4.QUADRATIC EQUATIONS

What is a Quadratic Equation?

A Quadratic Equation in one variable is an equation in which the greatest exponent/index of the variable is two.

Examples:

$$6x^2 - x - 2 = 0$$

$$y^2 - 5y + 4 = 0$$

$$(\text{number})(\text{variable})^2 \pm (\text{number})(\text{variable}) \pm (\text{number}) = 0$$

General Form:

$$ax^2 + bx + c = 0$$

x : variable

a, b, c: Real Numbers

$a \neq 0$

Permitted:

$b = 0$

$c = 0$

$b = 0$ & $c = 0$

Exercise: Create QE with variable 'x'

$a = 1,\ b = 2,\ c = 3$

$$1x^2 + 2x + 3 = 0$$

$$x^2 + 2x + 3 = 0$$

Exercise: Create QE with variable 'α'

$a = 5,\ b = 0,\ c = -25$

$$\alpha^2 + b\alpha \quad + c = 0$$

Quadratics

A quadratic equation in x is an equation that can be written in the standard form:

$$ax^2 + bx + c = 0$$

Where a,b,and c are real numbers and $a \neq 0$.

STANDARD FORM OF A QUADRATIC EQUATION

The standard form of a quadratic equation is $ax^2 + bx + c = 0$ where a,b and c are real numbers and $a \neq 0$.

'a' is the coefficient of x^2. It is called the quadratic coefficient. 'b' is the coefficient of x. It is called the linear coefficient. 'c' is the constant term.

SOLVING QUADRATIC EQUATION BY FACTORISATION

Roots of a Quadratic equation:

- The values of x for which a quadratic equation is satisfied are called the roots of the quadratic equation.

- If α is a root of the quadratic equation ax2+bx+c=0,then, aα2+bα+c=0.

- A quadratic equation can have two distinct roots, two equal roots or real roots may not exist.

QUADRATIC FORMUL A

The Quadratic Formula ...

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For Quadratic Equations

$ax^2 + bx + c = 0$

SOLVING QUADRATIC EQUATIONS USING QUADRATIC FORMUL A

- Quadratic Formula is used to directly obtain the roots of a quadratic equation from the standard form of the equation.

- For the quadratic equation ax2+bx+c=0,
- x= [-b± √(b2-4ac)]/2a

- By substituting the values of a,b and c, we can directly get the roots of the equation.

Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

(or)

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad , \quad \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

DISCRIMINANT

- For a quadratic equation of the form ax2+bx+c=0, the expression b2−4ac is called the discriminant, (denoted by D), of the quadratic equation.
- The discriminant determines the nature of roots of the quadratic equation based on the coefficients of the quadratic polynomial.

NATURE OF ROOTS

Based on the value of the discriminant, $D=b2-4ac$, the roots of a quadratic equation can be of three types.

- Case 1: If $D>0$, the equation has two distinct real roots.
- Case 2: If $D=0$, the equation has two equal real roots.
- Case 3: If $D<0$, the equation has no real roots.

EX: 4.1 :

1. Check whether the following are quadratic equations:

(i) $(x+1)^2=2(x-3)$

Solution:

(i) Given : $(x + 1)^2 = 2(x - 3)$
By using the formula for $(a+b)^2 = a^2+2ab+b^2$

$\Rightarrow x^2 + 2x + 1 = 2x - 6$
$\Rightarrow x^2 + 7 = 0$

Since the above equation is in the form of $ax^2 + bx + c = 0$.

Therefore, the given equation is quadratic equation.

(ii) $x - 2x = (-2)(3-x)$

(ii)Given, $x^2 - 2x = (-2)(3 - x)$

By using the formula for $(a+b)^2 = a^2+2ab+b^2$

$\Rightarrow x^2 - 2x = -6 + 2x$

$\Rightarrow x^2 - 4x + 6 = 0$

Since the above equation is in the form of $ax^2 + bx + c = 0$.

Therefore, the given equation is quadratic equation.

EX: 4.1 :

1. Check whether the following are quadratic equations:

(iii) $(x - 2)(x + 1) = (x - 1)(x + 3)$

Given, $(x - 2)(x + 1) = (x - 1)(x + 3)$

By using the formula for $(a+b)^2 = a^2+2ab+b^2$

$\Rightarrow x^2 - x - 2 = x^2 + 2x - 3$

$\Rightarrow 3x - 1 = 0$

Since the above equation is not in the form of $ax^2 + bx + c = 0$.

Therefore, the given equation is not a quadratic equation.

(iv) $(x - 3)(2x + 1) = x(x + 5)$

Given, $(x - 3)(2x + 1) = x(x + 5)$

By using the formula for $(a+b)^2=a^2+2ab+b^2$

$\Rightarrow 2x^2 - 5x - 3 = x^2 + 5x$

$\Rightarrow x^2 - 10x - 3 = 0$

Since the above equation is in the form of $ax^2 + bx + c = 0$.

Therefore, the given equation is quadratic equation.

EX: 4.1 :

1. Check whether the following are quadratic equations:

(v) $(2x - 1)(x - 3) = (x + 5)(x - 1)$

Given, $(2x - 1)(x - 3) = (x + 5)(x - 1)$

By using the formula for $(a+b)^2=a^2+2ab+b^2$

$\Rightarrow 2x^2 - 7x + 3 = x^2 + 4x - 5$

$\Rightarrow x^2 - 11x + 8 = 0$

Since the above equation is in the form of $ax^2 + bx + c = 0$.

Therefore, the given equation is quadratic equation.

(vi) $x^2 + 3x + 1 = (x - 2)^2$

(vi)Given, $x^2 + 3x + 1 = (x - 2)^2$

By using the formula for $(a+b)^2=a^2+2ab+b^2$

$\Rightarrow x^2 + 3x + 1 = x^2 + 4 - 4x$

$\Rightarrow 7x - 3 = 0$

Since the above equation is not in the form of $ax^2 + bx + c = 0$.

Therefore, the given equation is not a quadratic equation.

EX: 4.1 :

1. Check whether the following are quadratic equations:

(vii) $(x + 2)^3 = 2x(x^2 - 1)$

Given, $(x + 2)^3 = 2x(x^2 - 1)$
By using the formula for $(a+b)^2 = a^2+2ab+b^2$

$\Rightarrow x^3 + 8 + x^2 + 12x = 2x^3 - 2x$

$\Rightarrow x^3 + 14x - 6x^2 - 8 = 0$

Since the above equation is not in the form of $ax^2 + bx + c = 0$.

Therefore, the given equation is not a quadratic equation.

(viii) $x^3 - 4x^2 - x + 1 = (x-2)^3$

Given, $x^3 - 4x^2 - x + 1 = (x - 2)^3$

By using the formula for $(a+b)^2 = a^2+2ab+b^2$
$\Rightarrow x^3 - 4x^2 - x + 1 = x^3 - 8 - 6x^2 + 12x$

$\Rightarrow 2x^2 - 13x + 9 = 0$

Since the above equation is in the form of $ax^2 + bx + c = 0$.

Therefore, the given equation is quadratic equation.

EX: 4.1 :

2. Represent the following situations in the form of quadratic equations:

The area of a rectangular plot is 528 m^2. The length of the plot (in metres) is one more than twice its breadth. We need to find the length and breadth of the plot.

Solution:

Let us consider,

Breadth of the rectangular plot = x m

Thus, the length of the plot = (2x + 1) m.

Area of rectangle = length × breadth = 528 m^2

Putting the value of length and breadth of the plot in the formula, we get,

$$(2x + 1) \times x = 528$$

$$\Rightarrow 2x^2 + x = 528$$

$$\Rightarrow 2x^2 + x - 528 = 0$$

Therefore, the length and breadth of plot, satisfies the quadratic equation, $2x^2 + x - 528 = 0$, which is the required representation of the problem mathematically.

EX: 4.1 :

2. Represent the following situations in the form of quadratic equations:

(ii)The product of two consecutive positive integers is 306. We need to find the integers.

Let us consider,

The first integer number = x

Thus, the next consecutive positive integer will be = x + 1

Product of two consecutive integers = $x \times (x + 1) = 306 \Rightarrow x^2 + x = 306$

$\Rightarrow x^2 + x - 306 = 0$

Therefore, the two integers x and x+1, satisfies the quadratic equation, $x^2 + x - 306 = 0$, which is the required representation of the problem mathematically.

EX: 4.1 :

2. Represent the following situations in the form of quadratic equations:

(iii) Rohan's mother is 26 years older than him. The product of their ages (in years) 3 years from now will be 360. We would like to find Rohan's present age.

3) Let us consider,

Age of Rohan's = x years

Therefore, as per the given question,

Rohan's mother's age = $x + 26$

After 3 years,

Age of Rohan's = $x + 3$

Age of Rohan's mother will be = $x + 26 + 3 = x + 29$

The product of their ages after 3 years will be equal to 360, such that

$(x + 3)(x + 29) = 360$

$\Rightarrow x^2 + 29x + 3x + 87 = 360$

$\Rightarrow x^2 + 32x + 87 - 360 = 0$

$\Rightarrow x^2 + 32x - 273 = 0$

Therefore, the age of Rohan and his mother, satisfies the quadratic equation, $x^2 + 32x - 273 = 0$, which is the required representation of the problem mathematically.

EX: 4.1 :

2. Represent the following situations in the form of quadratic equations:

(iv) A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km/h less, then it would have taken 3 hours more to cover the same distance. We need to find the speed of the train.

Let us consider,

The uniform speed of train = x km/h

Time taken to travel 480 km = 480/x km/hr

As per second condition,

If the speed had beenn8km/hr less, then the

speed of train = $(x - 8)$ km/h

Time taken to travel 480 km = 480/(x-8) km/hr

Also given, the train will take 3 hours to cover

the same distance.

ATQ

480/(x-8) =480/x +3

Therefore,

$$\frac{480}{(x-8)} - \frac{480}{x} = 3$$

$\Rightarrow 480(x) - 480(x-8) = 3x(x-8)$

480x - 480x + 3840 = 3(x2-8x)

$3840 = 3x^2 - 24x$

$3x^2 - 24x = 3840$

$3x^2 - 24x - 3840 = 0$ (divide by 3)

$x^2 - 8x - 1280 = 0$

Therefore, the speed of the train, satisfies the quadratic equation, $3x^2 - 8x - 1280 = 0$, which is the required representation of the problem mathematically.

EX:4.2

EX: 4.2 :

1. **Find the roots of the following quadratic equations by factorisation:**

   (i) $x^2 - 3x - 10 = 0$

   (ii) $2x^2 + x - 6 = 0$

(i) Given: $x^2 - 3x - 10 = 0$

$\Rightarrow \quad x^2 - 5x + 2x - 10 = 0$

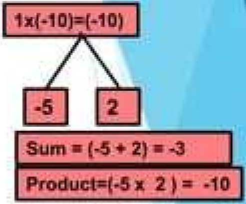
$\Rightarrow \quad x(x-5) + 2(x-5) = 0$

$\Rightarrow \quad (x-5)(x+2) = 0$

Either $x - 5 = 0$ or $x + 2 = 0$

$\Rightarrow \quad x = 5$ or $x = -2$

Hence, the roots are **5** and **–2**.



(ii) Given: $2x^2 + x - 6 = 0$

$\Rightarrow \quad 2x^2 + 4x - 3x - 6 = 0$

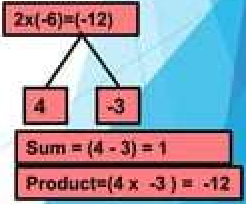
$\Rightarrow \quad 2x(x+2) - 3(x+2) = 0$

$\Rightarrow \quad (x+2)(2x-3) = 0$

Either $x + 2 = 0$ or $2x - 3 = 0$

$\Rightarrow \quad x = -2$ or $x = \frac{3}{2}$

Hence, the roots are **–2** and $\mathbf{\frac{3}{2}}$.



EX: 4.2 :

1. **Find the roots of the following quadratic equations by factorisation:**

   (iii) $\sqrt{2}\,x^2 + 7x + 5\sqrt{2} = 0$

   (iv) $2x^2 - x + 1/8 = 0$

(iii) Given: $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

$\Rightarrow \quad \sqrt{2}x^2 + 5x + 2x + 5\sqrt{2} = 0$

$\Rightarrow \quad x(\sqrt{2}x + 5) + \sqrt{2}(\sqrt{2}x + 5) = 0$

$\Rightarrow \quad (\sqrt{2}x + 5)(x + \sqrt{2}) = 0$

Either $\sqrt{2}x + 5 = 0$ or $x + \sqrt{2} = 0$

$\Rightarrow x = -\frac{5}{\sqrt{2}}$ or $x = -\sqrt{2}$

Hence, the roots are $-\frac{5}{\sqrt{2}}$ and $-\sqrt{2}$.

$\sqrt{2}\times5\sqrt{2}=5\times2=10$

5 2

Sum = (5 + 2) = 7

Product=(5 x 2) = 10

(iv) Given: $2x^2 - x + \frac{1}{8} = 0$

$\Rightarrow \quad 16x^2 - 8x + 1 = 0$

16x1=(16)

-4 -4

Sum = (-4-4) = -8

Product=(-4 x -4) = 16

EX: 4.2 :

1. Find the roots of the following quadratic equations by factorisation:

(v) $100x^2 - 20x + 1 = 0$

$\Rightarrow \quad 16x^2 - 4x - 4x + 1 = 0$

$\Rightarrow \quad 4x(4x - 1) - 1(4x - 1) = 0$

$\Rightarrow \quad (4x - 1)(4x - 1) = 0$

Either $4x - 1 = 0$ or $4x - 1 = 0$

$\Rightarrow x = \frac{1}{4}$ or $x = \frac{1}{4}$

Hence, the roots are $\mathbf{\frac{1}{4}}$ and $\mathbf{\frac{1}{4}}$.

(v) Given: $100x^2 - 20x + 1 = 0$

$\Rightarrow \quad 100x^2 - 10x - 10x + 1 = 0$

$\Rightarrow \quad 10x(10x - 1) - 1(10x - 1) = 0$

$\Rightarrow \quad (10x - 1)(10x - 1) = 0$

Either $10x - 1 = 0$ or $10x - 1 = 0$

$\Rightarrow x = \frac{1}{10}$ or $x = \frac{1}{10}$

Hence, the roots are $\mathbf{\frac{1}{10}}$ and $\mathbf{\frac{1}{10}}$.

100x1=100

-10 -10

Sum = (-10-10) = -20

Product=(-10 x -10) = 100

EX: 4.2 :

2. Solve the problems given in Example 1.Represent the following situations mathematically:

(i).John and Jivanti together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 124. We would like to find out how many marbles they had to start with.

Solution:

Let us say, the number of marbles John have = x.

Therefore, number of marble Jivanti have = $45 – x$

After losing 5 marbles each,

Number of marbles John have = $x – 5$

Number of marble Jivanti have = $45 – x – 5 = 40 – x$

Given that the product of their marbles is 124.

$\therefore (x – 5)(40 – x) = 124$

$40x - x^2 - 200 + 5x = 124$

$-x^2 + 45x - 200 - 124 = 0$

$- x^2 + 45x - 324 = 0$

$\Rightarrow x^2 – 45x + 324 = 0$

$\Rightarrow x^2 – 36x – 9x + 324 = 0$

$\Rightarrow x(x – 36) - 9(x – 36) = 0$

$\Rightarrow (x – 36)(x – 9) = 0$

Thus, we can say,

$x – 36 = 0$ or $x – 9 = 0$

$\Rightarrow x = 36$ or $x = 9$

Therefore,

If, John's marbles = 36,

Then, Jivanti's marbles = $45 – 36 = 9$

And if John's marbles = 9,

Then, Jivanti's marbles = $45 – 9 = 36$

EX: 4.2 :

(ii).A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of toys produced in a day. On a particular day, the total cost of production was Rs.750. We would like to find out the number of toys produced on that day.

(ii) Let us say, number of toys produced in a day be x.

Therefore, cost of production of each toy = Rs$(55 - x)$

Given, total cost of production of the toys = Rs 750

$\therefore x(55 - x) = 750$

$\Rightarrow x^2 - 55x + 750 = 0$

$\Rightarrow x^2 - 25x - 30x + 750 = 0$

$\Rightarrow x(x - 25) -30(x - 25) = 0$

$\Rightarrow (x - 25)(x - 30) = 0$

Thus, either $x -25 = 0$ or $x - 30 = 0$

$\Rightarrow x = 25$ or $x = 30$

Hence, the number of toys produced in a day, will be either 25 or 30.

EX: 4.2 :

3. Find two numbers whose sum is 27 and product is 182.

<u>Solution:</u>

Let us say, first number be x and the second number is 27 – x.

Therefore, the product of two numbers

$x(27 - x) = 182$

$\Rightarrow x^2 - 27x - 182 = 0$

$\Rightarrow x^2 - 13x - 14x + 182 = 0$

$\Rightarrow x(x - 13) -14(x - 13) = 0$

$\Rightarrow (x - 13)(x -14) = 0$

Thus, either, $x = -13 = 0$ or $x - 14 = 0$

$\Rightarrow x = 13$ or $x = 14$

Therefore, if first number = 13, then second number = 27 – 13 = 14

And if first number = 14, then second number = 27 – 14 = 13

Hence, the numbers are 13 and 14.

EX: 4.2 :

4. Find two consecutive positive integers, sum of whose squares is 365.

Solution:

Let us say, the two consecutive positive integers be *x* and *x* + 1.

Therefore, as per the given questions,

Given that $x^2 + (x+1)^2 = 365$

$\Rightarrow x^2 + x^2 + 1 + 2x = 365$

$\Rightarrow 2x^2 + 2x - 364 = 0$

$\Rightarrow x^2 + x - 182 = 0$

$\Rightarrow x^2 + 14x - 13x - 182 = 0$

$\Rightarrow x(x+14) - 13(x+14) = 0$

$\Rightarrow (x+14)(x-13) = 0$

Either $x + 14 = 0$ or $x - 13 = 0$, i.e., $x = -14$ or $x = 13$

Since the integers are positive, x can only be 13.

$\therefore x + 1 = 13 + 1 = 14$

Therefore, two consecutive positive integers will be 13 and 14.

EX: 4.2 :

5. The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides.

Solution:

Let the base of the right triangle be x cm.
Its altitude = $(x - 7)$ cm
From pythagoras theorem,

$$\text{Base}^2 + \text{Altitude}^2 = \text{Hypotenuse}^2$$

$$\therefore x^2 + (x-7)^2 = 13^2$$

$$\Rightarrow x^2 + x^2 + 49 - 14x = 169$$

$$\Rightarrow 2x^2 - 14x - 120 = 0$$

$$\Rightarrow x^2 - 7x - 60 = 0$$

$$\Rightarrow x^2 - 12x + 5x - 60 = 0$$

$$\Rightarrow x(x - 12) + 5(x - 12) = 0$$

$$\Rightarrow (x - 12)(x + 5) = 0$$

Either $x - 12 = 0$ or $x + 5 = 0$, i.e., $x = 12$ or $x = -5$
Since sides are positive, x can only be 12.
Therefore, the base of the given triangle is 12 cm and the altitude of this triangle will be $(12 - 7)$ cm $= 5$ cm.

Concept Insights: Apply Pythagoras Theorem to form the equation

EX: 4.2 :

6. A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was Rs.90, find the number of articles produced and the cost of each article.

<u>Solution:</u>

Let the number of artides produced be x.
Therefore, cost of production of each article = Rs $(2x + 3)$
It is given that the total production is Rs 90.

$\therefore x(2x+3) = 90$

$\Rightarrow 2x^2 + 3x - 90 = 0$

$\Rightarrow 2x^2 + 15x - 12x - 90 = 0$

$\Rightarrow x(2x+15) - 6(2x+15) = 0$

$\Rightarrow (2x+15)(x-6) = 0$

Either $2x + 15 = 0$ or $x - 6 = 0$, i.e., $x = \dfrac{-15}{2}$ or $x = 6$

As the number of artides produced can only be a positive integer, therefore, x can only be 6.

Hence, number of articles produced = 6
Cost of each artide = $2 \times 6 + 3$ = Rs 15

Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

(or)

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad , \quad \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

EX:4.3

EX: 4.3:

2. Find the roots of the quadratic equations given in Q.1 above by applying the quadratic formula.

(i) $2x^2 - 7x + 3 = 0$

Solution:

$a = 2, b = -7$ and $c = 3$

$$\therefore \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{7 \pm \sqrt{49 - 4(2)(3)}}{2 \times 2}$$

$$= \frac{7 \pm \sqrt{49 - 24}}{4} = \frac{7 \pm \sqrt{25}}{4} = \frac{7 \pm 5}{4}$$

Either $x = \frac{7 + 5}{4} = \frac{12}{4} = 3$

or $\quad x = \frac{7 - 5}{4} = \frac{2}{4} = \frac{1}{2}$

Hence, the required roots are **3** and $\mathbf{\frac{1}{2}}$.

(ii) $2x^2 + x - 4 = 0$

$a = 2, b = 1$ and $c = -4$

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

$$\therefore \quad x = \frac{-1 \pm \sqrt{1 - 4(2)(-4)}}{2} = \frac{-1 \pm \sqrt{33}}{2}$$

$$\Rightarrow \text{Either } x = \frac{-1 + \sqrt{33}}{2} \text{ or } x = \frac{-1 - \sqrt{33}}{2}$$

EX: 4.3:

2. Find the roots of the quadratic equations given in Q.1 above by applying the quadratic formula.

(iii) $4x^2 + 4\sqrt{3}x + 3 = 0$

Comparing it with $ax^2 + bx + c = 0$, we get:

$a = 4$, $b = 4\sqrt{3}$ and $c = 3$ $\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$

$$\therefore \quad x = \frac{-4\sqrt{3} \pm \sqrt{\left(4\sqrt{3}\right)^2 - 4(4)(3)}}{2 \times 4}$$

$$= \frac{-4\sqrt{3} \pm \sqrt{48 - 48}}{8}$$

$$= \frac{-4\sqrt{3}}{8} = \frac{-\sqrt{3}}{2}$$

Hence, the required roots are $\frac{-\sqrt{3}}{2}$ and $\frac{-\sqrt{3}}{2}$.

(iv) $2x^2 + x + 4 = 0$

Comparing it with $ax^2 + bx + c = 0$, we get:

$a = 2$, $b = 1$ and $c = 4$ $\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$

$$\therefore \quad x = \frac{-1 \pm \sqrt{1 - 4(2)(4)}}{2 \times 2}$$

$$= \frac{-1 \pm \sqrt{1 - 32}}{4} = \frac{-1 \pm \sqrt{(-31)}}{4}$$

Since $\sqrt{-31}$ is an imaginary value,

hence, roots do **not exist**.

EX: 4.3:

3. Find the roots of the following equations:

(i) $x - \frac{1}{x} = 3, x \neq 0$

(ii) $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}, x \neq -4, 7$

(i) Given: $x - \frac{1}{x} = 3$ $\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$

Multiplying both sides by x, we get:

$$x^2 - 1 = 3x$$

$$\Rightarrow \quad x^2 - 3x - 1 = 0$$

This is a quadratic equation.

Here, $a = 1$, $b = -3$ and $c = -1$

$$\therefore \quad x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \times 1 \times (-1)}}{2 \times 1}$$

$$= \frac{3 \pm \sqrt{9+4}}{2} = \frac{3 \pm \sqrt{13}}{2}$$

$$\Rightarrow \text{Either } x = \frac{\mathbf{3 + \sqrt{13}}}{\mathbf{2}} \text{ or } x = \frac{\mathbf{3 - \sqrt{13}}}{\mathbf{2}}$$

(ii) Given: $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}, x \neq -4, 7$

Multiplying both sides by $(x + 4)(x - 7)$, we get:

$$(x - 7) - (x + 4) = \frac{11}{30} \times (x + 4)(x - 7)$$

$$\Rightarrow \quad x - 7 - x - 4 = \frac{11}{30}(x^2 + 4x - 7x - 28)$$

$$\Rightarrow \quad -11 = \frac{11}{30}(x^2 - 3x - 28)$$

$$\Rightarrow \quad -11 \times 30 = 11(x^2 - 3x - 28)$$

$$\Rightarrow \quad x^2 - 3x - 28 = -30$$

$$\Rightarrow \quad x^2 - 3x + 2 = 0$$

EX: 4.3:

3. Find the roots of the following equations:

(i) $x - \frac{1}{x} = 3, x \neq 0$

(ii) $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}, x \neq -4, 7$

This is a quadratic equation.

Here, $a = 1$, $b = -3$ and $c = 2$ $\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$

$$\therefore \quad x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \times 1 \times 2}}{2 \times 1}$$

$$= \frac{3 \pm \sqrt{9-8}}{2}$$

$$= \frac{3 \pm \sqrt{1}}{2} = \frac{3 \pm 1}{2}$$

Either $x = \frac{3+1}{2}$ or $x = \frac{3-1}{2}$

$\Rightarrow \quad x = 2$ or $x = 1$.

EX: 4.3:

4. The sum of the reciprocals of Rehman's ages, (in years) 3 years ago and 5 years from now is 1/3. Find his present age.

Solution:

Let us say, present age of Rahman is x years.

Three years ago, Rehman's age was $(x - 3)$ years.

Five years after, his age will be $(x + 5)$ years.

Given, the sum of the reciprocals of

Rehman's ages 3 years ago and after 5 years

is equal to 1/3

EX: 4.3:

4. The sum of the reciprocals of Rehman's ages, (in years) 3 years ago and 5 years from now is 1/3. Find his present age.

According to the question, we have:

$$\frac{1}{x-3}+\frac{1}{x+5}=\frac{1}{3}$$

$$\Rightarrow \frac{x+5+x-3}{(x-3)(x+5)}=\frac{1}{3}$$

$$\Rightarrow \frac{2x+2}{x^2+2x-15}=\frac{1}{3}$$

$$\Rightarrow 6x+6=x^2+2x-15$$

$$\Rightarrow x^2-4x-21=0$$

This is a quadratic equation.

Here, $a=1$, $b=-4$ and $c=-21$

$$\boxed{x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}}$$

$$\therefore \quad x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(-21)}}{2}$$

$$=\frac{4\pm\sqrt{16+84}}{2}=\frac{4\pm\sqrt{100}}{2}$$

$$=\frac{4\pm 10}{2}=2\pm 5$$

Either $x=2+5$ or $x=2-5$

$\Rightarrow \quad x=7 \quad$ or $x=-3$

Since the age cannot be negative, so we ignore the root $x=-3$.

Hence, the present age of Rehman is **7 years**.

EX: 4.3:

5. In a class test, the sum of Shefali's marks in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks less in English, the product of their marks would have been 210. Find her marks in the two subjects.

Let Shefali's marks in Mathematics be x.

Then, her marks in English will be $30 - x$.

According to the question, we have:

$$(x + 2)(30 - x - 3) = 210$$

$$\Rightarrow \quad (x + 2)(27 - x) = 210$$

$$\Rightarrow \quad -x^2 + 25x + 54 = 210$$

$$\Rightarrow \quad x^2 - 25x + 156 = 0$$

$$\Rightarrow \quad x^2 - 12x - 13x + 156 = 0$$

$$\Rightarrow \quad (x - 12)(x - 13) = 0$$

$\Rightarrow x = 12$ or $x = 13 = 0$

If $x = 12$, then marks in Mathematics = **12** and in English = 30 – 12 = **18**.

If $x = 13$, then mark in Mathematics = **13** and in English = 30 – 13 = **17**.

EX: 4.3:

6. The diagonal of a rectangular field is 60 metres more than the shorter side. If the longer side is 30 metres more than the shorter side, find the sides of the field.

Let ABCD be the rectangular field with shorter side AB = x m.

Then, longer side BC = $(x + 30)$ m

and diagonal AC = $(x + 60)$ m

In ΔABC, by Pythagoras' theorem, we have:

$$AB^2 + BC^2 = AC^2$$

$$\Rightarrow \quad x^2 + (x + 30)^2 = (x + 60)^2$$

$$\Rightarrow \quad x^2 + x^2 + 60x + 900 = x^2 + 120x + 3600$$

$$\Rightarrow \quad x^2 - 60x - 2700 = 0$$

$$\Rightarrow \quad x^2 - 90x + 30x - 2700 = 0$$

$$\Rightarrow \quad (x - 90)(x + 30) = 0$$

$$\Rightarrow \quad x = 90 \quad \text{or } x = -30$$

Since the length cannot be negative, so $x = -30$ is rejected.

$\because x = 90$

$\therefore$ AB = 90 m, BC = (90 + 30) m = 120 m

Hence, the sides of the rectangular field are **90** m and **120 m**.

EX: 4.3:

7. The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers.

Let the larger and smaller number be x and y respectively.
According to the given question,

$x^2 - y^2 = 180$ and $y^2 = 8x$

$\Rightarrow x^2 - 8x = 180$

$\Rightarrow x^2 - 8x - 180 = 0$

$\Rightarrow x^2 - 18x + 10x - 180 = 0$

$\Rightarrow x(x - 18) + 10(x - 18) = 0$

$\Rightarrow (x - 18)(x + 10) = 0$

$\Rightarrow x = 18, -10$

However, the larger number cannot be negative as 8 times of the larger number will be negative and hence, the square of the smaller number will be negative which is not possible.
Therefore, the larger number will be 18 only.

$x = 18$

$\therefore\ y^2 = 8x = 8 \times 18 = 144$

$\Rightarrow\ y = \pm\sqrt{144} = \pm 12$

$\therefore$ Smaller number $= \pm 12$

Therefore, the numbers are 18 and 12 or 18 and −12.

EX: 4.3:

8. A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.

Solution Let the speed of the train be x km/h.

Distance covered by train = 360 km.

Then time taken to cover 360 km $= \frac{360}{x}$ h

When speed is increased by 5 km/h, the time taken $= \frac{360}{x+5}$ h.

According to the question, we have:

$$\frac{360}{x} - \frac{360}{x+5} = 1$$

$$\Rightarrow \quad \frac{360(x+5) - 360(x)}{x(x+5)} = 1$$

$$\Rightarrow \quad 360(x + 5 - x) = x(x + 5)$$

$$\Rightarrow \quad 360 \times 5 = x^2 + 5x$$

$$\Rightarrow \quad x^2 + 5x - 1800 = 0$$

Here, $a = 1$, $b = 5$ and $c = -1800$.

$\therefore \quad D = b^2 - 4ac$

$= (5)^2 - 4 \times 1 \times (-1800)$

$= 25 + 7200 = 7225$

$$\therefore \quad x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-5 \pm \sqrt{7225}}{2 \times 1} = \frac{-5 \pm 85}{2}$$

$\Rightarrow \quad x = 40$ or $x = -45$

Since the speed cannot be negative, so $x = -45$ is rejected.

Hence, the required speed of the train is **40 km/h.**

EX: 4.3:

9. Two water taps together can fill a tank in 9 3/8 hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

Solution:

Let the time taken by the smaller pipe to fill the tank = x hr.

Time taken by the larger pipe = (x – 10) hr

Part of tank filled by smaller pipe in 1 hour = 1/x
Part of tank filled by larger pipe in 1 hour = 1/(x – 10)

As given, the tank can be filled in 9 3/8 = 75/8 hours by both the pipes together.

$$\frac{1}{x}+\frac{1}{x-10}=9\frac{3}{8}$$

$$\Rightarrow \quad \frac{x-10+x}{x(x-10)}=\frac{8}{75}$$

$\left[\because \text{The amount of water flowing in 1 h} = \frac{1}{\frac{75}{8}}\right]$

$$\Rightarrow \quad 75(2x-10)=8x(x-10)$$

$$\Rightarrow \quad 150x-750=8x^2-80x$$

$$\Rightarrow \quad 8x^2-230x+750=0$$

Here, $a = 8$, $b = -230$ and $c = 750$.

$$\therefore \quad D = b^2-4ac$$

$$= (-230)^2-4\times 8\times 750$$

$$= 52900-24000 = 28900$$

$$\therefore \quad x=\frac{-b\pm\sqrt{D}}{2a}=\frac{-(-230)\pm\sqrt{28900}}{2\times 8}$$

$$=\frac{230\pm 170}{16}$$

EX: 4.3:

9. Two water taps together can fill a tank in 9 3/8 hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

Either $x = \frac{230 + 170}{16}$ or $\frac{230 - 170}{16}$

$\Rightarrow x = 25$ or $x = \frac{15}{4}$

Neglecting $x = \frac{15}{4}$, we have $x = 25$.

Hence, the smaller tap takes **25 h** and the larger tap takes **15 h** to fill the tank.

EX: 4.3:

10. An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore (without taking into consideration the time they stop at intermediate stations). If the average speeds of the express train is 11 km/h more than that of the passenger train, find the average speed of the two trains.

Let the average speed of passenger train be x km/h.
Average speed of express train = $(x + 11)$ km/h
It is given that the time taken by the express train to cover 132 km is 1 hour less than the passenger train to cover the same distance.

$$\therefore \frac{132}{x} - \frac{132}{x+11} = 1$$

$$\Rightarrow 132\left[\frac{x+11-x}{x(x+11)}\right] = 1$$

$$\Rightarrow \frac{132 \times 11}{x(x+11)} = 1$$

$$\Rightarrow 132 \times 11 = x(x+11)$$

$$\Rightarrow x^2 + 11x - 1452 = 0$$

$$\Rightarrow x^2 + 44x - 33x - 1452 = 0$$

$$\Rightarrow x(x+44) - 33(x+44) = 0$$

$$\Rightarrow (x+44)(x-33) = 0$$

$$\Rightarrow x = -44, 33$$

Speed cannot be negative. Therefore, the speed of the passenger train will be 33 km/h and thus, the speed of the express train will be 33 + 11 = 44 km/h.

EX: 4.3:

11. Sum of the areas of two squares is 468 m². If the difference of their perimeters is 24 m, find the sides of the two squares.

Let the side of smaller square be x and that of larger square be y.

Then, perimeter of smaller square will be $4x$ and that of larger square will be $4y$.

According to the question, we have:

$$4y - 4x = 24$$

$$\Rightarrow \quad y - x = 6 \quad \text{[Dividing both sides by 4]}$$

$$\Rightarrow \quad y = x + 6 \quad \text{...(i)}$$

Also given that the sum of squares of areas is 468 m².

$$\therefore \quad x^2 + y^2 = 468$$

$$\Rightarrow \quad x^2 + (x+6)^2 = 468 \quad \text{[From (i)]}$$

$$\Rightarrow \quad x^2 + x^2 + 12x + 36 = 468$$

$$\Rightarrow \quad 2x^2 + 12x - 432 = 0$$

$$\Rightarrow \quad x^2 + 6x - 216 = 0$$

Here, $a = 1$, $b = 6$ and $c = -216$.

$$\therefore \quad D = b^2 - 4ac$$

$$= (6)^2 - 4 \times 1 \times (-216) = 36 + 864$$

$$= 900$$

$$\therefore \quad x = \frac{-b \pm \sqrt{D}}{2a}$$

$$= \frac{-6 \pm \sqrt{900}}{2 \times 1} = \frac{-6 \pm 30}{2}$$

Either $x = \frac{-6+30}{2}$ or $\frac{-6-30}{2}$

$$\Rightarrow \quad x = 12 \text{ or } x = -18$$

Since the length of the side of a square cannot be negative, so $x = -18$ is rejected.

$\therefore$ $x = 12$ and $y = 12 + 6 = 18$

Hence, the sides of the two squares are **12 m** and **18 m**.

DISCRIMINANT

- For a quadratic equation of the form ax2+bx+c=0, the expression b2−4ac is called the discriminant, (denoted by D), of the quadratic equation.
- The discriminant determines the nature of roots of the quadratic equation based on the coefficients of the quadratic polynomial.

NATURE OF ROOTS

Based on the value of the discriminant, D=b2−4ac, the roots of a quadratic equation can be of three types.

- Case 1: If D>0, the equation has two distinct real roots.
- Case 2: If D=0, the equation has two equal real roots.
- Case 3: If D<0, the equation has no real roots.

EX:4.4

EX: 4.4:

1. Find the nature of the roots of the following quadratic equations. If the real roots exist, find them;

(i) $2x^2 - 3x + 5 = 0$

(ii) $3x^2 - 4\sqrt{3}x + 4 = 0$

(iii) $2x^2 - 6x + 3 = 0$

Solutions:

(i) Given: $2x^2 - 3x + 5 = 0$

Here, $a = 2$, $b = -3$ and $c = 5$.

$\therefore$ Discriminant, $D = b^2 - 4ac$

$$= (-3)^2 - 4 \times 2 \times 5$$

$$= 9 - 40 = -31 < 0$$

Hence, the roots are **imaginary**.

(ii) Given: $3x^2 - 4\sqrt{3}x + 4 = 0$

Here, $a = 3$, $b = -4\sqrt{3}$ and $c = 4$.

$\therefore$ Discriminant, $D = b^2 - 4ac$

$$= \left(-4\sqrt{3}\right)^2 - 4 \times 3 \times 4$$

$$= 48 - 48 = 0$$

Hence, the roots are **real** and **equal**.

Now using the formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \text{ we get:}$$

$$x = \frac{-\left(-4\sqrt{3}\right) \pm \sqrt{\left(-4\sqrt{3}\right)^2 - 4 \times 3 \times 4}}{2 \times 3}$$

$$= \frac{4\sqrt{3} \pm \sqrt{48 - 48}}{6} = \frac{4\sqrt{3}}{6} = \frac{2}{\sqrt{3}}$$

Hence, the **equal roots** are $\mathbf{\frac{2}{\sqrt{3}}}$ and $\mathbf{\frac{2}{\sqrt{3}}}$.

EX: 4.4:

1. Find the nature of the roots of the following quadratic equations. If the real roots exist, find them;

(iii) $2x^2 - 6x + 3 = 0$

Solution:

Here, $a = 2$, $b = -6$ and $c = 3$.

$\therefore$ Discriminant, $D = b^2 - 4ac$

$$= (-6)^2 - 4 \times 2 \times 3$$

$$= 36 - 24 = 12 > 0$$

Hence, the roots are **distinct** and **real**.

Now using the formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \text{ we get:}$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(2)(3)}}{2 \times 2}$$

$$= \frac{6 \pm \sqrt{36 - 24}}{4} = \frac{6 \pm \sqrt{12}}{4}$$

$$= \frac{6 \pm 2\sqrt{3}}{4} = \frac{3 \pm \sqrt{3}}{2}$$

i.e., $x = \mathbf{\frac{3 + \sqrt{3}}{2}}$ or $x = \mathbf{\frac{3 - \sqrt{3}}{2}}$.

EX: 4.4:

2. Find the values of k for each of the following quadratic equations, so that they have two equal roots.

(i) $2x^2 + kx + 3 = 0$
(ii) $kx(x - 2) + 6 = 0$

Solution:

(i) Given: $2x^2 + kx + 3 = 0$

Here, $a = 2$, $b = k$ and $c = 3$.

Since the roots are real and equal

$\therefore \quad D = b^2 - 4ac = 0$

$\Rightarrow \quad k^2 - 4 \times 2 \times 3 = 0$

$\Rightarrow \quad k^2 = 24$

$\Rightarrow \quad \mathbf{k = \pm 2\sqrt{6}}$

(ii) Given: $kx(x - 2) + 6 = 0$

$\Rightarrow \quad kx^2 - 2kx + 6 = 0$

Here, $a = k$, $b = -2k$ and $c = 6$.

Since the roots are real and equal

$\therefore \quad D = b^2 - 4ac = 0$

$\Rightarrow \quad (-2k)^2 - 4 \times k \times 6 = 0$

$\Rightarrow \quad 4k^2 - 24k = 0$

$\Rightarrow \quad 4k(k - 6) = 0$

Either $4k = 0$ or $k - 6 = 0$

$\Rightarrow k = 0$ or $k = 6$

$k \neq 0$, because coefficient of x^2 and x cannot be zero in the given equation.

Hence, k = 6.

EX: 4.4:

3. Is it possible to design a rectangular mango grove whose length is twice its breadth, and the area is 800 m^2? If so, find its length and breadth.

Let breadth of the rectangular be x m

Then, the length of rectangular will be 2x m.

According to question, we have

$$\text{Length} \times \text{Breadth} = \text{Area}$$

$$\Rightarrow \quad x \times 2x = 800$$

$$\Rightarrow \quad 2x^2 = 800$$

$$\Rightarrow \quad x^2 = 400 = (20)^2$$

$$\Rightarrow \quad x = 20$$

Hence, the rectangular mango grove is **possible** to design whose breadth is **20 m** and length is **40 m**.

EX: 4.4:

4. Is the following situation possible? If so, determine their present ages. The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48.

Solution:

Let's say, the age of one friend be x years.

Then, the age of the other friend will be (20 – x) years.

Four years ago,

Age of First friend = $(x - 4)$ years

Age of Second friend = $(20 - x - 4) = (16 - x)$ years

As per the given question, we can write,

$(x - 4)(16 - x) = 48$

$16x - x^2 - 64 + 4x = 48$

$-x^2 + 20x - 112 = 0$

$x^2 - 20x + 112 = 0$

Here, $a = 1$, $b = -20$ and $c = 112$.

Discriminant, $D = b^2 - 4ac$

$= (-20)^2 - 4 \times 1 \times 112$

$= 400 - 448 = -48 < 0$

$\therefore$ No real roots exist for x.

Hence, the given situation is **not possible.**

EX: 4.4:

5. Is it possible to design a rectangular park of perimeter 80 and area 400 m2? If so find its length and breadth.

Solution: Let the length and of the park be x.

Then, the perimeter of rectangular park

$$= 2(\text{Length} + \text{Breadth})$$

$$\Rightarrow \quad 2(x + \text{Breadth}) = 80$$

$$\Rightarrow \quad \text{Breadth} = 40 - x$$

$\therefore$ Area of rectangular park $=$ Length $\times$ Breadth

$$\Rightarrow \quad x(40 - x) = 400$$

$$\Rightarrow \quad 40x - x^2 = 400$$

$$\Rightarrow \quad x^2 - 40x + 400 = 0$$

$$\Rightarrow \quad x^2 - 20x - 20x + 400 = 0$$

$$\Rightarrow \quad (x - 20)(x - 20) = 0$$

$$\Rightarrow \quad x = 20$$

Thus, the rectangular park is **possible** to design. So, length of park $=$ **20 m** and its breadth $= 40 - 20 =$ **20 m**.

4.QUADRATIC EQUATIONS COMPLETED