Trigonometric Identities

1.Prove the following identity: $(\sin^2\theta - 1)(\tan^2\theta + 1) + I = 0$ [2023]

Step-by-step Explanation:

$$LHS$$
$$= (\sin^2\theta - 1)(\tan^2\theta + 1) + I$$
$$= (-\cos^2\theta)(sec^2\theta) + 1$$
$$= (-\cos^2\theta) \times \frac{1}{\cos^2\theta} + 1$$
$$= -1 + 1$$
$$= 0$$
$$= RHS$$
$$proved$$

2. $(1 + \sin A)(1 - \sin A)$ is equal to:
(a) $\text{cosec}^2 A$
(b) $\sin^2 A$
(c) $\sec^2 A$
(d) $\cos^2 A$ [2023]

Solution: (d)

Step-by-step Explanation:

$(1 + \sin A)(1 - \sin A)$

$= 1 - \sin^2 A$

$= \cos^2 A$

3. *Prove that :* $1+\dfrac{\tan^2\theta}{1+sec\theta}=sec\theta$ [2022 *Semester* – 2]

Step-by-step Explanation:

$$
\begin{aligned}
&LHS\\
&= 1+\frac{\tan^2\theta}{1+sec\theta}\\
&= 1+\frac{sec^2\theta-1}{sec\theta+1}\\
&= 1+\frac{(sec\theta+1)(sec\theta-1)}{(sec\theta+1)}\\
&= 1+sec\theta-1\\
&= sec\theta\\
&= RHS\\
&proved
\end{aligned}
$$

4. Prove that: [2022 Semester-2]

$$\frac{(1+\sin\theta)^2+(1-\sin\theta)^2}{2\cos^2\theta}=sec^2\theta+\tan^2\theta$$

Step-by-step Explanation:

$$LHS$$

$$= \frac{(1+\sin\theta)^2 + (1-\sin\theta)^2}{2\cos^2\theta}$$

$$= \frac{1+\sin^2\theta + 2\sin\theta + 1 + \sin^2\theta - 2\sin\theta}{2\cos^2\theta}$$

$$= \frac{2+2\sin^2\theta}{2\cos^2\theta}$$

$$= \frac{2(1+\sin^2\theta)}{2\cos^2\theta}$$

$$= \frac{1+\sin^2\theta}{\cos^2\theta}$$

$$= \frac{1}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta}$$

$$= sec^2\theta + \tan^2\theta$$

$$= RHS$$

Proved.

5.*Prove that* : $\frac{1}{1+sin\theta} + \frac{1}{1-sin\theta} = 2sec^2\theta$ [2022*Semester* – 2]

Step-by-step Explanation:

$$LHS$$

$$= \frac{1}{1+sin\theta} + \frac{1}{1-sin\theta}$$

$$= \frac{1-sin\theta+1+sin\theta}{(1+sin\theta)\,(1-sin\theta)}$$

$$= \frac{2}{1-sin^2\theta}$$

$$= \frac{2}{\cos^2\theta}$$

$$= 2sec^2\theta$$

$$= RHS$$

Proved.

6. *Prove the identity* : $\left(\frac{1-\tan\theta}{1-cot\theta}\right)^2 = tan^2\theta$ [2020]

Step-by-step Explanation:

$$LHS$$

$$= \left(\frac{1-\tan\theta}{1-cot\theta}\right)^2$$

$$= \left(\frac{1-\tan\theta}{1-\frac{1}{\tan\theta}}\right)^2$$

$$= \left(\frac{1-\tan\theta}{\frac{\tan\theta-1}{\tan\theta}}\right)^2$$

$$= \left(\frac{(1-\tan\theta)\times\tan\theta}{\tan\theta-1}\right)^2$$

$$= (-\ \tan\theta)^2$$

$$= \tan^2\theta$$

$$= RHS$$

Proved.

7. *Prove that* : $\frac{\sin A}{1+cotA} - \frac{\cos A}{1+\tan A} = \sin A - \cos A$ [2020]

Step-by-step Explanation:

$$LHS$$

$$= \frac{\sin A}{1+cotA} - \frac{\cos A}{1+\tan A}$$

$$= \frac{\sin A}{1+\frac{\cos A}{\sin A}} - \frac{\cos A}{1+\frac{\sin A}{\cos A}}$$

$$= \frac{\sin A}{\frac{Sin A+\cos A}{\sin A}} - \frac{\cos A}{\frac{\cos A+\sin A}{\cos A}}$$

$$= \frac{\sin^2 A}{\sin A+\cos A} - \frac{\cos^2 A}{\sin A+\cos A}$$

$$= \frac{\sin^2 A-\cos^2 A}{\sin A+\cos A}$$

$$= \frac{(\sin A+\cos A)(\sin A-\cos A)}{\sin A+\cos A}$$

$$= \sin A - \cos A$$

$$= RHS$$

Proved.

8. Prove that: (cosec θ – sin θ) (sec θ – cos θ) (tan θ + cot θ) = 1 [2019]

Step-by-step Explanation:

$$LHS$$
$$= (\cos ec\,\theta - \sin\ \theta)\ (sec\,\theta - \cos\ \theta)\ (\tan\ \theta\ +\ cot\ \theta)$$
$$= \left(\frac{1}{\sin\ \theta} - \sin\ \theta\right)\ \left(\frac{1}{\cos\ \theta} - \cos\ \theta\right)\ \left(\frac{\sin\ \theta}{\cos\ \theta} + \frac{\cos\ \theta}{\sin\ \theta}\right)$$
$$= \left(\frac{1 - \sin^2\theta}{\sin\ \theta}\right)\left(\frac{1 - \cos^2\theta}{\cos\ \theta}\right)\left(\frac{\sin^2\theta + \cos^2\theta}{\sin\ \theta.\cos\ \theta}\right)$$
$$= \left(\frac{\cos^2\theta}{\sin\ \theta}\right)\left(\frac{\sin^2\theta}{\cos\ \theta}\right)\left(\frac{1}{\sin\ \theta.\cos\ \theta}\right)$$
$$= 1$$
$$= RHS$$
$$Proved.$$

9. *Prove that* : $\sqrt{sec^2\theta + \cos ec^2\theta} = tan\theta + cot\theta$ [3] [2018]

Step – by – step Explanation :

$$LHS$$
$$= \sqrt{sec^2\theta + \cos ec^2\theta}$$
$$= \sqrt{\frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta}}$$
$$= \sqrt{\frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta.\ \sin^2\theta}}$$
$$= \sqrt{\frac{1}{\cos^2\theta.\ \sin^2\theta}}$$
$$= \frac{1}{\cos\theta\ \sin\theta}$$
$$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta\ \sin\theta}$$

$$= \frac{\sin^2\theta}{\cos\theta\ \sin\theta} + \frac{\cos^2\theta}{\cos\theta\ \sin\theta}$$

$$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \tan\theta + cot\theta$$

$$= \ RHS$$

$$Proved.$$

10. Prove that (1 + cot θ – cosec θ) (1 + tan θ + sec θ) = 2 [4] [2018]

Step-by-step Explanation:

$$LHS$$

$$= (1\ +\ cot\ \theta - \cos ec\ \theta)\ (1\ +\ \tan\ \theta\ +\ sec\ \theta)$$

$$= \left(1 + \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)\ \left(1 + \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)$$

$$= \left(\frac{\sin\theta + \cos\theta - 1}{\sin\theta}\right)\ \left(\frac{\cos\theta + \sin\theta + 1}{\cos\theta}\right)$$

$$= \left(\frac{(\sin\theta + \cos\theta)^2 - (1)^2}{\sin\theta\ \cos\theta}\right)$$

$$= \left(\frac{\sin^2\theta + \cos^2\theta + 2\ \sin\theta\ \cos\theta - 1}{\sin\theta\ \cos\theta}\right)$$

$$= \left(\frac{1 + 2\ \sin\theta\ \cos\theta - 1}{\sin\theta\ \cos\theta}\right)$$

$$= \left(\frac{2\ \sin\theta\ \cos\theta}{\sin\theta\ \cos\theta}\right)$$

$$= 2$$

$$= \ RHS$$

$$Proved.$$

$$= \ RHS$$

$$Proved.$$

11. *Prove that :* $\dfrac{sin\theta - 2sin^3\theta}{2cos^3\theta - cos\theta} = tan\theta$ [2017]

$$Step - by - step\ Explanation:$$

$$LHS$$

$$= \frac{sin\theta - 2sin^3\theta}{2cos^3\theta - cos\theta}$$

$$= \frac{sin\theta\,(1 - 2sin^2\theta)}{cos\theta\,(2cos^2\theta - 1)}$$

$$= \frac{sin\theta\,(sin^2\theta + \cos^2\theta - 2sin^2\theta)}{cos\theta\,(2cos^2\theta - sin^2\theta - \cos^2\theta)}$$

$$= \tan\theta\frac{(\cos^2\theta - sin^2\theta)}{(cos^2\theta - sin^2\theta)}$$

$$= \tan\theta$$

$$= RHS$$

Proved.

12. *Prove that :* $\dfrac{\cos A}{1 + \sin A} + \tan A = secA$ [3] [2016]

$$Step - by - step\ Explanation:$$

$$LHS$$

$$= \frac{\cos A}{1 + \sin A} + \tan A$$

$$= \frac{\cos A}{1 + \sin A} + \frac{\sin A}{\cos A}$$

$$= \frac{\cos^2 A + \sin A\,(1 + \sin A)}{\cos A\,(1 + \sin A)}$$

$$= \frac{\cos^2 A + \sin A + \sin^2 A}{\cos A\,(1 + \sin A)}$$

$$= \frac{1 + \sin A}{\cos A\,(1 + \sin A)}$$

$$= \frac{1}{\cos A}$$

$$= secA$$

$$= RHS$$

Proved.

13. *Prove that* $\frac{\sin\theta}{1-\cot\theta}+\frac{\cos\theta}{1-\tan\theta}=\cos\theta+\sin\theta$ [3] [2015]

Step – by – step Explanation :

LHS

$$=\frac{\sin\theta}{1-\cot\theta}+\frac{\cos\theta}{1-\tan\theta}$$

$$=\frac{\sin\theta}{1-\frac{\cos\theta}{\sin\theta}}+\frac{\cos\theta}{1-\frac{\sin\theta}{\cos\theta}}$$

$$=\frac{\sin\theta}{\frac{\sin\theta-\cos\theta}{\sin\theta}}+\frac{\cos\theta}{\frac{\cos\theta-\sin\theta}{\cos\theta}}$$

$$=\frac{\sin^2\theta}{\sin\theta-\cos\theta}+\frac{\cos^2\theta}{\cos\theta-\sin\theta}$$

$$=\frac{\sin^2\theta}{\sin\theta-\cos\theta}-\frac{\cos^2\theta}{\sin\theta-\cos\theta}$$

$$=\frac{\sin^2\theta-\cos^2\theta}{\sin\theta-\cos\theta}$$

$$=\frac{(\sin\theta+\cos\theta)(\sin\theta-\cos\theta)}{\sin\theta-\cos\theta}$$

$$=\sin\theta+\cos\theta$$

$$=RHS$$

Proved.

14. Prove the identity: (sin θ + cos θ) (tan θ + cot θ) = sec θ + cosec θ [3] [2014]

Step-by-step Explanation:

13. *Prove that* $\frac{\sin\theta}{1-\cot\theta}+\frac{\cos\theta}{1-\tan\theta}=\cos\theta+\sin\theta$ [3] [2015]

Step – by – step Explanation :

$$LHS$$

$$=(\sin\ \theta\ +\ \cos\ \theta)\ (\tan\ \theta\ +\ \cot\ \theta)$$

$$=(\sin\ \theta\ +\ \cos\ \theta)\left(\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}\right)$$

$$=(\sin\ \theta\ +\ \cos\ \theta)\left(\frac{\sin^2\theta+\cos^2\theta}{\cos\theta\ \sin\theta}\right)$$

$$=(\sin\ \theta\ +\ \cos\ \theta)\left(\frac{1}{\cos\theta\ \sin\theta}\right)$$

$$=\frac{\sin\theta}{\cos\theta\ \sin\theta}+\frac{\cos\theta}{\cos\theta\ \sin\theta}$$

$$=\frac{1}{\cos\theta}+\frac{1}{\sin\theta}$$

$$=sec\,\theta+\cos ec\,\theta$$

$$=RHS$$

Proved.

15. *Show that :* $\sqrt{\frac{1-\cos A}{1+\cos A}}=\frac{\sin A}{1+\cos A}$ [3] [2013]

Step – by – step Explanation :

$$LHS$$

$$=\sqrt{\frac{1-\cos A}{1+\cos A}}$$

$$=\sqrt{\frac{1-\cos A}{1+\cos A}\times\frac{1+\cos A}{1+\cos A}}$$

$$=\sqrt{\frac{1-\cos^2 A}{(1+\cos A)^2}}$$

$$=\sqrt{\frac{\sin^2 A}{(1+\cos A)^2}}$$

$$=\frac{\sin A}{1+\cos A}$$

$$=RHS$$

Proved.

16. *Prove that* : $\dfrac{\tan^2\theta}{(sec\theta-1)^2}=\dfrac{1+cos\theta}{1-cos\theta}$ [3] [2012]

Step–by–step Explanation

$$LHS$$

$$=\frac{\tan^2\theta}{(sec\theta-1)^2}$$

$$=\frac{sec^2\theta-1}{(sec\theta-1)\,(sec\theta-1)}$$

$$=\frac{(sec\theta+1)\,(sec\theta-1)}{(sec\theta-1)\,(sec\theta-1)}$$

$$=\frac{(sec\theta+1)}{(sec\theta-1)}$$

$$=\frac{\frac{1}{\cos\theta}+1}{\frac{1}{\cos\theta}-1}$$

$$=\frac{\frac{1+\cos\theta}{\cos\theta}}{\frac{1-\cos\theta}{\cos\theta}}$$

$$=\frac{1+\cos\theta}{1-\cos\theta}$$

$$=RHS$$

Proved.

17. Prove that (cosec A – sin A) (sec A – cos A) sec²A = tan A. [4] [2011]

Step-by-step Explanation:

$$
\begin{aligned}
& LHS \\
&= (\cos ec\ A - \sin\ A)\ (sec\ A - \cos\ A\ )\ sec^2 A \\
&= \left(\frac{1}{\sin A} - \sin A\right) \left(\frac{1}{\cos A} - \cos A\right) \frac{1}{\cos^2 A} \\
&= \left(\frac{1 - \sin^2 A}{\sin A}\right) \left(\frac{1 - \cos^2 A}{\cos A}\right) \frac{1}{\cos^2 A} \\
&= \left(\frac{\cos^2 A}{\sin A}\right) \left(\frac{\sin^2 A}{\cos A}\right) \cdot \frac{1}{\cos^2 A} \\
&= \frac{\sin A}{\cos A} \\
&= \tan A \\
&= RHS
\end{aligned}
$$

Proved.