BOARD PAPER QUESTIONS

1. Mr R K Nair gets ₹6,455 at the end of one year at the rate of 14% per annum in a recurring deposit account. Find the monthly instalment. **[2005]**

2. Mohan deposits ₹80 per month in a cumulative deposit account for six years. Find the amount payable to him on maturity, if the rate of interest is 6% per annum. **[2006]**

3. Saloni deposited ₹150 per month in her bank for eight months under the Recurring Deposit Scheme. What will be the maturity value of her deposit, if the rate of interest is 8% per annum and the interest is calculated at the end of every month? **[2007]**

4. David opened a Recurring Deposit Account in a bank and deposited ₹300 per month for two years. If he received ₹7725 at the time of maturity, find the rate of interest per annum. **[2008]**

5. Mrs Goswami deposits ₹1000 every month in a recurring deposit account for 3 years at 8% interest per annum. Find the matured value. **[2009]**

6. Mr Gupta opened a recurring deposit account in a bank. He deposited ₹2500 per month for two years. At the time of maturity he got ₹67,500. Find

(*i*) the total interest earned by Mr Gupta.

(*ii*) the rate of interest per annum. **[2010]**

7. Ahmed has a recurring deposit account in a bank. He deposits ₹2,500 per month for 2 years. If he gets ₹66,250 at the time of maturity, find

(*i*) The interest paid by the bank.

(*ii*) The rate of interest. **[2011]**

8. Kiran deposited ₹200 per month for 36 months in a bank's recurring deposit account. If the bank pays interest at the rate of 11% per annum, find the amount she gets on maturity. **[2012]**

9. Mr Britto deposits a certain sum of money each month in a Recurring Deposit Account of a bank. If the rate of interest is of 8% per annum and Mr Britto gets ₹8088 from the bank after 3 years, find the value of his monthly instalment. **[2013]**

10. Shahrukh opened a Recurring Deposit Account in a bank and deposited ₹800 per month for 1½ years. If he received ₹15,084 at the time of maturity, find the rate of interest per annum. **[2014]**

11. Katrina opened a recurring deposit account with a Nationalised Bank for a period of 2 years. If the bank pays interest at the rate of 6% per annum and the monthly instalment is ₹1000, find the:

(*i*) interest earned in 2 years.

(*ii*) matured value. **[2015]**

12. Mr. Richard has a recurring deposit account in a bank for 3 years at 7.5% p. a. simple interest. If he gets ₹8325 as interest at the time of maturity, find

(*i*) the monthly deposit

(*ii*) the maturity value. **[2018]**

COMMON ERRORS

1. When interest is ₹$\frac{365}{2}$, the answer can be wrongly expressed as ₹182.5 instead of ₹182.50, or writing ₹$92\frac{1}{3}$ as ₹92.30 instead of ₹92.33. Answer for *money* if in decimal should be expressed correct to 2 decimal places, unless it has to be expressed to nearest rupee.

2. Forgetting to take *time* as $\frac{1}{12}$ while calculating interest.

BOARD PAPER QUESTIONS

1. Solve the following inequation, and graph the solution set, on the number line:
   $2x - 3 < x + 2 \le 3x + 5,\ x \in R.$ **[1998]**

2. Solve the inequation:
   $12 + 1\frac{5}{6}\ x \le 5 + 3x,\ x \in R.$
   Represent the solution on a number line. **[1999]**

3. Solve the inequation: $-3 \le 3 - 2x < 9,\ x \in R.$ Represent your solution on a number line. **[2000]**

4. Solve the following inequation, and graph the solution on the number line:
   $2x - 5 \le 5x + 4 < 11,\ x \in R$ **[2002]**

5. Solve $2 \le 2x - 3 \le 5, x \in R$ and mark it on a number line. **[2003]**

6. Given that $x \in I$, solve the inequation and graph the solution on the number line:
   $3 \ge \frac{x-4}{2} + \frac{x}{3} \ge 2$ **[2004]**

7. $A = \{x : 11x - 5 > 7x + 3,\ x \in R\}$ and
   $B = \{x : 18x - 9 \ge 15 + 12x,\ x \in R\}$.
   Find the range of set $A \cap B$ and represent it on a number line. **[2005]**

8. Given that $x \in R$, solve the following inequality and graph the solution on the number line:
   $-1 \le 3 + 4x < 23$ **[2006]**

9. Solve the following inequation and graph the solution on the number line.
   $-2\frac{2}{3} \le x + \frac{1}{3} < 3\frac{1}{3}; x \in R$ **[2007]**

10. Solve the given inequation and graph the solution on the number line:
    $2y - 3 < y + 1 \le 4y + 7;\ y \in R.$ **[2008]**

11. Solve the following inequation and represent the solution set on the number line:
    $-3 < -\frac{1}{2} - \frac{2x}{3} \le \frac{5}{6},\ x \in R.$ **[2010]**

12. Solve the following inequation and represent the solution set on the number line:
    $2x - 5 \le 5x + 4 < 11$, where $x \in I$ **[2011]**

13. Solve the following inequation, write the solution set and represent it on the number line:
    $-\frac{x}{3} \le \frac{x}{2} - 1\frac{1}{3} < \frac{1}{6},\ x \in R$ **[2013]**

14. Find the values of x, which satisfy the inequation
    $-2\frac{5}{6} < \frac{1}{2} - \frac{2x}{3} \le 2,\ x \in W$. Graph the solution set on the number line. **[2014]**

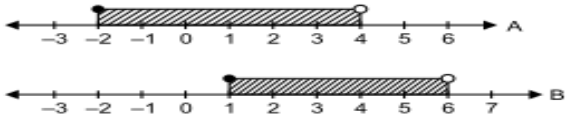
15. Solve the following inequation and write the solution set:
    $-3(x-7) \ge 15 - 7x > \frac{x+1}{3},\ x \in R$
    Represent the solution on a real number line.
    [2016]

3. $-2 \le \frac{1}{2} - \frac{2x}{3} \le 1\frac{5}{6},\quad x \in N$ **[2001]**

Example 5: Given $P = \{x : 5 < 2x - 1 \le 11, x \in R\}$ and $Q = \{x : -1 \le 4x + 3 < 23, x \in I\}$. Represent P and Q on number lines. Write down the elements of $P \cap Q$. **[1996]**

Fig. 4.4

Example 4: The diagram represents two inequations A and B on real number lines.



(*i*) Write down A and B in set-builder notation.

(*ii*) Represent $A \cap B$ and $A \cap B'$ on two different number lines. **[1995]**

7. $x - 3 \le \frac{8x}{3} + 2 \le 2x + \frac{14}{3},\ x \in I$ **[2009]**

8. $13x - 5 < 15x + 4 < 7x + 12,\ x \in R$ **[2015]**

BOARD PAPER QUESTIONS

1. A train covers a distance of 600 km at x km/h. Had the speed been $(x + 20)$ km/h, the time taken to cover the distance would have been reduced by 5 hours. Write down an equation in x and solve it to evaluate x. **[1996]**

2. Car A travels x km for every litre of petrol, while car B travels $(x + 5)$ km for every litre of petrol.

(*i*) Write down the number of litres of petrol used by car A and car B in covering a distance of 400 km.

(*ii*) If car A uses 4 litres of petrol more than car B in covering the 400 km, write down an equation in x and solve it to determine the number of litres of petrol used by car B for the journey. **[1997]**

3. The distance by road between two towns A and B, is 216 km, and by rail it is 208 km. A car travels at a speed of x km/h and the train travels at a speed which is 16 km/h faster than the car. Calculate:

(*i*) The time taken by the car to reach town B from A, in terms of x;

(*ii*) The time taken by the train, to reach town B, from A, in terms of x;

(*iii*) If the train takes 2 hours less than the car, to reach town B, obtain an equation in x, and solve it.

(*iv*) Hence, find the speed of the train. **[1998]**

4. A trader buys x articles for a total cost of ₹600.

(*i*) Write down the cost of one article in terms of x. If the cost per article were ₹5 more, the number of articles that can be bought for ₹600 would be four less.

(*ii*) Write down the equation in x for the above situation and solve it to find x. **[1999]**

5. The hotel bill for a number of people for overnight stay is ₹4800/- If there were 4 more, the bill each person had to pay would have reduced by ₹200/-. Find the number of people staying overnight. **[2000]**

6. Solve for x and give your answers correct to 2 decimal places.

$$x^2 - 10x + 6 = 0$$ **[2001]**

7. An aeroplane travelled a distance of 400 km at an average speed of x km/h. On the return journey, the speed was increased by 40 km/h. Write down an expression for the time taken for:-

(*i*) The onward journey;

(*ii*) The return journey.

If the return journey took 30 minutes less than the onward journey, write down an equation in x and find its value. **[2002]**

8. In an auditorium, seats were arranged in rows and columns. The number of rows was equal to the number of seats in each row. When the number of rows was doubled and the number of seats in each row was reduced by 10, the total number of seats increased by 300. Find:

(*i*) The number of rows in the original arrangement.

(*ii*) The number of seats in the auditorium after re-arrangement. **[2003]**

9. Solve the equation $3x^2 - x - 7 = 0$ and give your answer correct to two decimal places. **[2004]**

10. P and Q are centres of circles of radius 9 cm and 2 cm respectively. PQ = 17 cm. R is the centre of a circle of radius x cm, which touches the above circles externally. Given that $\angle PRQ = 90°$, write an equation in x and solve it. **[2004]**

11. By increasing the speed of a car by 10 km/h, the time of journey for a distance of 72 km is reduced by 36 minutes. Find the original speed of the car. **[2005]**

12. A shopkeeper buys a certain number of books for ₹720. If the cost per book was ₹5 less, the number of books that could be bought for ₹720 would be two more. Taking the original cost of each book to be ₹x, write an equation in x and solve it. **[2006]**

13. Five years ago, a woman's age was the square of her son's age. Ten years hence her age will be twice that of her son's age. Find:

(*i*) The age of the son five years ago

(*ii*) The present age of the woman. **[2007]**

14. Some students planned a picnic. The budget for the food was ₹480. As eight of them failed to join the party, the cost of the food for each member increased by ₹10. Find how many students went for the picnic. **[2008]**

15. The speed of an express train is x km/h and the speed of an ordinary train is 12 km/h less than that of the express train. If the ordinary train takes

one hour longer than the express train to cover a distance of 240 km, find the speed of the express train. **[2009]**

16. Without solving the following quadratic equation, find the value of 'p' for which the roots are equal.

$$px^2 - 4x + 3 = 0$$ **[2010]**

17. ₹480 is divided equally among 'x' children. If the number of children were 20 more then each would have got ₹12 less. Find 'x'. **[2011]**

18. A car covers a distance of 400 km at a certain speed. Had the speed been 12 km/h more, the time taken for the journey would have been 1 hour 40 minutes less. Find the original speed of the car. **[2012]**

19. Without solving the following quadratic equation, find the value of 'm' for which the given equation has real and equal roots.

$x^2 + 2(m - 1)x + (m + 5) = 0$. **[2012]**

20. Without solving the following quadratic equation, find the value of 'p' for which the given equation has real and equal roots:

$$x^2 + (p - 3)x + p = 0.$$ **[2013]**

21. A shopkeeper purchases a certain number of books for ₹960. If the cost per book was ₹8 less, the number of books that could be purchased for ₹960 would be 4 more. Write an equation, taking the original cost of each book to be ₹x, and solve it to find the original cost of the books. **[2013]**

22. A two digit positive number is such that the product of its digits is 6. If 9 is added to the number, the digits interchange their places. Find the number. **[2014]**

23. Sum of two natural numbers is 8 and the difference of their reciprocals is $\frac{2}{15}$. Find the numbers. **[2015]**

24. ₹7500 were divided equally among a certain number of children. Had there been 20 less children, each would have received ₹100 more. Find the original number of children. **[2018]**

25. The product of two consecutive natural numbers which are multiples of 3, is equal to 810. Find the two numbers. **[2019]**

POINTS AT A GLANCE

1. Standard form of Quadratic Equation is $ax^2 + bx + c = 0$.

The roots are $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

(*i*) Use powers and factorial page to find the square root of the numbers from 1–99 and round off only in the end as asked in the question.

(*ii*) If it is a 3-digit number, see square root page (1st line).

(*iii*) If asked to leave in surd (root) form, simplify $\sqrt{63} = \sqrt{9 \times 7} = 3\sqrt{7}$.

(*iv*) In significant figures, remember the last zero in a decimal number is significant. e.g., 2.703 answer to 3 significant figures is 2.70.

(*v*) When one of the answers is negative, the minus sign should not be forgotten in the final answer.

BOARD PAPER QUESTIONS

1. Given $\frac{a}{b} = \frac{c}{d}$, prove that $\frac{3a-5b}{3a+5b} = \frac{3c-5d}{3c+5d}$ **[2000]**

2. Two numbers are in the ratio of 3 : 5. If 8 is added to each number, the ratio becomes 2 : 3. Find the numbers. **[2001]**

3. If $\frac{3a+4b}{3c+4d} = \frac{3a-4b}{3c-4d}$, prove that $\frac{a}{b} = \frac{c}{d}$. **[2003]**

4. What number must be added to each of the numbers 6, 15, 20 and 43 to make them proportional? **[2005, 2013]**

5. If $\frac{8a-5b}{8c-5d} = \frac{8a+5b}{8c+5d}$, prove that $\frac{a}{b} = \frac{c}{d}$. **[2008]**

6. What least number must be added to each of the numbers 5, 11, 19 and 37 so that they are in proportion? **[2009]**

7. Given that $\frac{a^3+3ab^2}{b^3+3a^2b} = \frac{63}{62}$.

Using Componendo and Dividendo, find $a : b$. **[2009]**

8. If x, y, z are in continued proportion, prove that $\frac{(x+y)^2}{(y+z)^2} = \frac{x}{z}$. **[2010]**

9. Using componendo and dividendo, find the value of x: $\frac{\sqrt{3x+4}+\sqrt{3x-5}}{\sqrt{3x+4}-\sqrt{3x-5}} = 9$ **[2011]**

10. 6 is the mean proportion between two numbers x and y, and 48 is the third proportional to x and y. Find the numbers. **[2011]**

11. The monthly pocket money of Ravi and Sanjeev are in the ratio 5 : 7. Their expenditures are in the ratio 3 : 5. If each saves ₹80 every month, find their monthly pocket money. **[2012]**

12. If $x = \frac{\sqrt{a+1}+\sqrt{a-1}}{\sqrt{a+1}-\sqrt{a-1}}$, using properties of proportion show that $x^2 - 2ax + 1 = 0$. **[2012]**

13. Using the properties of proportion, solve for x, given $\frac{x^4+1}{2x^2} = \frac{17}{8}$. **[2013]**

14. If $(3a + 2b) : (5a + 3b) = 18 : 29$, find $a : b$. **[2016]**

15. If b is the mean proportion between a and c, show that: $\frac{a^4+a^2b^2+b^4}{b^4+b^2c^2+c^4} = \frac{a^2}{c^2}$ **[2017]**

16. If $\frac{7m+2n}{7m-2n} = \frac{5}{3}$, use properties of proportion to find the value of

(*i*) $m : n$ (*ii*) $\frac{m^2+n^2}{m^2-n^2}$ **[2018]**

17. Using properties of proportion, solve for x. Given that x is positive: $\frac{2x+\sqrt{4x^2-1}}{2x-\sqrt{4x^2-1}} = 4$ **[2018]**

COMMON ERRORS

1. Students forget to use componendo and dividendo on both sides.

2. While using componendo and dividendo, care is not taken by putting brackets nor the signs of the terms at right place, especially in the denominator.

3. When two different ratios are given, e.g., 5 : 4 as income ratio and 3 : 2 as expenditure ratio, both ratios are taken with same factor x as $5x$, $4x$ and $3x$, $2x$ instead of $3y$ and $2y$.

MISCELLANEOUS EXERCISE

1. If $2x^3 - 3x^2 - 4x + 5$ is divided by $(2x + 1)$, find the remainder.
2. What number should be added to $3x^3 - 4x^2 - 5x$ so that $(x - 2)$ is a factor?
3. What number should be subtracted from $4x^3 + 6x^2 - 5x$ so that $(x - 1)$ is a factor?
4. Find the value of k in the given expression if $(x + 3)$ is a factor of $2x^3 + kx^2 - x + 24$.
5. When $ax^3 + 5x^2 - 2x + 27$ is divided by $(x + 4)$, the remainder is -13. Find the value of a.
6. Find the values of a and b if $(x - 1)$ is a factor of $ax^3 - x^2 + bx + 6$ and when this polynomial is divided by $(x - 3)$, the remainder is 30.
7. Find the values of a and b if $(x + 2)$ is a factor of $f(x) = ax^3 - x^2 - 20x + b$ and $f(4) = 84$.
8. Prove that $(x - 5)$ is a factor of $2x^3 - 3x^2 - 29x - 30$. Hence, factorise the expression fully.
9. Using the factor theorem, factorise the polynomial $3x^3 + 19x^2 + 16x - 20$ completely.
10. Find the value of p if $5x^3 + px^2 - x - 3$ and $3x^3 - 4x^2 - 3x + p$ have the same remainder when divided by $(x - 2)$.
11. Find the value of a if $(x + 4)$ is a factor of $3x^3 + ax^2 - 18x + 8$. Hence, verify if $(x - 1)$ is also a factor.
12. Find the value of k if $(x + 2)$ is a factor of $3x^3 + kx^2 - 18x + 40$. Hence, factorise the expression.

BOARD PAPER QUESTIONS

1. Find the remainder when $2x^3 - 3x^2 + 7x - 8$ is divided by $x - 1$. **[2000]**
2. Find the value of the constants a and b, if $(x - 2)$ and $(x + 3)$ are both factors of the expression $x^3 + ax^2 + bx - 12$. **[2001]**
3. Using Factor theorem, show that $(x - 3)$ is a factor of $x^3 - 7x^2 + 15x - 9$. Hence, factorise the given expression completely. **[2002]**
4. Find the value of a, if $(x - a)$ is a factor of $x^3 - a^2x + x + 2$. **[2003]**
5. Use the factor theorem to factorise completely
   $x^3 + x^2 - 4x - 4$. **[2004]**
6. $(x - 2)$ is a factor of the expression $x^3 + ax^2 + bx + 6$. When this expression is divided by $(x - 3)$, it leaves the remainder 3. Find the values of a and b. **[2005]**
7. Show that $2x + 7$ is a factor of $2x^3 + 5x^2 - 11x - 14$. Hence factorise the given expression completely, using the factor theorem. **[2006]**
8. Show that $(x - 1)$ is a factor of $x^3 - 7x^2 + 14x - 8$. Hence, completely factorise the above expression. **[2007]**
9. If $(x - 2)$ is a factor of $2x^3 - x^2 - px - 2$
   (*i*) find the value of p.
   (*ii*) with the value of p, factorize the above expression completely. **[2008]**
10. Given that $x + 2$ and $x + 3$ are factors of $2x^3 + ax^2 + 7x - b$. Determine the values of a and b. **[2009]**
11. Use the Remainder Theorem to factorise the following expression:
    $2x^3 + x^2 - 13x + 6$ **[2010]**
12. Find the value of 'k' if $(x - 2)$ is a factor of
    $x^3 + 2x^2 - kx + 10$
    Hence determine whether $(x + 5)$ is also a factor. **[2011]**
13. Using the Remainder Theorem, factorise completely the following polynomial.
    $3x^3 + 2x^2 - 19x + 6$ **[2012]**

14. If $(x - 2)$ is a factor of the expression $2x^3 + ax^2 + bx - 14$ and when the expression is divided by $(x - 3)$, it leaves a remainder 52, find the values of a and b. **[2013]**

15. Using the Remainder and Factor Theorem, factorise the following polynomial:

$x^3 + 10x^2 - 37x + 26$ **[2014]**

16. Find 'a' if the two polynomials $ax^3 + 3x^2 - 9$ and $2x^3 + 4x + a$, leave the same remainder when divided by $x + 3$. **[2015]**

17. Using remainder theorem, find the value of k if on dividing $2x^3 + 3x^2 - kx + 5$ by $x - 2$ leave a remainder 7. **[2016]**

18. Use remainder theorem to factorize the following polynomial: $2x^3 + 3x^2 - 9x - 10$ **[2018]**

COMMON ERRORS

1. While finding a and b when it is given that $(x + 2)$ is a factor of the given polynomial, after substituting $x = -2$ in the polynomial students forget to equate the expression to zero.

   or

   When the remainder is given, equating the expression with the remainder is forgotten.
2. Putting commas between the factors is wrong.

POINTS AT A GLANCE

1. To prove that $(x - a)$ is a factor of a polynomial $f(x)$, substitute $x = a$ in $f(x)$ and show that $f(a) = 0$.

2. If a polynomial $f(x)$ is divided by $(x + a)$ and the remainder 'r' is given, then substitute $x = -a$ in $f(x)$ and equate it to remainder 'r'.

3. After finding the first factor of $f(x)$, use division method or synthetic division method to find the quotient after dividing by the factor. Then, further factorise the quotient by splitting the middle term of the trinomial.

4. In synthetic division, when you divide by a fraction, the third line does not give the coefficients of the quotient. Divide these numbers in the third line by the denominator of the fraction (divisor) to get the coefficients of quotient.

5. (*i*) While factorising a trinomial, if the last term (constant term) is positive, the middle term is split into two terms of same sign, i.e., both positive or both negative.

e.g., $x^2 - 9x + 18 = x^2 - 6x - 3x + 18$ or $x^2 + 7x + 12 = x^2 + 4x + 3x + 12$

(*ii*) If the last term is negative, the middle term is split into one positive and the other negative number.

e.g., $x^2 - 2x - 24 = x^2 - 6x + 4x - 24$ or $x^2 + x - 6 = x^2 + 3x - 2x - 6$

DO YOU KNOW?

POLYNOMIAL

The word polynomial was introduced in Latin by Franciscus Vieta. A polynomial is an expression of finite length constructed from variables and constants, using only the operations of addition, subtraction, multiplication and non-negative integer exponents.

Babylonians used base 60 numbers system called sexagesimal system for their calculations. It is because of this 1 hour is 60 minutes and 1 minute is 60 seconds. **60** is the smallest number with **12 factors** (or divisors) and **12** is the smallest number with **6 factors**.

Franciscus Vieta

BOARD PAPER QUESTIONS

1. Evaluate without using tables:

$$\begin{pmatrix} 2\cos 60° & -2\sin 30° \\ -\tan 45° & \cos 0° \end{pmatrix} \cdot \begin{pmatrix} \cot 45° & \operatorname{cosec} 30° \\ \sec 60° & \sin 90° \end{pmatrix}$$

[1997]

2. Find the value of x and y, if

$$\begin{bmatrix} 1 & 2 \\ 3 & 3 \end{bmatrix} \begin{bmatrix} x & 0 \\ 0 & y \end{bmatrix} = \begin{bmatrix} x & 0 \\ 9 & 0 \end{bmatrix}.$$ **[1998]**

3. Find the 2 × 2 matrix X which satisfies the equation.

$$\begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ 5 & 3 \end{bmatrix} + 2\,X = \begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix}$$ **[1999]**

4. Given $A = \begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix}$, evaluate $A^2 - 4A$. **[2000]**

5. Find x and y, if:

$$\begin{bmatrix} -3 & 2 \\ 0 & -5 \end{bmatrix} \begin{bmatrix} x \\ 2 \end{bmatrix} = \begin{bmatrix} -5 \\ y \end{bmatrix}$$ **[2001]**

6. Find x and y, if:

$$\begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} 2x \\ 1 \end{bmatrix} + 2\begin{bmatrix} -4 \\ 5 \end{bmatrix} = 4\begin{bmatrix} 2 \\ y \end{bmatrix}$$ **[2003]**

7. Find the value of x given that $A^2 = B$

$$A = \begin{bmatrix} 2 & 12 \\ 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 4 & x \\ 0 & 1 \end{bmatrix}$$ **[2005]**

8. If $2\begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$

find the values of x and y. **[2007]**

9. Let $A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$. Find $A^2 + AB + B^2$.

[2007]

10. If $\begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix} + 2\,M = 3\begin{bmatrix} 3 & 2 \\ 0 & -3 \end{bmatrix}$, find the matrix M.

[2008]

11. Given $A = \begin{bmatrix} p & 0 \\ 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 0 & -q \\ 1 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix}$ and

$BA = C^2$. Find the values of p and q. **[2008]**

12. Find x and y, if $\begin{bmatrix} 2x & x \\ y & 3y \end{bmatrix} \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 16 \\ 9 \end{bmatrix}$. **[2009]**

13. Given $A = \begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 6 \\ 1 \end{bmatrix}$, $C = \begin{bmatrix} -4 \\ 5 \end{bmatrix}$ and

$D = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$. Find AB + 2C – 4D. **[2010]**

14. Evaluate

$$\begin{bmatrix} 4\sin 30° & 2\cos 60° \\ \sin 90° & 2\cos 0° \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$$ **[2010]**

15. If $A = \begin{bmatrix} 3 & 5 \\ 4 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$, is the product AB

possible? Give a reason. If yes, find AB. **[2011]**

16. If $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$ and I is the identity

matrix of the same order and A^t is the transpose of matrix A, and $A^t.B + BI$. **[2011]**

17. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find $A^2 - 5A + 7I$.

[2012]

18. Given $A = \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$.

Find the matrix X such that A + 2X = 2B + C.

[2013]

19. Find x and y if $\begin{bmatrix} x & 3x \\ y & 4y \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$. **[2013]**

20. Find x, y if $\begin{bmatrix} -2 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 2x \end{bmatrix} + 3\begin{bmatrix} -2 \\ 1 \end{bmatrix} = 2\begin{bmatrix} y \\ 3 \end{bmatrix}$.

[2014]

21. Let $A = \begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix}$ and $C = \begin{bmatrix} -3 & 2 \\ -1 & 4 \end{bmatrix}$.

Find $A^2 + AC - 5B$. **[2014]**

22. If $A = \begin{bmatrix} 3 & x \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 9 & 16 \\ 0 & -y \end{bmatrix}$, find x and y when $A^2 = B$. **[2015]**

23. If $A = \begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 2 \\ 5 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix}$, find $AB - 5C$. **[2015]**

24. Given $A = \begin{bmatrix} 2 & 0 \\ -1 & 7 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $A^2 = 9A + mI$. Find m. **[2016]**

25. If $A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix}$, find $AC + B^2 - 10C$. **[2018]**

26. Simplify:

$$\sin A \begin{bmatrix} \sin A & -\cos A \\ \cos A & \sin A \end{bmatrix} + \cos A \begin{bmatrix} \cos A & \sin A \\ -\sin A & \cos A \end{bmatrix}$$

[2019]

COMMON ERRORS

1. While multiplying matrices, forgetting that any number multiplied by zero is zero.
2. When asked to find A – B + C, students tend to add B and C and subtract this from A, which is wrong. Either, find the difference A – B first and then add C to this difference or add A and C and from this sum, subtract B.
3. When A and B are two given matrices and to find X when AX = B. After finding the elements of X, in the final answer, students forget to write X in matrix form.

POINTS AT A GLANCE

1. $A_{m \times \boxed{n}} \times B_{\boxed{p} \times q} = P_{m \times q}$

A and B can be multiplied only if $n = p$ (No. of columns in A = No. of rows in B)

2. Square Matrix × Column Matrix = Column Matrix

e.g., $\begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix} \begin{bmatrix} 7 \\ 8 \end{bmatrix} = \begin{bmatrix} 21 + 32 \\ 35 + 48 \end{bmatrix} = \begin{bmatrix} 53 \\ 83 \end{bmatrix}$

3. Row Matrix × Matrix = Row Matrix

e.g., $[5 \quad 2] \begin{bmatrix} 4 & 8 \\ 7 & 3 \end{bmatrix} = [20 + 14 \quad 40 + 6] = [34 \quad 36]$

4. $AI = A = IA$ where I is identity matrix $I_{2 \times 2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

5. $(A + B)^2 \neq A^2 + 2AB + B^2$ in matrices because $AB \neq BA$.

$(A - B)^2 \neq A^2 - 2AB + B^2$ and $A^2 - B^2 \neq (A + B)(A - B)$

Only in some cases where $AB = BA$, these will be equal.

BOARD PAPER QUESTIONS

1. Attempt this question on graph paper.

(*i*) Plot A(3, 2) and B(5, 4) on the graph paper. Take 2 cm = 1 unit on both axes.

(*ii*) Reflect A and B in the *x*-axis to A′, B′. Plot these on the same graph paper.

(*iii*) Write down

(*a*) the geometrical name of the figure ABB′A′

(*b*) the axis of symmetry of ABB′A′

(*c*) the measure of the angle ABB′

(*d*) the image A″ of A, when A is reflected in the origin

(*e*) the single transformation that maps A′ to A″. **[1995]**

2. (*i*) Point P(*a*, *b*) is reflected in the *x*-axis to P′ (5, –2). Write down the values of *a* and *b*.

(*ii*) P″ is the image of P when reflected in the *y*-axis. Write down the coordinates of P″.

(*iii*) Name a single transformation that maps P′ to P″. **[1997]**

3. A point P(*a*, *b*) is reflected in the X axis to P′ (2, –3). Write down the values of *a* and *b*. P″ is the image of P, when reflected in the Y axis. Write down the coordinates of P″. Find the coordinates of P‴, when P is reflected in the line, parallel to the Y axis, such that $x = 4$. **[1998]**

4. Use graph paper for this question.

(*i*) Plot the points A(3, 5) and B(–2, –4). Use 1 cm = 1 unit on both axes.

(*ii*) A′ is the image of A when reflected in the *x* axis. Write down the coordinates of A′ and plot it on the graph paper.

(*iii*) B′ is the image of B when reflected in the *y* axis, followed by reflection in the origin. Write down the coordinates of B′ and plot it on the graph paper.

(*iv*) Write down the geometrical name of the figure AA′BB′.

(*v*) Name two invariant points under reflection in the *x* axis. **[1999]**

5. Write down the coordinates of the image of the point (3, –2) when:

(*i*) reflected in the *x* axis,

(*ii*) reflected in the *y* axis,

(*iii*) reflected in the *x* axis followed by reflection in the *y* axis,

(*iv*) reflected in the origin. **[2000]**

6. The point P(3, 4) is reflected to P′ in the *x* axis and O′ is the image of O (the origin) when reflected in the line P P′. Using graph paper, give:

(*i*) The coordinates of P′ and O′.

(*ii*) The lengths of the segments PP′ and OO′

(*iii*) The perimeter of the quadrilateral POP′O′

(*iv*) The geometrical name of the figure POP′O′ **[2002]**

7. Use a graph paper for this question. (Take 10 small divisions = 1 unit on both axes).

Plot the points P(3, 2) and Q(–3, –2). From P and Q, draw perpendiculars PM and QN on the *x* axis.

(*a*) Name the image of P on reflection in the origin.

(*b*) Assign the special name to the geometrical figure PMQN and find its area.

(*c*) Write the co-ordinates of the point to which M is mapped on reflection in (*i*) *x* axis; (*ii*) *y* axis; (*iii*) origin. **[2003]**

8. Use a graph paper for this question.

A(1, 1), B(5, 1), C(4, 2) and D(2, 2) are the vertices of a quadrilateral. Name the quadrilateral ABCD. A, B, C and D are reflected in the origin on to A′, B′, C′ and D′ respectively. Locate A′, B′, C′ and D′ on the graph sheet and write their co-ordinates. Are D, A, A′ and D′ collinear? **[2004]**

9. Use a graph paper for this question. (Take 10 small divisions = 1 unit on both axes).

P and Q have co-ordinates (0, 5) and (–2, 4).

(*i*) P is invariant when reflected in an axis. Name the axis.

(*ii*) Find the image of Q on reflection in the axis found in (*i*).

(*iii*) (0, *k*) on reflection in the origin is invariant. Write the value of *k*.

(*iv*) Write the co-ordinates of the image of Q, obtained by reflecting it in the origin followed by reflection in *x*-axis. **[2005]**

10. Use graph paper for this question.

The points A(2, 3), B(4, 5) and C(7, 2) are the vertices of ΔABC.

(*i*) Write down the coordinates of A′, B′, C′ if ΔA′B′C′ is the image of ΔABC, when reflected in the origin.

(*ii*) Write down the co-ordinates of A″, B″, C″ if ΔA″B″C″ is the image of ΔABC, when reflected in the *x*-axis.

(*iii*) Mention the special name of the quadrilateral BCC″B″ and find its area. **[2006]**

11. Use a graph paper for this question.

(*i*) The point P(2, –4) is reflected about the line $x = 0$ to get the image Q. Find the co-ordinates of Q.

(*ii*) Point Q is reflected about the line $y = 0$ to get the image R. Find the co-ordinates of R.

(*iii*) Name the figure PQR.

(*iv*) Find the area of figure PQR. **[2007]**

12. Use graph paper to answer this question.

(*i*) Plot the points A(4, 6) and B(1, 2).

(*ii*) A′ is the image of A when reflected in X-axis.

(*iii*) B′ is the image of B when B is reflected in the line AA′.

(*iv*) Give the geometrical name for the figure ABA′B′. **[2009]**

13. Use graph paper for this question.

A(0, 3), B(3, –2) and O(0, 0) are the vertices of triangle ABO.

(*i*) Plot the triangle on a graph sheet taking 2 cm = 1 unit on both the axes.

(*ii*) Plot D the reflection of B in the Y axis, and write its co-ordinates.

(*iii*) Give the geometrical name of the figure ABOD.

(*iv*) Write the equation of the line of symmetry of the figure ABOD. **[2010]**

14. Use a graph paper to answer the following questions. (Take 1 cm = 1 unit on both axes).

(*i*) Plot A(4, 4), B(4, – 6) and C(8, 0), the vertices of a triangle ABC.

(*ii*) Reflect ABC on the *y*-axis and name it as A′B′C′.

(*iii*) Write the coordinates of the images A′, B′ and C′.

(*iv*) Give a geometrical name for the figure AA′C′B′BC. **[2011]**

15. Using a graph paper, plot the points A(6, 4) and B(0, 4).

(*i*) Reflect A and B in the origin to get the images A′ and B′.

(*ii*) Write the co-ordinates of A′ and B′.

(*iii*) State the geometrical name for the figure ABA′B′.

(*iv*) Find its perimeter. **[2013]**

16. Use graph paper for this question.

(Take 2 cm = 1 unit along both *x* and *y*-axis.)

Plot the points O (0, 0), A (–4, 4), B (–3, 0) and C (0,–3).

(*i*) Reflect points A and B on the *y*-axis and name them A′ and B′ respectively. Write down their co-ordinates.

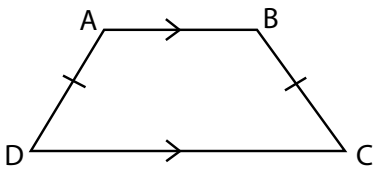
(*ii*) Name the figure OABCB′A′.

(*iii*) State the line of symmetry of this figure. **[2016]**

COMMON ERRORS

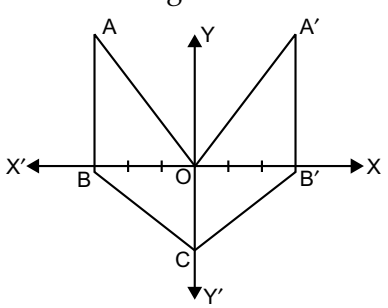
Errors are made in naming the figure formed after reflection.

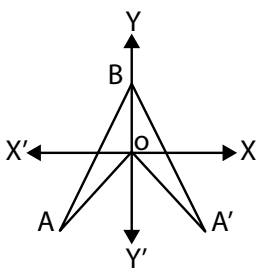
e.g.



(i) In the figure ABCD is an isosceles trapezium but named as trapezium.

(ii) Arrowhead is often confused. In the figure, OABA′ is an arrowhead but OABCB′A′ is a hexagon with 6 sides and not an arrowhead which has only 4 sides.





2. (*i*) Find the co-ordinates of a point R when AR : RB = 2 : 3 with A(1, 3) and B(6, 8).

(*ii*) If P(–2, 3) divides AB in the ratio 1 : 2, find the co-ordinates of A and B in the figure.

3. Midpoint of AB is (–3, 2*b*). If A is (*a*, 5) and B(4, –9). Find the values of *a* and *b*.

4. P and Q are two points on the line joining A(2, 4) and B(8, 1) such that AP = PQ = QB. Find the co-ordinates of P and Q.

5. In ΔABC, A ≡ (7, –5) and centroid G ≡ (3, –1). Find the midpoint of BC.

6. If origin is the centroid of ΔPQR where P(4, *k*), Q(*m*, 9) and R ≡ (1, –6). Find the values of *k* and *m*.

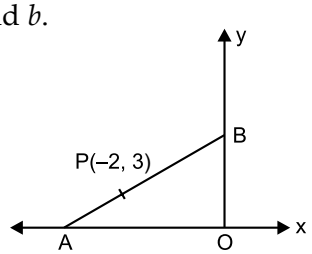


Figure for Q.2 (ii)

7. PQ is a diameter of a circle. P ≡ (7, – 4) and Q ≡ (–9, 8)

(*i*) Find the co-ordinates of the centre C.

(*ii*) If AB is another diameter of the circle and A = (1, 4), find the co-ordinates of B.

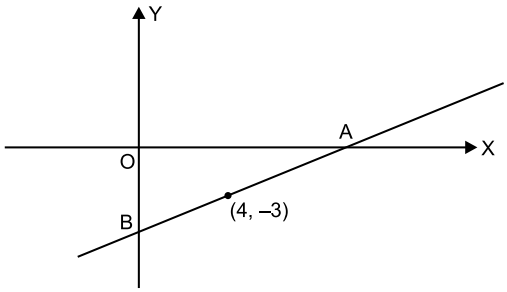
BOARD PAPER QUESTIONS

1. (*i*) Write down the co-ordinates of the point P that divides the line joining A(– 4, 1) and B(17, 10) in the ratio 1 : 2.

(*ii*) Calculate the distance OP, where O is the origin.

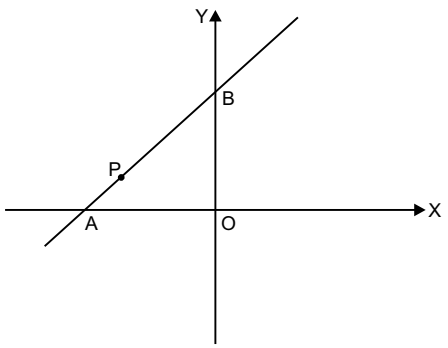
(*iii*) In what ratio does the *y*-axis divide the line AB? **[1995]**

2. The midpoint of the line segment AB shown in the diagram is (4, –3). Write down the co-ordinates of A and B. **[1996]**



3. The centre O, of a circle has the co-ordinates (4, 5) and one point on the circumference is (8, 10). Find the co-ordinates of the other end of the diameter of the circle through this point. **[1998]**

4.



In figure given above, line APB meets the X axis at A, Y axis at B. P is the point (– 4, 2) and AP : PB = 1 : 2. Write down the co-ordinates of A and B. **[1999]**

5. Calculate the ratio in which the line joining A(6, 5) and B(4, –3) is divided by the line *y* = 2. **[2000]**

6. The line joining P(– 4, 5) and Q(3, 2), intersects the *y* axis at R.

PM and QN are perpendiculars from P and Q on the X axis. Find:

(*i*) the ratio PR : RQ.

(*ii*) the co-ordinates of R.

(*iii*) the area of the quadrilateral PMNQ. **[2004]**

7. The line segment joining A(2, 3) and B(6, –5) is intercepted by the X axis at the point K. Write the ordinate of the point K. Hence, find the ratio in which K divides AB. **[2006]**

8. Find the co-ordinates of the centroid of a triangle whose vertices are:

A(–1, 3), B(1, –1) and C(5, 1) **[2006]**

9. The midpoint of the line segment joining (2*a*, 4) and (–2, 2*b*) is (1, 2*a* + 1). Find the values of *a* and *b*. **[2007]**

10. If the line joining the points A(4, –5) and B(4, 5) is divided by the point P such that $\frac{AP}{AB} = \frac{2}{5}$, find the co-ordinates of P. **[2007]**

11. If A = (– 4, 3) and B = (8, – 6). In what ratio is the line joining AB, divided by the *x*-axis? **[2008]**

12. ABC is a triangle and G(4, 3) is the centroid of the triangle. If A = (1, 3), B = (4, b) and C = (a, 1), find 'a' and 'b'. Find the length of side BC. **[2011]**

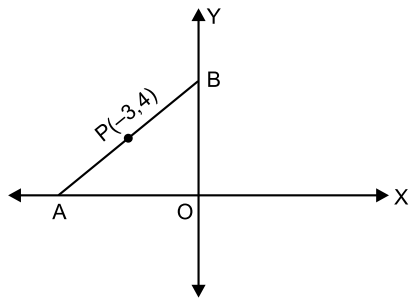
13. Given a line segment AB joining the points A(– 4, 6) and B(8, –3). Find:

(i) the ratio in which AB is divided by the y-axis.

(ii) find the co-ordinates of the point of intersection. **[2012]**

14. AB is a diameter of a circle with centre C = (–2, 5). If A = (3, –7), find the coordinates of B. **[2013]**

15. In the given figure, the line segment AB meets X-axis at A and Y-axis at B. The point P(–3, 4) on AB divides it in the ratio 2 : 3. Find the co-ordinates of A and B.

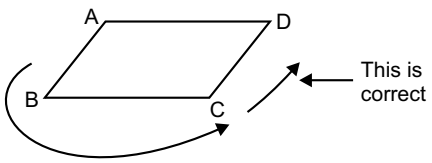


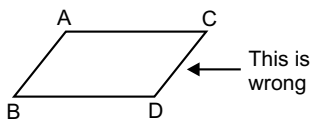
[2013]

16. Calculate the ratio in which the line joining A(– 4, 2) and B(3, 6) is divided by point P(x, 3). Also find x. **[2014]**

COMMON ERRORS

1. Labelling the vertices of a parallelogram wrongly. Either go clockwise or anti-clockwise while labelling.





2. Equation of x-axis and y-axis are confused often. Remember the equation of x-axis is $y = 0$ and y-axis is $x = 0$.

3. When $y = 2$ divides the join of AB and the ratio is to be found, it is wrongly assumed that the point of intersection of AB with $y = 2$ is (0, 2).

Take y-co-ordinate = 2 and find the ratio, but do not assume that $x = 0$ when $y = 2$.

4. There is confusion between circumcentre and centroid. For circumcentre, O of triangle ABC use distance formula in OA = OB = OC.

For centroid, use $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

6. Line $4x - 3y = 6$ meets y-axis at A. Write the co-ordinates of A. Find the equation of a line through A and perpendicular to the line $4x - 3y = 6$.

7. A(2, 3), B(6, –1) and C(–4, –3) are the vertices of a triangle. Write down the equations of the (*i*) median through C (*ii*) altitude through B (*iii*) perpendicular bisector of B and C.

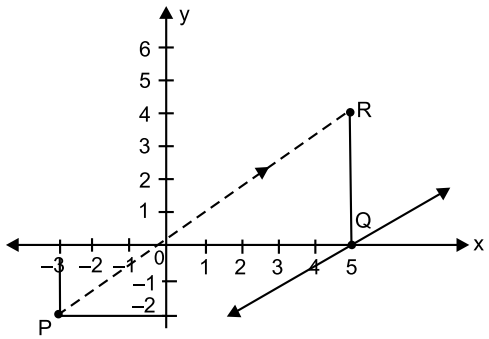
8. (*i*) Find the equation of a line passing through R(3, –2) and S(– 4, 5).

(*ii*) If the line RS passes through the point $(2a, 3 - a)$, find the value of a.

9. (*i*) Write down the co-ordinates of P, Q and R in the figure. Find:

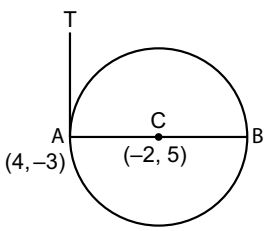
(*ii*) the equations of PR and QR

(*iii*) the equation of a line through Q and parallel to PR.



10. Find the equation of a line with x-intercept 4 and passing through the point (–2, 5).

11. AB is diameter of a circle with centre C (–2, 5). If A is (4, –3), find (*i*) the coordinates of B (*ii*) equation of tangent AT.



12. PQRS is a parallelogram. P(–2, 0), Q(6, 3), S(8, 5). Find the

(*i*) coordinates of R (*ii*) equation of QS.

BOARD PAPER QUESTIONS

1. (*i*) Write down the equation of the line AB, through (3, 2), perpendicular to the line $2y = 3x + 5$.

(*ii*) AB meets the x-axis at A and the y-axis at B. Write down the coordinates of A and B. Calculate the area of triangle OAB, where O is the origin. **[1995]**

2. Write down the equation of the line whose gradient is $\frac{3}{2}$ and which passes through P, where P divides the line segment joining A(–2, 6) and B(3, – 4) in the ratio 2 : 3. **[1996, 2001]**

3. Find the equation of a line, which has the y intercept 4, and is parallel to the line $2x - 3y = 7$. Find the coordinates of the point, where it cuts the x-axis. **[1998]**

4. Find the equation of the line passing through (0, 4) and parallel to the line $3x + 5y + 15 = 0$. **[1999]**

5. ABCD is a rhombus. The co-ordinates of A and C are (3, 6) and (–1, 2) respectively. Write down the equation of BD. **[2000]**

6. Find the equation of a line passing through the point (–2, 3) and having the x-intercept of 4 units. **[2002]**

7. A(1, 4), B(3, 2) and C(7, 5) are the vertices of a ΔABC. Find:

(*i*) the coordinates of the centroid G of ΔABC.

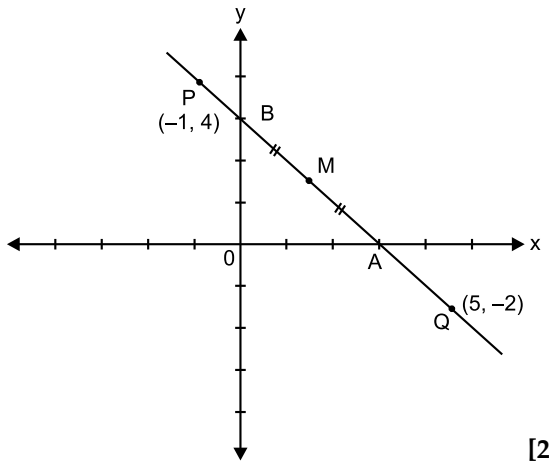
(*ii*) the equation of a line, through G and parallel to AB. **[2002]**

8. A straight passes through the points P(–1, 4) and Q(5, –2). It intersects the co-ordinate axes at points A and B. M is the midpoint of the segment AB. Find:

(*i*) the equation of the line.

(*ii*) the co-ordinates of A and B.

(*iii*) the co-ordinates of M.

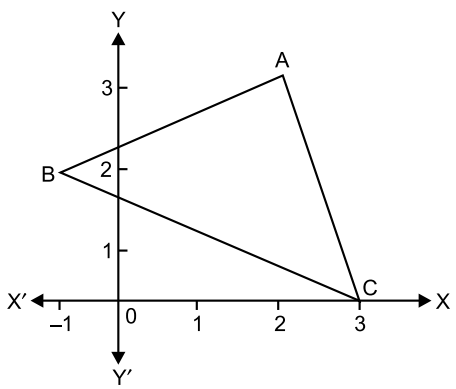


[2003]

9. Find the value of k for which the lines $kx - 5y + 4 = 0$ and $4x - 2y + 5 = 0$ are perpendicular to each other. **[2003]**

10. P(3, 4), Q(7, –2) and R(–2, –1) are the vertices of triangle PQR. Write down the equation of the median of the triangle, through R. **[2004]**

11. In the adjoining figure, write



(*i*) the co-ordinates of A, B and C.

(*ii*) the equation of the line through A and parallel to BC. **[2005]**

12. If the lines $y = 3x + 7$ and $2y + px = 3$ are perpendicular to each other, find the value of p. **[2006]**

13. Find the equation of the line parallel to the line $3x + 2y = 8$ and passing through the point (0, 1). **[2007]**

14. Points A and B have coordinates (7, –3) and (1, 9) respectively. Find

(*i*) the slope of AB.

(*ii*) the equation of the perpendicular bisector of the line segment AB.

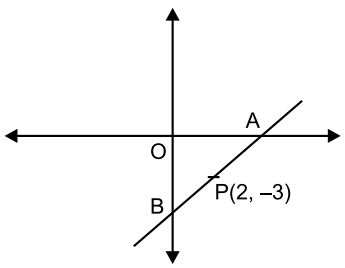
(*iii*) the value of 'p' if (–2, p) lies on it. **[2008]**

15. Find the value of p for which the lines

$2x + 3y - 7 = 0$ and $4y - px - 12 = 0$ are perpendicular to each other. **[2009]**

16. Find the equation of a line with x intercept = 5 and passing through the point (4, –7). **[2009]**

17. A and B are two points on the x-axis and y-axis respectively. P(2, –3) is the midpoint of AB. Find the



(*i*) co-ordinates of A and B

(*ii*) slope of line AB.

(*iii*) equation of line AB. **[2010]**

18. The equation of a line is $3x + 4y - 7 = 0$. Find

(*i*) the slope of the line.

(*ii*) the equation of a line perpendicular to the given line and passing through the intersection of the lines $x - y + 2 = 0$ and $3x + y - 10 = 0$. **[2010]**

19. ABCD is a parallelogram where A(x, y), B(5, 8), C(4, 7) and D(2, – 4). Find

(*i*) co-ordinates of A

(*ii*) equation of diagonal BD. **[2011]**

20. The line through A(–2, 3) and B(4, b) is perpendicular to the line $2x - 4y = 5$. Find the value of b. **[2012]**

21. In ΔABC, A(3, 5), B(7, 8) and C(1, –10). Find the equation of the median through A. **[2013]**

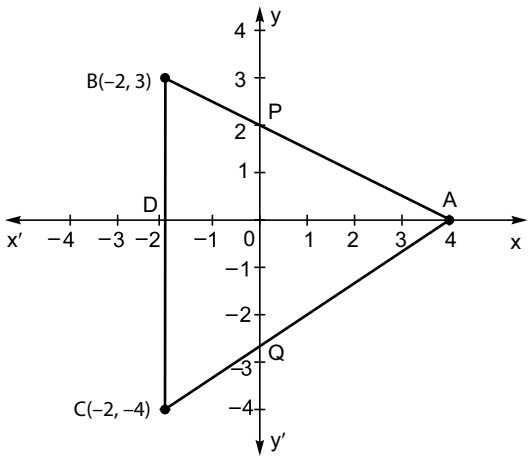
22. Find the value of 'a' for which the following points A(a, 3), B(2, 1) and C(5, a) are collinear. Hence find the equation of the line AB. **[2014]**

23. Three vertices of a parallelogram ABCD taken in order are A (3, 6), B (5, 10) and C (3, 2), find:

(*i*) the co-ordinates of the fourth vertex D.

(*ii*) equation of side AB of the parallelogram ABCD. **[2015]**

24. In the given figure ABC is a triangle and BC is parallel to the y-axis. AB and AC intersect the y-axis at P and Q respectively.



(*i*) Write the co-ordinates of A.

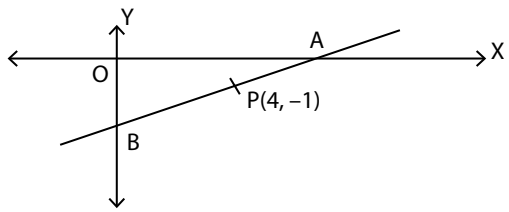
(*ii*) Find the ratio in which Q divides AC.

(*iii*) Find the equation of the line AC. **[2015]**

25. A line AB meets X-axis at A and Y-axis at B. P(4,–1) divides AB in the ratio 1 : 2.

(*i*) Find the co-ordinates of A and B.

(*ii*) Find the equation of the line through P and perpendicular to AB. **[2016]**



26. The slope of a line joining P (6, k) and Q(1–3k, 3) is $\frac{1}{2}$. Find

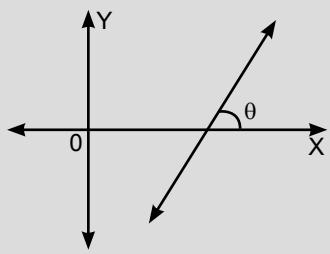
(*i*) k

(*ii*) Midpoint of PQ, using the value of 'k'found in (*i*). **[2016]**

POINTS AT A GLANCE

1. Slope of AB = m(AB) = $\frac{y_2 - y_1}{x_2 - x_1}$, where A = (x_1, y_1) and B = (x_2, y_2)

2. When the equation of a line is of the form $ax + by + c = 0$, change it into $y = mx + c$ form (making y as the subject), to get slope m and y-intercept c.



3. When inclination of a line is given, slope = tan θ

4. Equation of a line $y - y_1 = m(x - x_1)$, where m = slope; (x_1, y_1) is a point on the line.

5. If a **point P lies on a given line,** substitute the co-ordinates of point P in the given equation of the line.

6. To verify **if a line bisects AB,** find the midpoint of AB. Put these coordinates in the equation of the line. If midpoint's co-ordinates satisfy the equation of the line, then the line bisects AB.

7. Any point on x-axis is $(x, 0)$ and equation of x-axis is $y = 0$.

Any point on y-axis is $(0, y)$ and equation of y-axis is $x = 0$.

8. To find ***x*-intercept** put $y = 0$ in the given equation.

9. To find points of **trisection,** P and Q of line segment AB.

BOARD PAPER QUESTIONS

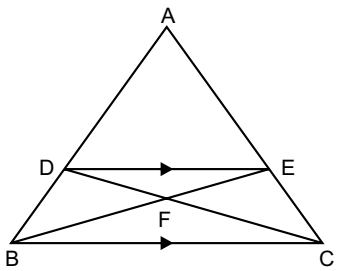
1. (*a*) A model of a ship is made to a scale of 1 : 200.

(*i*) The length of the model is 4 m. Calculate the length of the ship.

(*ii*) The area of the deck of the ship is 160000 m^2. Find the area of the deck of the model.

(*iii*) The volume of the model is 200 litres. Calculate the volume of the ship in m^3.

(*b*) In the diagram, ABC is a triangle, DE is parallel to BC and $\frac{AD}{DB} = \frac{3}{2}$.



(*i*) Write down $\frac{AD}{AB}$.

(*ii*) Prove $\Delta ADE \sim \Delta ABC$ and write down the ratio $\frac{DE}{BC}$.

(*iii*) Prove $\Delta DEF \sim \Delta CFB$. Write down the ratio $\frac{\text{area of } \Delta DFE}{\text{area of } \Delta CFB}$. **[1995]**

2. (*a*) The scale of a map is 1 : 200000. A plot of land of area 20 km^2 is to be represented on the map. Find:

(*i*) The number of kilometres on the ground which is represented by 1 centimetre on the map.

(*ii*) The area in km^2, that can be represented by 1 cm^2.

(*iii*) The area on the map that represents the plot of land.

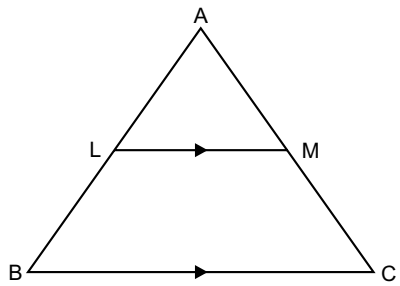
(*b*) In the figure (not drawn to scale), LM is parallel to BC.

AB = 6 cm, AL = 2 cm and AC = 9 cm

Calculate:

(*i*) The length of CM;

(*ii*) The value of the ratio = $\frac{\text{Area of triangle ALM}}{\text{Area of the trapezium LBCM}}$



[1996]

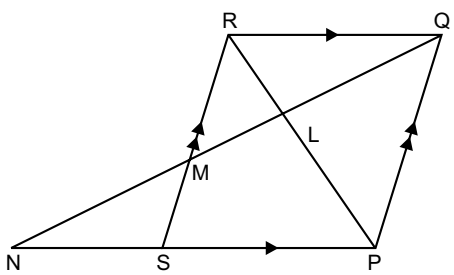
3. On a map drawn to a scale of 1 : 250000, a triangular plot of land has the following measurements:

AB = 3 cm, BC = 4 cm, angle ABC = 90°. Calculate:

(*i*) the actual length of AB in km;

(*ii*) the area of the plot in sq. km. **[1997]**

4.



In the figure, PQRS is a parallelogram; PQ = 16 cm, QR = 10 cm. L is a point on PR such that RL : LP = 2 : 3. QL produced meets RS at M and PS produced at N.

(*i*) Prove that triangle RLQ is similar to triangle PLN. Hence find PN.

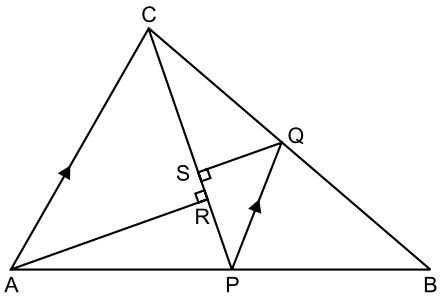
(*ii*) Name a triangle similar to triangle RLM. Evaluate RM as a fraction. **[1997]**

5. On a map drawn to a scale of 1 : 25000, a rectangular plot of land, ABCD has the following measurements. AB = 12 cm and BC = 16 cm. Angles A, B, C and D are all 90° each. Calculate :

(*i*) The diagonal distance of the plot in km,

(*ii*) The area of the plot in sq. km. **[1998]**

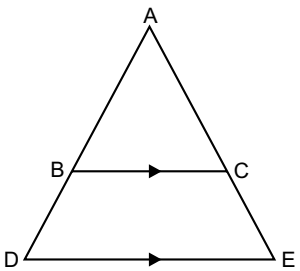
6. In the figure given below, P is a point on AB such that AP : PB = 4 : 3. PQ is parallel to AC.



(*i*) Calculate the ratio PQ : AC, giving reasons for your answer;

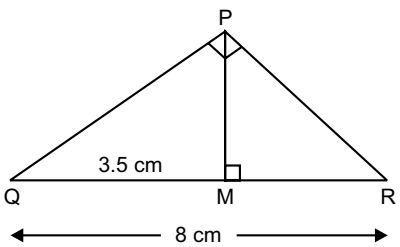
(*ii*) In ΔARC, ∠ARC = 90° and in ΔPQS, ∠PSQ = 90°. Given QS = 6 cm. Calculate the length of AR. **[1999]**

7.



In the figure above, BC is parallel to DE. Area of triangle ABC = 25 cm^2, area of trapezium BCED = 24 cm^2, DE = 14 cm. Calculate the length of BC. **[2000]**

8. In the right angled ΔQPR, PM is an altitude. Given that QR = 8 cm and MQ = 3.5 cm, calculate the value of PR.



[2002]

9. In a triangle PQR, L and M are two points on the base QR, such that ∠LPQ = ∠QRP and ∠RPM = ∠RQP. Prove that:

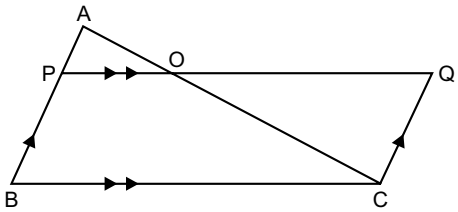
(*i*) ΔPQL ~ ΔRPM (*ii*) QL . RM = PL . PM

(*iii*) PQ^2 = QR . QL **[2003]**

10. In ΔABC, AP : PB = 2 : 3. PO is parallel to BC and is extended to Q so that CQ is parallel to BA. Find:

(*i*) area Δ APO : area Δ ABC

(*ii*) area Δ APO : area Δ CQO



[2008]

11. The model of a building is constructed with scale factor 1 : 30.

(*i*) If the height of the model is 80 cm, find the actual height of the building in metres.

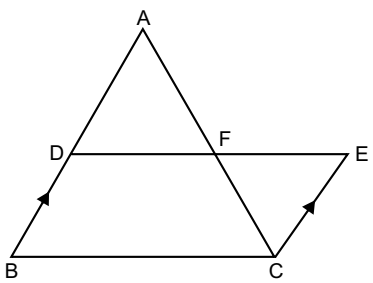
(*ii*) If the actual volume of a tank at the top of the building is 27 m^3, find the volume of the tank on the top of the model. **[2009]**

12. In the given figure, ABC and CEF are two triangles where BA is parallel to CE and AF : AC = 5 : 8.

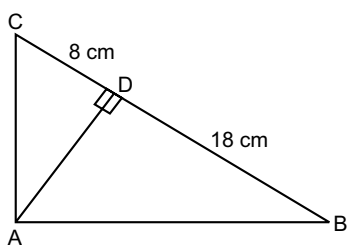
(*i*) Prove that ΔADF ~ ΔCEF.

(*ii*) Find AD if CE = 6 cm.

(*iii*) If DF is parallel to BC, find area of ΔADF : area of ΔABC. **[2009]**



13. In the adjoining figure, ABC is a right angled triangle with ∠BAC = 90°.

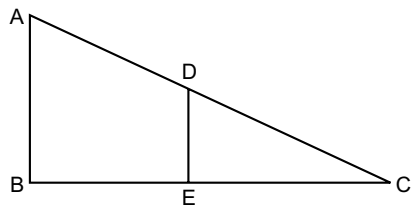


(*i*) Prove ΔADB ~ ΔCDA.

(*ii*) If BD = 18 cm, CD = 8 cm, find AD.

(*iii*) Find the ratio of the area of ΔADB is to area ΔCDA. **[2011]**

14. In the given figure, AB and DE are perpendiculars to BC.

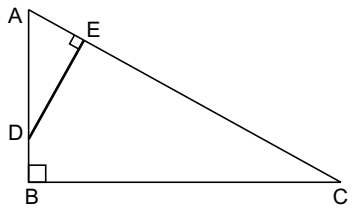


(*i*) Prove that ΔABC ~ ΔDEC.

(*ii*) If AB = 6 cm, DE = 4 cm and AC = 15 cm. Calculate CD.

(*iii*) Find the ratio of the area of ΔABC : area of ΔDEC. **[2013]**

15. ABC is a right-angled triangle with ∠ABC = 90°. D is any point on AB and DE is perpendicular to AC.



Prove that:

(*i*) ΔADE ~ ΔACB.

(*ii*) If AC = 13 cm, BC = 5 cm and AE = 4 cm, find DE and AD.

(*iii*) Find area of ΔADE : area of quadrilateral BCED. **[2015]**

16. A model of a ship is made to a scale 1 : 300.

(*i*) The length of the model of the ship is 2 m. Calculate the length of the ship.

(*ii*) The area of the deck ship is 180,000 m^2. Calculate the area of the deck of the model.

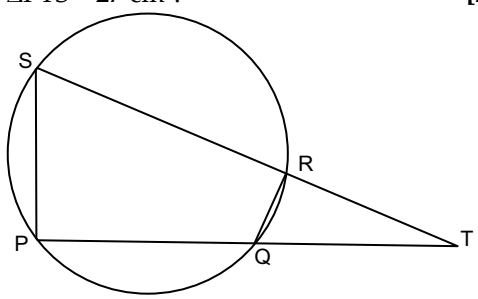
(*iii*) The volume of the model is 6.5 m^3. Calculate the volume of the ship. **[2016]**

17. In the given figure PQRS is a cyclic quadrilateral PQ and SR produced meet at T.

(*i*) Prove ΔTPS ~ ΔTRQ.

(*ii*) Find SP if TP = 18 cm, RQ = 4 cm and TR = 6 cm.

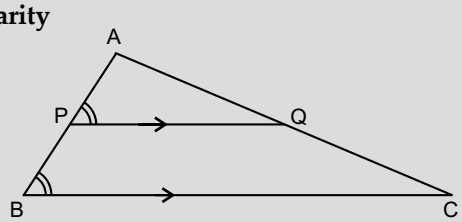
(*iii*) Find area of quadrilateral PQRS if area of ΔPTS = 27 cm^2. **[2016]**

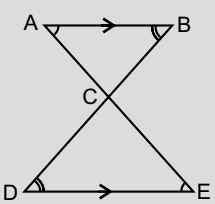


POINTS AT A GLANCE

Two similar figures are different in size but have same shape.

1. AA test of similarity





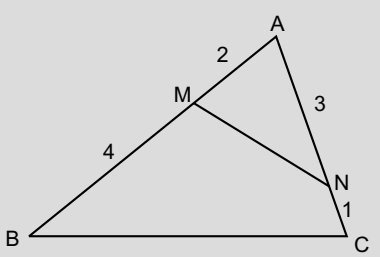
2. SAS test of similarity

$$\angle A = \angle A$$

$$\frac{AM}{AC} = \frac{2}{4} = \frac{1}{2}$$

$$\frac{AN}{AB} = \frac{3}{6} = \frac{1}{2}$$

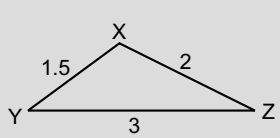
$$\therefore \quad \Delta AMN \sim \Delta ACB$$

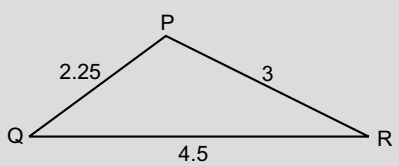


3. SSS test of similarity

The sides of triangles are proportional

$$\frac{XY}{PQ} = \frac{YZ}{QR} = \frac{XZ}{PR} = \frac{2}{3}$$

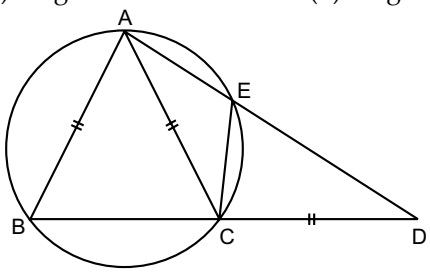




BOARD PAPER QUESTIONS

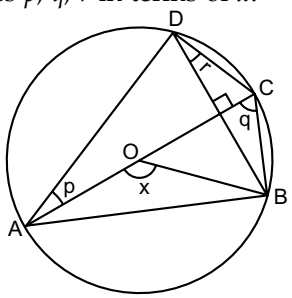
1. In the figure AB = AC = CD, angle ADC = 38°. Calculate

(*i*) angle ABC. (*ii*) angle BEC.



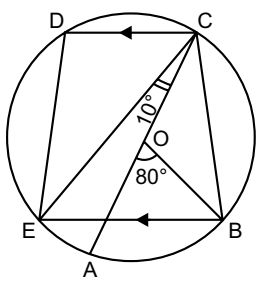
[1995]

2. In the figure, AC is the diameter of circle, centre O. Chord BD is perpendicular to AC. Write down the angles *p*, *q*, *r* in terms of *x*.



[1996]

3.

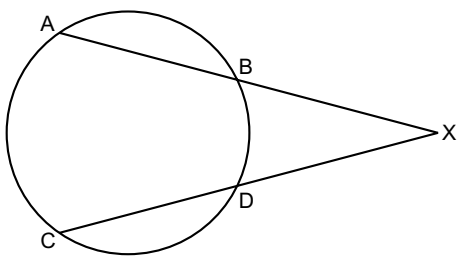


In the diagram given above, AC is the diameter of the circle, with centre O. CD and BE are parallel. Angle AOB = 80° and angle ACE = 10°. Find:

(*i*) Angle BEC, (*ii*) Angle BCD,

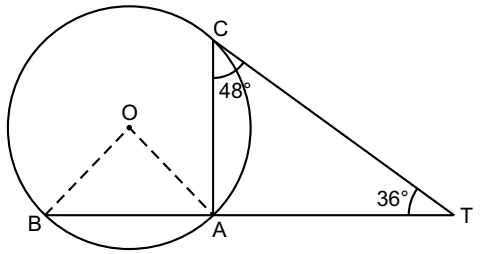
(*iii*) Angle CED. **[1998]**

4.



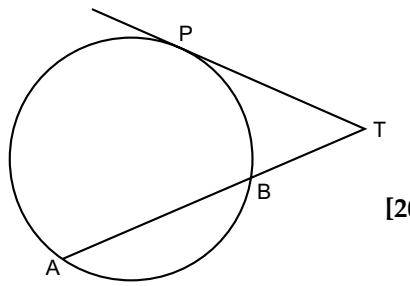
In the figure, chords AB and CD when extended meet at X. Given AB = 4 cm, BX = 6 cm, XD = 5 cm, calculate the length of CD. **[2000]**

5. A, B and C are three points on a circle. The tangent at C meets BA produced at T. Given that ∠ATC = 36° and that the ∠ACT = 48°, calculate the angle subtended by AB at centre of the circle.



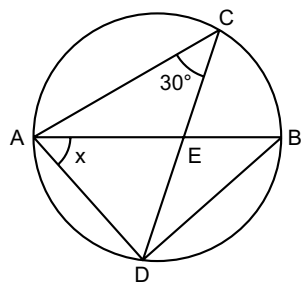
[2001]

6. In the given figure, find TP if AT = 16 cm and AB = 12 cm.



[2001, 2007]

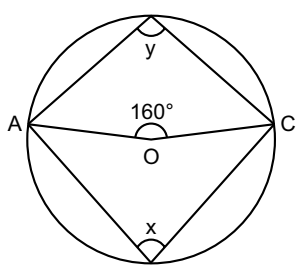
7. In the given circle with diameter AB, find the value of *x*.



[2003]

8. PQR is a right-angled triangle with PQ = 3 cm and QR = 4 cm. A circle which touches all the sides of the triangle is inscribed in the triangle. Calculate the radius of the circle. **[2005]**

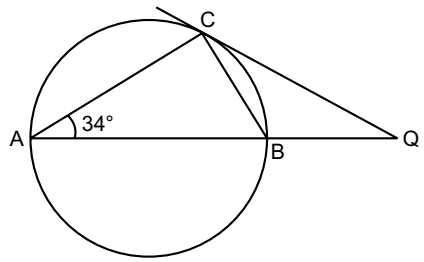
9.



In the above figure, O is the centre of the circle and ∠AOC = 160°. Prove that $3\angle y - 2\angle x = 140°$. **[2005]**

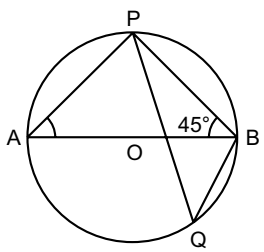
10. In the given figure, AB is a diameter. The tangent at C meets AB produced at Q. If ∠CAB = 34°, find:

(*i*) ∠CBA (*ii*) ∠CQA



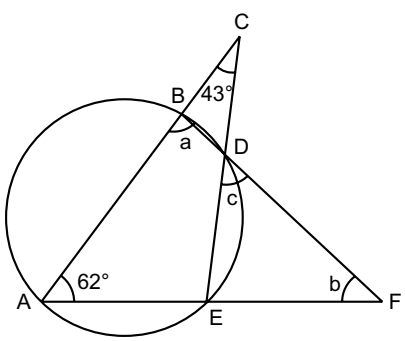
[2006]

11. In the given figure, O is the centre of the circle and ∠PBA = 45°. Calculate the value of ∠PQB.



[2007]

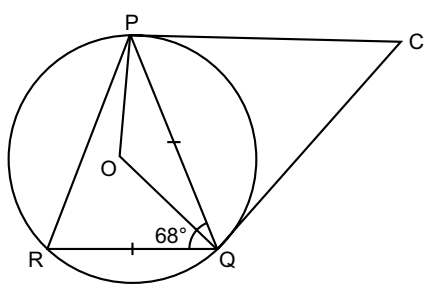
12. In the given figure, if ∠ACE = 43° and ∠CAF = 62°, find the values of *a*, *b* and *c*.



[2007]

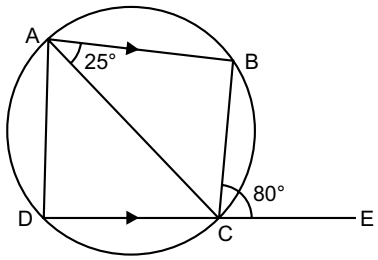
13. In the figure given below, PQ = QR, ∠RQP = 68°, PC and CQ are tangents to the circle with centre O. Calculate the values of:

(*i*) ∠QOP (*ii*) ∠QCP



[2008]

14.

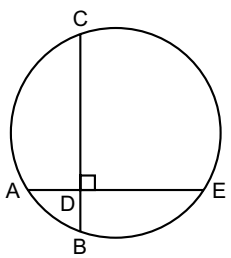


In the above figure, AB is parallel to DC, ∠BCE = 80° and ∠BAC = 25°. Find:

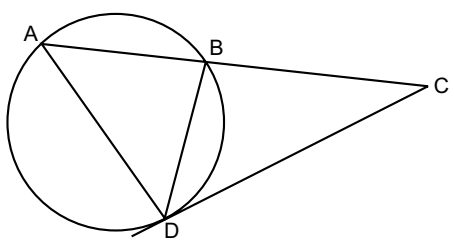
(*i*) ∠CAD (*ii*) ∠CBD

(*iii*) ∠ADC **[2008]**

15. In the given figure, AE and BC intersect each other at point D. If ∠CDE = 90°, AB = 5 cm, BD = 4 cm and CD = 9 cm, find DE. **[2008]**



16.

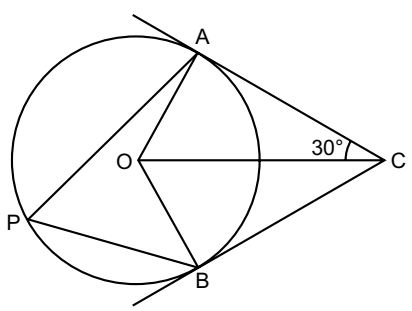


In the above figure, AB = 7 cm and BC = 9 cm.

(*i*) Prove ΔACD ~ ΔDCB.

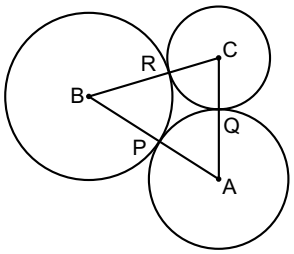
(*ii*) Find the length of CD. **[2009]**

17. In the given figure, O is the centre of the circle. Tangents at A and B meet at C. If ∠ACO = 30°, find (*i*) ∠BCO (*ii*) ∠AOB (*iii*) ∠APB



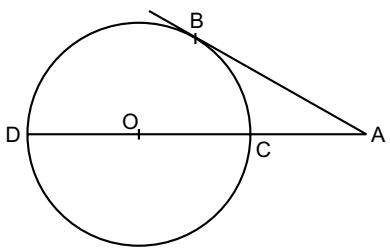
[2011]

18. ABC is a triangle with AB = 10 cm, BC = 8 cm and AC = 6 cm (not drawn to scale). Three circles are drawn touching each other with the vertices as centres. Find the radii of the three circles.



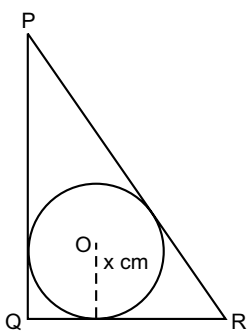
[2011]

19. In the given figure, O is the centre of the circle and AB is a tangent at B. If AB = 15 cm and AC = 7.5 cm. Calculate the radius of the circle.



[2012]

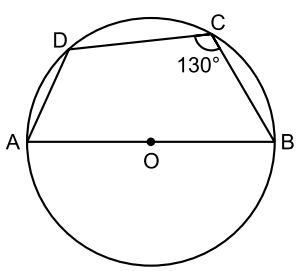
20. In triangle PQR, PQ = 24 cm, QR = 7 cm and ∠PQR = 90°. Find the radius of the inscribed circle.



[2012]

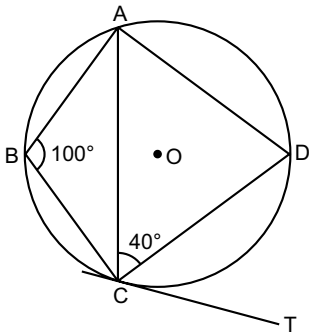
21. In the given figure, AB is the diameter of a circle with centre O. ∠BCD = 130°. Find:

(*i*) ∠DAB (*ii*) ∠DBA



[1998, 2002, 2012]

22. In the given circle with centre O, ∠ABC = 100°, ∠ACD = 40° and CT is a tangent to the circle at C. Find ∠ADC and ∠DCT.

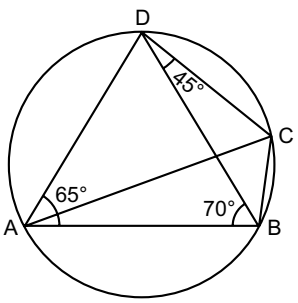


[2013]

23. In the given figure, ∠BAD = 65°, ∠ABD = 70° and ∠BDC = 45°.

(*i*) Prove that AC is a diameter of the circle.

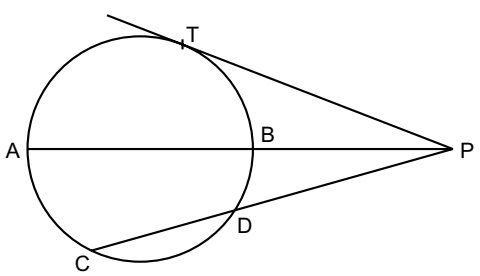
(*ii*) Find ∠ACB.



[2006, 2013]

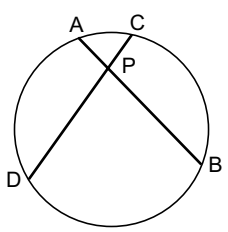
24. In the figure given below, diameter AB and chord CD of a circle meet at P. PT is a tangent to the circle at T. CD = 7.8 cm, PD = 5 cm, PB = 4 cm. Find:

(*i*) AB. (*ii*) the length of tangent PT.



[2014]

25. AB and CD are two chords of a circle intersecting at P. Prove that AP × PB = CP × PD.



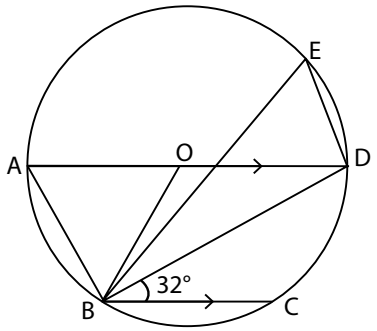
[2015]

26. In the adjoining figure, AD is a diameter. O is the centre of the circle. AD is parallel to BC and CBD = 32°. Find:

(*i*) ∠OBD

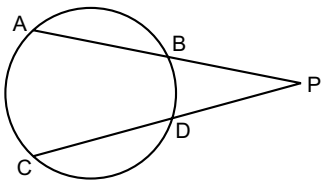
(*ii*) ∠AOB

(*iii*) ∠BED **[2016]**



COMMON ERRORS

1. When chords AB and CD intersect at P externally, instead of writing PA × PB = PC × PD an error is made by writing as PA × AB = PC × CD when the length of AB or CD is given.



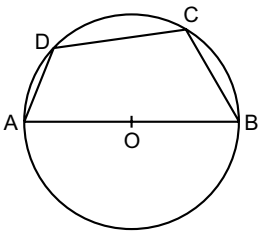
2. When a semicircle is drawn, one often fails to recognise the angle in semicircle.

e.g., The error is assuming that

$$\angle ADC = 90° \quad \text{or} \quad \angle BCD = 90°$$

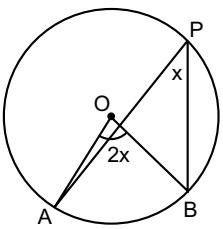
Remember $\angle \overset{\frown}{ADB} = 90°$ and $\angle \overset{\frown}{ACB} = 90°$

The two end points of the angle should be A and B, the end points of the diameter.



3. (*i*) To recognise angle at the centre and angle at the circumference by the same arc (or chord) AB, again the two end points of the angle should be A and B, only the middle letter is centre O or point P on the circumference.

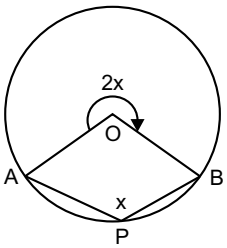
$$\angle \overset{\frown}{AOB} = 2\angle \overset{\frown}{APB}$$



(*ii*) When ∠APB is in the minor arc, it is obtuse.

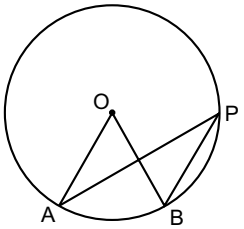
reflex ∠AOB = 2 ∠APB

Here, ∠AOB is *not double* of ∠APB but the reflex ∠AOB is **double** of ∠APB.



4. In this figure, often error of equating ∠AOB and ∠APB is made assuming both the angles are in the same segment.

But ∠AOB = 2 ∠APB. Remember ∠AOB is at the centre and ∠APB is at circumference.



Steps:

1. With the given length as side, construct the regular hexagon ABCDEF.
2. Draw the bisectors of angles at A and B. Let these intersect at point I.
3. From point I, draw IP perpendicular to side AB.
4. With I as centre and IP as radius, draw a circle which touches all the sides of the regular hexagon.
   This is the required incircle of regular hexagon ABCDEF.

EXERCISE 16

1. Draw a circle of radius 2.5 cm. Mark a point P at a distance of 6.5 cm from the centre of the circle. Draw two tangents to the circle from P and measure the length of each.

2. Draw a circle of diameter 9 cm. Take a point P at a distance of 7.5 cm from the centre of the circle. Draw tangents PA and PB to the circle and measure their length.

3. Draw a circle of radius 4 cm. Draw two tangents to this circle so that the angle between the tangents is 60°.

4. Draw a circle of radius 3 cm. Draw the tangents to the circle so that the angle between them is 45°.

5. (*i*) Construct a triangle ABC, given AB = 4 cm, BC = 6 cm and ∠ABC = 90°.

(*ii*) Construct a circle which passes through the points A, B and C. Mark its centre as O.

6. Construct a triangle PQR, given PQ = 5 cm, QR = 6.5 cm and ∠PQR = 120°. Construct a circumcircle of ΔPQR.

7. Construct a triangle DEF, with DE = 6 cm, ∠FDE = 60° and ∠FED = 45°. Circumscribe a circle about the triangle.

8. Construct a triangle ABC, given that AB = 4 cm, BC = 6 cm and median AM = 3 cm. Circumscribe a circle about the triangle.

9. Construct a ΔPQR, given PQ = 5 cm, QR = 7 cm and ∠PQR = 60°. Inscribe a circle in the triangle.

10. Construct a ΔABC, given that AB = 5 cm, BC = 6 cm and ∠ABC = 120°. Inscribe a circle in the triangle.

11. Construct a ΔPMN, given that MN = 5 cm, PN = 7 cm and ∠PMN = 90°. Construct an incircle of the triangle.

12. Construct a triangle ABC, given that AB = 6 cm, BC = 8 cm and median AM = 5 cm. Construct an incircle of triangle ABC and measure its radius.

13. Construct a regular hexagon of side 4 cm. Construct a circle circumscribing the hexagon.

14. Construct a regular hexagon of side 3.5 cm. Construct a circumcircle of the hexagon.

15. Construct a regular hexagon of side 5 cm. Inscribe a circle in it.

16. Construct a regular hexagon of side 4.5 cm. Inscribe a circle in it.

17. Construct ΔPQR, QR = 5 cm, ∠PQR = 60° and perpendicular from P to QR is 3 cm. Draw an incircle of ΔPQR.

18. Construct ΔABC, BC = 6 cm, ∠ABC = 45° and perpendicular from A to BC is 3.5 cm. Construct a circumcircle of the triangle.

BOARD PAPER QUESTIONS

1. Use a ruler and compass only in this question.

(*i*) Draw a circle, centre O and radius 4 cm.

(*ii*) Mark a point P such that OP = 7 cm.

Construct the two tangents to the circle from P. Measure and record the length of one of the tangents. **[1999]**

2. Only ruler and compass may be used in this question.

(*i*) Construct ΔABC such that AB = AC = 7 cm and BC = 5 cm.

(*ii*) Draw AX, the perpendicular bisector of side BC.

(*iii*) Draw a circle with centre A and radius 3 cm cutting AX at Y.

(*iv*) Construct another circle to touch the circle with centre A externally at Y and passing through B and C. **[1998, 2003]**

3. Using a ruler and compass only, construct a triangle ABC such that AB = 5 cm, ABC = 75° and the radius of the circumicircle of triangle ABC is 3.5 cm. **[2004]**

4. Using a ruler and compass construct a triangle ABC with BC = 6.4 cm, CA = 5.8 cm and ∠ABC = 60°. Draw its incircle. Measure and record the radius of the incircle. **[2007]**

5. Using a ruler and a pair of compasses only, construct:

(*i*) a triangle ABC, given AB = 4 cm, BC = 6 cm and ∠ABC = 90°.

(*ii*) a circle which passes through the points A, B and C and mark its centre as O. **[2008]**

6. Construct a triangle ABC in which base BC = 6 cm, AB = 5.5 cm and ∠ABC = 120°.

(*i*) Construct a circle circumscribing the triangle ABC.

(*ii*) Draw a cyclic quadrilateral ABCD so that D is equidistant from B and C. **[2012]**

7. Construct a Δ ABC with BC = 6.5 cm, AB = 5.5 cm, AC = 5 cm. Construct the incircle of the triangle. Measure and record the radius of the incircle. **[2014]**

8. Construct a regular hexagon of side 5 cm. Construct a circle circumscribing the hexagon. All traces of construction must be clearly shown. **[2010, 2015]**

9. Construct a regular hexagon of side 5 cm. Hence construct all its lines of symmetry and name them. **[2016]**

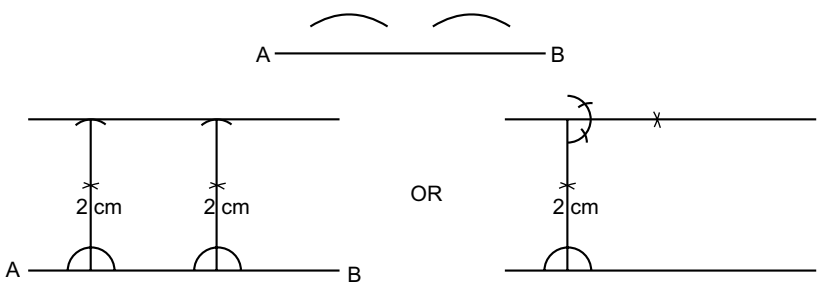
10. Draw a line AB = 5 cm. Mark a point C on AB such that AC = 3 cm. Using a ruler and a compass only, construct:

(*i*) A circle of radius 2.5 cm, passing through A and C.

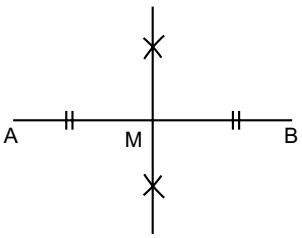
(*ii*) Construct two tangents to the circle from the external point B. Measure and record the length of the tangents. **[2016]**

COMMON ERRORS

1. To construct a parallel line to AB at 2 cm distance, do not use 'eye-brow method'. Draw two perpendiculars and cut off 2 cm on the perpendiculars.



2. To find the midpoint of a line segment, do not just mark the midpoint using a ruler. Draw the perpendicular bisector of the line segment.
e.g., when asked to construct a circle on diameter AB, find the centre by constructing ⊥ bisector of AB.

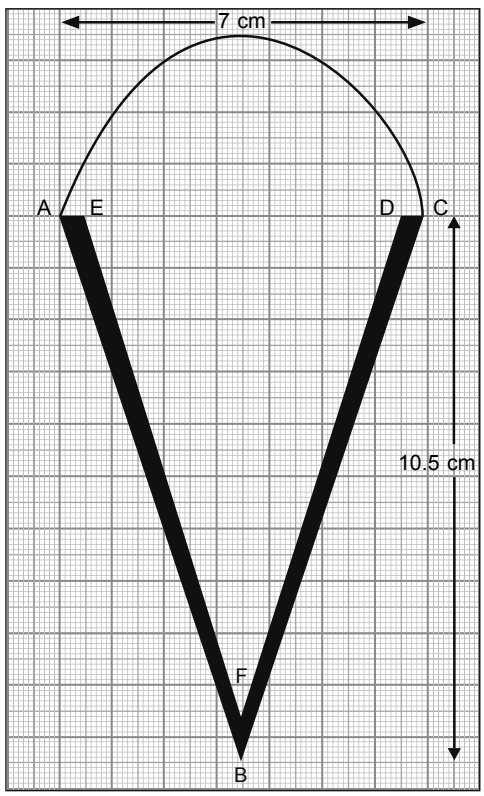


3. Forgetting to drop a perpendicular from the incentre of a triangle to one side, before drawing the incircle.

4. Forgetting to measure the tangent or the radius of the circle constructed when asked to do so.

BOARD PAPER QUESTIONS

1. The figure shows the cross-section of an ice-cream cone consisting of a cone surmounted by a hemisphere. The radius of the hemisphere is 3.5 cm and the height of the cone is 10.5 cm. The outer shell ABCDEF is shaded and is not filled with ice-cream. AE = DC = 0.5 cm, AB ∥ EF and BC ∥ FD.



Calculate:

(*i*) the volume of the ice cream in the cone (the unshaded portion including the hemisphere) in cm^3;

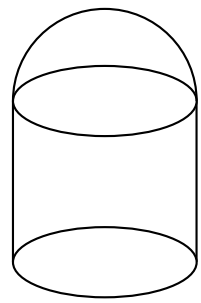
(*ii*) the volume of the outer shell (the shaded portion) in cm^3.

Give your answers correct of the nearest cm^3.

[1996]

2. A solid consisting of a right circular cone, standing on a hemisphere, is placed upright, in a right circular cylinder, full of water, and touches the bottom. Find the volume of water left in the cylinder, having given that the radius of the cylinder is 3 cm and its height is 6 cm; the radius of the hemisphere is 2 cm and the height of the cone is 4 cm. Give your answer to the nearest cubic centimetre. [Take $\pi = \frac{22}{7}$]. **[1998]**

3.



With reference to the figure given above, a metal container in the form of a cylinder is surmounted by a hemisphere of the same radius. The internal height of the cylinder is 7 m and the internal radius is 3.5 m.

Calculate:

(*i*) the total area of the internal surface, excluding the base;

(*ii*) the internal volume of the container in m^3. (Take π to be $\frac{22}{7}$.) **[1999]**

4. The surface area of a solid metallic sphere is 1256 cm^2. It is melted and recast into solid right circular cones of radius 2.5 cm and height 8 cm. Calculate:

(*i*) the radius of the solid sphere,

(*ii*) the number of cones recast.

Take $\pi = 3.14$ **[2000]**

5. A hollow sphere of internal and external diameters 4 cm and 8 cm respectively, is melted into a cone of base diameter 8 cm. Find the height of the cone.

[2002]

6. A vessel is in the form of an inverted cone. Its height is 11 cm and the radius of its top which is open, is 2.5 cm. It is filled with water up to the rim. When lead shots, each of which is a sphere of radius 0.25 cm are dropped into the vessel, $\frac{2}{5}$ of the water flows out. Find the number of lead shots dropped into the vessel. **[2003]**

7. A girl fills a cylindrical bucket 32 cm in height and 18 cm in radius with sand. She empties the bucket on the ground and makes a conical heap of the sand. If the height of the conical heap is 24 cm. Find:

(*i*) its radius and

(*ii*) its slant height. (Leave your answer in square root form). **[2004]**

8. A metallic sphere of radius 10.5 cm is melted and then recast into small cones, each of radius 3.5 cm and height 3 cm. Find the number of cones thus obtained. **[2005]**

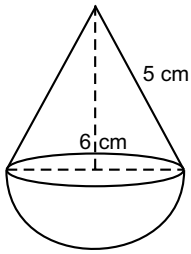
9. A vessel in the form of an inverted cone is filled with water to the brim. Its height is 20 cm and diameter is 16.8 cm. Two equal solid cones are dropped in it so that they are fully submerged. As a result, one third of the water in the original cone overflows. What is the volume of each of the solid cones submerged? **[2006]**

10. The surface area of a solid metallic sphere is 616 cm^2. It is melted and recast into smaller spheres of diameter 3.5 cm. How many such spheres can be obtained? **[2007]**

11. The given figure represents a hemisphere surmounted by a conical block of wood. The diameter of their bases is 6 cm each and the slant height of the cone is 5 cm. Calculate:

(*i*) the height of the cone.

(*ii*) the volume of the solid.



[2009]

12. A solid cone of radius 5 cm and height 8 cm is melted and made into small spheres of radius 0.5 cm. Find the number of spheres formed.

[2011]

13. A solid sphere of radius 15 cm is melted and recast into solid right circular cones of radius 2.5 cm and height 8 cm. Calculate the number of cones recast. **[2013]**

14. The surface area of a solid metallic sphere is 2464 cm^2. It is melted and recast into solid right circular cones of radius 3.5 cm and height 7 cm. Calculate:

(*i*) the radius of the sphere.

(*ii*) the number of cones recast. $\left(\text{Take } \pi = \frac{22}{7}\right)$

[2014]

15. Two solid spheres of radii 2 cm and 4 cm are melted and recast into a cone of height 8 cm. Find the radius of the cone so formed. **[2015]**

16. A certain number of metallic cones, each of radius 2 cm and height 3 cm are melted and recast into a solid sphere of radius 6 cm. Find the number of cones. **[2016]**

17. A conical tent is to accommodate 77 persons. Each person must have 16 m^3 of air to breathe. Given the radius of the tent as 7 m, find the height of the tent and also its curved surface area. **[2017]**

POINTS AT A GLANCE

Solids		Volume	Curved Surface Area	Total Surface Area
Cylinder		$\pi r^2 h$	$2\pi rh$	$2\pi r(r + h)$
Cone		$\frac{\pi r^2 h}{3}$	πrl	$\pi r(r + l)$
Sphere		$\frac{4\pi r^3}{3}$	$4\pi r^2$	$4\pi r^2$

(*iv*) $\frac{1+\tan\theta}{\sec\theta}-\frac{\sec\theta}{1+\tan\theta}=\frac{2\sin\theta\cos\theta}{\sin\theta+\cos\theta}$ (*v*) $\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}+\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}=\frac{2}{1-2\sin^2\theta}$

(*vi*) $\sqrt{\sec^2\theta+\operatorname{cosec}^2\theta}=\tan\theta+\cot\theta=\sec\theta\operatorname{cosec}\theta$

(*vii*) $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}}+\sqrt{\frac{1-\sin\theta}{1+\sin\theta}}=2\sec\theta$

(*viii*) $\frac{1}{\cos\theta+\sin\theta-1}+\frac{1}{\cos\theta+\sin\theta+1}=\sec\theta+\operatorname{cosec}\theta$

(*ix*) $\frac{\tan^3\theta+\tan\theta}{\cot^3\theta+\cot\theta}=\tan^4\theta$ (*x*) $\frac{\operatorname{cosec}\theta}{\tan\theta+\cot\theta}=\cos\theta$

(*xi*) $\sin A(1+\tan A)+\cos A(1+\cot A)=\sec A+\operatorname{cosec} A$

BOARD PAPER QUESTIONS

1. Given $5\cos A-12\sin A=0$, evaluate without using tables:
$\frac{\sin A+\cos A}{2\cos A-\sin A}$. **[1995]**

2. If $2\sin A-1=0$, show that:
$\sin 3A=3\sin A-4\sin^3 A$. **[2001]**

3. Prove that: $1-\frac{\cos^2\theta}{1+\sin\theta}=\sin\theta$. **[2001]**

4. If $\sin x=\frac{3}{5}$ and $\cos y=\frac{12}{13}$; evaluate
(*a*) $\text{secant}^2 x$ (*b*) $\tan x+\tan y$. **[2003]**

5. Without using tables, find the value of
$14\sin 30°+6\cos 60°-5\tan 45°$. **[2004]**

6. Prove that $(1+\tan A)^2+(1-\tan A)^2=2\sec^2 A$. **[2005]**

7. Prove that $\frac{\sin\theta\tan\theta}{1-\cos\theta}=1+\sec\theta$. **[2006]**

8. Prove that identity: $\frac{\sec A-1}{\sec A+1}=\frac{1-\cos A}{1+\cos A}$. **[2007]**

9. Prove the identity:
$\frac{\sin A}{1+\cos A}=\operatorname{cosec} A-\cot A$. **[2008]**

10. Prove that following identity:
$\frac{\sin A}{1+\cos A}+\frac{1+\cos A}{\sin A}=2\operatorname{cosec} A$. **[2009]**

11. Prove that $\frac{\tan^2\theta}{(\sec\theta-1)^2}=\frac{1+\cos\theta}{1-\cos\theta}$. **[2012]**

12. Show that $\sqrt{\frac{1-\cos A}{1+\cos A}}=\frac{\sin A}{1+\cos A}$. **[2000, 2013]**

13. Prove the identity
$(\sin\theta+\cos\theta)(\tan\theta+\cot\theta)=\sec\theta+\operatorname{cosec}\theta$.
[2014]

14. Prove that $\frac{\cos A}{1+\sin A}+\tan A=\sec A$. **[2016]**

15. Prove that
$(1+\cot\theta-\operatorname{cosec}\theta)(1+\tan\theta+\sec\theta)=2$ **[2018]**

16. Prove that
$(\operatorname{cosec}\theta-\sin\theta)(\sec\theta-\cos\theta)(\tan\theta+\cot\theta)=1$
[2019]

BOARD PAPER QUESTIONS

1. In triangle ABC, AD = 12 cm, angle B = 58°, the perpendicular from A to BC meets it at D. The bisector of angle ABC meets AD at E. Calculate:

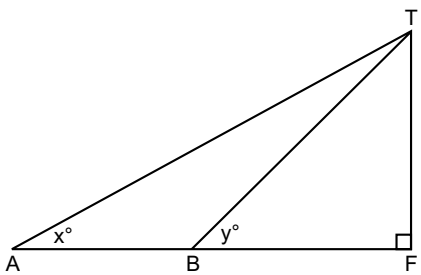
(*i*) the length of BD;

(*ii*) the length of ED.

Give your answers correct to one decimal place.

[1996]

2.



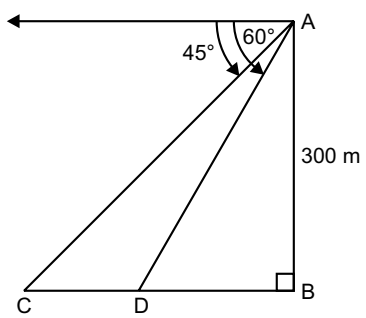
In the figure, not drawn to scale, TF is a tower. The elevation of T from A is $x°$, where $\tan x = \frac{2}{5}$ and AF = 200 m. The elevation of T from B, where AB = 80 m, is $y°$. Calculate:

(*i*) The height of the tower TF;

(*ii*) The angle y, correct to the nearest degree.

[1997]

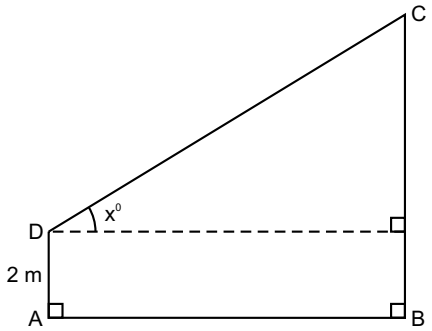
3.



The figure drawn above is not to the scale. AB is a tower, and two objects C and D are located on the ground, on the same side of AB. When observed from the top A of the tower, their angles of depression are 45° and 60°. Find the distance between the two objects, if the height of the tower is 300 m. Give your answer to the nearest metre.

[1998]

4.

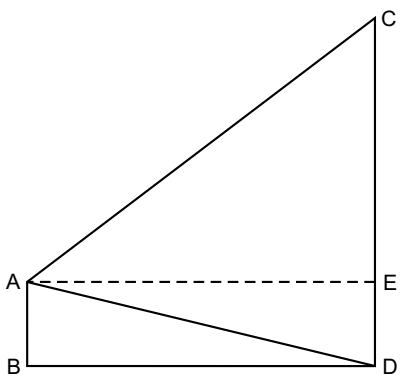


With reference to the figure given above, a man stands on the ground at a point A, which is on the same horizontal plane as B, the foot of a vertical pole BC. The height of the pole is 10 m. The man's eye is 2 m above the ground. He observes the angle of elevation at C, the top of the pole as $x°$, where $\tan x° = 2/5$. Calculate:

(*i*) the distance AB in m,

(*ii*) the angle of elevation of the top of the pole when he is standing 15 m from the pole. Give your answer to the nearest degree. See the figure above. **[1999]**

5.



From a window A, 10 m above ground the angle of elevation of the top C of a tower is $x°$, where $\tan x = 5/2$ and the angle of depression of the foot D of the tower is $y°$, where $\tan y = 1/4$. See the figure given above.

Calculate the height CD of the tower in metres.

[2000]

6. A vertical tower is 20 m high. A man standing at some distance from the tower knows that the cosine of the angle of elevation of the top of the tower is 0.53. How far is he standing from the foot of the tower? **[2001]**

7. A man standing on the bank of a river observes that the angle of elevation of a tree on the opposite bank is 60°. When he moves 50 m away from the bank, he finds the angle of elevation to be 30°. Calculate:

(*i*) the width of the river and

(*ii*) the height of the tree. **[2003]**

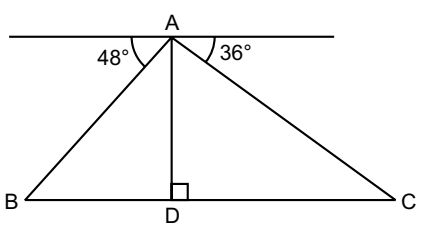
8. Two people standing on the same side of a tower in a straight line with it, measure the angles of elevation of the top of the tower as 25° and 50° respectively. If the height of the tower is 70 m, find the distance between the two people. **[2004]**

9. From the top of a cliff 92 m high, the angle of depression of a buoy is 20°. Calculate to the nearest metre, the distance of the buoy from the foot of the cliff. **[2005]**

10. A vertical pole and a vertical tower are on the same level ground. From the top of the pole the angle of elevation of the top of the tower is 60° and the angle of depression of the foot of the tower is 30°. Find the height of the tower if the height of the pole is 20 m. **[2008]**

11. From two points A and B on the same side of a building, the angles of elevation of the top of the building are 30° and 60° respectively. If the height of the building is 10 m, find the distance between A and B correct to two decimal places. **[2009]**

12. From the top of a light house 100 m high the angles of depression of two ships on opposite sides of it are 48° and 36° respectively. Find the distance between the two ships to the nearest metre.



[2010]

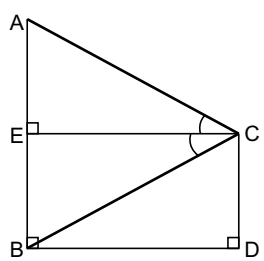
13. A man observes the angle of elevation of the top of a building to be 30°. He walks towards it in horizontal line through its base. On covering 60 m, the angle of elevation changes to 60°. Find the height of the building to the nearest metre.

[2011]

14. As observed from the top of a 80 m tall lighthouse, the angles of depression of two ships on the same side of the light house in horizontal line with its base are 30° and 40° respectively. Find the distance between the two ships. Give your answer correct to the nearest metre. **[2012]**

15. An aeroplane at an altitude of 250 m observes the angle of depression of two boats on the opposite banks of a river to be 45° and 60° respectively. Find the width of the river. Write the answer correct to the nearest whole number. **[2014]**

16. The horizontal distance between two towers is 120 m. The angle of elevation of the top and angle of depression of the bottom of the first tower as observed from the second tower is 30° and 24° respectively.



Find the height of the two towers. Give your answer correct to 3 significant figures. **[2015]**

17. An aeroplane at an altitude of 1500 metres finds that two ships are sailing towards it in the same direction. The angles of depression as observed from the aeroplane are 45° and 30° respectively. Find the distance between the two ships.

[2016]

39. In a school the money spent in the canteen by 60 students is as follows.

Money spent (in ₹)	20–30	30–40	40–50	50–60	60–70
No. of students	6	16	22	13	3

Draw a histogram and find the mode.

40. The heights of boys in a group were recorded as follows.

Height (in cm)	120–129	130–139	140–149	150–159	160–169	170–179
No. of boys	2	12	16	10	6	2

Estimate the mode by drawing a histogram for the above frequency distribution.

BOARD PAPER QUESTIONS

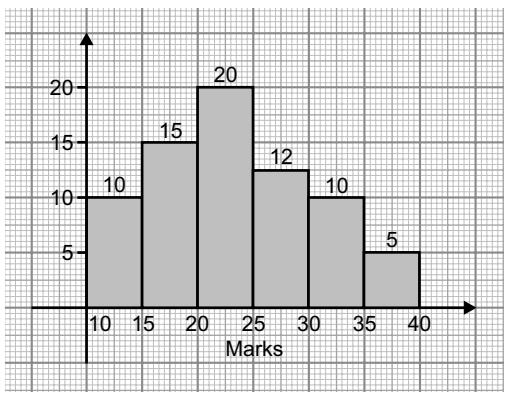
1.

Category	A	B	C	D	E	F	G
Wages in ₹/day	50	60	70	80	90	100	110
Number of Workers	2	4	8	12	10	6	8

(*i*) Calculate the mean wage, correct to the nearest rupee.

(*ii*) If the number of workers in each category is doubled, what would be the new mean wage? **[1995]**

2. The histogram alongside represents the marks obtained by some candidates in an examination. Using the data in the diagram, calculate the mean mark. **[1996]**



3. Attempt this question on a graph paper.

The table shows the distribution of marks gained by a group of 400 students in an examination:

Marks less than	10	20	30	40	50	60	70	80	90	100
No. of Students	5	10	30	60	105	180	270	355	390	400

Using a scale of 2 cm to represent 10 marks and 2 cm to represent 50 students, plot these values and draw a smooth curve through the points. Estimate from the graph (*i*) the median mark (*ii*) the quartile marks. **[1997]**

4. The daily profit in Rupees, of 100 shops in a department store, are distributed as follows:

Profit per shop (in ₹)	0—100	100—200	200—300	300—400	400—500	500—600
No. of shops	12	18	27	20	17	6

Draw a histogram of the data given above, on graph paper, and estimate the mode. **[1998]**

5. Attempt this question on a graph paper. The table shows the distribution of the daily wages, earned by 160 workers in a building site.

Wages in ₹ per day	0—10	10—20	20—30	30—40	40—50	50—60	60—70	70—80
No. of workers	12	20	30	38	24	16	12	8

Using a scale of 2 cm to represent ₹10 and 2 cm to represent 20 workers, plot these values, and draw a smooth ogive, through the points. Estimate from the graph—

(*i*) The median wage;

(*ii*) The upper and lower quartile wages earned by the workers. **[1998]**

6. Given below are the weekly wages of 200 workers in a small factory:

Weekly wages in ₹	80—100	110—120	120—140	140—160	160—180
No. of workers	20	30	20	40	90

Calculate the mean weekly wage of the workers. **[1998]**

7. The marks obtained by a set of students in an examination are given below:

Marks:	5	10	15	20	25	30
No. of students:	6	4	6	12	y	4

Given that the mean mark of the set is 18, calculate the numerical value of y. **[1999]**

8. (*i*) Using the table, calculate the cumulative frequencies of workers.

(*ii*) Draw the cumulative frequency curve.

Use 2 cm = ₹500, starting the origin at ₹6500 on x axis, and 2 cm = 10 workers on the y axis.

(*iii*) Use your graph to write down the median wage in ₹.

Wages in ₹	No. of workers	Cumulative Frequency
6500–7000	10	.....
7000–7500	18	.....
7500–8000	22	.....
8000–8500	25	.....
8500–9000	17	.....
9000–9500	10	.....
9500–10000	8	.....

[1999]

9. Calculate the mean, the median and the mode of the following numbers:

3, 1, 5, 6, 3, 4, 5, 3, 7, 2 **[2000]**

10. The marks of 20 students in a test were as follows:

5, 6, 8, 9, 10, 11, 11, 12, 13, 13, 14, 14, 15, 15, 15, 16, 16, 18, 19, 20.

Calculate:

(*i*) The mean (*ii*) The median

(*iii*) The mode **[2002]**

11. The following table gives the weekly wages of workers in a factory:

Weekly wages in ₹	50–55	55–60	60–65	65–70	70–75	75–80	80–85	85–90
No. of workers	5	20	10	10	9	6	12	8

Calculate:

(*i*) The mean. (*ii*) The modal class.

(*iii*) The number of workers getting weekly wages, below ₹80/-.

(*iv*) The number of workers getting ₹65/- or more, but less than ₹85/- as weekly wages. **[2002]**

12. The marks obtained by 120 students in a Mathematics test is given below:

Marks	0—10	10—20	20—30	30—40	40—50	50—60	60—70	70—80	80—90	90—100
No. of Students	5	9	16	22	26	18	11	6	4	3

Draw an Ogive for the given distribution on a graph sheet. Use a suitable scale for your Ogive. Use your Ogive to estimate:

(*i*) The median. (*ii*) The lower quartile.

(*iii*) The number of students who obtained more than 75% in the test;

(*iv*) The number of students who did not pass in the test if the pass percentage was 40. **[2002]**

13. Draw a histogram and hence estimate the mode for the following frequency distribution:

Class	0–10	10–20	20–30	30–40	40–50	50–60
Frequency	2	8	10	5	4	3

[2003]

14. Find the mean of the following frequency distribution: **[2003]**

Class Interval	0–50	50–100	100–150	150–200	200–250	250–300
Frequency	4	8	16	13	6	3

15. For the following set of numbers, find the median:

10, 75, 3, 81, 17, 27, 4, 48, 12, 47, 9, 15 **[2004]**

16. For the following frequency distribution, draw a histogram. Hence, calculate the mode.

Class	0–5	5–10	10–15	15–20	20–25	25–30
Frequency	2	7	18	10	8	5

[2004]

17. The marks obtained by 200 students in an examination are given below:

Marks	0—10	10—20	20—30	30—40	40—50	50—60	60—70	70—80	80—90	90—100
No. of Students	5	10	11	20	27	38	40	29	14	6

Using a graph paper, draw an ogive for the above distribution. Use your ogive to estimate:

(*i*) the median; (*ii*)the lower quartile;

(*iii*) the number of students who obtained more than 80% marks in the examination and

(*iv*) the number of students who did not pass, if the pass percentage was 35.

Use the scale as 2 cm = 10 marks on one axis and 2 cm = 20 students on the other axis. **[2004]**

18. If the mean of the following distribution is 7.5, find the missing frequency '*f*': **[2005]**

Variable:	5	6	7	8	9	10	11	12
Frequency:	20	17	*f*	10	8	6	7	6

19. The median of the following observations 11, 12, 14, 18, (x + 4), 30, 32, 35, 41 arranged in ascending order is 24. Find x. **[2006]**

20. Draw a histogram to represent the following data:

Pocket money in ₹	150–200	200–250	250–300	300–350	350–400
No. of Students	10	5	7	4	3

21. The daily wages of 160 workers in a building project are given below:

Wages in ₹ per day	0—10	10—20	20—30	30—40	40—50	50—60	60—70	70—80
No. of workers	12	20	30	38	24	16	12	8

Using a graph paper, draw an ogive for the above distribution.

Use your ogive to estimate:

(*i*) the median wage of the workers

(*ii*) the upper quartile wage of the workers

(*iii*) the lower quartile wage of the workers

(*iv*) the percentages of workers who earn more than ₹45 a day. **[2006]**

22. The table below shows the distribution of the scores obtained by 120 shooters in a shooting competition. Using a graph sheet, draw an ogive for the distribution.

Scores obtained	0—10	10—20	20—30	30—40	40—50	50—60	60—70	70—80	80—90	90—100
No. of shooters	5	9	16	22	26	18	11	6	4	3

Use your ogive to estimate:

(*i*) The median (*ii*) The inter quartile range

(*iii*) The number of shooters who obtained more than 75% scores. **[2007]**

23. Find the mean of the following distribution:

Class Interval	0–10	10–20	20–30	30–40	40–50
Frequency	10	6	8	12	5

[2007]

24. The weights of 50 apples were recorded as given below. Calculate the mean weight, to the nearest gram, by the step-deviation method. **[2008]**

Weight in grams	80–85	85–90	90–95	95–100	100–105	105–110	110–115
No. of apples	5	8	10	12	8	4	3

25. Using a graph paper, draw an *ogive* for the following distribution which shows the marks obained in the General Knowledge paper by 100 students.

Marks	0—10	10—20	20—30	30—40	40—50	50—60	60—70	70—80
No. of students	5	10	20	25	15	12	9	4

Use the ogive to estimate:

(*i*) the median

(*ii*) the number of students who score marks above 65. **[2008]**

26. Attempt this question on graph paper.

Marks obtained by 200 students in examination are given below:

Marks	0—10	10—20	20—30	30—40	40—50	50—60	60—70	70—80	80—90	90—100
No. of students	5	10	14	21	25	34	36	27	16	12

Draw an ogive for the given distribution taking 2 cm = 10 marks on one axis and 2 cm = 20 students on the other axis.

From the graph find:

(*i*) the median (*ii*) the upper quartile

(*iii*) Number of students scoring above 65 marks.

(*iv*) If 10 students qualify for merit scholarship, find the minimum marks required to qualify. **[2009]**

27. In a school the weekly pocket money of 50 students is as follows:

Weekly pocket money in ₹	40—50	50—60	60—70	70—80	80—90	90—100
No. of students	2	8	12	14	8	6

Draw a histogram and a frequency polygon on the same graph. Find the mode from the same graph. **[2009]**

28. Find the mean, median and mode of the following distribution:

8, 10, 7, 6, 10, 11, 6, 13, 10 **[2009]**

29. The following table gives the wages of workers in a factory:

Wages in ₹	45–50	50–55	55–60	60–65	65–70	70–75	75–80
No. of workers	5	8	30	25	14	12	6

Calculate the mean by the short cut method. **[2009]**

30. The mean of the following distribution is 52 and the frequency of class interval 30-40 is '*f*'. Find '*f*'.

Class Interval	10—20	20—30	30—40	40—50	50—60	60—70	70—80
Frequency	5	3	*f*	7	2	6	13

[2010]

31. The monthly income of a group of 320 employees in a company is given below:

Monthly Income	**No. of Employees**
6000–7000	20
7000–8000	45
8000–9000	65
9000–10000	95
10000–11000	60
11000–12000	30
12000–13000	5

Draw an ogive of the given distribution on a graph sheet taking 2 cm = ₹1000 on one axis and 2 cm = 50 employees on the other axis. From the graph, determine:

(*i*) the median wage

(*ii*) the number of employees whose income is below ₹8500.

(*iii*) If the salary of a senior employee is above ₹11,500, find the number of senior employees in the company.

(*iv*) the upper quartile. **[2010]**

32. Marks obtained by 200 students in an examination are given below:

Marks	0—10	10—20	20—30	30—40	40—50	50—60	60—70	70—80	80—90	90—100
No. of students	5	11	10	20	28	37	40	29	14	6

Draw an ogive for the given distribution taking 2 cm = 10 marks on one axis and 2 cm = 20 students on the other axis. Using the graph, determine

(*i*) The median marks.

(*ii*) The number of students who failed if minimum marks required to pass is 40.

(*iii*) If scoring 85 and more marks is considered as grade one, find the number of students who secured grade one in the examination. **[2011]**

33. A Mathematics aptitude test of 50 students was recorded as follows:

Marks	50–60	60–70	70–80	80–90	90–100
No. of students	4	8	14	19	5

Draw a histogram for the above data using graph paper and locate the mode. **[2011]**

34. (*i*) Using step-deviation method, calculate the mean marks of the following distribution.

(*ii*) State the modal class.

Class Interval	50–55	55–60	60–65	65–70	70–75	75–80	80–85	85–90
Frequency	5	20	10	10	9	6	12	8

[2011]

35. Marks obtained by 40 students in a short assessment is given below, where a and b are two missing data.

Marks	5	6	7	8	9
Number of Students	6	a	16	13	b

If the mean of the distribution is 7.2, find a and b. **[2012]**

36. Find the mode and median of the following frequency distribution:

x	10	11	12	13	14	15
f	1	4	7	5	9	3

[2012]

37. The following distribution represent the height of 160 students of a school.

Height (in cm)	**No. of Students**
140–145	12
145–150	20
150–155	30
155–160	38
160–165	24
165–170	16
170–175	12
175–180	8

Draw an ogive for the given distribution taking 2 cm = 5 cm of height on one axis and 2 cm = 20 students on the other axis. Using the graph, determine:

(*i*) The median height.

(*ii*) The interquartile range.

(*iii*) The number of student whose height is above 172 cm. **[2012]**

38. The marks obtained by 120 students in a test are given below:

Marks	0—10	10—20	20—30	30—40	40—50	50—60	60—70	70—80	80—90	90—100
No. of students	5	9	16	22	26	18	11	6	4	3

Draw an ogive for the given distribution on a graph sheet.

Use suitable scale for ogive to estimate the following:

(*i*) The median.

(*ii*) The number of students who obtained more than 75% marks in the test.

(*iii*) The number of students who did not pass the test if minimum marks required to pass is 40. **[2013]**

39. Draw a histogram from the following frequency distribution and find the mode from the graph:

Class	0–5	5–10	10–15	15–20	20–25	25–30
Frequency	2	5	18	14	8	5

[2013]

40. Find the mean of the following distribution by step-deviation method:

Class Interval	20—30	30—40	40—50	50—60	60—70	70—80
Frequency	10	6	8	12	5	9

[2006, 2013]

41. The median of the following observations 11, 12, 14, $(x - 2)$, $(x + 4)$, $(x + 9)$, 32, 38, 47 arranged in ascending order is 24. Find the value of x and hence find the mean. **[2013]**

42. The numbers 6, 8, 10, 12, 13 and x are arranged in an ascending order. If the mean of the observations is equal to the median, find the value of x.

[2014]

43. The marks obtained by 30 students in a class assessment of 5 marks is given below:

Marks	0	1	2	3	4	5
No. of students	1	3	6	10	5	5

Calculate the mean, median and mode of the above distribution. **[2015]**

44. Calculate the mean of the following distribution:

Class interval	**Frequency**
0—10	8
10—20	5
20—30	12
30—40	35
40—50	24
50—60	16

[2015]

45. The weight of 50 workers is given below:

Weight in kg	No. of workers
50—60	4
60—70	7
70—80	11
80—90	14
90—100	6
100—110	5
110—120	3

Draw an ogive of the given distribution using a graph sheet. Take 2 cm = 10 kg on one axis and 2 cm = 5 workers along the other axis. Use a graph to estimate the following:

(*i*) the upper and lower quartiles.

(*ii*) if weighing 95 kg and above is considered overweight, find the number of workers who are overweight. **[2015]**

46. The mean of following numbers is 68. Find the value of '*x*'.

45, 52, 60, *x*, 69, 70, 26, 81, and 94.

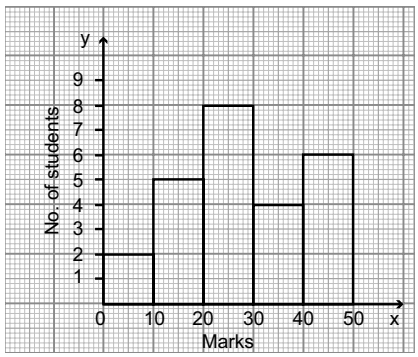
Hence estimate the median. **[2016]**

47. The histogram below represents the scores obtained by 25 students in a Mathematics mental test. Use the data to:

(*i*) Frame a frequency distribution table.

(*ii*) To calculate mean.

(*iii*) To determine the Modal class. **[2016]**



48. If the mean of the following distribution is 24, find the value of '*a*'. **[2018]**

Marks	0—10	10—20	20—30	30—40	40—50
Number of students	7	*a*	8	10	5

COMMON ERRORS

1. Forgetting to put kink for histogram or ogive when not starting at (0, 0).
2. Assuming wrongly that inter-quartile range is $\frac{3N}{4}-\frac{N}{4}$.
3. In calculation of mean by step-deviation method assuming wrongly that $i = 9$ when classes are 0 – 9, 10 – 19, 20 – 29, etc. [$i = 10$ here].
4. While finding the median from Raw Data, forgetting to arrange the given numbers in ascending (or descending) order.

POINTS AT A GLANCE

1. Raw data

- Range = Highest value – Lowest value
- Mean = $\frac{\text{Sum of observations}}{\text{No. of observations}}$
- Mode = Most common number
- Median = The middlemost number after arranging the data in ascending or descending order

(*i*) If n is odd,

$$\text{Median} = \frac{n+1}{2}\text{th observation or Middle observation}$$

3. Identical cards are marked 1 to 100. When a card is drawn at random, what is the probability that it is

(*i*) a multiple of 11 (*ii*) a perfect square

(*iii*) a number in which one digit is twice the other (*iv*) a multiple of 2 and 3

(*v*) a prime number containing the digit 9

4. On a square paper of side 20 cm, a circle of diameter 10 cm is drawn. Assuming that it falls on the paper, what is the probability that a tiny cube dropped on the paper falls $\left[\text{Take } \pi = \frac{22}{7}\right]$

(*i*) within the circle (*ii*) outside the circle?

5. When 3 unbiased coins are tossed, what is the probability of getting

(*i*) no heads (*ii*) at least 2 tails

6. When a deck of playing cards is well shuffled and a card is randomly picked up, what is the probability that it is

(*i*) a red card or a queen (*ii*) a multiple of 3

(*iii*) a red face card (*iv*) a nine and an ace

7. From the letters of the word MATHEMATICS, a letter is chosen randomly. What is the probability that it is a

(*i*) Vowel (*ii*) Consonant

(*iii*) M or T

8. When two dice are rolled, what is the probability that on the uppermost faces

(*i*) the sum of two numbers is less than 5 (*ii*) the product of the numbers is 6

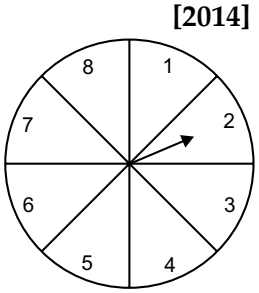
(*iii*) the sum is divisible by 5

9. A dice has the following numbers marked on its 6 faces : 1, 2, 3, –1, –2, –3. What is the probability of getting **[2014]**

(*i*) a positive integer (*ii*) an integer less than –1

(*iii*) the smallest integer

10. A spinning arrow pivoted at the centre of a dial is shown in the figure. When turned, the arrow points to any of the 8 numbers. What is the probability that the arrow points at



(*i*) a perfect cube (*ii*) an odd number

(*iii*) a factor of 8 (*iv*) the smallest prime number

11. Odd numbered cards 1, 3, 5, 7, 9, 11, ..., 27, 29 were shuffled and put in a box. If a card is picked up from this box, what is the probability that the number on the card is a

(*i*) prime number (*ii*) factor of 21 (*iii*) multiple of 5

BOARD PAPER QUESTIONS

1. A dice is thrown once. What is the probability that the

(*i*) number is even

(*ii*) number is greater than 2? **[2009]**

2. Cards marked with numbers 1, 2, 3, 4, ..., 20 are well shuffled and a card is drawn at random. What is the probability that the number on the card is

(*i*) a prime number (*ii*) divisible by 3

(*iii*) a perfect square? **[2010]**

3. A box contains some black balls and 30 white balls. If the probability of drawing a black ball is two-fifths of a white ball, find the number of black balls in the box. **[2013]**

4. A bag contains 5 white balls, 6 red balls and 9 green balls. A ball is drawn at random from the bag. Find the probability that the ball drawn is:
(*i*) a green ball (*ii*) a white or a red ball
(*iii*) is neither a green ball nor a white ball. **[2015]**

5. A game of numbers has cards marked with 11, 12, 13, ..., 40. A card is drawn at random. Find the Probability that the number on the card drawn is:
(*i*) A perfect square
(*ii*) Divisible by 7 **[2016]**

6. Sixteen cards are labelled as a, b, c, d,..., m, n, o, p. They are put in a box and shuffled. A boy is asked to draw a card from the box. What is the probability that the card drawn is:
(*i*) A vowel? (*ii*) A consonant?
(*iii*) None of the letters of the word 'median'. **[2017]**

7. Cards bearing numbers 2, 4, 6, 8, 10, 12, 14, 16, 18 and 20 are kept in a bag. A card is drawn at random from the bag. Find the probability of getting a card which is:
(*i*) a prime number.
(*ii*) a number divisible by 4.
(*iii*) a number that is a multiple of 6.
(*iv*) an odd number. **[2018]**

8. There are 25 discs numbered 1 to 25. They are put in a closed box and shaken thoroughly. A disc is drawn at random from the box.
Find the probability that the number on the disc is:
(*i*) an odd number
(*ii*) divisible by 2 and 3 both.
(*iii*) a number less than 16. **[2019]**

POINTS AT A GLANCE

1. Probability of an event or an outcome = $\frac{\text{No. of favourable outcomes}}{\text{No. of all possible outcomes}}$

2. P(event) + P(not event) = 1 **3.** $0 \le$ Probability of an event ≤ 1

4. If the probability of an event = 0, the event is an impossible event.

5. If the probability of an event = 1, the event is a certain event or a sure event.

6. If a coin is tossed twice or 2 coins are tossed simultaneously, the total no. of possible outcomes = $2^2 = 4$

7. If a coin is tossed n times or n coins are tossed together = 2^n outcomes are possible.

8. If 2 dice are rolled simultaneously or one dice two times, the total no. of outcomes = $6^2 = 36$

9. (*i*) At least 3 $\Rightarrow$ 3 or more (*ii*) At most 2 $\Rightarrow$ 0 or 1 or 2

10. (*i*) multiples of 2 and 3 $\Rightarrow$ multiples of 6 $\Rightarrow$ 6, 12 18, etc.
(*ii*) multiples of 2 or 3 $\Rightarrow$ multiples of 2 as well as multiples of 3 $\Rightarrow$ 2, 3, 4, 6, 8, 9, 10, 12, etc.

11. Favourable outcomes should be listed in each case.

DO YOU KNOW?

BLAISE PASCAL (1623–1662)

Blaise Pascal, a French mathematician, was a child prodigy. He invented the first digital calculator and hydraulic press. He gave a tabular presentation for binomial coefficients in the expansion of $(a + b)^n$. In Pascal's triangle, each number is the sum of the numbers above it.

Probability had its origin in the games of chance. In 1654, prompted by a friend interested in gambling problems, Blaise Pascal corresponded with Fermat and from that collaboration was born the **Theory of Probability.** Blaise Pascal helped in creating two major new areas of research. He wrote a significant treatise on the subject of **Projective Geometry** and later on **Probability Theory** strongly influencing Modern Economics and Social Science.

Blaise Pascal