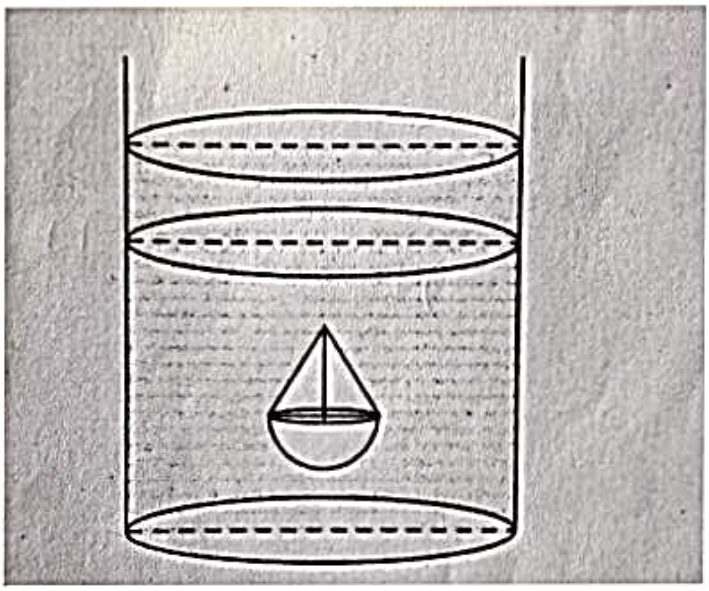
Mensuration

Q1. A solid is in the shape of a hemisphere of radius 7 cm, surmounted by a cone of height 4 cm. The solid is immersed completely in a cylindrical container filled with water to a certain height. If the radius of the cylinder is 14 cm, find the rise in the water level. [2023]



Solution: 1.5 cm

Step-by-step Explanation:

$$\textit{Volume of the solid} = \textit{Volume of cone} + \textit{Volume of hemisphere}$$

$$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$

$$= \frac{1}{3}\pi r^2 (h + 2r)$$

$$= \frac{1}{3} \times \frac{22}{7} \times 7^2 (4 + 14)$$

$$= \frac{1}{3} \times \frac{22}{7} \times 49 \times 18$$

Water will raise in the shape of cylinder.
Volume of the water raised = volume of the solid

$$\pi R^2 H = \frac{1}{3} \times \frac{22}{7} \times 49 \times 18$$

$$\Rightarrow \frac{22}{7} \times 14 \times 14 \times H = \frac{1}{3} \times \frac{22}{7} \times 49 \times 18$$

$$\Rightarrow 14 \times 14 \times H = 49 \times 6$$

$$\Rightarrow H = \frac{49 \times 6}{14 \times 14}$$

$$\Rightarrow H = \frac{3}{2}$$

$$\Rightarrow H = 1.5\text{cm}$$

Height of water raised is 1.5cm.

Q2. Volume of a cylinder of height 3 cm is 48 π. Radius of the cylinder is:
(a) 48 cm
(b) 16 cm
(c) 4 cm
(d) 24 cm [2023]

Solution: (c) 4 cm

Step-by-step solution:

$$\textit{Volume of the cylinder} = \pi r^2 h$$

$$\pi r^2 \times 3 = 48\pi$$

$$r^2 = \frac{48}{3}$$

$$r^2 = 16$$

$$r = \sqrt{16}$$

$$r = 4 \text{ cm}$$

Q3. A solid cone of radius 5 cm and height 9 cm is melted and made into small cylinders of radius of 0.5 cm and height 1.5 cm. find the number of cylinders so formed. [2022 Semester-2]

Solution: 200

Step-by-step Explanation:

Let the number of small cylinders be n.

volume of all the small cylinders = Volume of cone

$$n \times \pi r^2 h = \frac{1}{3}\pi R^2 H$$

$$n \times (0.5)^2 \times 1.5 = \frac{1}{3} \times 5^2 \times 9$$

$$n = \frac{25 \times 9}{3 \times 0.25 \times 1.5}$$

$$n = \frac{25 \times 9 \times 1000}{3 \times 25 \times 15}$$

$$n = 200$$

Total number of small cylinders formed is 200.

Q4. The radius of a roller 100 cm long is 14 cm. The curved surface area of the roller is: (Take $\pi = 22/7$)

(a) 13200 cm^2
(b) 15400 cm^2
(c) 4400 cm^2
(d) 8800 cm^2 [2022 Semester-2]

Solution: (d) 8800 cm^2

Step-by-step explanation:

curved surface area of the roller = curved surface area of the cylinder

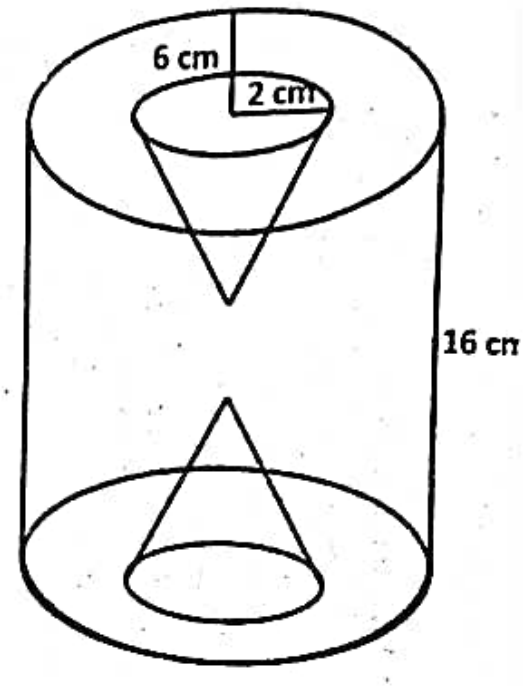
$$= 2\pi rh$$

$$= 2 \times \frac{22}{7} \times 14 \times 100$$

$$= 8800 \text{ cm}^2$$

Q5. A solid wooden cylinder is of radius 6cm and height 16cm. Two cones each of radius 2cm and height 6cm are drilled out of the cylinder. Find the volume of the remaining solid. [Take π = 22/7]

[2022 Semester-2]



Solution: 1760 cm^3

Step-by-step Explanation:

$$\text{Volume of the remaining solid}$$
$$= \text{Volume of Cylinder} - \text{Volume of 2 cones}$$
$$= \pi R^2 H - 2\left(\frac{1}{3}\pi r^2 h\right)$$
$$= \pi\left(R^2 H - \frac{2}{3} r^2 h\right)$$
$$= \frac{22}{7}\left(6 \times 6 \times 16 - \frac{2}{3} \times 2 \times 2 \times 6\right)$$
$$= \frac{22}{7}\left(576 - 16\right)$$
$$= \frac{22}{7} \times 560$$
$$= 22 \times 80$$
$$= 1760 \text{ cm}^3$$

Q6. The volume of a conical tent is 462 m^3 and the area of the base is 154 m^2. The height of the cone is:
(a) 15 m
(b) 12 m
(c) 9 m
(d) 24 m [2022 Semester-2]

Solution: (c) 9 m

Step-by-step Explanation:

Area of the base of the cone $= \pi r^2$

$$\Rightarrow \pi r^2 = 154$$

$$\Rightarrow r^2 = 154 \times \frac{7}{22}$$

$$\Rightarrow r = \sqrt{7 \times 7}$$

$$\Rightarrow r = 7$$

Volume of conical tent $= 462\ m^3$

$$\frac{1}{3}\pi r^2 h = 462$$

$$\Rightarrow \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times h = 462$$

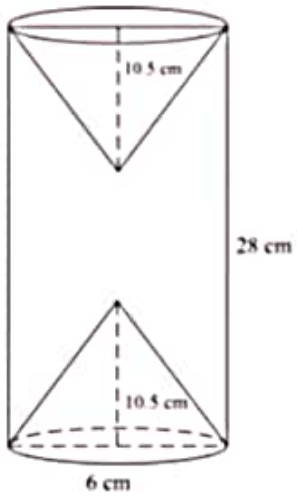
$$\Rightarrow \frac{22 \times 7 \times h}{3} = 462$$

$$\Rightarrow h = \frac{462 \times 3}{22 \times 7}$$

$$\Rightarrow h = 9$$

Hence, height of the cone is 9 m.

Q7. From a solid wooden cylinder of height 28 cm and diameter 6 cm, two conical cavities are hollowed out. The diameters of the cones are also of 6 cm and height 10.5 cm. Taking $\pi = 22/7$ find the volume of the remaining solid. [2020]



Solution: 594 cm^3

Step-by-step Explanation:

$$\begin{aligned}
&\textit{Volume of the remaining solid}\\
&= \textit{Volume of Cylinder} - \textit{Volume of 2 cones}\\
&= \pi r^2 H - 2 \times \frac{1}{3}\pi r^2 h\\
&= \pi r^2 \left(H - \frac{2}{3}h\right)\\
&= \frac{22}{7} \times 3 \times 3 \left(28 - \frac{2}{3} \times 10.5\right)\\
&= \frac{198}{7}(28 - 7)\\
&= \frac{198}{7} \times 21\\
&= 594
\end{aligned}$$

Hence, volume of the remaining solid is 594 cm^3.

Q8. A solid spherical ball of radius 6 cm is melted and recast into 64 identical spherical marbles. Find the radius of each marble. [2020]

Solution: 1.5 cm

Step-by-step Explanation:

$$\begin{aligned}
\textit{Volume of 64 identical spheres} &= \textit{Volume of big sphere}\\
64 \times \frac{4}{3}\pi r^3 &= \frac{4}{3}\pi R^3\\
\Rightarrow 64 \times r^3 &= 6 \times 6 \times 6\\
\Rightarrow r^3 &= \frac{6 \times 6 \times 6}{64}\\
\Rightarrow r &= \sqrt[3]{\frac{6 \times 6 \times 6}{64}}\\
\Rightarrow r &= \frac{6}{4}\\
\Rightarrow r &= 1.5
\end{aligned}$$

Hence, radius of each identical hemisphere is 1.5 cm.

Q9. A solid metallic sphere of radius 6 cm is melted and made into a solid cylinder of height 32 cm. Find the : [4]

(i) radius of the cylinder
(ii) curved surface area of the cylinder [Take $\pi = 3.1$] [2019]

Solution: (i) 3 cm (ii) 595.2 cm^2

Step-by-step Explanation:

$$Volume\ of\ solid\ cylinder\ =\ Volume\ of\ sphere$$

$$\pi r^2 h\ =\ \frac{4}{3}\pi R^3$$

$$\Rightarrow r^2 \times 32\ =\ \frac{4}{3} \times 6 \times 6 \times 6$$

$$r^2 = \frac{4 \times 6 \times 6 \times 6}{3 \times 32}$$

$$\Rightarrow r^2 = 3 \times 3$$

$$\Rightarrow r = 3$$

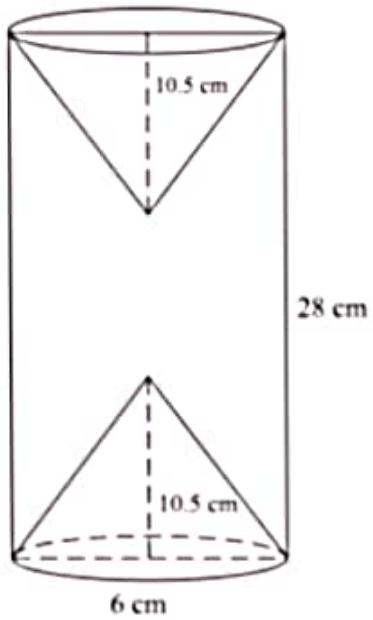
Curved surface area of the cylinder

$$= 2\pi rh$$

$$= 2 \times 3.1 \times 3 \times 32$$

$$= 595.2\ cm^2$$

Q10. A hemispherical and conical hole is scooped out of a solid wooden cylinder. Find the volume of the remaining solid where the measurements are as follows: [4] [2019]



The height of the solid cylinder is 7 cm, radius of each of hemisphere, cone and cylinder is 3 cm. Height of cone is 3 cm. Give your answer correct to the nearest whole number. [Take $\pi = 22/7$]

Solution: 113 cm^3

Step-by-step Explanation:

(i) *Circumference of the base of the cylinder* $= 132\ cm$

$$\Rightarrow 2\pi r = 132$$

$$\Rightarrow r = \frac{132 \times 7}{22 \times 2}$$

$$\Rightarrow r = 21$$

Hence, radius of the cylinder is $21\ cm$.

(ii) *Volume of the cylinder* $= \pi r^2 h$

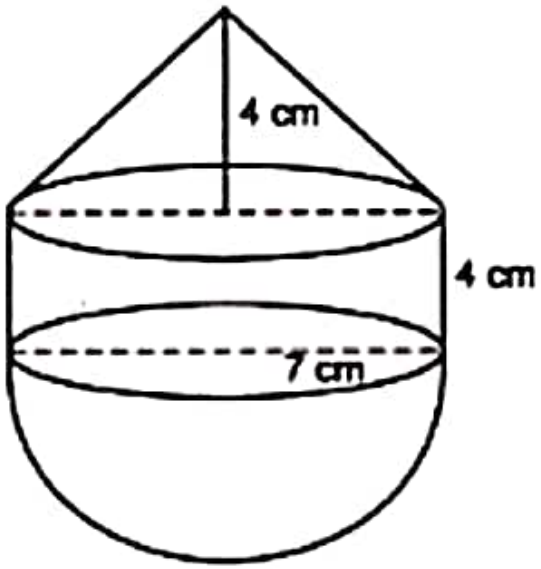
$$= \frac{22}{7} \times 21 \times 21 \times 25$$

$$= 22 \times 3 \times 21 \times 25$$

$$= 34650\ cm^3$$

Hence, volume of the cylinder is $34650\ cm^3$.

Q12. The following figure represents a solid consisting of a right circular cylinder with a hemisphere at one end and a cone at the other. Their common radius is 7. The height of the cylinder and cone are each of 4 cm. Find the volume of the solid. [4] [2018]



Solution: 1540 cm^3

Step-by-step Explanation:

$$\textit{Volume of the solid} =$$
$$\textit{Volume of cone} + \textit{volume of cylinder} + \textit{volume of hemisphere}$$
$$= \frac{1}{3}\pi r^2 h + \pi r^2 h + \frac{2}{3}\pi r^3$$
$$= \pi r^2 \left(\frac{1}{3}h + h + \frac{2}{3}r\right)$$
$$= \frac{22}{7} \times 7 \times 7\left(\frac{1}{3} \times 4 + 4 + \frac{2}{3} \times 7\right)$$
$$= 154 \left(\frac{4}{3} + 4 + \frac{14}{3}\right)$$
$$= 154 \left(\frac{4 + 12 + 14}{3}\right)$$
$$= 154 \times \frac{30}{3}$$
$$= 1540 \text{ cm}^3$$

Q13. A conical tent is to accommodate 77 persons. Each person must have 16 m^3 of air to breathe. Given the radius of the tent as 7 m, find the height of the tent and also its curved surface area. [2017]

Solution: 24 m ; 550 m^2

Step-by-step Explanation:

In a conical tent,

1 person can breathe $16\ m^3$ *of air.*

Hence, amount of air taken by 77 *persons* $= 16 \times 77 = 1232\ m^3$

Hence, volume of the conical tent $= 1232\ m^3$

$$\frac{1}{3}\pi r^2 h = 1232$$

$$\Rightarrow h = \frac{1232 \times 3 \times 7}{22 \times 7 \times 7}$$

$$\Rightarrow h = 24$$

Therefore, height of the cone is 24 m

$$\text{slant height (l)} = \sqrt{h^2 + r^2}$$

$$l = \sqrt{24^2 + 7^2}$$

$$l = \sqrt{625}$$

$$l = 25$$

$$\text{curved surface area of the cone} = \pi r l$$

$$= \frac{22}{7} \times 7 \times 25$$

$$= 550\ m^2$$

Hence, curved surface area of the conical tent is $550\ m^2$.

Q14. A certain number of metallic cones, each of radius 2 cm and height 3 cm are melted and recast into a solid sphere of radius 6 cm. Find the number of cones. [3] [2016]

Solution: 72

Step-by-step Explanation:

Let the number of cones melted be x.

Hence, volume of x cones = volume of sphere

$$\Rightarrow x \times \frac{1}{3}\pi r^2 h = \frac{4}{3}\pi R^3$$

$$\Rightarrow x \times \frac{1}{3} \times 2 \times 2 \times 3 = \frac{4}{3} \times 6 \times 6 \times 6$$

$$\Rightarrow x \times 4 = 8 \times 36$$

$$\Rightarrow x = \frac{8 \times 36}{4}$$

$$\Rightarrow x = 72$$

Hence total number of cones melted is 72.

Q15. Two solid spheres of radii 2 cm and 4 cm are melted and recast into a cone of height 8 cm. Find the radius of the cone so formed. [3] [2015]

Solution: 6 cm

Step-by-step Explanation:

Volume of cone = Volume of 1st sphere + volume of 2nd sphere

$$\frac{1}{3}\pi R^2 H = \frac{4}{3}\pi (r_1)^3 + \frac{4}{3}\pi (r_2)^3$$

$$\Rightarrow \frac{1}{3}\pi R^2 H = \frac{4}{3}\pi \left((r_1)^3 + (r_2)^3\right)$$

$$\Rightarrow R^2 H = 4\left((r_1)^3 + (r_2)^3\right)$$

$$\Rightarrow R^2 \times 8 = 4\left(2^3 + 4^3\right)$$

$$\Rightarrow R^2 \times 8 = 4 \times 72$$

$$\Rightarrow R^2 = 4 \times 9$$

$$\Rightarrow R = \sqrt{36}$$

$$\Rightarrow R = 6$$

Hence, radius of the cone is 6 cm.

Q16. The surface area of a solid metallic sphere is 2464 cm^2. It is melted and recast into solid right circular cones of radius 3.5 cm

and height 7 cm. Calculate :
(i) the radius of the sphere.
(ii) the number of cones recast. (Take $\pi = 22/7$) [4] [2014]

Solution: (i) 14 cm (ii) 128

Step-by-step Explanation:

(i) *Surface area of solid metallic sphere* $= 2464\ cm^2$

$$4\pi r^2 = 2464$$
$$\Rightarrow r^2 = \frac{2464 \times 7}{22 \times 4}$$
$$\Rightarrow r^2 = 28 \times 7$$
$$\Rightarrow r = \sqrt{28 \times 7}$$
$$\Rightarrow r = 14$$

Hence, radius of the sphere is 14 cm.

(ii) let the number of cones recast by the sphere be x.

So, volume of the cones = volume of sphere

$$x \times \frac{1}{3}\pi r^2 h = \frac{4}{3}\pi R^3$$
$$\Rightarrow x \times (3.5)^2 \times 7 = 4 \times 14 \times 14 \times 14$$
$$\Rightarrow x = \frac{4 \times 14 \times 14 \times 14}{3.5 \times 3.5 \times 7}$$
$$\Rightarrow x = 128$$

Hence , 128 cones are recast.

Q17. A solid sphere of radius 15 cm is melted and recast into solid right circular cones of radius 2.5 cm and height 8 cm. Calculate the number of cones recast. [2013]

Solution: 270

Step-by-step Explanation:

let the number of cones recast by the sphere be x.

So, volume of the cones = volume of sphere

$$x \times \frac{1}{3}\pi r^2 h = \frac{4}{3}\pi R^3$$

$$\Rightarrow x \times (2.5)^2 \times 8 = 4 \times 15 \times 15 \times 15$$

$$\Rightarrow x = \frac{4 \times 15 \times 15 \times 15}{2.5 \times 2.5 \times 8}$$

$$\Rightarrow x = 270$$

Hence , 270 cones are recast.

Q18. A hollow sphere of internal and external radii 6 cm and 8 cm respectively is melted and recast into small cones of base radius 2 cm and height 8 cm. Find the number of cones. [3] [2012]

Solution: 37

Step-by-step Explanation:

let the number of cones recast by the hollow sphere be x.

So, volume of the cones = volume of hollow sphere

$$x \times \frac{1}{3}\pi r^2 h = \frac{4}{3}\pi \left(R^3 - r^3\right)$$

$$\Rightarrow x \times 2 \times 2 \times 8 = 4 \times \left(8^3 - 6^3\right)$$

$$\Rightarrow x \times 32 = 4 \times 296$$

$$\Rightarrow x = \frac{4 \times 296}{32}$$

$$\Rightarrow x = 37$$

Hence , 37 cones are recast.

Q19. A solid cone of radius 5 cm and height 8 cm is melted and made into small spheres of radius 0.5 cm. Find the number of spheres formed. [2011]

Solution: 400

Step-by-step Explanation:

let the number of spheres recast by the cone be x.

So, volume of spheres = volume of the cone

$$x \times \frac{4}{3}\pi r^3 = \frac{1}{3}\pi R^2 h$$

$$\Rightarrow x \times 4 \times (0.5)^3 = 5^2 \times 8$$

$$\Rightarrow x = \frac{25 \times 8}{4 \times 0.5 \times 0.5 \times 0.5}$$

$$\Rightarrow x = 400$$

Hence , 400 spheres are formed.

Q20. A hemispherical bowl of diameter 7.2 cm is filled completely with chocolate sauce. This sauce is poured into an inverted cone of radius 4.8 cm. Find the height of the cone. [3] [2010]

Solution: 4.05 cm

Step-by-step Explanation:

Volume of the chocolate sauce = volume of the hemisphere

$$\textit{Hence, Volume of the chocolate sauce} = \frac{2}{3}\pi r^3$$

$$= \frac{2}{3} \times \frac{22}{7} \times \frac{7.2}{2} \times \frac{7.2}{2} \times \frac{7.2}{2}$$

let the height of the sauce in the cone be h cm.

$$\text{Therefore, } \frac{1}{3}\pi r^2 h = \frac{2}{3} \times \frac{22}{7} \times \frac{7.2}{2} \times \frac{7.2}{2} \times \frac{7.2}{2}$$

$$\Rightarrow \frac{1}{3} \times \frac{22}{7} \times 4.8 \times 4.8 \times h = \frac{2}{3} \times \frac{22}{7} \times \frac{7.2}{2} \times \frac{7.2}{2} \times \frac{7.2}{2}$$

$$\Rightarrow h = \frac{2 \times 3.6 \times 3.6 \times 3.6}{4.8 \times 4.8}$$

$$\Rightarrow h = 4.05$$

height of the chocolate sauce in the cone is 4.05 cm.