Quadratic Equations

Definition

- In mathematics, a **quadratic equation** is a polynomial equation of the second degree. The general form is

$$ax^2 + bx + c = 0$$

- where *x* represents a variable or an unknown, and *a*, *b*, and *c* are constants with $a \neq 0$. (If $a = 0$, the equation is a linear equation.)
- The constants *a*, *b*, and *c* are called respectively, the quadratic coefficient, the linear coefficient and the constant term or free term.

Quadratic & Roots

Quadratic: A polynomial of degree=2

$$y= ax^2+bx+c$$

$ax^2 + bx + c = 0$ is a quadratic equation. ($a \neq 0$)

Here is an example of one:

this makes it Quadratic

$$5x^2 - 3x + 3 = 0$$

- The name **Quadratic** comes from "quad" meaning square, because the variable gets squared (like $\mathbf{x^2}$).
- It is also called an "Equation of Degree 2" (because of the "2" on the **x**)

Roots

- A real number α is called a *root of the quadratic equation* $ax^2 + bx + c = 0, a \neq 0$ *if* $a\alpha^2 + b\alpha^2 + c = 0$.
- If α is a root of $ax^2 + bx + c = 0$, then we say that:

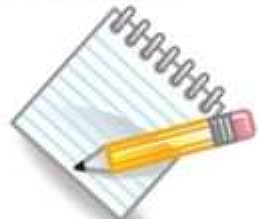
(i) $x = \alpha$ *satisfies the equation* $ax^2+bx+c=0$

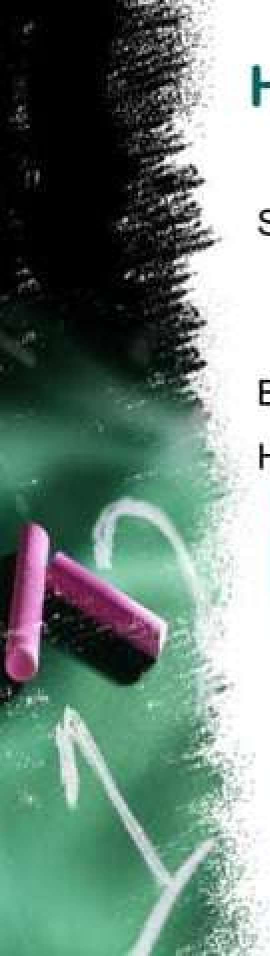
Or (ii) $x = \alpha$ is a solution of the equation $ax^2+bx+c=0$

- The Root of a quadratic equation $ax^2+bx+c=0$ are called *zeros* of the polynomial ax^2+bx+c.

More Examples of Quadratic Equations

- $2x^2 + 5x + 3 = 0$ In this one **a=2**, **b=5** and **c=3.**
- $x^2 - 3x = 0$ This one is a little more tricky: Where is **a**? In fact **a=1**, as we don't usually write "$1x^2$" **b = -3** and where is **c**? Well, **c=0**, so is not shown.
- $5x - 3 = 0$ **Oops!** This one is **not** a quadratic equation, because it is missing x^2 (in other words **a=0**, and that means it can't be quadratic)





Hidden Quadratic Equations!

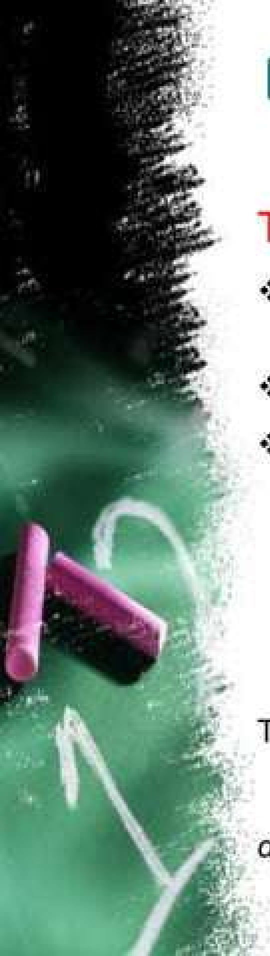
So far we have seen the "Standard Form" of a Quadratic Equation:

$$ax^2 + bx + c = 0$$

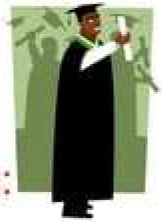
But sometimes a quadratic equation doesn't look like that..!

Here are some examples of different form:

In disguise		In Standard Form	a, b and c
$x^2 = 3x - 1$	*Move all terms to left hand side*	$x^2 - 3x + 1 = 0$	a=1, b=-3, c=1
$2(w^2 - 2w) = 5$	*Expand (undo the brackets), and move 5 to left*	$2w^2 - 4w - 5 = 0$	a=2, b=-4, c=-5
$z(z-1) = 3$	*Expand, and move 3 to left*	$z^2 - z - 3 = 0$	a=1, b=-1, c=-3
$5 + 1/x - 1/x^2 = 0$	*Multiply by x^2*	$5x^2 + x - 1 = 0$	a=5, b=1, c=-1



How To Solve It?



There are 3 ways to find the solutions:

- We can Factor the Quadratic (find what to multiply to make the Quadratic Equation)
- We can Complete the Square, or
- We can use the special **Quadratic Formula**:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Thus $ax^2+bx+c=0$ *has two roots α and β, given by*

$$\alpha = \frac{-b+\sqrt{b^2-4ac}}{2a} \qquad \beta = \frac{-b-\sqrt{b^2-4ac}}{2a}$$

Discriminant

- The expression $b^2 - 4ac$ in the formula $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
- It is called the **Discriminant**, because it can "discriminate" between the possible types of answer.It can be denoted by "D"
- when $b^2 - 4ac$, D is positive, you get two real solutions
- when it is zero you get just ONE real solution (both answers are the same)
- when it is negative you get two Complex solutions

Value of D	*Nature of Roots*	*Roots*
$D > 0$	**Real and Unequal**	$[(-b \pm \sqrt{D})/2a]$
$D = 0$	**Real and Equal**	**Each root =** $(-b/2a)$
$D < 0$	**No real roots**	**None**

Using the Quadratic Formula

Just put the values of a, b and c into the Quadratic Formula, and do the calculation

Example: **Solve $5x^2 + 6x + 1 = 0$**

Coefficients are: $a = 5$, $b = 6$, $c = 1$

Quadratic Formula: $x = [\,-b \pm \sqrt{(b^2-4ac)}\,] / 2a$

Put in a, b and c:

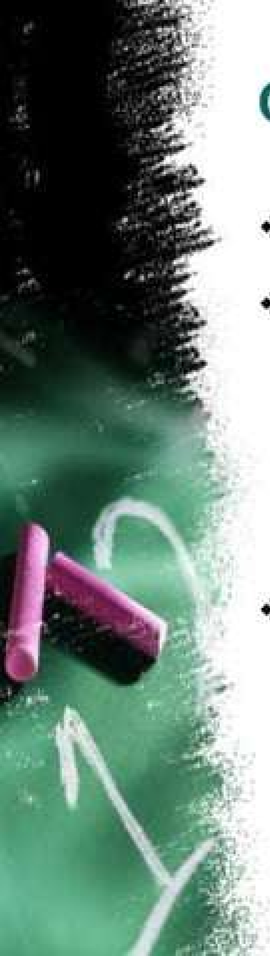
$$x = \left[\frac{-6 \pm \sqrt{6^2 - 4\times5\times1}}{2\times5}\right]$$

Solve: $$x = \left[\frac{-6 \pm \sqrt{36-20}}{10}\right]$$

$$x = \left[\frac{-6 \pm \sqrt{16}}{10}\right]$$

$$x = \left[\frac{-6 \pm 4}{10}\right]$$

$x = -0.2$ **or** -1



Continue..

- **Answer:** $x = -0.2$ **or** $x = -1$
- Check **-0.2**: $5 \times (\mathbf{-0.2})^2 + 6 \times (\mathbf{-0.2}) + 1$
  $= 5 \times (0.04) + 6 \times (-0.2) + 1$
  $= 0.2 - 1.2 + 1$
  $\mathbf{= 0}$
- Check **-1**: $5 \times (\mathbf{-1})^2 + 6 \times (\mathbf{-1}) + 1$
  $= 5 \times (1) + 6 \times (-1) + 1$
  $= 5 - 6 + 1$
  $\mathbf{= 0}$

Factoring Quadratics

- To "Factor" (or "Factorize") a Quadratic is to find what to multiply to get the Quadratic

  It is called "Factoring" because you find the factors (a factor is something you multiply by)

- **Example**

The factors of $x^2 + 3x - 4$ are:

$(x+4)$ and $(x-1)$

Why? Well, let us multiply them to see:

$(x+4)(x-1)$

$= x(x-1) + 4(x-1)$

$= x^2 - x + 4x - 4$

$= x^2 + 3x - 4$

- Multiplying **(x+4)(x-1)** together is called Expanding.
- In fact, Expanding and Factoring are opposites:

$$(x+4)(x-1) \underset{\text{Factor}}{\overset{\text{Expand}}{\rightleftarrows}} x^2 + 3x - 4$$

Examples of Factor

To solve by factoring:

1. Set the equation equal to zero.
2. Factor. The factors will be linear expressions.
3. Set each linear factor equal to zero.
4. Solve both linear equations.

Example: Solve by factoring $x^2 + 3x = 0$

$x^2 + 3x = 0$ → set equation to zero

$x(x + 3) = 0$ → factor

$x = 0$, $x + 3 = 0$

$x = -3$ set the linear factors equal to zero and solve the linear equation

Completing the Square

Solving General Quadratic Equations by Completing the Square:

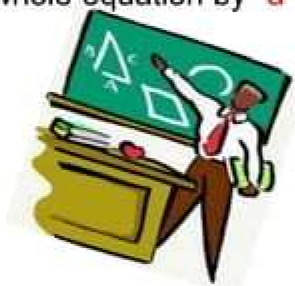
"**Completing the Square**" is where we take a Quadratic Equation : $ax^2 + bx + c = 0$ and turn into $a(x+d)^2 + e = 0$

We can use that idea to **solve** a Quadratic Equation (find where it is equal to zero).

But a general Quadratic Equation can have a coefficient of a in front of x^2:

$$ax^2 + bx + c = 0$$

But that is easy to deal with ... just divide the whole equation by "a" first, then carry on.



Steps

Now we can solve Quadratic Equations in 5 steps:

- **Step 1** Divide all terms by **a** (the coefficient of x^2).

- **Step 2** Move the number term (**c/a**) to the right side of the equation.

- **Step 3** Complete the square on the left side of the equation and balance this by adding the same value to the right side of the equation.

- **Step 4** Take the square root on both sides of the equation.

- **Step 5** Add or subtract the number that remains on the left side of the equation to find **x**.

Example

Example 1: Solve $x^2 + 4x + 1 = 0$

Step 1 can be skipped in this example since the coefficient of x^2 is 1

Step 2 Move the number term to the right side of the equation:

$$x^2 + 4x = -1$$

Step 3 Complete the square on the left side of the equation and balance this by adding the same number to the right side of the equation:

$$x^2 + 4x + 4 = -1 + 4$$

$$(x + 2)^2 = 3$$

Step 4 Take the square root on both sides of the equation:

$$x + 2 = \pm\sqrt{3} = \pm 1.73 \text{ (to 2 decimals)}$$

Step 5 Subtract 2 from both sides:

$$x = \pm 1.73 - 2 = -3.73 \text{ or } -0.27$$

BIBLIOGRAPHY

- Internet (Wikipedia,www.mathsisfun.com)
- Secondary School Mathematics (R.S. Aggarwal)