ICSE 2000 : MATHEMATICS

MATHEMATICS- 2000

(Two hours and a half)

SECTION - A (40 Marks)

*Answer **all** questions from this Section.*

Question 1

(a) A colour T.V. is marked for sale for Rs. 17,600 which includes sales tax at 10%. Calculate the sales tax in rupees. [2]

(b) Calculate the compound interest for the second year on Rs. 8,000 invested for 3 years at 10% p.a. [3]

Question 2

(a) Find the remainder when $2x^3 - 3x^2 + 7x - 8$ is divided by $x - 1$. [2]

(b) Given $a/b = b/c$, prove that [2]

$$\frac{3a - 5b}{3a + 5b} = \frac{3c - 5d}{3c + 5d}$$

Question 3

(a) Evaluate, without the use of trignometrical tables: [3]

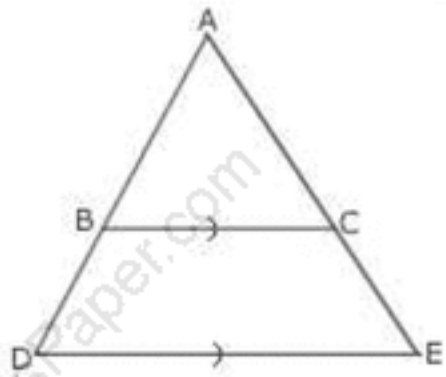
(b) In the figure: $\angle PSR = 90°$, PQ = 10 cm, QS = 6 cm, RQ = 9 cm. Calculate rhe length of PR. [2]

$$3\,\frac{\sin 72°}{\cos 18°} - \frac{\sec 32°}{\text{cosec } 58}$$

Question 4

(a) Solve the inequation: $-3 \leq 3 - 2x < 9$, $x \in R$. Represent your solution on a number line. [3]

(b) In the figure alongside, BC is parallel to DE. Area of triangle ABC = 25 cm^2, area of trapezium BCED = 24 cm^2, DE = 14 cm. Calculate the length of BC. [3]



Question 5

(a) Calculate the ratio in which the line joining A (6, 5) andB (4, − 3) is divided by the line y = 2. [2]

(b) Write down the coordinates of the image of the point (3, − 2) when: [1]
(i) reflected in the x-axis.

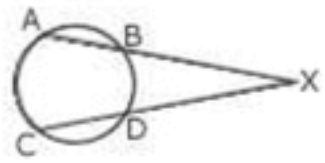
(b) Write down the coordinates of the image of the point (3, − 2) when: [1]
(ii) reflected in the y-axis.

(b) Write down the coordinates of the image of the point (3, − 2) when: [1]
(iii) reflected in the x-axis followed by reflection in the y-axis.

(b) Write down the coordinates of the image of the point (3, − 2) when: [1]
(iv) reflected in the origin.

Question 6

(a) In the figure, chords AB and CD when extended meet at X. Given AB = 4cm, BX = 6cm, XD = 5cm, calculate the length of CD. [3]



(b) Construct triangle ABC with AB = 7 cm, BC = 8 cm and ∠ ABC = 60°. Locate by construction the point P such that: [1]
(i) P is equidistant from B and C.

(b) Construct triangle ABC with AB = 7 cm, BC = 8 cm and ∠ ABC = 60°. Locate by construction the point P such that: [1]
(ii) P is equidistantfrom AB and BC.

(b) Construct triangle ABC with AB = 7 cm, BC = 8 cm and ∠ ABC = 60°. Locate by construction the point P such that: [1]
(iii) Measure and record the length of PB.

Question 7

(a) Calculate the mean, the median and the mode of the following numbers: 3,1,5, 6,3,4,5,3, 7,2. [3]

(b) Given A = [3]

$$\begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix}$$

, evaluate A2 − 4A

(c) Use graph paper for this question. Plot the points A (8, 2) and B (6, 4). These two points are the vertices of a figure which is symmetrical about x = 6 and y = 2. Complete the figure on the graph. Write down the geometrical name of the figure. [3]

SECTION - B (40 Marks)

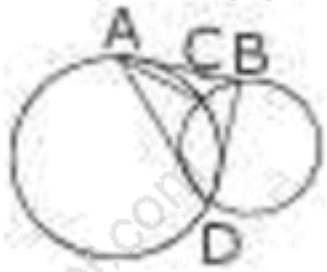
*Answer any **four** questions from this Section.*

Question 8

Show that [3]

$$\frac{\sqrt{1-\cos A}}{\sqrt{1+\cos A}} = \frac{\sin A}{1+\cos A}$$

(b) In the figure, AB is a common tangent to two circles intersecting at C and D. Write down the measure of ($\angle$ ACB + $\angle$ ADB), Justify your answer. [3]



(c) Solve graphically the simultaneous equations: [4]

$x - 2y = 1; x + y = 4.$

Use 2 cm = 1 unit on both axes and plot only three points per line.

Question 9 [10]

(a) Only ruler and compass may be used in this question:

(i) Construct Δ ABC such that AB = CA = 7 cm and BC = 5 cm.

(a) Only ruler and compass may be used in this question:

(ii) Draw AX, the perpendicular bisector of side BC.

(a) Only ruler and compass may be used in this question:

(iii) Draw a circle with centre A and radius 3 cm cutting AX at Y.

(a) Only ruler and compass may be used in this question:

(iv) Construct another circle to touch the circle with centre A externally at Y and passing through B and C.

(b) The surface area of a solid metallic sphere is 1256 cm^2. It is melted and recast into solid right circular cones of radius 2·5 cm and height 8 cm. Calculate:

(i) The radius of the solid sphere.

(b) The surface area of a solid metallic sphere is 1256 cm^2. It is melted and recast into solid right circular cones of radius 2·5 cm and height 8 cm. Calculate:

(ii) The number of cones recast. Take $\pi = 3{\cdot}14$.

Question 10

(a) A dividend of 9% was declared on Rs. l00 shares selling at a certain prire. If the rate of return is 7½%, Calculate: [2]

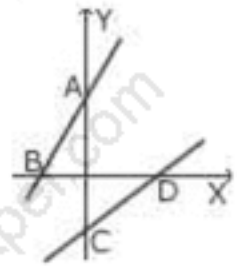
(i) The market value of the share.

(a) A dividend of 9% was declared on Rs. l00 shares selling at a certain prire. If the rate of return is 7½%, Calculate: [2]

(ii) The amount to be invested to obtain an annual dividend of Rs. 630.

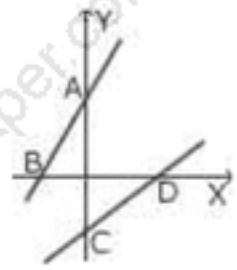
(b) In the figure AB and CD are the lines $2x - y + 6 = 0$ and $x - 2y = 4$ respectively: [2]

(i) Write down the co-ordinates of A, B, C and D.



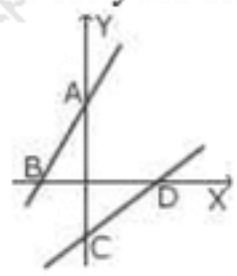
(b) In the figure AB and CD are the lines $2x - y + 6 = 0$ and $x - 2y = 4$ respectively: [2]

(ii) Prove that triangles OAB and ODC are similar.



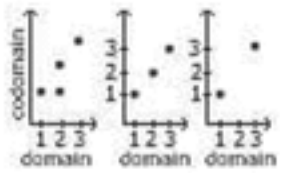
(b) In the figure AB and CD are the lines $2x - y + 6 = 0$ and $x - 2y = 4$ respectively: [2]

(iii) Is figure ABCD cyclic? Give reasons for your answer.



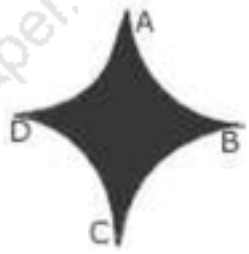
Question 11

(a) The figure given below represent relations on A × A, where A = {1, 2, 3}. The ordered pairs are represented by the points shown. For each diagram state whether it represents a relation or a function. Justify your answer. [3]



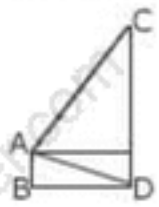
(b) A = {real number}. On A a relation R is dejined by: For all a, b ∈ A, a R b holds if and only if the difference between a and b less than 2. Is R an equivalence relation? Justify your answer. [3]

(c) Calculate the area of the shaded portion. The quadrants shown in the figure are each of radius 7 cm. Take π = 22/7 [3]



Question 12

(a) From a window A 10m above ground the angle of elevation of the top C of a tower is x°, where tan x° = 5/2 and angle of depression of the foot D of the tower is y°, where tan y° = 1/4. See the figure given alongside. Calculate the height CD of the tower in metres. [5]



(b) The following are the details of income and investments of Mr. Mathur for a particular year. [5]

Annual Salary	Rs. 1,50,000
L.I.C Premium	Rs. 18,000 per annum
Provident Fund	Rs. 1,500 per month
Tax Deducted at source	Rs. 500 per month

Calculate the tax payable at the end of the year. You may use the following:

Tax slabs: Re. 1 - Rs. 40,000 - No tax

Rs. 40,000 - Rs. 60,000 - 10% of income exceeding Rs. 40,000

Rs. 60,001 - Rs. 1,50,000 - Rs. 2,000 + 20% of income exceeding Rs. 60,000

Above Rs. 1,50,000 - Rs. 20,000 + 30% of income exceeding Rs. 1,50,000

Standard deduction : Rs. 20,000

Tax rebate : 20% of all investments.

Question 13 [10]

(a) The hotel bill for a number of people for overnight stay is Rs. 4,800. If there were 4 more, the bill each person had to pay would have reduced by Rs. 200. Find the number of people staying overnight.

(b) The following table shows the distribution of the heights of a group of factory workers:

Height (cm)	150-155	I55-160	160-165	165-170	170-175	175-180	180-185
No. of workers	6	12	18	20	13	8	6

(i) Determine the cumulative frequencies.

(b) The following table shows the distribution of the heights of a group of factory workers:

Height (cm)	150-155	I55-160	160-165	165-170	170-175	175-180	180-185
No. of workers	6	12	18	20	13	8	6

(ii) Draw the cumulative frequency curve on a graph paper.

Use 2 cm = 5 cm height on one axis and 2 cm = 10 workers on the other.

(b) The following table shows the distribution of the heights of a group of factory workers:

Height (cm)	150-155	I55-160	160-165	165-170	170-175	175-180	180-185

No. of workers	6	12	18	20	13	8	6

(iii) From your graph, write down the median height in cm.

Question 14

(a) ABCD is a rhombus. The coordinates of A and C are (3, 6) and (−1, 2) respectively. [4]
Write down the equation of BD.

(b) Given below are the entries in a Saving bank a/c pass book: [6]

Date	Particulars	Withdrawal	Deposit	Balance
Feb. 8	B/F	—	—	Rs. 8,500
Feb. 18	To self	Rs. 4,000	—	—
April 12	By cash	—	Rs. 2,238	—
June 15	To self	Rs. 5000	—	—
July 8	By cash	—	Rs. 6,000	

Calculate the interest for the six months February to July, at 4½% p.a. on minimum balance on or after the 10th day of each month.