ICSE 2003 : MATHEMATICS

Answer key / correct responses on:

MATHEMATICS
(Two hours and a half)

Answers to this Paper must be written on the paper provided separately.
*You will **not** be allowed to write during the first **15** minutes.*
This time is to be spent in reading the question paper.
The time given at the head of this Paper is the time allowed for writing the answers.

*Attempt **all** questions from **Section A** and **any four** questions from **Section B**.*
All working, including rough work, must be clearly shown and must be done on the same sheet as the rest of the answer. Omission of essential working will result in the loss of marks.
The intended marks for questions or parts of questions are given in brackets [ ].
Mathematical tables are provided.

SECTION A (40 Marks)
*Attempt **all** questions from this Section.*

Question 1

(a) The price of a washing machine, inclusive of sales tax is Rs. 13,530/-. If the sales tax is 10%, find its basic price. [3]

(b) What sum of money will amount to Rs. 3,630/- in two years at 10% per annum compound interest? [3]

(c) Solve using the quadratic formula $x^2 - 4x + 1 = 0$. [3]
(Type in the answers separated by commas. Copy and paste the √ (square root) symbol from the question. Leave no spaces in your answer.)

Question 2

(a) If $\dfrac{3a + 4b}{3c + 4d} = \dfrac{3a - 4b}{3c - 4d}$, prove that $\dfrac{a}{b} = \dfrac{c}{d}$ [3]

(b) Find the value of a, if $(x - a)$ is a factor of $x^3 - a^2x + x + 2$. [3]

Question 3 [6]

Use a graph paper for this question. (Take 10 small divisions = 1 unit on both axes).

Plot the points P (3, 2) and Q (−3, −2) from P and Q, draw perpendiculars PM and QN on the x-axis.

(a) Name the image of P on reflection in the origin.

(b) Assign the special name to the geometrical figure PMQN and find its area.

(c) Write the co-ordinates of the point to which M is mapped on reflection in

(i) x-axis.

(c) Write the co-ordinates of the point to which M is mapped on reflection in

(ii) y-axis.

(c) Write the co-ordinates of the point to which M is mapped on reflection in

(iii) origin.

Question 4

(a) Find the value of $\dfrac{\cos 75°}{\sin 15°} + \dfrac{\sin 12°}{\cos 78°} - \dfrac{\cos 18°}{\sin 72°}$ [3]

(b) Solve $2 \leq 2x - 3 \leq 5$, $x \in R$ and mark it on a number line. [3]

Question 5

Given the following details, calculate the simple interest at the rate of 6% per annum up to June 30:- [5]

Date	Debit Rs.	Credit Rs.	Balance Rs.
Jan 1	--	24000.00	24000.00
Jan 20	5000.00	--	19000.00
Jan 29	--	10000.00	29000.00
March 15	--	8000.00	37000.00

April 3	--	7653.00	44653.00
May 6	3040.00	--	41613.00
May 8	--	5087.00	46700.00

Question 6 [5]

If sin x = 3 / 5 and cos y = 12 / 13; evaluate
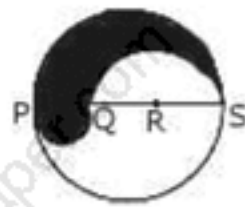
(a) secant^2x

(b) tan x + tan y.

(*Type in the answers separated by commas. Leave no spaces in your answer.*)

Question 7

PS is a diameter of a circle of radius 6 cm. Q and R are points on the diameter such that PQ, [3]
QR and RS are equal. Semicircles are drawn with PQ and QS as diameters, as shown in the
figure. Find the perimeter of the shaded region.
(π = 3.14)

SECTION B (40 Marks)

*Attempt any **four** questions from this Section.*

Question 8

(a) The work done by $(x - 3)$ men in $(2x + 1)$ days and the work done by $(2x + 1)$ men in $(x$ [3]
$+ 4)$ days are in the ratio of 3 : 10. Find the value of x.

(b) If $f(x)$ = 2x − 5 / 4x + 1, find:- [1]
(i) $f(3)$

(b) If $f(x)$ = 2x − 5 / 4x + 1, find:- [1]
(ii) $f(x^2)$

(b) If $f(x)$ = 2x − 5 / 4x + 1, find:- [1]

(iii) $f(2x + 1)$

(c) Find the mean of the following frequency distribution:- [4]

Class Interval	Frequency
0 - 50	4
50 - 100	8
100 - 150	16
150 - 200	13
200 - 250	6
250 - 300	3

Question 9 [10]

(a) A man invests Rs. 20,020/- in buying shares of nominal value Rs. 26/- at 10% premium. The dividend on the shares is 15% per annum. Calculate:-

(i) The number of shares he buys.

(a)(ii) The dividend he receives annually.

(a)(iii) The rate of interest he gets on his money.

(*Type in the answers separated by commas. Leave no spaces around the commas.*)

(b) Prove that $\dfrac{\cos A}{1 - \tan A} + \dfrac{\sin A}{1 - \cot A} = \cos A + \sin A$
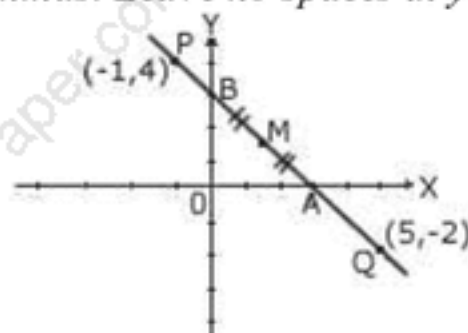
Question 10 [10]

(a) A straight line passes through the points P (−1, 4) and Q (5, −2). It intersects the co-ordinate axes at points A and B. M is the mid point of the segment AB. Find:-

(i) The equation of the line.

(a) (ii) The co-ordinates of A and B.

(a) (iii) The co-ordinates of M.

(*Type in the answers separated by commas. Leave no spaces in your answer.*)

(b) In an auditorium, seats were arranged in rows and columns. The number of rows was equal to the number of seats in each row. When the number of rows was doubled and the number of seats in each row was reduced by 10, the total number of seats increased by 300. Find:-

(i) The number of rows in the original arrangement.

(b) (ii) The number of seats in the auditorium after re-arrangement.

(*Type in the answers separated by commas. Leave no spaces around the commas.*)

Question 11 [5]

(a) Draw a histogram and hence estimate the mode for the following frequency distribution:-

Class	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60
Frequency	2	8	10	5	4	3

(b) A man standing on the bank of a river observes that the angle of elevation of a tree on the opposite bank is 60°. When he moves 50 m away from the bank. He finds the angle of elevation to be 30°. Calculate:-

(i) the width of the river and

(b) (ii) the height of the tree.

(*Type in the answers separated by commas. Leave no spaces around the commas.*)

Question 12

(a) Find x and y if:- [4]

$$\begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} 2x \\ 1 \end{bmatrix} + 2 \begin{bmatrix} -4 \\ 5 \end{bmatrix} = 4 \begin{bmatrix} 2 \\ y \end{bmatrix}$$

(*Type in the answers separated by commas. Leave no spaces around the commas.*)

(b) A vessel is in the form of an inverted cone. Its height is 11 cm and the radius of its top which is open, is 2.5 cm. It is filled with water upto the rim. When lead shots, each of which is a sphere of radius 0.25 cm are dropped into the vessel, 2 / 5 of the water flows out. Find the number of lead shots dropped into the vessel. [4]

(c) In the given circle with diameter AB, find the value of x. [2]
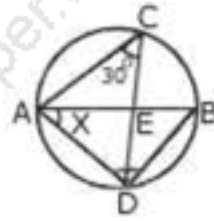
Question 13

(a) Construct an angle PQR = 45°. Mark a point S on QR such that QS = 4.5 cm. [4]
Construct a circle to touch PQ at Q and also to pass through S.

(b) Find the value of k for which the lines $kx - 5y + 4 = 0$ and $4x - 2y + 5 = 0$ are [4]
perpendicular to each other.
(*Leave no spaces in your answer.*)

(c) If (a, b) ∈ R, name the kind of relation between a and b if a R b => b R a. [2]
Does also show a relation of this kind? Explain.

Question 14 [10]

(a) The annual income of Mrs. Sharma (excluding HRA) is Rs. 1,68,000. She contributes Rs. 4,500 per
month to her P.F. account and pays an annual insurance premium of Rs. 8,000. Calculate the income
tax including surcharge Mrs. Sharma has to pay in the last month of the year if her earlier deductions as
income tax for the first 11 months were at the rate of Rs. 600 per month. Assume the following for
calculating income tax.
Standard deduction : 1/3rd of the total income subject to a maximum of Rs. 20,000
Tax slab:-

Upto Rs. 50,000	: No tax.
Rs. 50,001 to Rs. 60,000	: 10% of the income exceeding Rs. 50,000.
Rs. 60,001 to Rs. 1,50,000	: Rs. 1,000 + 20% of the income exceeding Rs. 60,000.
Above Rs. 1,50,000	: Rs. 19,000 + 30% of the income exceeding Rs. 1,50,000.
Tax Rebate	: 20% of the total savings subject to a maximum of Rs 12,000.
Surcharge	: 10% of the total tax payable after rebate.

(b) In a triangle PQR, L and M are two points on the base QR, such that ∠ LPQ = ∠ QRP and
∠ RPM = ∠ RQP. Prove that:-
(i) ΔPQL ~ ΔRPM

(b) In a triangle PQR, L and M are two points on the base QR, such that $\angle LPQ = \angle QRP$ and $\angle RPM = \angle RQP$. Prove that:-

(ii) QL·RM = PL·PM

(b) In a triangle PQR, L and M are two points on the base QR, such that $\angle LPQ = \angle QRP$ and $\angle RPM = \angle RQP$. Prove that:-

(iii) $PQ^2 = QR·QL$