Matrices

1. If
$$A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$
, $B = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$, $C = \begin{bmatrix} 4 & 1 \\ 1 & 5 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

find A (B + C) - 14I. [2023]

$$(B+C) = \begin{bmatrix} 1+4 & 2+1 \\ 2+1 & 4+5 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 3 \\ 3 & 9 \end{bmatrix}$$

$$A(B+C) = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 5 & 3 \\ 3 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \times 5 + 3 \times 3 & 1 \times 3 + 3 \times 9 \\ 2 \times 5 + 4 \times 3 & 2 \times 3 + 4 \times 9 \end{bmatrix}$$

$$= \begin{bmatrix} 5+9 & 3+27 \\ 10+12 & 6+36 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & 30 \\ 22 & 42 \end{bmatrix}$$

$$A(B+C) - 14I$$

$$= \begin{bmatrix} 14 & 30 \\ 22 & 42 \end{bmatrix} - 14 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & 30 \\ 22 & 42 \end{bmatrix} - \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix}$$

$$= \begin{bmatrix} 14 - 14 & 30 - 0 \\ 22 - 0 & 42 - 14 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 30 \\ 22 & 28 \end{bmatrix}$$

2. If
$$\begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -8 \end{bmatrix}$$
, the value of x and y respectively are
$$(a) \ 1, \ -2$$

$$(b) \ -2, \ 1$$

$$(c) \ 1, \ 2$$

$$(d) \ -2, \ -1 \quad [2023]$$

Solution: (a)

$$\begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -8 \end{bmatrix}$$

$$\begin{bmatrix} 2 \times x + 0 \times y \\ 0 \times x + 4 \times y \end{bmatrix} = \begin{bmatrix} 2 \\ -8 \end{bmatrix}$$

$$\begin{bmatrix} 2x + 0 \\ 0 + 4y \end{bmatrix} = \begin{bmatrix} 2 \\ -8 \end{bmatrix}$$

$$\begin{bmatrix} 2x \\ 4y \end{bmatrix} = \begin{bmatrix} 2 \\ -8 \end{bmatrix}$$

$$2x = 2 \text{ and } 4y = -8$$

$$x = 1 \qquad y = -2$$

3. If
$$A = \begin{bmatrix} 3 & 5 \\ 1 & 4 \end{bmatrix}$$
, $B = \begin{bmatrix} 2 & 4 \\ 0 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}$, then $5A - BC$ is equal to:

$$(a) \begin{bmatrix} -5 & -23 \\ 1 & 17 \end{bmatrix}$$

$$(b) \begin{bmatrix} 5 & 23 \\ 1 & 17 \end{bmatrix}$$

$$(c) \begin{bmatrix} -2 & 8 \\ -3 & 3 \end{bmatrix}$$

$$(d) \begin{bmatrix} 5 & 23 \\ -1 & 17 \end{bmatrix} \quad [2021 \ semester - 1]$$

Solution: (d)

$$5A - BC$$

$$= 5 \begin{bmatrix} 3 & 5 \\ 1 & 4 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 15 & 25 \\ 5 & 20 \end{bmatrix} - \begin{bmatrix} 2 \times 1 + 4 \times 2 & 2(-1) + 4 \times 1 \\ 0 \times 1 + 3 \times 2 & 0(-1) + 3 \times 1 \end{bmatrix}$$

$$= \begin{bmatrix} 15 & 25 \\ 5 & 20 \end{bmatrix} - \begin{bmatrix} 2 + 8 & -2 + 4 \\ 0 + 6 & 0 + 3 \end{bmatrix}$$

$$= \begin{bmatrix} 15 & 25 \\ 5 & 20 \end{bmatrix} - \begin{bmatrix} 10 & 2 \\ 6 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 15 - 10 & 25 - 2 \\ 5 - 6 & 20 - 3 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 23 \\ -1 & 17 \end{bmatrix}$$

4. If
$$A = \begin{bmatrix} 5 & 10 \\ 3 & -4 \end{bmatrix}$$
 and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$,

then AI is equal to:

$$(a) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$(b) \begin{bmatrix} 5 & 10 \\ -3 & 4 \end{bmatrix}$$

$$(c) \begin{bmatrix} 5 & 10 \\ 3 & -4 \end{bmatrix}$$

$$(d) \begin{bmatrix} 15 & 15 \\ -1 & -1 \end{bmatrix} [2021 \ semester - 1]$$

Solution: (c)

$$AI = \begin{bmatrix} 5 & 10 \\ 3 & -4 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 \times 1 + 10 \times 0 & 5 \times 0 + 10 \times 1 \\ 3 \times 1 + (-4) \times 0 & 3 \times 0 + (-4) \times 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 + 0 & 0 + 10 \\ 3 + 0 & 0 - 4 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 10 \\ 3 & -4 \end{bmatrix}$$

5. The product AB of two matrices A and B is possible if:

- (a) A and B have the same number of rows.
- (b) The number of columns of A is equal to the number of rows of B.
- (c) The number of rows of A is equal to the number of columns of B.
- (d) A and B have the same number of columns. [2021 Semester-I]

Solution: (b) The number of columns of A is equal to the number of rows of B.

6. If
$$A = \begin{bmatrix} 3 & 0 \\ 5 & 1 \end{bmatrix}$$
 and $B = \begin{bmatrix} -4 & 2 \\ 1 & 0 \end{bmatrix}$, find $A^2 - 2AB + B^2$. [2020]

$$A^{2} = \begin{bmatrix} 3 & 0 \\ 5 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 5 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \times 3 + 0 \times 5 & 3 \times 0 + 0 \times 1 \\ 5 \times 3 + 1 \times 5 & 5 \times 0 + 1 \times 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 0 \\ 20 & 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 3 & 0 \\ 5 & 1 \end{bmatrix} \begin{bmatrix} -4 & 2 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 3(-4) + 0 \times 1 & 3 \times 2 + 0 \times 0 \\ 5(-4) + 1 \times 1 & 5 \times 2 + 1 \times 0 \end{bmatrix}$$

$$= \begin{bmatrix} -12 & 6 \\ -19 & 10 \end{bmatrix}$$

$$B^{2} = \begin{bmatrix} -4 & 2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} -4 & 2 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} (-4)(-4) + 2 \times 1 & (-4) \times 2 + 2 \times 0 \\ 1(-4) + 0 \times 1 & 1 \times 2 + 0 \times 0 \end{bmatrix}$$

$$= \begin{bmatrix} 18 & -8 \\ -4 & 2 \end{bmatrix}$$

$$Hence, A^{2} - 2AB + B^{2}$$

$$= \begin{bmatrix} 9 & 0 \\ 20 & 1 \end{bmatrix} - 2 \begin{bmatrix} -12 & 6 \\ -19 & 10 \end{bmatrix} + \begin{bmatrix} 18 & -8 \\ -4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 0 \\ 20 & 1 \end{bmatrix} - \begin{bmatrix} -24 & 12 \\ -38 & 20 \end{bmatrix} + \begin{bmatrix} 18 & -8 \\ -4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 9 + 24 + 18 & 0 - 12 - 8 \\ 20 + 38 - 4 & 1 - 20 + 2 \end{bmatrix}$$

$$= \begin{bmatrix} 51 & -20 \\ 54 & -17 \end{bmatrix}$$

7. Given,
$$A = \begin{bmatrix} x & 3 \\ y & 3 \end{bmatrix}$$
. If $A^2 = 3I$, [2020]

where I is identity matrix of order 2, find x and y.

Solution: x = -3, y = -2

Step-by-step Explanation:

$$A^{2} = 3I$$

$$\begin{bmatrix} x & 3 \\ y & 3 \end{bmatrix} \begin{bmatrix} x & 3 \\ y & 3 \end{bmatrix} = 3 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x^{2} + 3y & 3x + 9 \\ xy + 3y & 3y + 9 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$\therefore 3x + 9 = 0$$

$$\Rightarrow 3x = -9$$

$$\Rightarrow x = -3$$
and $3y + 9 = 3$

$$\Rightarrow 3y = -6$$

$$\Rightarrow y = -2$$

8. Simplify:

$$\sin A \begin{bmatrix} \sin A & -\cos A \\ \cos A & \sin A \end{bmatrix} + \cos A \begin{bmatrix} \cos A & \sin A \\ -\sin A & \cos A \end{bmatrix} [2019]$$

$$\sin A \begin{bmatrix} \sin A & -\cos A \\ \cos A & \sin A \end{bmatrix} + \cos A \begin{bmatrix} \cos A & \sin A \\ -\sin A & \cos A \end{bmatrix} \\
\begin{bmatrix} \sin^2 A & -\sin A \cdot \cos A \\ \sin A \cdot \cos A & \sin^2 A \end{bmatrix} + \begin{bmatrix} \cos^2 A & \sin A \cdot \cos A \\ -\sin A \cdot \cos A & \cos^2 A \end{bmatrix} \\
\begin{bmatrix} \sin^2 A + \cos^2 A & -\sin A \cdot \cos A + \sin A \cdot \cos A \\ \sin A \cdot \cos A - \sin A \cdot \cos A & \sin^2 A + \cos^2 A \end{bmatrix}
\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

9. Given,
$$\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} imes M = 6I$$
, where M is a matrix and

I is unit matrix of order 2×2 .

- State the order of matrix M.
- (ii) Find the matrix M. [2019]

(i)
$$(m \times n) (n \times p) = (m \times p)$$

 $\Rightarrow (2 \times 2) (n \times p) = (2 \times 2)$
 $\Rightarrow \text{ order of matrix } M = (2 \times 2)$

(ii) Let
$$M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} \times M = 6I$$

$$\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = 6 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 4a + 2c & 4b + 2d \\ -a + c & -b + d \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$$

$$4a + 2c = 6...(i)$$

-a + c = 0....(ii)

Multiplying (ii) by 4 and adding (i) and (ii)

$$4a + 2c = 6$$
$$+ - 4a + 4c = 0$$
$$6c = 6$$

$$c = 1$$

Putting
$$c = 1$$
 in (ii)

$$-a + 1 = 0$$

$$a = 1$$

Now,

$$4b + 2d = 0 \dots (iii)$$

$$-b + d = 6 ... (iv)$$

Multiplying (iv) by4 and adding (iii) and (iv)

$$4b + 2d = 0$$
$$+ -4b + 4d = 24$$
$$6d = 24$$

$$d = 4$$

Putting d = 4 in (iv)

$$-b+4=6$$

$$b = -2$$

$$M = \begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix}$$

10. Find the value of 'x' and 'y' if :

$$2\begin{bmatrix} x & 7 \\ 9 & y-5 \end{bmatrix} + \begin{bmatrix} 6 & -7 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 10 & 7 \\ 22 & 15 \end{bmatrix}$$

Solution: x=2 and y=10

Step-by-step Explanation:

$$2\begin{bmatrix} x & 7 \\ 9 & y - 5 \end{bmatrix} + \begin{bmatrix} 6 & -7 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 10 & 7 \\ 22 & 15 \end{bmatrix}$$

$$\begin{bmatrix} 2x & 14 \\ 18 & 2y - 10 \end{bmatrix} + \begin{bmatrix} 6 & -7 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 10 & 7 \\ 22 & 15 \end{bmatrix}$$

$$\begin{bmatrix} 2x + 6 & 14 - 7 \\ 18 + 4 & 2y - 10 + 5 \end{bmatrix} = \begin{bmatrix} 10 & 7 \\ 22 & 15 \end{bmatrix}$$

$$\begin{bmatrix} 2x + 6 & 7 \\ 22 & 2y - 5 \end{bmatrix} = \begin{bmatrix} 10 & 7 \\ 22 & 15 \end{bmatrix}$$

$$2x + 6 = 10 \text{ and } 2y - 5 = 15$$

$$x = 2 \text{ and } y = 10$$

11.
$$A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}, B = \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix},$$
find $AC + B^2 - 10C$. [2018]

$$AC = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix}$$
$$\begin{bmatrix} 2 \times 1 + 3(-1) & 2 \times 0 + 3 \times 4 \\ 5 \times 1 + 7(-1) & 5 \times 0 + 7 \times 4 \end{bmatrix}$$
$$\begin{bmatrix} -1 & 12 \\ -2 & 28 \end{bmatrix}$$

$$B^{2} = \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix} \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}$$

$$\begin{bmatrix} 0 \times 0 + 4(-1) & 0 \times 4 + 4 \times 7 \\ (-1) \times 0 + 7(-1) & (-1) \times 4 + 7 \times 7 \end{bmatrix}$$

$$\begin{bmatrix} -4 & 28 \\ -7 & 45 \end{bmatrix}$$

$$AC + B^{2} - 10C$$

$$= \begin{bmatrix} -1 & 12 \\ -2 & 28 \end{bmatrix} + \begin{bmatrix} -4 & 28 \\ -7 & 45 \end{bmatrix} - 10 \begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 12 \\ -2 & 28 \end{bmatrix} + \begin{bmatrix} -4 & 28 \\ -7 & 45 \end{bmatrix} - \begin{bmatrix} 10 & 0 \\ -10 & 40 \end{bmatrix}$$

$$= \begin{bmatrix} -1 - 4 - 10 & 12 + 28 - 0 \\ -2 - 7 + 10 & 28 + 45 - 40 \end{bmatrix}$$

$$= \begin{bmatrix} -15 & 40 \\ 1 & 33 \end{bmatrix}$$

12. If
$$A = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$$
 and $B = \begin{bmatrix} -2 & 1 \\ -3 & 2 \end{bmatrix}$ and $A^2 - 5B^2 = 5C$,

Find matrix C where C is a 2 by 2 matrix. [2017]

$$Let C = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$A^{2} = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 \times 1 + 3 \times 3 & 1 \times 3 + 3 \times 4 \\ 3 \times 1 + 4 \times 3 & 3 \times 3 + 4 \times 4 \end{bmatrix}$$

$$\begin{bmatrix} 10 & 15 \\ 15 & 25 \end{bmatrix}$$

$$B^2 = \begin{bmatrix} -2 & 1 \ -3 & 2 \end{bmatrix} \begin{bmatrix} -2 & 1 \ -3 & 2 \end{bmatrix}$$

$$\begin{bmatrix} (-2)(-2) + 1(-3) & (-2)1 + 1 \times 2 \ (-3)(-2) + 2(-3) & (-3)1 + 2 \times 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \ 0 & 1 \end{bmatrix}$$

$$A^2 - 5B^2 = 5C$$

$$A^{2} - 5B^{2} = 5C$$

$$\begin{bmatrix} 10 & 15 \\ 15 & 25 \end{bmatrix} - 5 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 5 \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\begin{bmatrix} 10 & 15 \\ 15 & 25 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 5a & 5b \\ 5c & 5d \end{bmatrix}$$

$$\begin{bmatrix} 10 - 5 & 15 - 0 \\ 15 - 0 & 25 - 5 \end{bmatrix} = \begin{bmatrix} 5a & 5b \\ 5c & 5d \end{bmatrix}$$

$$\begin{bmatrix} 5 & 15 \\ 15 & 20 \end{bmatrix} = \begin{bmatrix} 5a & 5b \\ 5c & 5d \end{bmatrix}$$

$$5a = 5$$

 $a = 1$
 $5b = 15$

$$b = 3$$

$$5c = 15$$

$$c = 3$$

$$5d = 20$$

$$d = 4$$

$$matrix C = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$$

13. Given matrix
$$B = \begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix}$$
.

Find matrix X if $X = B^2 - 4B$.

Hence, solve for a and b, given
$$X \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 5 \\ 50 \end{bmatrix}$$
 [2017]

$$X = B2 - 4B$$

$$X = \begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix} - 4 \begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix}$$
$$X = \begin{bmatrix} 1 \times 1 + 1 \times 8 & 1 \times 1 + 1 \times 3 \\ 8 \times 1 + 3 \times 8 & 8 \times 1 + 3 \times 3 \end{bmatrix} - \begin{bmatrix} 4 & 4 \\ 32 & 12 \end{bmatrix}$$

$$X = \begin{bmatrix} 9 & 4 \\ 32 & 17 \end{bmatrix} - \begin{bmatrix} 4 & 4 \\ 32 & 12 \end{bmatrix}$$

$$X = \begin{bmatrix} 9 - 4 & 4 - 4 \\ 32 - 32 & 17 - 12 \end{bmatrix}$$

$$X = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$Now, X \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 5 \\ 50 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 5 \\ 50 \end{bmatrix}$$

$$\begin{bmatrix} 5a \\ 5b \end{bmatrix} = \begin{bmatrix} 5 \\ 50 \end{bmatrix}$$

$$5a = 5$$
 and $5b = 50$
 $a = 1$ and $b = 10$

14. Given
$$A = \begin{bmatrix} 2 & 0 \\ -1 & 7 \end{bmatrix}$$
 and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $A^2 = 9A + mI$. Find m . [2016]

Solution: m = -14

Sep-by-step Explanation:

$$A^{2} = 9A + mI$$

$$\begin{bmatrix} 2 & 0 \\ -1 & 7 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ -1 & 7 \end{bmatrix} = 9 \begin{bmatrix} 2 & 0 \\ -1 & 7 \end{bmatrix} + m \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 \times 2 + 0(-1) & 2 \times 0 + 0 \times 7 \\ (-1) \times 2 + 7(-1) & (-1)0 + 7 \times 7 \end{bmatrix} = \begin{bmatrix} 18 & 0 \\ -9 & 63 \end{bmatrix} + \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}$$

$$\begin{bmatrix} 4 & 0 \\ -9 & 49 \end{bmatrix} = \begin{bmatrix} 18 + m & 0 \\ -9 & 63 + m \end{bmatrix}$$

$$18 + m = 4$$

$$\Rightarrow m = -14$$

15. Given matrix
$$A = \begin{bmatrix} 4\sin 30^{\circ} & \cos 0^{\circ} \\ \cos 0^{\circ} & 4\sin 30^{\circ} \end{bmatrix}$$
 and $B = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$,

If AX = B

- (i) write the order of matrix X.
- (ii) Find the matrix 'X'. [2016]

(i)
$$(m \times n) (n \times p) = (m \times p)$$

 $(2 \times 2) (n \times p) = (2 \times 1)$