

Hence by AA postulate, we can say that, ΔARC ~ ΔPSQ

$$\therefore \frac{AC}{PQ} = \frac{AR}{QS}$$

We know that $\frac{PQ}{AC} = \frac{3}{7}$, QS = 6 cm

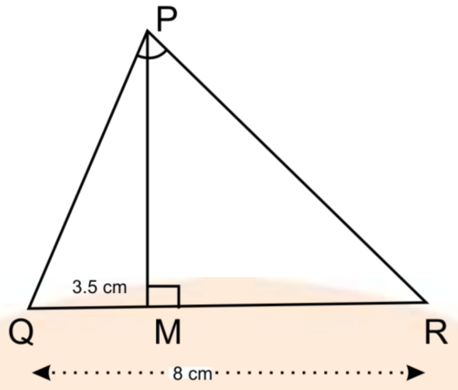
$$\frac{AC}{PQ} = \frac{AR}{QS}$$

$$\frac{7}{3} = \frac{AR}{6}$$

AR = 14 cm.

The length of side AR is 14 cm

27. In the right-angled triangle QPR, PM is an altitude.



Given that QR = 8 cm and MQ = 3.5 cm. Calculate the value of PR.

Given: In right-angled ΔQPR, ∠P = 90° PM ⊥ QR, QR = 8 cm, MQ = 3.5 cm. Calculate PR

Ans: Let us consider ΔPQM and ΔQPR

∠PMQ = ∠QPR (each angle is equal to 90°)

∠Q = ∠Q

Hence by AA postulate, we can say that, ΔPQM ~ ΔQPR

$\therefore \frac{PQ}{QR} = \frac{QM}{PQ} = \frac{PM}{PR}$ let us consider this as an equation (i)

$PQ^2 = QR \times QM = 8 \times 3.5 = 28$

$PQ = \sqrt{28}$ let us consider this as an equation (ii)

In ΔPQR, ∠P = 90° and we know that PM ⊥ QR

$\therefore PM^2 = MR \times QM = 3.5 \times 4.5$

$PM = \sqrt{3.5 \times 4.5}$ let us consider this as an equation (iii)

Let us consider equation (i)

PQ/QR=QM/PQ=PM/PR

$$\frac{\sqrt{28}}{8} = \frac{\sqrt{3.5 \times 4.5}}{PR^2}$$

On squaring both sides and solving it we get

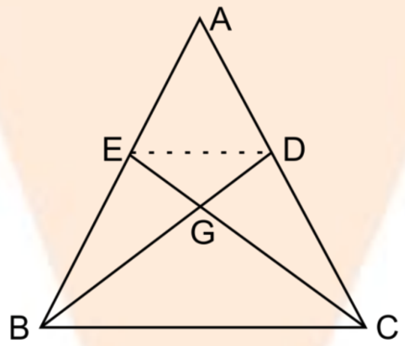
$PR^2 = 36$

PR = 6 cm.

28. In the figure given below, the medians BD and CE of a triangle ABC meet at G. Prove that

(i) ΔEGD ~ ΔCGB

(ii) BG = 2 GD from (i) above.



Ans: It is given that In ΔABC, BD and CE are the medians of sides AC and AB respectively which intersect each at G.

To Prove that

(i) ΔEGD ~ ΔCGB

From the figure, we can say that D and E are the midpoints of AC and AB respectively.

∴ ED || BC and ED = ½ BC or it can be also represented as

ED/BC= ½, let this be equation (i)

Now let us consider ΔEGD and ΔCGB

∠EGD = ∠BCG (As they are vertically opposite angle)

∠EDG = ∠BGG (Alternate angles)

Hence by AA postulate, we can say that, ΔEGD ~ ΔCGB

$\therefore \frac{GD}{BG} = \frac{ED}{BC} = \frac{1}{2}$

BG = 2 GD, Hence, proved.

Exercise 15(B)

1. In the following figure, point D divides AB in the ratio 3: 5. Find:

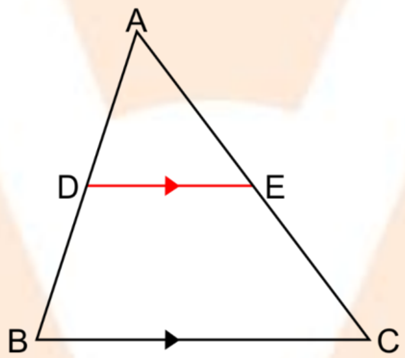
(i) AE/EC

(ii) AD/AB

(iii) AE/AC Also if,

(iv) DE = 2.4 cm, find the length of BC.

(v) BC = 4.8 cm, find the length of DE.



Ans: (i) It is given that $\frac{AD}{AB} = \frac{3}{5}$ and DE || BC.

By following the Basic Proportionality theorem, we have

$$\frac{AD}{AB} = \frac{AE}{EC}$$

$$\frac{AE}{EC} = \frac{3}{5}$$

(ii)Given that $\frac{AD}{AB} = \frac{3}{5}$

Then, $\frac{DB}{AD}=\frac{5}{3}$

Add 1 on both sides, we will get

$\frac{AB}{AD}+1=\frac{5}{3}+1$

(DB + AD)/ AD = (5 + 3)/ 3

$\frac{AB}{AD}=\frac{8}{3}$

Now, $\frac{AD}{AB}=\frac{3}{8}$

(iii) Let us consider ΔABC in which DE || BC

By following the Basic Proportionality theorem, we have

$\frac{AD}{AB}=\frac{AE}{AC}$

So, $\frac{AD}{AB}=\frac{AE}{AC}$

$\frac{AD}{AB}=\frac{3}{8}$

$\therefore \frac{AE}{AC}=\frac{3}{8}$

(iv) Let us consider ΔADE and ΔABC,

∠ADE = ∠ABC(As corresponding angles are equal also as DE || BC)

∠A = ∠A

Hence by AA criterion for similarity, we can say that ΔADE ~ ΔABC

Now we have,

$\frac{AD}{AB}=\frac{DE}{BC}$

$\frac{3}{8}=\frac{2.4}{BC}$

BC = 6.4 cm

(v) Since we have proven that ΔADE ~ ΔABC due to AA criterion for similarity

We have,

$\frac{AD}{AB}=\frac{DE}{BC}$

$\frac{3}{8}=\frac{DE}{4.8}$

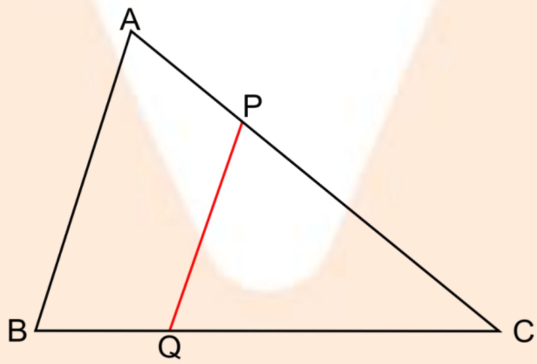
DE = 1.8 cm

2. In the given figure, PQ ∥ AB; CQ = 4.8 cm QB = 3.6 cm and AB = 6.3 cm. Find:

(i) CP/PA

(ii) PQ

(iii) If AP = x, then the value of AC in terms of x.



Ans:

It is given that PQ ∥ AB and CQ = 4.8 cm QB = 3.6 cm and AB = 6.3 cm then

(i) Let us consider ΔCPQ and ΔCAB,

As PQ || AB, and corresponding angles are equal we can say that ∠PCQ = ∠APQ

∠C = ∠C as its a common angle

Hence by the AA criterion for similarity, we can say that ΔCPQ ~ ΔCAB

Now we have,

$$\frac{CP}{AC}=\frac{CQ}{CB}$$

$$\frac{CP}{AC}=\frac{4.8}{8.4}=\frac{4}{7}$$

Thus, $\frac{CP}{PA}=\frac{4}{3}$

(ii) As we have proven that ΔCPQ ~ ΔCAB by AA criterion for similarity

We have,

$$\frac{PQ}{AB}=\frac{CQ}{CB}$$

$$\frac{PQ}{6.3}=\frac{4.8}{8.4}$$

PQ = 3.6 cm

(iii)As we have proven that ΔCPQ ~ ΔCAB by AA criterion for similarity

We have,

$$\frac{CP}{AC}=\frac{CQ}{CB}$$

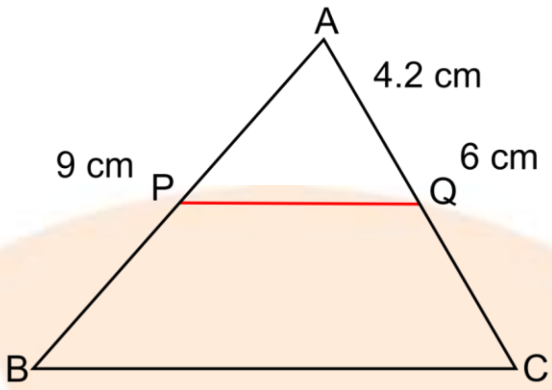
$$\frac{CP}{AC}=\frac{4.8}{8.4}=\frac{4}{7}$$

So, if CP is 4 parts and AC is 7 parts then we can say that PA is 3 parts

Hence, AC = 7/3 × PA = (7/3)x

3. A-line PQ is drawn parallel to the side BC of Δ ABC which cuts side AB at P and side AC at Q. If AB = 9.0 cm, CA = 6.0 cm and AQ = 4.2 cm, find the length of AP.

Ans:



Let us consider Δ APQ and Δ ABC,

As PQ || BC, and corresponding angles are equal we can say that, ∠APQ = ∠ABC

∠PAQ = ∠BAC (Common angles)

Hence by the AA criterion for similarity, we can say that ΔAPQ ~ ΔABC

Now we have,

$$\frac{AP}{AB} = \frac{AQ}{AC}$$

$$\frac{AP}{9} = \frac{4.2}{6}$$

Thus,

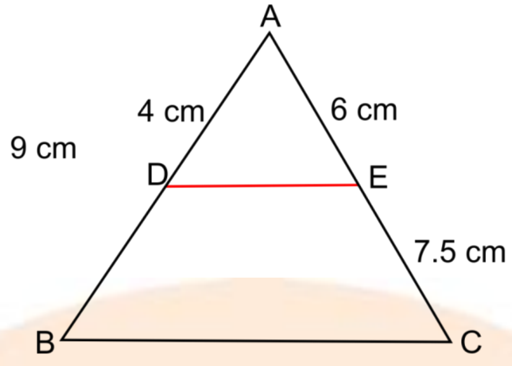
AP = 6.3 cm

4. In Δ ABC, D and E are the points on sides AB and AC respectively.

Find whether DE ∥ BC, if

(i) AB = 9cm, AD = 4cm, AE = 6cm and EC = 7.5cm.

(ii) AB = 6.3 cm, EC = 11.0 cm, AD =0.8 cm and EA = 1.6 cm.



Ans: (i) Let us consider Δ ADE and Δ ABC,

$$\frac{AE}{EC}=\frac{6}{7.5}=\frac{4}{5}$$

$$\frac{AD}{BD}=\frac{4}{5}\text{ (As BD = AB – AD = 9 – 4 = 5 cm)}$$

So, $\frac{AE}{EC}=\frac{AD}{BD}$

∴By the converse of Basic Proportionality theorem, DE || BC

(ii) Let us consider the Δ ADE and Δ ABC,

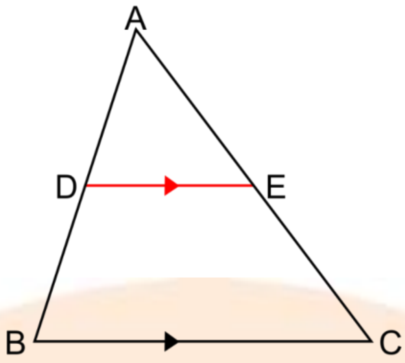
$$\frac{AE}{EC}=\frac{1.6}{11}=\frac{0.8}{5.5}$$

$$\frac{AD}{BD}=\frac{0.8}{5.5}\text{(As BD = AB – AD = 6.3 – 0.8 = 5.5 cm)}$$

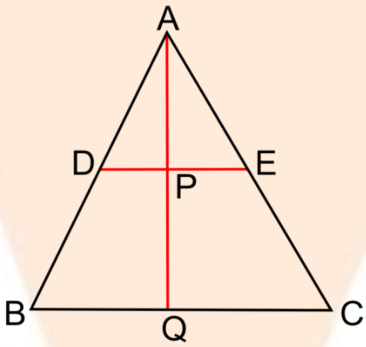
Hence, $\frac{AE}{EC}=\frac{AD}{BD}$

∴By the converse of Basic Proportionality theorem, DE || BC

5. In the given figure, Δ ABC ~ Δ ADE. If AE: EC = 4: 7 and DE = 6.6 cm, find BC. If 'x' is the length of the perpendicular from A to DE, find the length of the perpendicular from A to BC in terms of 'x'.



Ans:



It is given that, Δ ABC ~ Δ ADE (AA criterion for similarity)

So, we have,

$$\frac{AE}{EC} = \frac{DE}{BC}$$

$$\frac{4}{11} = \frac{6.6}{BC}$$

BC = (11 × 6.6)/ 4 = 18.15 cm

As Δ ABC ~ Δ ADE, we can say that ∠ABC = ∠ADE and ∠ACB = ∠AED

So ∴ DE || BC

And since $\frac{AE}{EC} = \frac{4}{7}$, we have $\frac{AB}{AD} = \frac{AC}{AE} = \frac{11}{4}$

Now let us consider Δ ADP and Δ ABQ,

∠ADP = ∠ABQ(Since DP || BQ, corresponding angles are equal)

∠APD = ∠AQB(Since DP || BQ, corresponding angles are equal)

By applying the AA criterion for similarity, we can say that ΔADP ~ ΔABQ.

$$\frac{AD}{AB} = \frac{AP}{AQ}$$

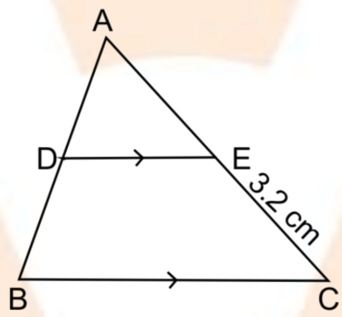
$$\frac{4}{11} = \frac{X}{AQ}$$

Thus,

AQ = (11/4)x

6. A line segment DE is drawn parallel to base BC of ΔABC which cuts AB at point D and AC at point E. If AB = 5 BD and EC = 3.2 cm, find the length of AE.

Ans:



Given that,

In ΔABC, DE || BC

AB = 5 BD and EC = 3.2 cm

Since it is given that DE || BC

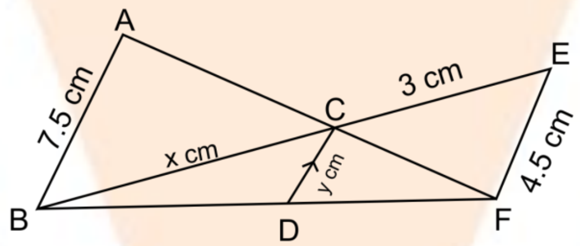
Hence by the AA criterion for similarity, we can say that ΔADE ~ ΔABC

$$\therefore \frac{AD}{AB} = \frac{AE}{EC}$$

$$\frac{4}{1}=\frac{AE}{3.2}$$

AE = 12.8/1 = 12.8 cm.

7. In the figure, given below, AB, CD and EF are parallel lines. Given AB = 7.5 cm, DC = y cm, EF = 4.5 cm, BC = x cm and CE = 3 cm, calculate the values of x and y.



Ans: Let us consider the ΔACB and ΔFCE, in which we have

$\angle ACB = \angle FCE$(As they are vertically opposite angles)

$\angle CBA = \angle CEF$(Alternate to each other)

Hence, by AA Axiom of similarity, we can say that $\Delta ACB \sim \Delta FCE$

Thus, the corresponding sides of the triangles are proportional to each other.

$$\therefore \frac{AB}{BC}=\frac{EF}{EC}$$

$$\Rightarrow \frac{7.5cm}{xcm}=\frac{4.5cm}{3cm}$$

On solving x= 5 cm

Now let us consider ΔEFB and ΔBCD, we have

$\angle EFB = \angle DBC$ (Corresponding angles)

$\angle BEF = \angle BCD$

$\therefore$ By AAA axiom of similarity, we can say that $\Delta EFB \sim \Delta BCD$.

Hence, $\frac{EB}{CB}=\frac{4EF}{CD}$

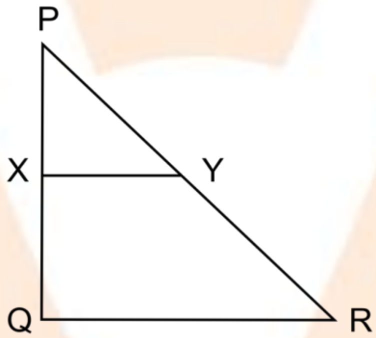
$$\frac{EC}{CB+CB}=\frac{4.5cm}{y}$$

$$3+\frac{x}{x}=\frac{4.5}{y}$$

On solving and by substituting the value of x= 5 cm here we get,

y= $\frac{45}{16}$ cm

8. In the figure, given below, PQR is a right angle triangle right angled at Q. XY is parallel to QR, PQ = 6 cm, PY = 4 cm and PX : XQ = 1 : 2. Calculate the lengths of PR and QR.



Ans: In the question it is given that PQ = 6 cm; PY = 4 cm, and PX : XQ = 1 : 2

From the figure given it is evident that the line drawn parallel to one side of the triangle divides the other two sides proportionally.

∴we can say that,

$$\frac{PX}{XQ}=\frac{PY}{YR}=\frac{1}{2}=\frac{4}{YR}$$

Hence, YR = 8 cm.

But PR = PY+ YR

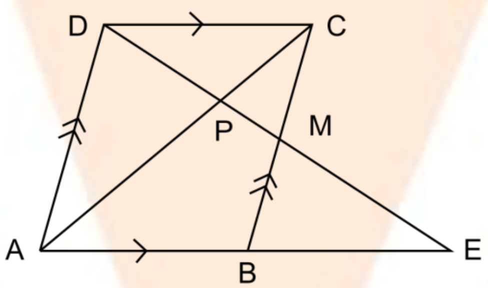
$= 4 + 8 = 12$ cm

Let us consider the right angled ΔPQR, and on using the Pythagorean theorem we have

$$QR^2 + PQ^2 = PR^2$$

$$QR = \sqrt{144 - 36} = 10.392\text{cm}$$

9. In the following figure, M is the midpoint of BC of a parallelogram ABCD. DM intersects the diagonal AC at P and AB produced at E. Prove that: PE = 2PD.



Ans: Given from the question that,

Figure, ABCD is a parallelogram

AB || CD, AD || BC

M is mid point of BC

DM intersect AB produced at E and AC at P

To prove that PE = 2PD

Proof: In ΔDEA,

AD || BC (As they are opposite sides of the parallelogram)

M is a mid-point of CB B is a mid-point of AE

$AB = BE \Rightarrow AE = 2AB$ or $2CD$

Let us consider ΔPAE and ΔPCD

$\angle APE = \angle CPD$ (They are vertically opposite angles)

$\angle PAE = \angle PCD$ (Alternate angles)

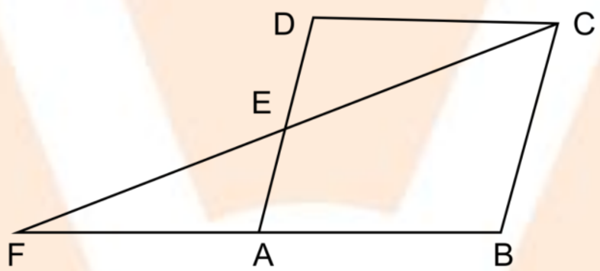
$\therefore$ Hence by the AA criterion for similarity, we can say that $\Delta PAE \sim \Delta PCD$

$$\therefore \frac{PE}{PD} = \frac{AE}{DC} \Rightarrow \frac{PE}{PD} = \frac{2DC}{DC}$$

$\therefore PE = 2PD$

Hence proved.

10. The given figure shows a parallelogram ABCD. E is a point in AD and CE produced meets BA produced at point F. If AE = 4 cm, AF = 8 cm and AB = 12 cm, find the perimeter of the parallelogram ABCD.



Ans: It is given that

In the given figure, ABCD is a parallelogram and E is a point on AD

CE is produced to meet BA produced at point F

AE = 4 cm, AF = 8 cm, AB = 12 cm

We have to find the perimeter of parallelogram ABCD

Let us consider the ΔFBC,

AD or AE || BC (As they are the opposite sides of the parallelogram)

$\therefore$ Hence by the AA criterion for similarity, we can say that $\Delta AFE \sim \Delta FBC$

$$\therefore \frac{FA}{FB} = \frac{AE}{BC} = \frac{8}{8} + 12 = \frac{4}{BC}$$

$\Rightarrow \frac{8}{20} = \frac{4}{BC}$

$\angle APC = \angle BPD$ (Vertically opp. angles) $BC = 4 \times 20 / 8 = 10$

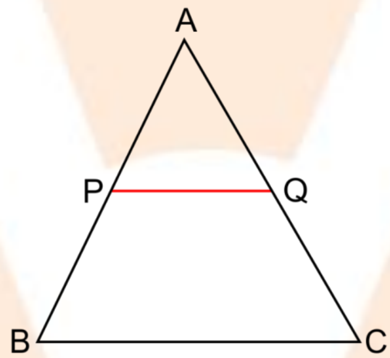
Perimeter of parallelogram ABCD = 2 (AB + BC) = 2 (12 + 10) cm = 2 × 22 = 44 cm.

Exercise 15(C)

1. (i) The ratio between the corresponding sides of two similar triangles is 2: 5. Find the ratio between the areas of these triangles.

(ii) Areas of two similar triangles are 98 sq. cm and 128 sq. cm. Find the ratio between the lengths of their corresponding sides.

Ans:



It is given that the ratio between the corresponding sides of two similar triangles is 2: 5.

As we all know, the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides. Hence, from the above statement, we can say that,

(i) The required ratio is given by,

$\frac{2^2}{5^2} = \frac{4}{25}$

(ii) It is given that areas of two similar triangles are 98 sq. cm and 128 sq. cm.

Now the required ratio is given by,

$$\sqrt{\frac{98}{128}} = \sqrt{\frac{49}{64}} = \sqrt{\frac{7}{8}}$$

2. A line PQ is drawn parallel to the base BC of Δ ABC which meets sides AB and AC at points P and Q respectively. If AP = 1/3 PB; find the value of:

(i) Area of Δ ABC/ Area of Δ APQ

(ii) Area of Δ APQ/ Area of Trapezium PBCQ

Ans: It is given that, AP = (1/3) PB

Thus, AP/PB = 1/3

Let us consider Δ APQ and Δ ABC,

As its mentioned that PQ || BC, we can say that corresponding angles are equal

$\angle APQ = \angle ABC$ and

$\angle AQP = \angle ACB$

Hence by AA criterion for similarity, we can say that ΔAPQ ~ ΔABC

So now,

(i) Area of Δ ABC/ Area of Δ APQ = $\frac{AB^2}{AP^2}$ = 16: 1

(ii) Area of Δ APQ/ Area of Trapezium PBCQ

= Area of Δ APQ/(Area of Δ ABC – Area of Δ APQ)

= 1/ (16/ 1) = 1: 16

3. The perimeters of two similar triangles are 30 cm and 24 cm. If one side of the first triangle is 12 cm, determine the corresponding side of the second triangle.

Ans: It is given that the perimeters of two similar triangles are 30 cm and 24 cm. So let the triangles be ΔABC and ΔDEF

Let $\Delta ABC \sim \Delta DEF$

As they are similar, $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{(AB + BC + AC)}{(DE + EF + DF)}$

= Perimeter of Δ ABC/Perimeter of Δ DEF

Perimeter of Δ ABC/Perimeter of Δ DEF = $\frac{AB}{DE}$

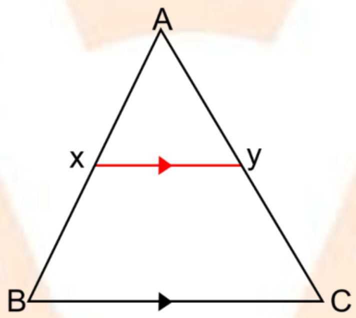
30/24 = 12/DE

DE = 9.6 cm

4. In the given figure, AX: XB = 3: 5.

Find: (i) the length of BC, if the length of XY is 18 cm.

(ii) the ratio between the areas of trapezium XBCY and triangle ABC.



Ans: It is given in the question that $\Rightarrow$

Now, $\frac{X}{AB} = \frac{3}{8}$……. (1)

(i) To find the length of BC, if the length of XY is 18 cm.

Let us consider Δ AXY and Δ ABC,

As BC || XY, then corresponding angles are also equal.

$\angle AXY = \angle ABC$ and $\angle AYX = \angle ACB$

Hence by AA criterion for similarity, we can say that $\Delta AXY \sim \Delta ABC$

So now we have,

$$\frac{AX}{AB} = \frac{XY}{BC}$$

$$\frac{3}{8} = \frac{18}{BC}$$

BC = 48 cm

(ii) To find the ratio between the areas of trapezium XBCY and triangle ABC.

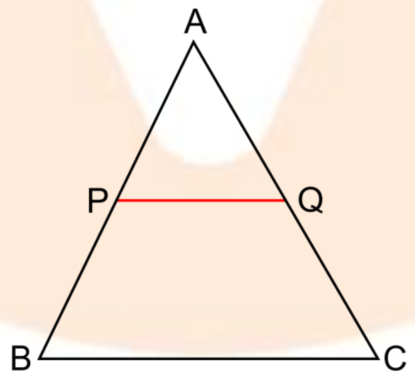
$$\frac{\text{Area of } \Delta AXY}{\text{Area of } \Delta ABC} = \frac{AX2}{AB2} = \frac{9}{64}$$

$$\frac{\text{Area of } \Delta ABC \text{ - Area of } \Delta AXY}{\text{Area of } \Delta ABC} = \frac{(64-9)}{64} = \frac{55}{64}$$

$$\frac{\text{Area of trapezium XBCY}}{\text{Area of } \Delta ABC} = \frac{55}{64}$$

5. ABC is a triangle. PQ is a line segment intersecting AB in P and AC in Q such that PQ || BC and divides triangle ABC into two parts equal in area. Find the value of ratio BP : AB.

Ans:



It is given that in ΔABC, PQ || BC in such a way that area APQ = area PQCB. We have to find- The ratio of BP : AB.

Area of (Δ APQ) = ½ Area of (Δ ABC) which could be also written as

Area of (Δ APQ)/Area of (Δ ABC) = ½

$$\frac{AP^2}{AB^2} = \frac{1}{2}$$

$$\frac{AP}{AB} = \frac{1}{\sqrt{2}}$$

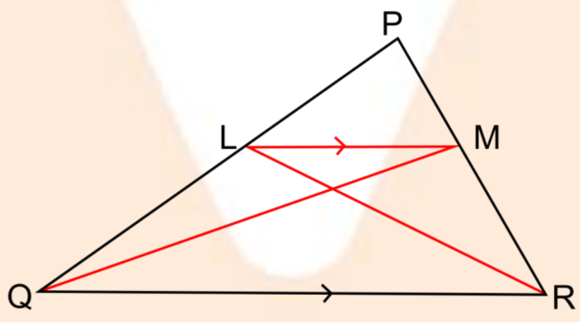
$$\frac{(AB - BP)}{AB} = \frac{1}{\sqrt{2}}$$

$$1 - (\frac{BP}{AB}) = \frac{1}{\sqrt{2}}$$

$$\frac{BP}{AB} = 1 - \frac{1}{\sqrt{2}}$$

$$\frac{BP}{AB} = \frac{2 - \sqrt{2}}{2}$$

6. In the given triangle PQR, LM is parallel to QR and PM: MR = 3: 4. Calculate the value of ratio:



(i) PL/PQ and then LM/QR

(ii) Area of Δ LMN/ Area of Δ MNR

(iii) Area of Δ LQM/ Area of Δ LQN

Ans: (i)Let us consider the Δ PLM and Δ PQR,

As it is mentioned that LM || QR, hence the corresponding points are equal

∠PLM = ∠PQR

∠PML = ∠PRQ

Hence by AA criterion for similarity, we can say that ΔPLM ~ ΔPQR.

Now we have,

$$\frac{PM}{PR} = \frac{LM}{QR}$$

$$\frac{3}{7} = \frac{LM}{QR}$$

By following the Basic Proportionality theorem, we have

$$\frac{PL}{LQ} = \frac{PM}{MR} = \frac{3}{4}$$

$$\frac{LQ}{PL} = \frac{4}{3}$$

$$1 + (\frac{LQ}{PL}) = 1 + \frac{4}{3}$$

$$\frac{(PL + LQ)}{PL} = \frac{(3 + 4)}{3}$$

$$\frac{PQ}{PL} = \frac{7}{3}$$

Hence, $\frac{PL}{PQ} = \frac{3}{7}$

(ii) We have to find the Area of Δ LMN/ Area of Δ MNR

ΔLMR and ΔMNR have the same vertex at M and their bases NR and LN are found to be along the same straight line.

The ratio of the areas of the triangles can be expressed as follows,

Area of Δ LMN/Area of Δ RNQ = LN/NR

Now by considering the Δ LMN and Δ RNQ we have,

∠NLM = ∠NRQ

∠LMN = ∠NQR (Alternate angles)

Hence, by AA criterion for similarity, we can say that ΔLNM ~ ΔRNQ

Thus now,

$$\frac{MN}{QN} = \frac{LN}{NR} = \frac{LM}{QR} = \frac{3}{7}$$

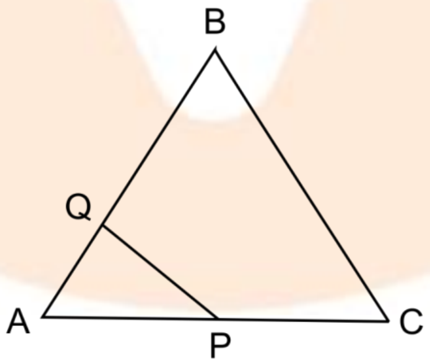
∴ Area of Δ LMN/Area of Δ RNQ = $\frac{LN}{NR} = \frac{3}{7}$

(iii) We have find Area of Δ LQM/ Area of Δ LQN

As it is mentioned that Δ LQM and Δ LQN have common vertices at L and their bases QM and QN are along the straight line.

Area of Δ LQM/ Area of Δ LQN = $\frac{QM}{QN} = \frac{10}{7}$

7. The given diagram shows two isosceles triangles which are similar also. In the given diagram, PQ and BC are not parallel: PC = 4, AQ = 3, QB = 12, BC = 15 and AP = PQ.



Calculate-

(i) the length of AP

(ii) the ratio of the areas of triangle APQ and triangle ABC.

Ans: As it is given in the triangle that the triangles are similar

$\Delta APQ \sim \Delta ABC$

$$\frac{AQ}{AC} = \frac{AP}{BC}$$

$$\Rightarrow \frac{3}{AP + PC} = \frac{AP}{15}$$

AP(AP+4)=45

Let us consider AP = x, then we have

x(x+4) = 45

$x^2 + 4x - 45 = 0$, on solving this we have

$\Rightarrow (x + 9)\ (x - 5 = 0)$

x= -9 is not possible and hence

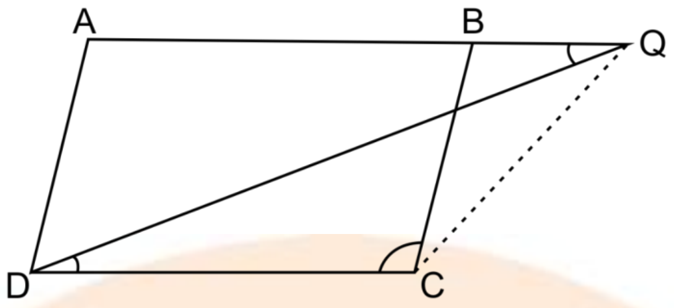
Value of x = 5

∴Now AC = AP+PC = 5+4 = 9.

8. In the figure, given below, ABCD is a parallelogram. P is a point on BC such that BP: PC = 1: 2. DP produced meets AB produced at Q. Given the area of triangle CPQ = 20 cm². Calculate

(i) area of triangle CDP

(ii) area of parallelogram ABCD



Ans: (i) To find the area of triangle CDP

Let us consider ΔBPQ and ΔCPD,

∠DPC = ∠BPQ (As they are vertically opposite angles)

∠PDC = ∠BQP(Alternate angles)

Hence by AA postulates, we can say that ΔBPQ ~ ΔCDP

$$\therefore \frac{\text{Area of }\Delta BPQ}{\text{Area of }\Delta CPD} = \frac{(BP^2)}{(CP^2)} = \frac{1}{4} \text{ ..... (i)}$$

⇒ Area ΔCDP = 4 (Area ΔBPQ)

⇒ 2 (2 Area ΔBPQ) = 2 x 20 = 40 cm²

(ii) To find the area of parallelogram ABCD

Area of parallelogram ABCD = Area ΔCPD + Area ΔADQ – Area ΔBPQ

Area of parallelogram ABCD = 40 + 9 (area BPQ) – area BPQ

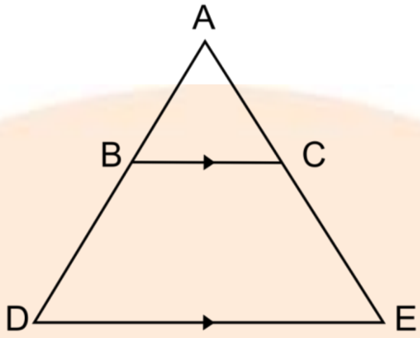
= 40 + 8 (area ΔBPQ)

= 40 + 8 (10) cm²

= 40 + 80

Area of parallelogram ABCD= 120 cm²

9. In the given figure. BC is parallel to DE. Area of triangle ABC = 25 cm². Area of trapezium BCED = 24 cm² and DE = 14 cm. Calculate the length of BC. Also, Find the area of triangle BCD.



Ans: It is given that, in ΔADE, BC || DE

Area of ΔABC = 25 cm²

and area of trapezium BCED = 24 cm²

Area of ΔADE = Area of ΔABC + Area of trapezium BCED

Area of ΔADE = 25 + 24 = 49 cm², DE = 14 cm.

Now let BC = x cm.

Let us consider ΔABC and ΔADE.

∠ABC = ∠ADE(corresponding angles)

Hence by AA postulates, we can say that ΔABC ~ ΔADE

$$\therefore \frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta ADE} = \frac{(BC^2)}{(DE^2)} = \frac{25}{49} = \frac{BC^2}{(14)^2}$$

$\Rightarrow BC^2 = 100$

BC = 10cm.

Let us consider the trapezium BCED,

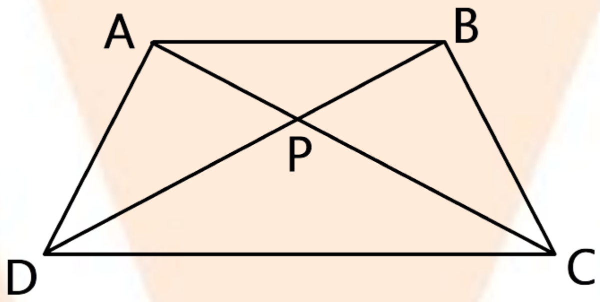
Area of trapezium BCED = ½ (Sum of parallel sides) ×h

But we know that area of trapezium BCED = 24 cm² and DE = 14 cm

$h = \dfrac{\text{Area} \times 2}{(BC + DE)} = \dfrac{48}{24} = 2\text{cm}.$

Now, the Area of $\Delta BCD = \dfrac{1}{2} \times 10 \times 2 = 10\text{cm}^2$

10. The given figure shows a trapezium in which AB is parallel to DC and diagonals AC and BD intersect at point P. If AP : CP = 3 : 5.



Find-

(i) ΔAPB : ΔCPB

(ii) ΔDPC : ΔAPB

(iii) ΔADP : ΔAPB

(iv) ΔAPB : ΔADB

Ans: It is given that AP : CP = 3 : 5 $\Rightarrow \dfrac{AD}{DG} = \dfrac{CF}{FG}$

(i) Now let us consider ΔAPB and ΔCPB,

Both of these triangles have the same vertex and their bases are in the same straight lines and hence

Area ΔAPB : area ΔCPB = AP : PC = 3 : 5 or

ΔAPB : ΔCPB = 3 : 5

(ii)Now let us consider ΔAPB and ΔDPC,

∠APB = ∠DPC (As they are vertically opposite angles)

∠PAB = ∠PCD (alternate angles)

Hence by AA postulates, we can say that ΔAPB ~ ΔDPC.

$$\therefore \frac{\text{Area of } \Delta DPC}{\text{Area of } \Delta APB} = \frac{CP^2}{AP^2} = \frac{25}{9}$$

⇒ Area ΔDPC : Area ΔAPB = 25 : 9 or

⇒ΔDPC : ΔAPB = 25 : 9

(iii) Now let us consider ΔADP and ΔAPB,

As both of the triangles have the same vertex and their bases arc in the same straight line, we can say that

Area ΔADP : Area ΔAPB = DP : PB

But PC : AP = 5 : 3

ΔADP : ΔAPB = 5 : 3

(iv) The ΔADB and ΔAPB have same vertex at A and their bases BP

and BD are along the same parallel line.

Therefore, the ratio of area of triangles is equal to the ratio of

corresponding sides.

$$\therefore \text{Area of } \Delta ADB/ \text{ Area of } \Delta APB = \frac{PB}{BD} PB/ BD = \frac{3}{8}.$$

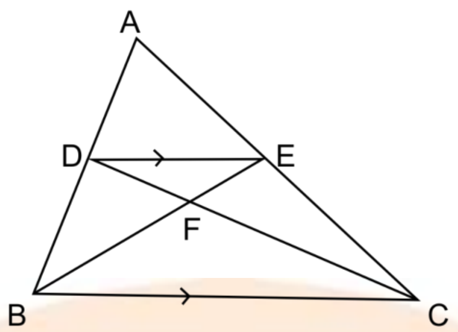
11. In the given figure, ARC is a triangle. DE is parallel to BC and AD/DB = 3/2.

(i) Determine the ratios AD/AB, DE/BC

(ii) Prove that ΔDEF is similar to ΔCBF.

Hence, find EF/FB

(iii) What is the ratio of the areas of ΔDEF and ΔBFC?



Ans: It is given that

ABC is a triangle, DE is parallel to BC and the ratio of AD and DB is 3:2.

(i) $\frac{AD}{DB}=\frac{3}{2}$

$\frac{DB}{AD}=\frac{2}{3}$ or

$\frac{DB}{AD}+1=\frac{2}{3}+1$ or it could be also written as

$\frac{AB}{AD}=\frac{5}{3}$ or

$\frac{AD}{AB}=\frac{3}{5}$

Now let us consider ΔADE and ΔABC

$\angle ADE = \angle B$ (corresponding angles)

$\angle AED = \angle C$ (corresponding angles)

Hence by AA similarity, we can say that $\Delta ADE \sim \Delta ABC$

$\frac{AD}{AB}=\frac{DE}{BC}$

$\therefore \frac{DE}{BC}=\frac{3}{5}$

(ii) We have to prove that ΔDEF is similar to ΔCBF.

In Δ DEF and ΔCBF

∠FDE= ∠FCB (Alternate angles)

∠DFE =∠BFC (Vertical opposite angles)

Hence by AA similarity, we can say that Δ DEF~ ΔCBF

The ratios of corresponding sides are equal.

$$\frac{EF}{FB} = \frac{DE}{BC}$$

$$\frac{EF}{FB} = \frac{3}{5}$$

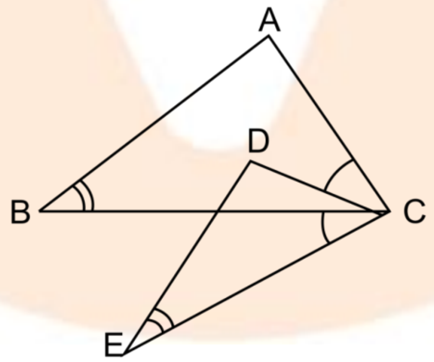
(iii) The ΔDFE and ΔCBF are similar. Therefore, areas of two similar

triangles are proportional to the squares of their corresponding sides.

$$\frac{\text{Area of } \Delta DFE}{\text{Area of } \Delta CBF} = \frac{DE^2}{BC^2} = \frac{9}{25}$$

12. In the given figure, ∠B = ∠E, ∠ACD = ∠BCE, AB = 10.4 cm and DE = 7.8 cm. Find the ratio between areas of the ΔABC and ΔDEC.

Ans:



Ans:

It is given that in the figure DE = 7.8 cm and AB = 10.4 cm.

∠ACD = ∠ BCE (Given in the question)

Add ∠DCB on both sides,

∠ACD + ∠DCB = ∠DCB + ∠BCE

∠ACB = ∠DCE

Now let us consider ΔABC and ΔDCE

∠B = ∠E(Given in the question)

∠ACB = ∠DCE

Hence by AA similarity, we can say that ΔABC ~ ΔDCE.

$$\therefore \frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DCE} = \frac{AB^2}{DE^2}$$

As areas of the two similar triangles are in proportional to their corresponding sides

$$= \frac{AB^2}{DE^2} = \frac{(10.4)^2}{(7.8)^2} = \frac{16}{9}$$

∴The ratio between areas of the ΔABC and ΔDEC= 16:9.

Exercise 15(D)

1. A triangle ABC has been enlarged by scale factor m = 2.5 to the triangle A’ B’ C’ Calculate:

(i) the length of AB, if A’ B’ = 6 cm.

(ii) the length of C’ A’ if CA = 4 cm.

Ans: It is given that, triangle ABC has been enlarged by scale factor m = 2.5 to the triangle A’ B’ C’.

(i) It is given that we have to find the length of AB if CA’B’ = 6 cm,

Now, AB(2.5) = A’B’ = 6 cm

AB = 2.4 cm.

(ii) To find the length of C’ A’ if CA = 4 cm.

We know that,

CA(2.5) = C'A'

C'A' = 4 x 2.5 = 10 cm

2. A triangle LMN has been reduced by a scale factor 0.8 to the triangle L' M' N'. Calculate:

(i) the length of M' N', if MN = 8 cm.

(ii) the length of LM, if L' M' = 5.4 cm.

Ans: It is given that Δ LMN has been reduced by a scale factor m = 0.8 to Δ L'M'N'.

(i) To find the length of M' N', if MN = 8 cm.

So, MN (0.8) = M'N'

(8)(0.8) = M'N'

M'N' = 6.4 cm

(ii) To find the length of LM, if L' M' = 5.4 cm.

So, LM (0.8) = L'M'

LM (0.8) = 5.4

LM = 6.75 cm

3. A triangle ABC is enlarged, about the point 0 as centre of enlargement, and the scale factor is 3. Find:

(i) A'B', if AB = 4 cm.

(ii) BC, if B'C' = 15 cm.

(iii) OA, if OA' = 6 cm

(iv) OC', if OC = 21 cm

Also, state the value of:

(a) OB’/OB (b) C’A’/CA

Ans: It has been given that Δ ABC is enlarged and the scale factor m = 3 to the Δ A’B’C’.

(i) To find A’B’, if AB = 4 cm.

So, AB(3) = A’B’

(4)(3) = A’B’

A’B’ = 12 cm

(ii) To find BC if B’C’ = 15 cm

So, BC(3) = B’C’

BC(3) = 15

BC = 5 cm

(iii) To find OA if OA’ = 6 cm

So, OA (3) = OA’

OA (3) = 6

OA = 2 cm

(iv) To find OC’, if OC = 21 cm

Now, OC(3) = OC’

21×3 = OC’

OC’ = 63 cm

Also, we have to find the values of OB’/OB and C’A’/CA and as we know the OB’, OB, C’A’, and CA we have.

(a) OB’/OB = 3

(b) C’A’/CA = 3

4. A model of an aeroplane is made to a scale of 1 : 400. Calculate:

(i) the length, in cm, of the model; if the length of the aeroplane is 40 m.

(ii) the length, in m, of the airplane, if the length of its model is 16 cm.

Ans: It is given that model of an aeroplane to the actual = 1: 400

∴ Scale factor = 400/1

(i) Length of aeroplane = 40 m

Then the length of model = 40 ×1/400 = 1/10 m.

$= \frac{1}{10} \times 100 = 10\text{cm}$

(ii) The length of aeroplane if the length of the model = 16 cm.

∴ Length of aeroplane = $\frac{16 \times 400}{1}$ = 6400 = 6400/100 = 64 m.

5. The dimensions of the model of a multi stored building are 1.2 m x 75 cm x 2 m. If the scale factor is 1 : 30; find the actual dimensions of the building.

Ans: It is been given that the dimensions of a model of multi stored building = 1.2 m × 75 cm × 2 m

Now scale factor is given as = 1: 30 = 1/30

∴ Actual length = 1.2 m × 30 = 36m.

Breath is given by = 75cm = $\frac{75 \times 30}{100}\text{m}$.

= 2250/100 = 22.5 m

Height = 2m = $2 \times \frac{30}{1} = 60\text{m}$.

Hence the actual dimensions of the building are 36m × 22.5m × 60m.

6.On a map drawn to a scale of 1 : 2,50,000; a triangular plot of land has the following measurements : AB = 3 cm, BC = 4 cm and angle ABC = 90°.

Calculate:

(i) the actual lengths of AB and BC in km.

(ii) the area of the plot in sq. km.

Ans: It is given that scale of a map drawn of a triangular plot = 1: 2,50,000 and also the measurement of plot AB = 3 cm, BC = 4 cm and and ∠ABC = 90°

Now let us consider the right-angled triangle ABC,

(i) We have to calculate the actual lengths of AB and BC in km.

The actual length of AB = $3 \times 25000\text{cm} = \frac{3 \times 250000}{100000}\text{km}$

= 15/2 = 7.5 km

Now the actual length of BC = $\frac{4 \times 250000}{100 \times 1000}$ = 10 km.

(ii) Area of the plot = ½ × BC × AB.

$= \frac{1}{2} \times 7.5 \times 10\text{km}^2$

= 37.5 km^2

7. A model of a ship of made to a scale 1 : 300

(i) The length of the model of ship is 2 m. Calculate the lengths of the ship.

(ii) The area of the deck ship is 180,000 m². Calculate the area of the deck of the model.

(iii) The volume of the model is 6.5 m3. Calculate the volume of the ship. (2016)

Ans: It is given that the model of a ship of made to a scale 1: 300

Now,

(i) Scale factor k = 1/300

Length of the model = k (Length of the ship)

2 = 1/300 length of the ship.

Length of the ship= 600 m.

(ii) Area of the deck of the model = k^2which is equal to the area of the deck of the ship.

$\Rightarrow$ Area of the deck of the model = $(\frac{1}{300})^2 \times (180000) = 2m^2$

(iii) The volume of the model = k^3 which is the volume of the ship

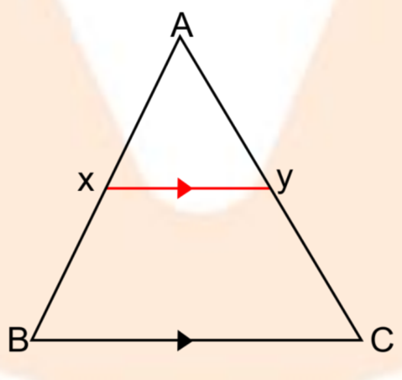
6.5 = $(1/300)^3$ × Volume of the ship

Volume of the ship = 6.5 × 27000000

Volume of the ship = 175500000 m^3

Exercise 15(E)

1. In the following figure, XY is parallel to BC, AX = 9 cm, XB = 4.5 cm and BC = 18 cm.



Find:

(i) AY/YC (ii) YC/AC (iii) XY

Ans: It is given that, XY || BC and AX = 9 cm, XB = 4.5 cm and BC = 18 cm.

Now let us consider Δ AXY and Δ ABC\

∠AXY = ∠ABC (Corresponding angles are equal)

∠AYX = ∠ACB (Corresponding angles are equal)

Hence by AA criterion for similarity we can say that $\Delta AXY \sim \Delta ABC$.

As we have now established that the corresponding sides of the similar triangles are proportional to each other we have,

(i) $\frac{AX}{AB} = \frac{AY}{AC}$

$$\frac{9}{13.5} = \frac{AY}{AC}$$

$$\frac{AY}{YC} = \frac{9}{4.5}$$

$$\frac{AY}{YC} = \frac{2}{1}$$

$$\frac{AY}{YC} = \frac{2}{1}$$

(ii) To find YC/AC,

Now we have,

$$\frac{AX}{AB} = \frac{AY}{AC}$$

$$\frac{9}{13.5} = \frac{AY}{AC}$$

$$\frac{YC}{AC} = \frac{4.5}{13.5} = \frac{1}{3}$$

(iii) To find XY

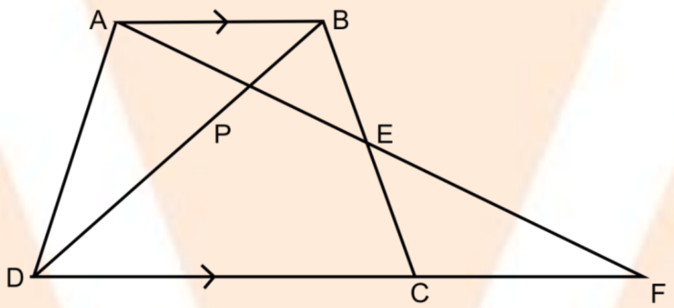
As we know that $\Delta AXY \sim \Delta ABC$

$$\frac{AX}{AB} = \frac{XY}{BC}$$

$$\frac{9}{13.5} = \frac{XY}{18}$$

$$XY = \frac{(9 \times 18)}{13.5} = 12 \text{ cm}$$

2. In the following figure, ABCD to a trapezium with AB ∥ DC. If AB = 9 cm, DC = 18 cm, CF= 13.5 cm, AP = 6 cm and BE = 15 cm,



Calculate:

(i) EC

(ii) AF

(iii) PE

Ans:(i) Let us consider Δ AEB and Δ FEC,

∠FEC = ∠AEB (Vertically opposite angles)

∠BAE = ∠CFE (As AB||DC)

Hence by considering the AA criterion for similarity we can say that ΔAEB ~ ΔFEC

Now we have,

$$\frac{AE}{FE} = \frac{BE}{EC} = \frac{AB}{FC}$$

15/EC = 9/13.5

EC = 22.5 cm

(ii) Let us consider the Δ APB and Δ FPD,

∠APB = ∠FPD (Vertically opposite angles)

∠BAP = ∠DFP (Since, AB||DF)

Hence by AA criterion for similarity we can say that ΔAPB ~ ΔFPD.

Now we have,

$$\frac{AP}{FP} = \frac{AB}{FD}$$

$$\frac{6}{FP} = \frac{9}{31.5}$$

FP = 21 cm

So, AF = AP + PF = 6 + 21 = 27 cm

(iii) We already have, ΔAEB ~ ΔFEC

So,

AE/FE = BE/CE = AB/FC

AE/FE = 9/13.5

(AF – EF)/ FE = 9/13.5

AF/EF – 1 = 9/13.5

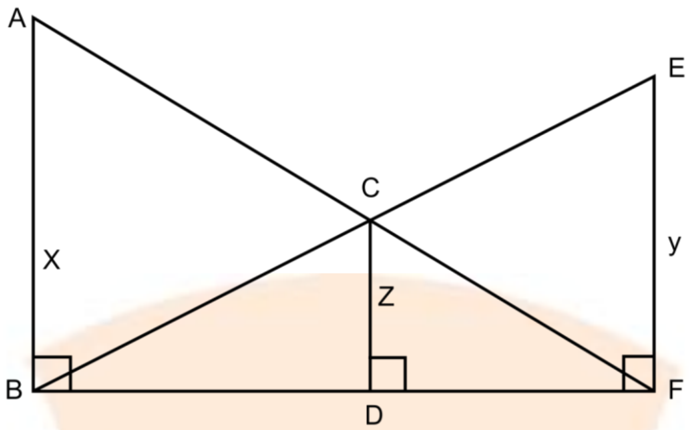
27/EF = 9/13.5 + 1 = 22.5/ 13.5

Therefore, EF = 16.2 cm.

Now, we have

PE = PF – EF = 21 – 16.2 = 4.8 cm

3.In the following figure, AB, CD and EF are perpendicular to the straight line BDF



If AB = x and; CD = z unit and EF = y unit, prove that:

$$\frac{1}{x}+\frac{1}{y}=\frac{1}{z}$$

Ans: Let us consider Δ FDC and Δ FBA,

∠FDC = ∠FBA (As DC || AB)

∠DFC = ∠BFA (common angle)

Hence by AA criterion for similarity we can say that ΔFDC ~ ΔFBA.

Thus, we have

DC/AB = DF/BF

$$\frac{z}{x}=\frac{DF}{BF} \ldots. (1)$$

Let us consider Δ BDC and Δ BFE,

∠DBC = ∠FBE (Common angle)

∠BDC = ∠BFE [As DC || FE]

Hence by AA criterion for similarity, we can say that ΔBDC ~ ΔBFE.

Now we have, $\frac{BD}{BF}=\frac{z}{y} \ldots.. (2)$

Add (1) and (2), we get

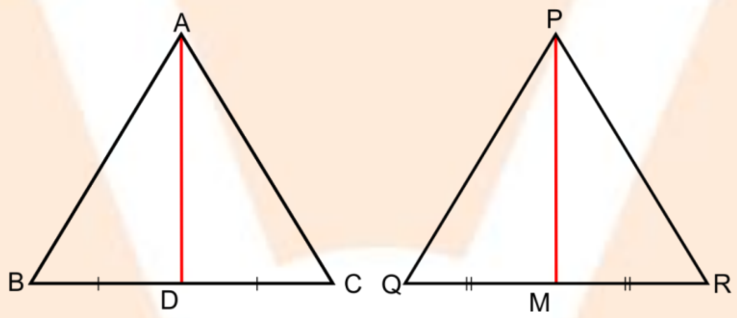
$$\frac{BD}{BF}+\frac{DF}{BF}=\frac{z}{y}+\frac{z}{x}$$

1 = z/y + z/x, divide both sides by z

1/z = 1/x + 1/y, hence it is proved.

4. Triangle ABC is similar to triangle PQR. If AD and PM are corresponding medians of the two triangles, prove that: $\frac{AB}{PQ}=\frac{AD}{PM}$

Ans:



It is given that ΔABC ~ ΔPQR and AD and PM are the medians, so BD = DC and QM = MR

As the corresponding sides of the similar triangles are proportional we have AB/PQ = BC/QR.

Now, AB/PQ = (BC/2)/ (QR/2) = BD/QM

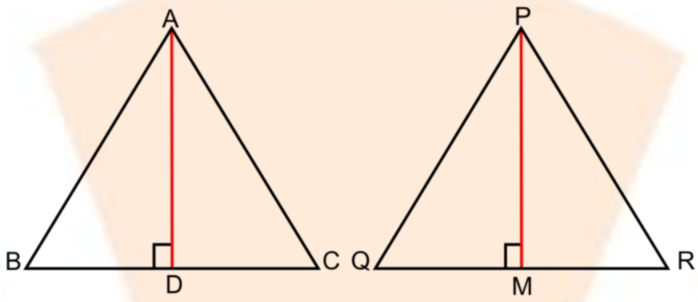
and, ∠ABC = ∠PQR i.e. ∠ABD = ∠PQM

Hence by SAS criterion for similarity we can say that ΔABD ~ ΔPQM

Therefore, $\frac{AB}{PQ}=\frac{AD}{PM}$

5.Triangle ABC is similar to triangle PQR. If AD and PM are altitudes of the two triangles, prove that: $\frac{AB}{PQ}=\frac{AD}{PM}$

Ans:



It is given that $\Delta ABC \sim \Delta PQR$

Now, $\angle ABC = \angle PQR$ (Corresponding angles)

$\angle ABD = \angle PQM$ (Corresponding angles)

$\angle ADB = \angle PMQ$ (Both are right angles)

Hence by AA criterion for similarity, we can say that $\Delta ABD \sim \Delta PQM$.

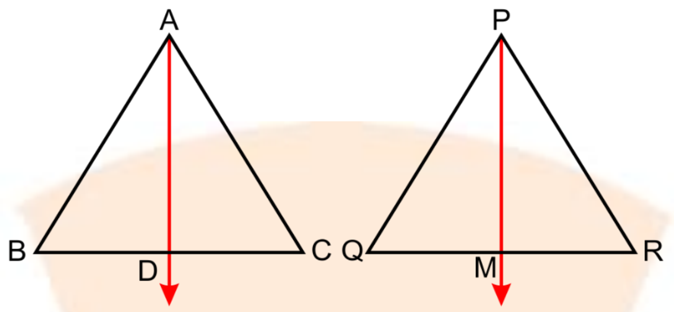
Thus, now we can say that,

$$\frac{AB}{PQ}=\frac{AD}{PM}$$

6. Triangle ABC is similar to triangle PQR. If bisector of angle BAC meets BC at point D and the bisector of angle QPR meets QR at point M, prove that:

$$\frac{AB}{PQ}=\frac{AD}{PM}$$

Ans:



It is given that $\Delta ABC \sim \Delta PQR$ and AD and PM are the angle bisectors.

So now we have,

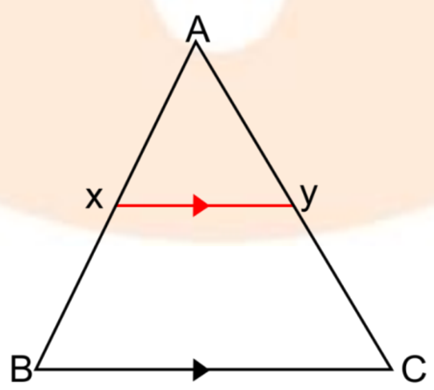
$\angle BAD = \angle QPM$ (Both are right angles)

$\angle ABC = \angle PQR$ i.e. $\angle ABD = \angle PQM$

Hence by AA criterion for similarity we can say that $\Delta ABD \sim \Delta PQM$.

Thus, we can say that $\frac{AB}{PQ} = \frac{AD}{PM}$

7. In the following figure, $\angle AXY = \angle AYX$. If BX/AX = CY/AY, show that triangle ABC is isosceles.



Ans: It is given that, $\angle AXY = \angle AYX$

So now we have AX = AY, as sides opposite to equal angles are equal.

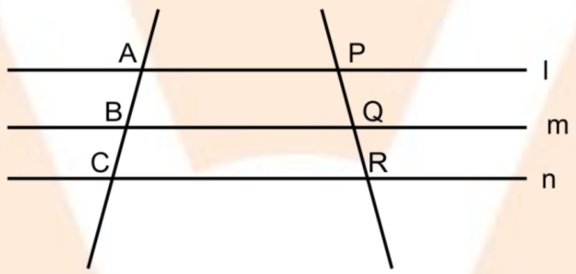
By considering BPT we have,

$$\frac{BX}{AX} = \frac{CY}{AY}$$

Hence, AX + BX = AY + CY

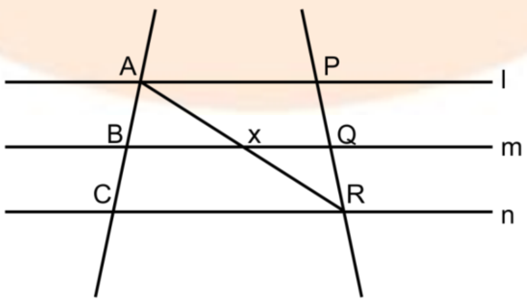
As AB = AC, we can say that ΔABC is an isosceles triangle.

8. In the following diagram, lines l, m and n are parallel to each other. Two transversals p and q intersect the parallel lines at points A, B, C and P, Q, R as shown.



Prove that: $\frac{AB}{BC} = \frac{PQ}{QR}$

Ans:



Consider the figure given in the question. Let join AR such that it intersects BQ at point X.

Let us consider ΔACR were BX || CR.By, BPT we have

$$\frac{AB}{BC}=\frac{AX}{XR} \text{......} (1)$$

Let us consider ΔAPR were XQ || AP.By BPT we have

$$\frac{PQ}{QR}=\frac{AX}{XR} \text{.. ...} (2)$$

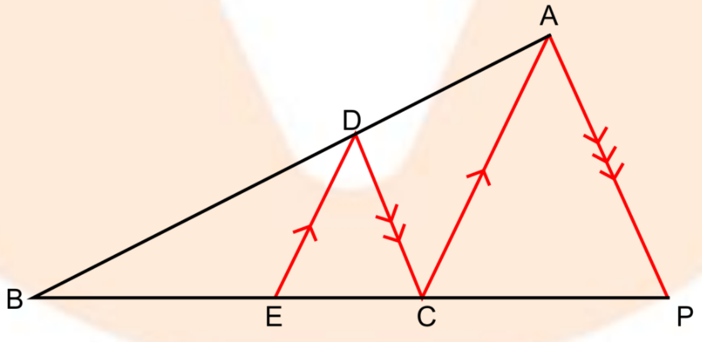
From (1) and (2),

$$\frac{AB}{BC}=\frac{PQ}{QR}$$

AB/BC = PQ/QR, Hence, it is proved.

9. In the following figure, DE || AC and DC || AP. Prove that: $\frac{BE}{EC}=\frac{BC}{CP}$

Ans:



It is given that DE || AC and DC || AP.

So now we have,

$\frac{BE}{EC}=\frac{BD}{DA}$(By basic proportionality theorem)

And as DC || AP we will have

$\frac{BE}{EC}=\frac{BD}{DA}$(By basic proportionality theorem)

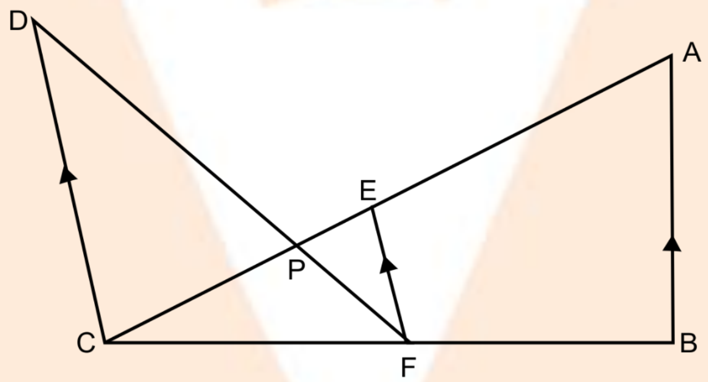
Therefore, $\frac{BE}{EC}=\frac{BC}{CP}$

10. In the figure given below, AB ∥ EF ∥ CD. If AB = 22.5 cm, EP = 7.5 cm, PC = 15 cm and DC = 27 cm.

Calculate:

(i) EF

(ii) AC



Ans: (i) It is given that AB ∥ EF ∥ CD and AB = 22.5 cm, EP = 7.5 cm, PC = 15 cm and DC = 27 cm.

Let us consider ΔPCD and ΔPEF,

∠CPD = ∠EPF (Vertically opposite angles)

∠DCE = ∠FEP (As DC || EF, alternate angles.)

Hence by considering the AA criterion for similarity we have ΔPCD ~ ΔPEF.

Now we have $\frac{27}{EF}=\frac{15}{7.5}$

EF = 13.5

(ii) As it is been given that EF || AB

We can say that by following the AA criterion for similarity we have ΔCEF ~ ΔCAB.

$\frac{EC}{AC}=\frac{EF}{AB}$

$\frac{22.5}{AC}=\frac{13.5}{22.5}$

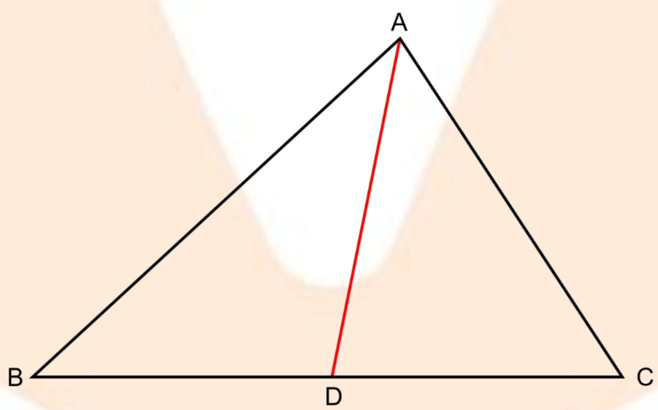
Thus, AC = 37.5 cm

11. In ΔABC, ∠ABC = ∠DAC, AB = 8 cm, AC = 4 cm and AD = 5 cm.

(i) Prove that ΔACD is similar to ΔBCA.

(ii) Find BC and CD

(iii) Find the area of ΔACD: area of ΔABC



Ans: (i) Let us consider ΔACD and ΔBCA,

It is given that ∠DAC = ∠ABC

∠ACD = ∠BCA (Common angles)

Hence by AA similarity, we can say that ΔACD ~ ΔBCA.

(ii) As we have established that $\Delta ACD \sim \Delta BCA$.

We have,

$$\frac{AC}{BC}=\frac{CD}{CA}=\frac{AD}{AB}$$

$$\frac{4}{BC}=\frac{CD}{4}=\frac{5}{8}$$

$$\frac{4}{BC}=\frac{5}{8}$$

BC = 6.4 cm, also

$$\frac{CD}{4}=\frac{5}{8}$$

CD = 2.5 cm

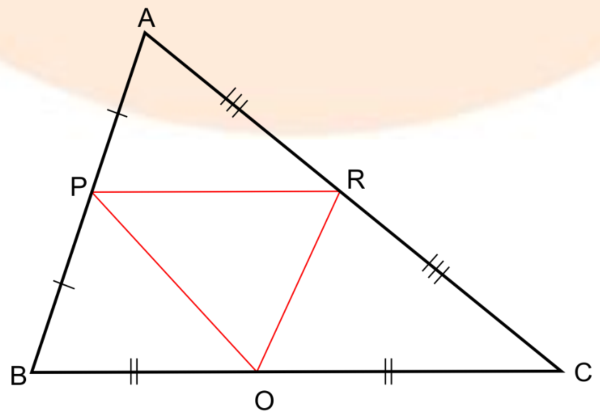
(iii) As we know that $\Delta ACD \sim \Delta BCA$

Now we have,

$$\frac{\text{Area of }(\Delta ACD)}{\text{Area of }(\Delta BCA)}=\frac{AD^2}{AB^2}=\frac{5^2}{8^2}=\frac{25}{64}$$

12. In the given triangle P, Q and R are mid-points of sides AB, BC and AC respectively. Prove that triangle QRP is similar to triangle ABC.

Ans:



Let us consider ΔABC and as PR || BC by BPT we have

$$\frac{AP}{PB} = \frac{AR}{RC}$$

Now let us consider ΔPAR and ΔBAC,

$\angle PAR = \angle BAC$ (Common angles)

$\angle APR = \angle ABC$ (Corresponding angles)

Hence by AA criteria for similarity, we can say that $\Delta PAR \sim \Delta BAC$.

Thus, now we have,

$$\frac{PR}{BC} = \frac{AP}{AB} = \frac{1}{2} \text{(As P is the midpoint of AB)}$$

PR = ½ BC

Similarly,

PQ = ½ AC

RQ = ½ AB

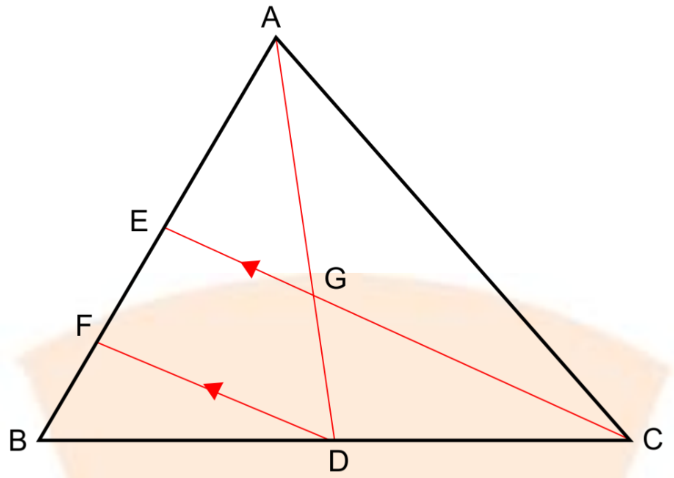
So,

$$\frac{PR}{BC} = \frac{PQ}{AC} = \frac{RQ}{AB}$$

Therefore,

By SSS similarity we can say that $\Delta QRP \sim \Delta ABC$

13. In the following figure, AD and CE are medians of ΔABC. DF is drawn parallel to CE. Prove that:



(i) EF = FB,

(ii) AG: GD = 2: 1

Ans: (i) Let us consider ΔBFD and ΔBEC,

∠BFD = ∠BEC (Corresponding angles)

∠FBD = ∠EBC (Common angles)

Hence by AA criteria for similarity, we can say that ΔBFD ~ ΔBEC.

$$\frac{BF}{BE} = \frac{BD}{BC}$$

$$\frac{BF}{BE} = \frac{1}{2} \text{ (As D is the midpoint of BC)}$$

BE = 2BF

BF = FE = 2BF

Thus,

EF = FB

(ii) Let us consider ΔAFD in which EG || FD. Now by using BPT we have

$$\frac{AE}{EF} = \frac{AG}{GD} \text{ .... (1)}$$

Now as AE = EB (As E is the midpoint of AB)

$AE = 2EF$

From equation 1 we have

$$\frac{AG}{GD} = \frac{2}{1}$$

Therefore, AG: GD = 2: 1

14. Two similar triangles are equal in area. Prove that the triangles are congruent.

Ans: As it is given that two similar triangles are equal, let us consider that $\Delta ABC \sim \Delta PQR$.

So now we have,

$$\frac{\text{Area of }(\Delta ABC)}{\text{Area of }(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$$

Since it is given that, Area of ΔABC = Area of ΔPQR, hence

$AB = PQ$

$BC = QR$

$AC = PR$

As we know that the respective sides of the two similar triangles are all of the same length, we can conclude that.

$\Delta ABC \cong \Delta PQR$ (By SSS rule)

Hence it is proved.

15. The ratio between the altitudes of two similar triangles is 3: 5; write the ratio between their:

(i) medians.

(ii) perimeters.

(iii) areas.

Ans: We know that the ratio between the altitudes of two similar triangles is the same as the ratio between their sides.

So now we have,

(i) The ratio between the medians of two similar triangles is the same as the ratio between their sides.

Thus, the required ratio = 3: 5

(ii) The ratio between the perimeters of two similar triangles is the same as the ratio between their sides.

Thus, the required ratio = 3: 5

(iii) The ratio between the areas of two similar triangles is the same as the square of the ratio between their corresponding sides.

Thus, the required ratio = (3)2: (5)2 = 9: 25

16. The ratio between the areas of two similar triangles is 16 : 25. Find the ratio between their:

(i) perimeters

(ii) altitudes

(iii) medians.

Ans: It is given that the ratio between the areas of two similar triangles is 16 : 25.

The ratio between the medians of the two triangles that are similar

to each other is the same as the ratio of their corresponding sides.

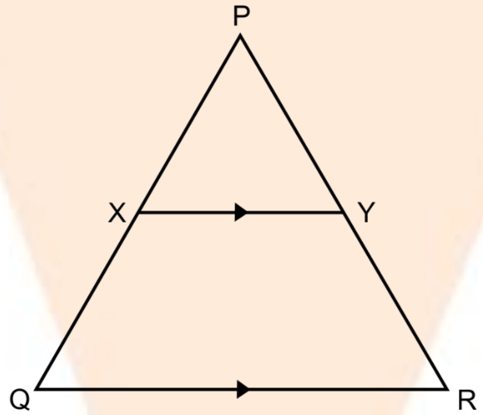
Ratio = $\sqrt{\frac{16}{25}}$ = 4:5.

17. The following figure shows a triangle PQR in which XY is parallel to QR. If PX: XQ = 1:3 and QR = 9 cm, find the length of XY.

Further, if the area of Δ PXY = x cm2; find in terms of x, the area of :

(i) triangle PQR.

(ii) trapezium XQRY.



Ans: Let us consider ΔPXY and Δ PQR in which XY is parallel to QR so in a way we can say that corresponding angles are equal.

$\angle PXY = \angle PQR$

$\angle PYX = \angle PRQ$

Hence by AA criteria for similarity, we can say that $\Delta PXY \sim \Delta PQR$

$$\frac{PX}{PQ} = \frac{XY}{QR}$$

$$\frac{1}{4} = \frac{XY}{QR}$$

$$\frac{1}{4} = \frac{XY}{9}$$

XY = 2.25 cm.

(i) We know that the ratio of the areas of two similar triangles are equal to the ratio of the squares of their corresponding sides.

$$\frac{\text{Area of }(\Delta PXY)}{\text{Area of }(\Delta PQR)} = (\frac{PX}{PQ})^2$$

$$\frac{x}{\text{Area of }(\Delta PQR)} = \frac{1}{16}$$

Area of $(\Delta PQR) = 16\ x cm^2$.

(ii) Area of trapezium XQRY = Area of ΔPQR - Area of Δ PXY

$= (16x-x)cm^2$

$= 15x\ cm^2$

18. On a map, drawn to a scale of 1 : 20000, a rectangular plot of land ABCD has AB = 24 cm, and BC = 32 cm. Calculate :

(i) The diagonal distance of the plot in kilometer

(ii) The area of the plot in sq. km.

Ans: It is given that the scale = 1:20000

1cm represents 20000 cm = $\frac{20000}{1000 \times 100}$ = 0.2 km.

(i) $AC^2 = AB^2 + BC^2$

It is given that AB = 24 cm, and BC = 32 cm

$AC^2 = 576+1024 = 1600$

AC = 40 cm

Actual length of the diagonal = 40×0.2 km = 8 km.

(ii) To find the area of the plot in sq. km.

1 cm represents 0.2 km

1 cm^2 represents $0.2 \times 0.2\ km^2$

Let us consider the rectangle ABCD = AB × BC = 24 × 32 = 768 cm^2

Actual area of the plot = 30.72 km²

19. The dimensions of the model of a multi stored building are lm by 60 cm by 1.20 m. If the scale factor is 1 : 50. Find the actual dimensions of the building. Also, find:

(i) the floor area of a room of the building, if the floor area of the corresponding room in the model is 50 sq cm.

(ii) the space (volume) inside a room of the model, if the space inside the corresponding room of the building is 90 m^3.

Ans: The dimensions of the building can be calculated by following the below given procedure.

Length = 50 m

Breadth = 0.60× 50 m = 30 m

Height =120 × 50 m = 60 m

Hence the actual dimensions of the building are 50m × 30m × 60m

(i) Floor area of the room of the building = $50\times(\frac{50}{1})^2$ =125000 cm² = 12.5 m²

(ii) Volume of the model of the building $=90\times(\frac{1}{50})^3 = 720$ cm³.

20. In a triangle PQR, L and M are two points on the base QR, such that ∠LPQ = ∠QRP and ∠RPM = ∠RQP. Prove that:

(i) ΔPQL ~ ΔRPM

(ii) QL x RM = PL x PM

(iii) PQ² = QR x QL

Ans: (i) Let us consider ΔPQL and ΔRPM

∠PQL = ∠RPM (Given)

$\angle LPQ = \angle MRP$ (Given)

Hence by the AA criterion of similarity, we can say that $\Delta PQL \sim \Delta RPM$

(ii) As it has been proved that $\Delta PQL \sim \Delta RPM$

$$\therefore \frac{QL}{PM} = \frac{PL}{RM}$$

$QL.RM = PL.PM$

(iii) Now let us consider ΔLQP and ΔPQR

$\angle Q = \angle Q$ (Common angle)

$\angle LPQ = \angle QRP$ (Given information)

By AA criterion of similarity, we have $\Delta LQP \sim \Delta PQR$

$$\therefore \frac{PQ}{QR} = \frac{QL}{PQ}$$

$PQ^2 = QR \times QL$

21. A triangle ABC with AB = 3 cm, BC = 6 cm and AC = 4 cm is enlarged to ΔDEF such that the longest side of $\Delta\frac{AB}{DE}$DEF = 9 cm. Find the scale factor and hence, the lengths of the other sides of ΔDEF.

Ans: It is given that triangle ABC is enlarged to DEF, So the two triangles are said to be similar.

$$\therefore \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

Longest side in the Δ ABC = BC = 6cm

Corresponding longest side in ΔDEF = EF = 9 cm

Scale factor = $\frac{EF}{BC} = \frac{3}{2} = 1.5$

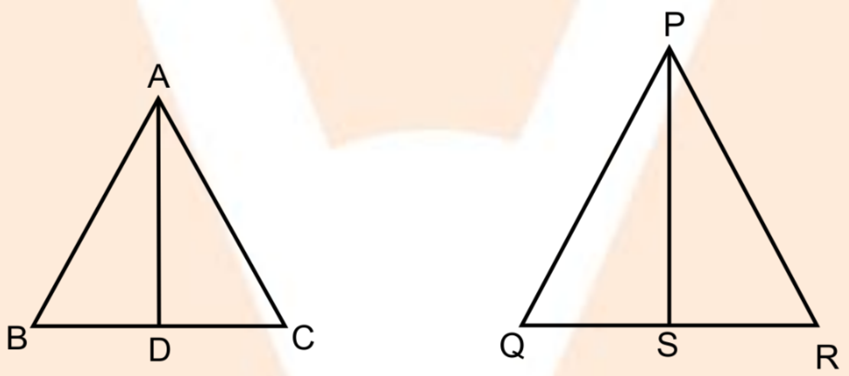
$$\frac{AB}{DE}=\frac{BC}{EF}=\frac{AC}{DF}=\frac{2}{3}$$

$$DE=\frac{3}{2}AB=4.5\text{ cm}$$

Similarly, $DF=\frac{3}{2}AC=6$ cm.

22. Two isosceles triangles have equal vertical angles. Show that the triangles are similar. If the ratio between the areas of these two triangles is 16 : 25, find the ratio between their corresponding altitudes.

Ans:



Let ABC and PQR be the two isosceles triangles which are mentioned.

Then we have $\frac{AB}{AC}=\frac{1}{1}=\frac{PQ}{PR}$

Also, it is given that $\angle A=\angle P$

Hence by SAS similarity, we can say that $\Delta ABC\sim\Delta PQR$

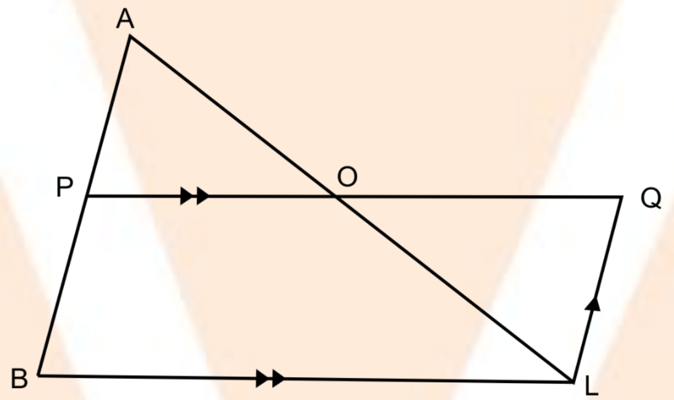
Let us consider that AD and PS are the altitudes of the respective triangles.

It is known that the ratio of areas of the two similar triangles are equal to the square of their corresponding altitudes, hence we have

$$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta PQR} = (\frac{AD}{PS})^2$$

$$\frac{AD}{PS} = \frac{4}{5}.$$

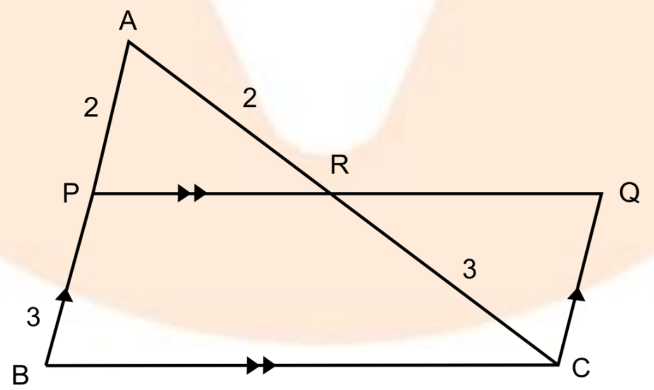
23.In ΔABC, AP : PB = 2 : 3. PO is parallel to BC and is extended to Q so that CQ is parallel to BA



Find: (i) area ΔAPO : area ΔABC.

(ii) area ΔAPO : area ΔCQO.

Ans:



In ΔABC,

AP : PB = 2 : 3

PQ || BC and CQ || BA

From the figure we can say that PQ || BC

$$\therefore \frac{AP}{PB} = \frac{AO}{OC} = \frac{2}{3}$$

(i) To find the area ΔAPO : area ΔABC.

As we know that ΔAPO ~ ΔABC

$$\therefore \frac{\text{Area } \Delta APO}{\text{Area } \Delta CQO} = \frac{AP^2}{AB^2} = \frac{AP^2}{(AP+PB)^2} = \frac{4}{25}$$

Area ΔAPO : Area ΔABC= 4 : 25

(ii) Let us consider ΔAPO and ΔCQO

∠APO = ∠OQC (Alternate angles)

∠AOP = ∠COQ (Vertically opposite angles)

Hence by AA axiom, we can say that ΔAPO~ ΔCQO

$$\therefore \frac{\text{Area } \Delta APO}{\text{Area } \Delta CQO} = \frac{AP^2}{PB^2} = \frac{4}{9}$$

∴Area ΔAPO :Area ΔCQO = 4:9

24. The following figure shows a triangle ABC in which AD and BE are perpendiculars to BC and AC respectively.

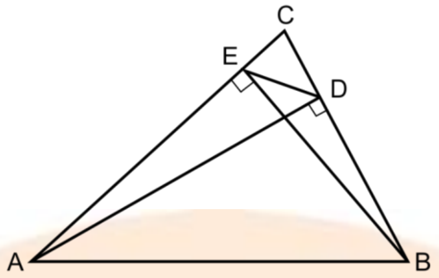
Show that:

(i) ΔADC ~ ΔBEG

(ii) CA × CE = CB × CD

(iii) ΔABC ~ ΔDEC

(iv) CD × AB = CA × DE



Ans: It is given that

In ΔABC, $AD \perp BC$, and $BE \perp AC$, DE is joined

We have to prove:

(i) $\Delta ADC \sim \Delta BEG$

(ii) $CA \times CE = CB \times CD$

(iii) $\Delta ABC \sim \Delta DEC$

(iv) $CD \times AB = CA \times DE$

Proof:

(i) In ΔADC and ΔBEC,

$\angle C = \angle C$ (common angle)

$\angle ABE = \angle BEC$ (as each angle is $90°$)

Hence by AA axiom we can say that $\Delta ADC \sim \Delta BEC$

(ii) As $\frac{CA}{CB} = \frac{CD}{CE}$

Hence, $CA \times CE = CB \times CD$

(iii)Let us consider ΔABC and ΔDEC

$\angle C = \angle C$ (common angle)

$\frac{CA}{CB} = \frac{CD}{CE}$ (already proven)

Hence by SAS axiom, we can say that $\Delta ABC \sim \Delta DEC$

(iv) $\frac{CA}{CD} = \frac{CB}{CE} = \frac{AB}{DE}$

$\frac{CA}{CD} = \frac{AB}{DE}$

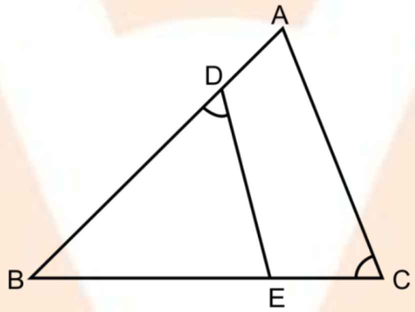
Now, $CD \times AB = CA \times DE$ hence, proved.

25. In the given figure, ABC is a triangle-with $\angle EDB = \angle ACB$. Prove that $\Delta ABC \sim \Delta EBD$.

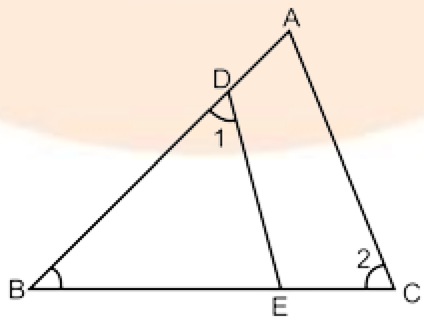
If BE = 6 cm, EC = 4 cm, BD = 5 cm and area of $\Delta BED = 9\ cm^2$. Calculate the

(i) length of AB

(ii) area of ΔABC



Ans:



Let us consider ΔABC and ΔEBD

$\angle 1 = \angle 2$ (given in the diagram)

$\angle B = \angle B$ (common angles)

$\Delta ABC \sim \Delta EBD$.

Now, $\dfrac{\text{Area of } \Delta ABC}{\text{Area of } \Delta EBD} = (\dfrac{BC}{BD})^2$

$\dfrac{\text{Area of } \Delta ABC}{9} = (\dfrac{10}{5})^2$

Area of $\Delta ABC = 36 \text{ cm}^2$

$\dfrac{\text{Area of } \Delta ABC}{\text{Area of } \Delta EBD} = (\dfrac{AB}{BE})^2$

$\dfrac{36}{9} = \dfrac{AB^2}{36}$

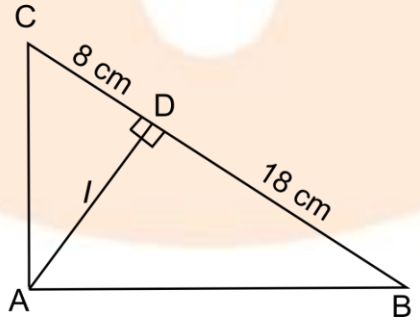
Now, AB = 12cm

26. In the given figure, ABC is a right-angled triangle with $\angle BAC = 90°$.

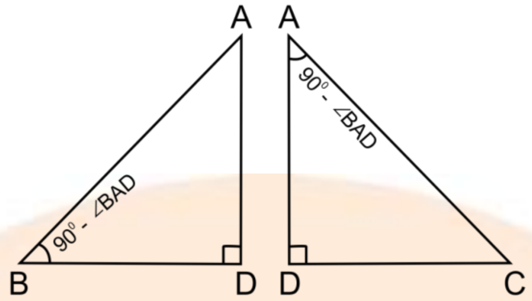
(i) Prove $\Delta ADB \sim \Delta CDA$.

(ii) If BD = 18 cm, CD = 8 cm, find AD.

(iii) Find the ratio of the area of ΔADB is to area of ΔCDA



Ans:



(i) Let us consider ΔADB and ΔCDA :

$\angle ADB = \angle ADC$ [As each angle = 90°]

$\angle ABD = \angle CAD$ [each = $90° - \angle BAD$]

Hence by the AA similarity axiom, we can say that $\Delta ADB \sim \Delta CDA$

(ii) Since, $\Delta ADB \sim \Delta CDA$

$$\therefore \frac{AD}{CD} = \frac{BD}{AD}$$

$AD = \sqrt{144} = 12cm$

(iii)To find the ratio of the area of ΔADB is to area of ΔCDA

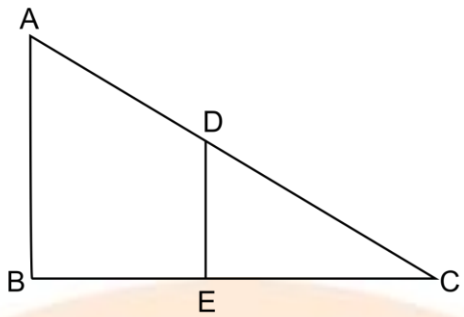
$$\frac{\text{area of } \Delta ADB}{\text{area of } \Delta CDA} = \frac{BD^2}{AD^2} = 9.4$$

27. In the given figure, AB and DE are perpendicular to BC.

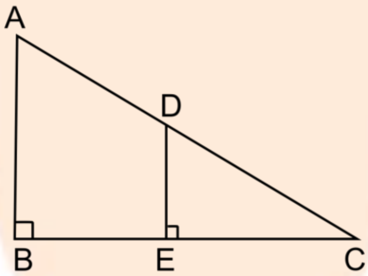
(i) Prove that $\Delta ABC \sim \Delta DEC$

(ii) If AB = 6 cm, DE = 4 cm and AC = 15 cm. Calculate CD.

(iii) Find the ratio of the area of ΔABC : area of ΔDEC.



Ans:



(i) To prove that $\Delta ABC \sim \Delta DEC$

Let us consider In ΔABC and ΔDEC

$\angle ABC = \angle DEC$ (As each = 90°)

$\angle C = \angle C$ (common angles)

$\Delta ABC \sim \Delta DEC$ (by AA axiom)

(ii) $\frac{AC}{DC} = \frac{AB}{DE}$

$\frac{15}{CD} = \frac{6}{4}$

CD = 10 cm

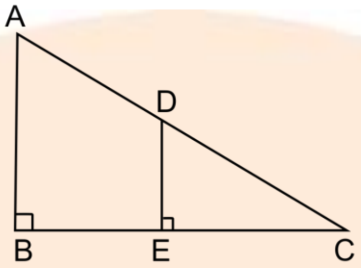
(iii) $\frac{\text{area of } \Delta ABC}{\text{area of } \Delta DEC} = (\frac{AB}{DE})^2 = \frac{36}{16} = 9.4$

28. ABC is a right angled triangle with ∠ABC = 90°. D is any point on AB and DE is perpendicular to AC. Prove that:

(i) ΔADE ~ ΔACB.

(ii) If AC = 13 cm, BC = 5 cm and AE = 4 cm. Find DE and AD.

(iii) Find, area of ΔADE : area of quadrilateral BCED.



Ans: From the given figure we can figure out that

ΔABC is a right-angled triangle at a right angle at B.

D is any point on AB and DE ⊥ AC

Proof:

(i) Let us consider ΔADE and ΔACB

∠A = ∠A (common angles)

∠E = ∠B (each angle is = 90°)

By AA axiom we can say that ΔADE ~ ΔACB.

(ii) It is given that AC = 13 cm, BC = 5 cm, AE = 4 cm

And we know that ΔADE ~ ΔACB.

$\frac{AD}{AC} = \frac{AE}{AB} = \frac{DE}{BC}$ (As corresponding sides are proportional)

$\frac{AD}{13} = \frac{4}{12} = \frac{DE}{5}$

AD = $4\frac{1}{3}$ cm and

DE = $\frac{5}{3}$cm

(iii) To find the area of ΔADE : area of quadrilateral BCED

Area of Δ ABC = ½ × BC × AB= ½ × 5 × 12 = 30 cm²

Now area of ΔADE = $\frac{1}{2}\times BC\times AB=\frac{1}{2}\times 4\times\frac{5}{3}=\frac{10}{3}$cm²

Area of quadrilateral BCED= Area of Δ ABC - area of ΔADE

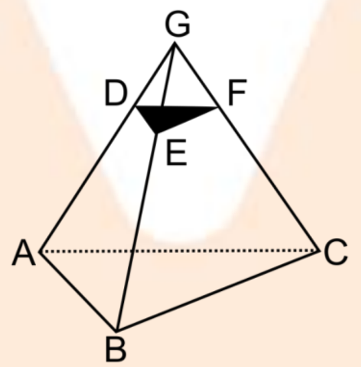
$= 30-\frac{10}{3}=\frac{80}{3}$

Thus the area of ΔADE : Area of quadrilateral BCED $=\frac{10}{3}:\frac{80}{3}$ = 1: 8

29. Given: AB // DE and BC // EF. Prove that:

(i) $\frac{AD}{DG}=\frac{CF}{FG}$

(ii) ΔDFG ~ ΔACG.



Ans: (i) Let us consider ΔAGB, in which DE // AB.

Now by the basic proportionality theorem, we have,

$\frac{GD}{DA}=\frac{GE}{EB}$ ………..(1)

In ΔGBC, in which EF // BC.

Now by the basic proportionality theorem, we have,

$$\frac{GD}{DA}=\frac{GE}{FC}$$

$$\frac{AD}{DG}=\frac{CF}{FG}$$

(ii) To show that $\Delta DFG \sim \Delta ACG$.

$$\frac{AD}{DG}=\frac{CF}{FG}$$

$\angle DGF = \angle AGC$ (Common angle)

Hence, by SAS similarity, we can say that $\Delta DFG \sim \Delta ACG$.