CHAPTER 1

Relations and Functions

CASE STUDY / PASSAGE BASED QUESTIONS

1

A relation R on a set A is said to be an equivalence relation on A iff it is

- Reflexive *i.e.*, $(a, a) \in R \ \forall\ a \in A$.
- Symmetric *i.e.*, $(a, b) \in R \Rightarrow (b, a) \in R \ \forall\ a, b \in A$.
- Transitive *i.e.*, $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R \ \forall\ a, b, c \in A$.

Based on the above information, answer the following questions.

(i) If the relation $R = \{(1, 1), (1, 2), (1, 3), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$ defined on the set $A = \{1, 2, 3\}$, then R is

(a) reflexive (b) symmetric (c) transitive (d) equivalence

(ii) If the relation $R = \{(1, 2), (2, 1), (1, 3), (3, 1)\}$ defined on the set $A = \{1, 2, 3\}$, then R is

(a) reflexive (b) symmetric (c) transitive (d) equivalence

(iii) If the relation R on the set N of all natural numbers defined as $R = \{(x, y) : y = x + 5$ and $x < 4\}$, then R is

(a) reflexive (b) symmetric (c) transitive (d) equivalence

(iv) If the relation R on the set $A = \{1, 2, 3, ..., 13, 14\}$ defined as $R = \{(x, y) : 3x - y = 0\}$, then R is

(a) reflexive (b) symmetric (c) transitive (d) None of these

(v) If the relation R on the set $A = \{1, 2, 3\}$ defined as $R = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$, then R is

(a) reflexive only (b) symmetric only
(c) transitive only (d) equivalence

2

Consider the mapping $f : A \to B$ is defined by $f(x) = \dfrac{x-1}{x-2}$ such that f is a bijection.

Based on the above information, answer the following questions.

(i) Domain of f is

(a) $R - \{2\}$ (b) R (c) $R - \{1, 2\}$ (d) $R - \{0\}$

Syllabus

Types of relations: reflexive, symmetric, transitive and equivalence relations. One to one and onto functions.

(ii) Range of f is

(a) R (b) $R-\{1\}$ (c) $R-\{0\}$ (d) $R-\{1, 2\}$

(iii) If $g: R-\{2\} \to R-\{1\}$ is defined by $g(x) = 2f(x) - 1$, then $g(x)$ in terms of x is

(a) $\frac{x+2}{x}$ (b) $\frac{x+1}{x-2}$ (c) $\frac{x-2}{x}$ (d) $\frac{x}{x-2}$

(iv) The function g defined above, is

(a) One-one (b) Many-one (c) into (d) None of these

(v) A function $f(x)$ is said to be one-one iff

(a) $f(x_1) = f(x_2) \Rightarrow -x_1 = x_2$ (b) $f(-x_1) = f(-x_2) \Rightarrow -x_1 = x_2$

(c) $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$ (d) None of these

HINTS & EXPLANATIONS

1. (i) (a): Clearly, (1, 1), (2, 2), (3, 3), $\in R$. So, R is reflexive on A.
Since, $(1, 2) \in R$ but $(2, 1) \notin R$. So, R is not symmetric on A.
Since, $(2, 3), \in R$ and $(3, 1) \in R$ but $(2, 1) \notin R$. So, R is not transitive on A.

(ii) (b): Since, (1, 1), (2, 2) and (3, 3) are not in R.
So, R is not reflexive on A.
Now, $(1, 2) \in R \Rightarrow (2, 1) \in R$
and $(1, 3) \in R \Rightarrow (3, 1) \in R$.
So, R is symmetric
Clearly, $(1, 2) \in R$ and $(2, 1) \in R$ but $(1, 1) \notin R$.
So, R is not transitive on A.

(iii) (c): We have, $R = \{(x, y) : y = x + 5 \text{ and } x < 4\}$, where $x, y \in N$.
$\therefore \quad R = \{(1, 6), (2, 7), (3, 8)\}$
Clearly, (1, 1), (2, 2) etc. are not in R. So, R is not reflexive.
Since, $(1, 6) \in R$ but $(6, 1) \notin R$. So, R is not symmetric.
Since, $(1, 6) \in R$ and there is no order pair in R which has 6 as the first element. Same is the case for (2, 7) and (3, 8).
So, R is transitive.

(iv) (d): We have, $R = \{(x, y) : 3x - y = 0\}$, where $x, y \in A = \{1, 2, ..., 14\}$
$\therefore \quad R = \{(1, 3), (2, 6), (3, 9), (4, 12)\}$
Clearly, $(1, 1) \notin R$. So, R is not reflexive on A.
Since, $(1, 3) \in R$ but $(3, 1) \notin R$. So, R is not symmetric on A.
Since, $(1, 3) \in R$ and $(3, 9) \in R$ but $(1, 9) \notin R$. So, R is not transitive on A.

(v) (d): Clearly, (1, 1), (2, 2), (3, 3) $\in R$. So, R is reflexive on A.
We find that the ordered pairs obtained by interchanging the components of ordered pairs in R are also in R. So, R is symmetric on A.
For $1, 2, 3 \in A$ such that (1, 2) and (2, 3) are in R implies that (1, 3) is also, in R. So, R is transitive on A.
Thus, R is an equivalence relation.

2. (i) (a): For $f(x)$ to be defined $x - 2 \neq 0$ *i.e.*, $x \neq 2$
$\therefore$ Domain of $f = R - \{2\}$

(ii) (b): Let $y = f(x)$, then $y = \frac{x-1}{x-2}$

$$\Rightarrow xy - 2y = x - 1 \Rightarrow xy - x = 2y - 1 \Rightarrow x = \frac{2y-1}{y-1}$$

Since, $x \in R - \{2\}$, therefore $y \neq 1$
Hence, range of $f = R - \{1\}$

(iii) (d): We have, $g(x) = 2f(x) - 1$

$$= 2\left(\frac{x-1}{x-2}\right) - 1 = \frac{2x-2-x+2}{x-2} = \frac{x}{x-2}$$

(iv) (a): We have, $g(x) = \frac{x}{x-2}$

Let $g(x_1) = g(x_2) \Rightarrow \frac{x_1}{x_1-2} = \frac{x_2}{x_2-2}$

$$\Rightarrow x_1x_2 - 2x_1 = x_1x_2 - 2x_2 \Rightarrow 2x_1 = 2x_2 \Rightarrow x_1 = x_2$$

Thus, $g(x_1) = g(x_2) \Rightarrow x_1 = x_2$
Hence, $g(x)$ is one-one.

(v) (c)