Factorisation of Polynomials

Q1. Factorize completely using factor theorem: $2x^3 - x^2 - 13x - 6$ [2023]

Answer: (x+2)(x-3)(2x+1)

Step-by-step Explanation:

$$P(x) = 2x^3 - x^2 - 13x - 6$$

$$Let\ x = -2,$$

$$the\ value\ of\ f(x)\ will\ be$$

$$f(-2) = 2(-2)^3 - (-2)^2 - 13(-2) - 6$$

$$= -16 - 4 + 26 - 6$$

$$= 0$$

$$As\ f(-2) = 0,\ so\ (x+2)\ is\ a\ factor\ of\ f(x).$$

$$Now,\ performing\ long\ division\ we\ have\ Thus,$$

$$\Rightarrow \quad x + 2 \overline{)\, 2x^3 - x^2 - 13x - 6 \,(}\ 2x^2 - 5x - 3$$

$$\begin{array}{l}
-2x^3 + 4x^2 \\
\quad\ \ - \\
\hline
-5x^2 - 13x - 6 \\
-5x^2 - 10x \\
+ \qquad\ + \\
\hline
\qquad -3x - 6 \\
\qquad -3x - 6 \\
\hline
\qquad\quad 0 \\
\hline
\end{array}$$

$$P(x) = 2x^3 - x^2 - 13x - 6$$

$$\text{Let } x = -2,$$

$$\text{the value of } f(x) \text{ will be}$$

$$f(-2) = 2(-2)^3 - (-2)^2 - 13(-2) - 6$$

$$= -16 - 4 + 26 - 6$$

$$= 0$$

$$\text{As } f(-2) = 0, \text{ so } (x+2) \text{ is a factor of } f(x).$$

$$\text{Now, performing long division we have Thus,}$$

$$f(x) = (x+2)(2x^2 - 5x - 3)$$

$$= (x+2)[2x^2 - 6x + x - 3]$$

$$= (x+2)[2x(x-3) + 1(x-3)]$$

$$= (x+2)[(2x+1)(x-3)]$$

$$= (x+2)(2x+1)(x-3)$$

Q2. Find the value of 'a' if x -a is a factor of the polynomial

$3x^3 + x^2 - ax - 81$. [4] [2023]

Answer: a=3

Step-by-step Explanation:

$$x - a = 0$$
$$x = a and,$$
$$p(x) = 3x^3 + x^2 - ax - 81$$
$$substituting\ x = a\ in\ p(x)\ we\ get,$$
$$3a^3 + a^2 - a^2 - 81 = 0$$
$$3a^3 - 81 = 0$$
$$3a^3 = 81$$
$$a^3 = 27$$
$$a = 3$$

Q3. If x -2 is a factor of x^3 – kx -12, then the value of k is:

(a) 3

(b) 2

(c) -2

(d) -3 [2023]

Answer: (c) -2

Step-by-step Explanation:

$$P(x) = x^3 - kx - 12$$

$$(x-2) is\ a\ factor\ of\ P(x)$$

$$So,\ 2\ is\ the\ zero\ of\ the\ polynomial$$

$$Substitute\ x = 2\ in\ P(x)$$

$$x^3 - kx - 12 = 0$$

$$2^3 - k.2 - 12 = 0$$

$$8 - 2k - 12 = 0$$

$$-2k - 4 = 0$$

$$-2k = 4$$

$$k = -2$$

Q4. If (x + 2) is a factor of the polynomial $x^3 - kx^2 - 5x + 6$ then the value of k is: [1]

(a) 1

(b) 2

(c) 3

(d) -2 [2021 Semester-1]

Answer: (b) 2

Step-by-step Explanation:

$$P(x) = x^3 - kx^2 - 5x + 6$$

$$(x+2) is\ a\ factor\ of\ P(x)$$

$$So,\ -2\ is\ the\ zero\ of\ the\ polynomial$$

$$Substitute\ x = -2\ in\ P(x)$$

$$x^3 - kx^2 - 5x + 6 = 0$$

$$(-2)^3 - k.(-2)^2 - 5.(-2) + 6 = 0$$

$$-8 - 4k + 10 + 6 = 0$$

$$-4k + 8 = 0$$

$$-4k = -8$$

$$k = 2$$

Q5. The polynomial $x^3 - 2x^2 + ax + 12$ when divided by $(x + 1)$ leaves a remainder 20, then 'a' is equal to: [1]

(a) – 31

(b) 9

(c) 11

(d) – 11 [2021 Semester-1]

Answer: (d) -11

Step-by-step Explanation:

$$x + 1 = 0$$
$$x = -1 and,$$
$$p(x) = x^3 - 2x^2 + ax + 12$$
$$By\ remainder\ theorem,$$
$$p(-1) = 20$$
$$substituting\ x = -1\ in\ p(x)\ we\ get,$$
$$(-1)^3 - 2.(-1)^2 + a.(-1) + 12 = 20$$
$$-1 - 2 - a + 12 = 20$$
$$9 - a = 20$$
$$-a = 11$$
$$a = -11$$

Q6. (x + 2) and (x + 3) are two factors of the polynomial $x^3 + 6x^2 + 11x + 6$. If this polynomial is completely factorised the result is: [2]
(a) (x – 2)(x + 3)(x + 1)

(b) (x + 2)(x – 3)(x – 1)

(c) (x + 2)(x + 3)(x – 1)

(d) (x + 2)(x + 3)(x + 1) [2021 Semester-1]

Answer: (d) (x+2)(x+3)(x+1)

Step-by-step Explanation:

$$
\begin{aligned}
(x+2)(x+3) &= x^2+2x+3x+6 \\
&= x^2+5x+6
\end{aligned}
$$

$$
\begin{array}{r}
x+1 \\
x^2+5x+6 \overline{)\, x^3+6x^2+11x+6} \\
-\ (x^3+5x^2+6x) \\
\hline
0+x^2+5x+6 \\
-\ (x^2+5x+6) \\
\hline
0
\end{array}
$$

$$Therefore,\ p(x) = (x+2)(x+3)\ (x+1)$$

Q7. What must be added to the polynomial $2x^3 - 3x^2 - 8x$, so that it leaves a remainder 10 when divided by 2x + 1? [2020]

Answer: 7

Step-by-step Explanation:

$$
\begin{gathered}
Let\ a\ must\ be\ added\ to\ the\ polynomial. \\
Therefore,\ p(x) = 2x^3 - 3x^2 - 8x + a \\
The\ polynomial\ is\ divided\ by\ (2x+1) \\
So,\ 2x+1=0 \\
x = -\frac{1}{2} \\
Therefore,\ by\ remainder\ theorem,
\end{gathered}
$$

$$p(-\frac{1}{2}) = 10$$

$$2(-\frac{1}{2})^3 - 3(-\frac{1}{2})^2 - 8(-\frac{1}{2}) + a = 10$$

$$2.(-\frac{1}{8}) - 3.\frac{1}{4} + \frac{8}{2} + a = 10$$

$$-\frac{1}{4} - \frac{3}{4} + 4 + a = 10$$

$$a = 10 - 4 + \frac{1}{4} + \frac{3}{4}$$

$$a = 6 + \frac{1}{4} + \frac{3}{4}$$

$$a = \frac{24 + 1 + 3}{4}$$

$$a = \frac{28}{4}$$

$$a = 7$$

Q8. Use factor theorem to factorise

$6x^3 + 17x^2 + 4x - 12$ completely. [2020]

Answer: (x+2)(2x+3)(3x-2)

Step-by-step Explanation:

$$p(x) = 6x^3 + 17x^2 + 4x - 12$$

Taking $x = -2$ *we have,*

$$p(-2) = 6.(-2)^3 + 17.(-2)^2 + 4.(-2) - 12$$
$$= -48 + 68 - 8 - 12$$
$$= -68 + 68$$
$$= 0$$

Therefore, $(x + 2)$ *is a factor of* $p(x)$.

dividing $p(x)$ *by* $(x + 2)$ *we have,*

$$\begin{array}{r l}
 & 6x^2 + 5x - 6 \\ \cline{2-2}
x + 2) & 6x^3 + 17x^2 + 4x - 12 \\
 & -\ (6x^3 + 12x^2) \\ \cline{2-2}
 & 0 + 5x^2 + 4x - 12 \\
 & -\ (5x^2 + 10x) \\ \cline{2-2}
 & 0 - 6x - 12 \\
 & -\ (-6x - 12) \\ \cline{2-2}
 & 0
\end{array}$$

$$6x^2 + 5x - 6$$
$$= 6x^2 + 9x - 4x - 6$$
$$= 3x(2x + 3) - 2(2x + 3)$$
$$= (3x - 2)(2x + 3)$$

Therefore $p(x) = (x + 2)(3x - 2)(2x + 3)$

Q9. Using the factor theorem, show that (x – 2) is a factor of $x^3 + x^2 - 4x - 4$. [3]
Hence, factorise the polynomial completely. [2019]

Answer: (x-2)(x+2)(x+1)

Step-by-step Explanation:

$$f(x) = x^3 + x^2 - 4x - 4.$$

$$Let\ x - 2 = 0$$

$$x = 2$$

$$Therefore,$$

$$f(2) = (2)^3 + (2)^2 - 4.2 - 4$$

$$= 8 + 4 - 8 - 4$$

$$= 0$$

$$Hence,\ x - 2\ is\ a\ factor\ of\ f(x).$$

$$Dividing\ f(x)\ by\ (x - 2),\ we\ have,$$

$$\begin{array}{r} x^2 + 3x + 2 \\ x - 2 \overline{\big) x^3 + x^2 - 4x - 4} \\ - \ (x^3 - 2x^2) \\ \hline 0 + 3x^2 - 4x - 4 \\ - \ (3x^2 - 6x) \\ \hline 0 + 2x - 4 \\ - \ (2x - 4) \\ \hline 0 \end{array}$$

$$f(x) = (x - 2)(x + 2)(x + 1)$$

Q10. Using the Remainder Theorem find the remainders obtained when $x^3 + (kx + 8)x + k$ is divided by $x + 1$ and $x - 2$. Hence, find k if the sum of the two remainders is 1. [3] [2019]

Answer: k=-2

Step-by-step Explanation:

$$f(x) = x^3 + (kx + 8)x + k$$

$$g(x) = x + 1$$

$$So,\ x = -1$$

$$using\ the\ remainder\ theorem,$$

$$f(-1) = Remainder_1$$

$$(-1)^3 + \{k.(-1) + 8\}.(-1) + k$$

$$-1 + k - 8 + k$$

$$Remainder_1 = 2k - 9$$

$$Now,\ h(x) = x - 2$$

$$Therefore,\ x = 2$$

$$f(2) = Remainder_2$$

$$(2)^3 + (k.2 + 8).2 + k$$

$$8 + 4k + 16 + k$$

$$Remainder_2 = 5k + 24$$

$$Given\ that,$$

$$(2k - 9) + (5k + 24) = 1$$

$$7k + 15 = 1$$

$$7k = -14$$

$$k = -2$$

Q11. If (x + 2) and (x + 3) are factors of $x^3 + ax + b$, find the values of 'a' and 'b'. [3] [2018]

Answer: a =-19 , b = -30.

Step-by-step Explanation:

$$f(x) = x^3 + ax + b$$
$$Given,\ (x+2)\ is\ a\ factor\ of\ f(x).$$
$$By\ factor\ theorem,$$
$$f(-2) = 0$$
$$(-2)^3 + a.(-2) + b = 0$$
$$-8 - 2a + b = 0$$
$$-2a + b = 8 \ldots\ldots.(1)$$
$$Also\ given\ (x+3)\ is\ a\ factor\ of\ f(x)$$
$$f(-3) = 0$$
$$(-3)^3 + a.(-3) + b = 0$$
$$-27 - 3a + b = 0$$
$$-3a + b = 27 \ldots\ldots.(2)$$
$$Subtracting\ (1)\ from\ (2)\ we\ have,$$
$$-a = 19$$
$$a = -19$$
$$substituting\ a = -19\ in\ (1)\ we\ have$$
$$-2 \times (-19) + b = 8$$
$$38 + b = 8$$
$$b = -30$$
$$Hence,\ a = -19\ and\ b = -30.$$

Q12. Use Remainder theorem to factorize the following polynomial: [3]
$2x^3 + 3x^2 - 9x - 10$. [2018]

Answer: (x-2)(x+1)(2x+5)

Step-by-step Explanation:

$$f(x) = 2x^3 + 3x^2 - 9x - 10$$
$$Taking\ x = 2\ we\ have,$$
$$2.(2)^3 + 3.(2)^2 - 9.(2) - 10$$
$$= 16 + 12 - 18 - 10$$
$$= 28 - 28$$
$$= 0$$
$$Therefore,\ (x-2)\ is\ a\ factor\ of\ f(x).$$
$$Dividing\ f(x)\ by\ (x-2),\ we\ have,$$

$$\begin{array}{r|l}
 & 2x^2 + 7x + 5 \\ \hline
x - 2 & 2x^3 + 3x^2 - 9x - 10 \\
 & -\ (2x^3 - 4x^2) \\ \hline
 & 0 + 7x^2 - 9x - 10 \\
 & -\ (7x^2 - 14x) \\ \hline
 & 0 + 5x - 10 \\
 & -\ (5x - 10) \\ \hline
 & 0
\end{array}$$

$$
\begin{aligned}
& 2x^2 + 7x + 5 \\
&= 2x^2 + 5x + 2x + 5 \\
&= x(2x+5) + 1(2x+5) \\
&= (2x+5)(x+1)
\end{aligned}
$$

$$Hence,\ f(x) = (x-2)(x+1)(2x+5)$$

Q13. What must be subtracted from $16x^3 - 8x^2 + 4x + 7$ so that the resulting expression has $2x + 1$ as a factor? [3] [2017]

Answer: 1

Step-by-step Explanation:

$$
\begin{aligned}
& Let\ a\ be\ subtracted.\ Therefore, \\
& f(x) = 16x^3 - 8x^2 + 4x + 7 - a \\
& g(x) = 2x + 1 \\
& So,\ x = -\frac{1}{2} \\
& By\ factor\ theorem, \\
& f(-\frac{1}{2}) = 0 \\
& 16(-\frac{1}{2})^3 - 8(-\frac{1}{2})^2 + 4(-\frac{1}{2}) + 7 - a = 0 \\
& \frac{-16}{8} - \frac{8}{4} - \frac{4}{2} + 7 - a = 0 \\
& -2 - 2 - 2 + 7 - a = 0 \\
& 1 - a = 0 \\
& a = 1
\end{aligned}
$$

Q14. Using remainder theorem, find the value of k, if on dividing $2x^3 + 3x^2 - kx + 5$ by x-2, leaves a remainder 7. [3] [2016]

Answer: k=13

Step-by-step Explanation:

$$Let\ a\ be\ subtracted.\ Therefore,$$

$$f(x) = 2x^3 + 3x^2 - kx + 5$$

$$g(x) = x - 2$$

$$So,\ x = 2$$

$$By\ remainder\ theorem,$$

$$f(2) = 7$$

$$2(2)^3 + 3(2)^2 - k.2 + 5 = 7$$

$$16 + 12 - 2k + 5 = 7$$

$$33 - 2k = 7$$

$$-2k = 7 - 33$$

$$-2k = -26$$

$$k = 13$$

Q15. Find 'a' if the two polynomials $ax^3 + 3x^2 - 9$ and $2x^3 + 4x + a$, leaves the same remainder when divided by x + 3. [3] [2015]

Answer: a=3

Step-by-step Explanation:

$$\textit{The given polynomials are}$$
$$p(x) = ax^3 + 3x^2 - 9 \textit{ and}$$
$$q(x) = 2x^3 + 4x + a$$

Given that p(x) and q(x) leave the same remainder when divided by x + 3.

Thus by remainder theorem,

p(−3) = q(−3)

$$\Rightarrow a(-3)^3 + 3(-3)^2 - 9 = 2(-3)^3 + 4(-3) + a$$
$$\Rightarrow -27a + 27 - 9 = -54 - 12 + a$$
$$\Rightarrow -27a - a = -54 - 12 - 27 + 9$$

⇒ −28a = −93 + 9

⇒ −28a = −84

⇒ a = 3

Q16. Using the Remainder and Factor Theorem, factorise the following polynomial:
$x^3 + 10x^2 – 37x + 26$. [3] [2014]

Answer: (x-1)(x-2)(x+13)

Step-by-step Explanation:

$$f(x) = x^3 + 10x^2 - 37x + 26.$$
$$\textit{Let } x = 1$$
$$f(1) = (1)^3 + 10(1)^2 - 37(1) + 26$$
$$= 1 + 10 - 37 + 26$$
$$= 0$$

Therefore, By factor theorem,

(x−1) is a factor off (x).

Dividing f(x) by x – 1 we have,

$$
\begin{array}{r|l}
 & x^2 + 11x - 26 \\
x - 1 & x^3 + 10x^2 - 37x + 26 \\
 & -\ (x^3 - x^2) \\ \hline
 & 0 + 11x^2 - 37x + 26 \\
 & -\ (11x^2 - 11x) \\ \hline
 & 0 - 26x + 26 \\
 & -\ (-26x + 26) \\ \hline
 & 0
\end{array}
$$

$$
\begin{aligned}
& x^2 + 11x - 26 \\
&= x^2 + 13x - 2x - 26 \\
&= x(x + 13) - 2(x + 13) \\
&= (x + 13)(x - 2)
\end{aligned}
$$

$$Therefore,\ f(x) = (x - 1)(x - 2)(x + 13)$$

Q17. If (x – 2) is a factor of the expression $2x^3 + ax^2 + bx - 14$ and when the expression is divided by (x – 3), it leaves a remainder 52, find the values of a and b. [3] [2013]

Answer: a = 5 ; b = -11

Step-by-step Explanation:

$$f(x) = 2x^3 + ax^2 + bx - 14$$

Given, $(x-2)$ is a factor of $f(x)$.

By factor theorem,

$$f(2) = 0$$

$$2(2)^3 + a(2)^2 + b(2) - 14 = 0$$

$$16 + 4a + 2b - 14 = 0$$

$$4a + 2b = -2$$

$$2a + b = -1 \ldots\ldots (1)$$

Given, when f(x) is divided by (x – 3), it leaves 52 as remainder.

Therefore, By remainder theorem,

$f(3) = 52$

$$2(3)^3 + a(3)^2 + b(3) - 14 = 52$$

$54 + 9a + 3b - 14 = 52$
$9a + 3b = 52 + 14 - 54$
$3(3a + b) = 12$
$3a + b = 4 \ldots\ldots (2)$
Subtracting (1) by (2) we get,
$a = 5$
Substituting $a = 5$ in (1)
$2a + b = -1$
$2 \times 5 + b = -1$
$10 + b = -1$
$b = -11$
Hence, $a = 5$ and $b = -11$

Q18. Using the Remainder Theorem factorise completely the following polynomial:

$3x^3 + 2x^2 - 19x + 6$. [3] [2012]

Answer: (x-2)(x+3)(3x-1)

Step-by-step Explanation:

$$f(x) = 3x^3 + 2x^2 - 19x + 6.$$

$$Taking\ x = 2\ we\ have,$$

$$\begin{aligned} f(2) &= 3(2)^3 + 2(2)^2 - 19 \times 2 + 6 \\ &= 24 + 8 - 38 + 6 \\ &= 38 - 38 \\ &= 0 \end{aligned}$$

$$Therefore,\ (x-2)\ is\ a\ factor\ of\ f(x).$$

$$Dividing\ f(x)\ by\ (x-2),\ we\ have,$$

$$\begin{array}{r} 3x^2 + 8x - 3 \\ x - 2 \overline{\big) 3x^3 + 2x^2 - 19x + 6} \\ -\ (3x^3 - 6x^2) \\ \hline 0 + 8x^2 - 19x + 6 \\ -\ (8x^2 - 16x) \\ \hline 0 - 3x + 6 \\ -\ (-3x + 6) \\ \hline 0 \end{array}$$

$$3x^2 + 8x - 3$$
$$= 3x^2 + 9x - x - 3$$
$$= 3x(x+3) - 1(x+3)$$
$$= (3x-1)(x+3)$$
$$Therefore,\ f(x) = (x-2)(x+3)(3x-1)$$

Q19. Find the value of 'k' if (x – 2) is a factor of $x^3 + 2x^2 - kx + 10$? [3] [2011]

Answer: k = 13

Step-by-step Explanation:

$$f(x) = x^3 + 2x^2 - kx + 10$$
$$(x-2)\ is\ a\ factor\ of\ f(x).$$
$$Therefore,\ f(2) = 0$$
$$(2)^3 + 2 \times (2)^2 - k \times 2 + 10 = 0$$
$$8 + 8 - 2k + 10 = 0$$
$$-2k + 26 = 0$$
$$-2k = -26$$
$$k = 13$$

Q20. When divided by x – 3 the polynomials $x^3 - px^2 + x + 6$ and $2x^3 - x^2 - (p + 3)x - 6$ leave the same remainder. Find the value of 'p'. [3] [2010]

Answer: p = 1

Step-by-step Explanation:

$$p(x) = x^3 - px^2 + x + 6 \text{ and}$$
$$q(x) = 2x^3 - x^2 - (p + 3)x - 6$$
$$\text{when } (x - 3) \text{ divides } p(x) \text{ and } q(x), \text{ the remainders are same.}$$
$$Therefore, \ p(3) = q(3)$$
$$(3)^3 - p \times (3)^2 + 3 + 6 = 2 \times (3)^3 - (3)^2 - (p + 3) \times 3 - 6$$
$$27 - 9p + 9 = 54 - 9 - 3p - 9 - 6$$
$$-9p + 3p + 36 = 54 - 24$$
$$-6p = 30 - 36$$
$$-6p = -6$$
$$p = 1$$

Q21. Use the Remainder Theorem to factorise the following expression:
$2x^3 + x^2 - 13x + 6$ [3] [2010]

Answer: (x-2)(x+3)(2x-1)

Step-by-step Explanation:

$$f(x) = 2x^3 + x^2 - 13x + 6$$
$$taking \ x = 2, \ we \ have,$$
$$f(2) = 2 \times (2)^3 + (2)^2 - 13 \times 2 + 6$$
$$= 16 + 4 - 26 + 6$$
$$= 26 - 26$$
$$= 0$$
$$Therefore, \ (x - 2) \text{ is a factor of } f(x).$$
$$\text{dividing } f(x) \text{ by } (x - 2) \text{ we have,}$$

$$
\begin{array}{r}
2x^2 + 5x - 3 \\
x - 2 \overline{\smash{\big)}\, 2x^3 + x^2 - 13x + 6} \\
-\ (2x^3 - 4x^2) \\
\hline
0 + 5x^2 - 13x + 6 \\
-\ (5x^2 - 10x) \\
\hline
0 - 3x + 6 \\
-\ (-3x + 6) \\
\hline
0
\end{array}
$$

$$
\begin{aligned}
&2x^2 + 5x - 3 \\
&= 2x^2 + 6x - x - 3 \\
&= 2x(x + 3) - 1(x + 3) \\
&= (x + 3)(2x - 1)
\end{aligned}
$$

$$Therefore,\ f(x) = (x - 2)(x + 3)(2x - 1)$$