KISA PREPARATORY EXAM
2023-2024

CLASS: X SUB: MATHEMATICS MARKS: 80

Question 1 (15 marks)

(i) d) 6

(ii) c) 3

(iii) c) $\frac{-2}{3}$

(iv) b) cosA

v) b) 25

vi) a) 30°

vii) c) 2:5

viii) c) ₹800

ix) b) 20:27

x) d) ₹5400

xi) d) both of (a) and (b)

xii) c) 11:3

xiii) a) {0,1,2,3}

xiv) b) (1,1)

xv) d) (A) is false but (R) is true

Question 2

i) To prove $\frac{\operatorname{cosec}\theta}{(\operatorname{cosec}\theta-1)} + \frac{\operatorname{cosec}\theta}{(\operatorname{cosec}\theta+1)} = 2\sec^2\theta$

LHS: $\frac{\operatorname{cosec}\theta}{(\operatorname{cosec}\theta-1)} + \frac{\operatorname{cosec}\theta}{(\operatorname{cosec}\theta+1)}$

$= \frac{\operatorname{cosec}\theta(\operatorname{cosec}\theta+1) + \operatorname{cosec}\theta(\operatorname{cosec}\theta-1)}{(\operatorname{cosec}\theta-1)(\operatorname{cosec}\theta+1)}$ (1)

$= \frac{\operatorname{cosec}^2\theta + \operatorname{cosec}\theta + \operatorname{cosec}^2\theta - \operatorname{cosec}\theta}{\operatorname{cosec}^2\theta - 1}$

$$= \frac{2\operatorname{cosec}^2\theta}{\cot^2\theta} \quad (1) \quad [\because \operatorname{cosec}^2\theta - \cot^2\theta = 1]$$

$$= 2 \times \frac{1}{\sin^2\theta} \times \frac{\sin^2\theta}{\cos^2\theta}$$

$$= \frac{2}{\cos^2\theta} \quad (1)$$

$$= 2\sec^2\theta \quad (1)$$

ii) $MV = ₹8088$, $n = 3$ years $= 36$ months, $r = 8\%$ pa

$$MV = P \times n + \frac{P \times n \times (n+1)}{2 \times 12} \times \frac{r}{100} \quad (1)$$

$$8088 = 36P + \frac{P \times 36 \times 37}{2 \times 12} \times \frac{8}{100}$$

$$8088 = 36P + \frac{111P}{25} \quad (1)$$

$$8088 = \frac{1011P}{25} \quad (1)$$

$$\therefore P = \frac{8088 \times 25}{1011}$$

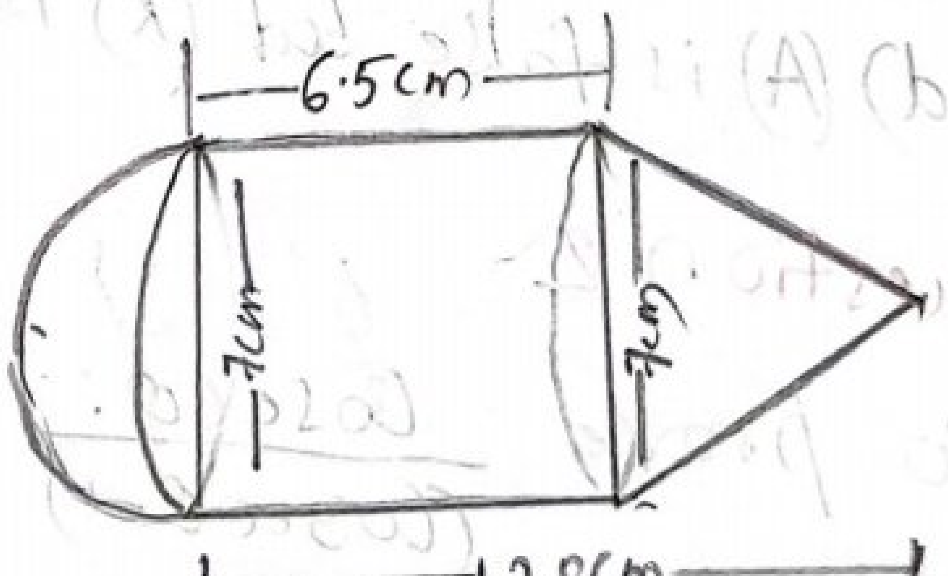
$$P = ₹200 \quad (1)$$

Monthly installment $= ₹200$

iii) For cone: $r = 3.5$cm, $h_1 = 6.3$cm

For cylinder: $r = 3.5$cm, $h_2 = 6.5$cm

For hemisphere: $r = 3.5$cm



Total Volume of solid = Vol. of cone + Vol. of cylinder + Vol. hemisphere

$$= \frac{1}{3}\pi r^2 h_1 + \pi r^2 h_2 + \frac{2}{3}\pi r^3$$

$$= \pi r^2\left(\frac{1}{3} \times h_1 + h_2 + \frac{2}{3} \times r\right)$$

$$= \frac{22}{7} \times 3.5 \times 3.5 \left(\frac{1}{3} \times 6.3 + 6.5 + \frac{2}{3} \times 3.5\right) \quad (1)$$

$$= 38.5 \left(2.1 + 6.5 + \frac{7}{3}\right) \quad (1)$$

$$= 38.5\left(\frac{6.3+19.5+7}{3}\right)$$

$$= 38.5 \times \frac{32.8}{3} \quad (1)$$

$$= \frac{1262.8}{3}$$

$$= 420.93\ cm^3 \quad (1)$$

Question 3

(i) $$\frac{x}{1} = \frac{\sqrt{a+2b}+\sqrt{a-2b}}{\sqrt{a+2b}-\sqrt{a-2b}}$$

Applying componendo and dividendo

$$\frac{x+1}{x-1} = \frac{\sqrt{a+2b}+\sqrt{a-2b}+\sqrt{a+2b}-\sqrt{a-2b}}{\sqrt{a+2b}+\sqrt{a-2b}-\sqrt{a+2b}+\sqrt{a-2b}}$$

$$\frac{x+1}{x-1} = \frac{2\sqrt{a+2b}}{2\sqrt{a-2b}} \quad (1)$$

$$\frac{x+1}{x-1} = \frac{\sqrt{a+2b}}{\sqrt{a-2b}}$$

Squaring on both side

$$\frac{x^2+2x+1}{x^2-2x+1} = \frac{a+2b}{a-2b} \quad (1)$$

Applying componendo and dividendo

$$\frac{x^2+2x+1+x^2-2x+1}{x^2+2x+1-x^2+2x-1} = \frac{a+2b+a-2b}{a+2b-a+2b}$$

$$\frac{2(x^2+1)}{4x} = \frac{2a}{4b} \quad (1)$$

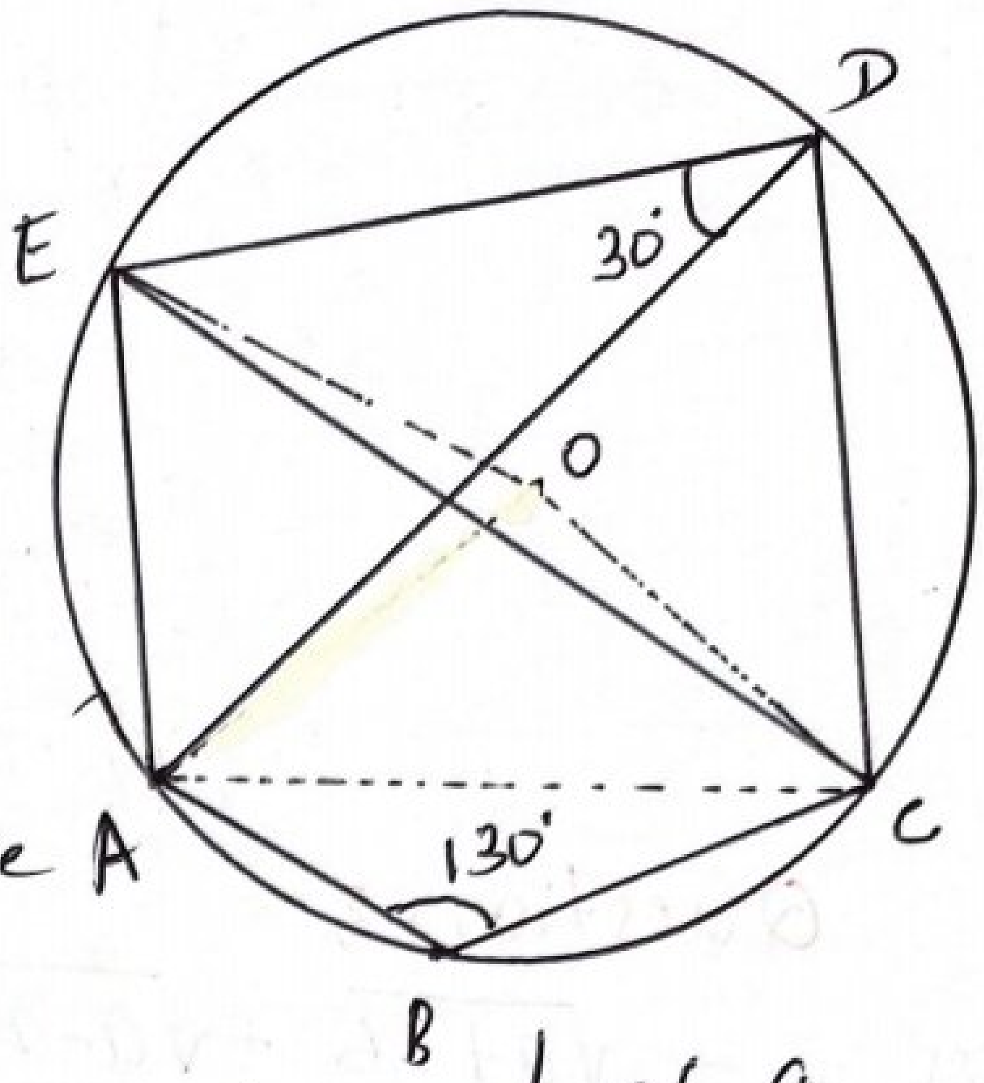
$$\frac{x^2+1}{x} = \frac{a}{b}$$

$$b(x^2+1) = ax$$

$$bx^2+b-ax=0 \quad (1)$$

ii) $\angle ACE = \angle ADE$ (Angles in the same segment of a circle are equal)

$\therefore \angle ACE = 30^\circ$



Next,

ABCD is a cyclic quadrilateral

$\therefore \angle ABC + \angle ADC = 180^\circ$ (opposite angles of a cyclic quadrilateral are supplementary)

$130^\circ + \angle ADC = 180^\circ$

$\angle ADC = 50^\circ$ (1)

Now, $\angle AEC = \angle ADC$ (Angles in the same segment of a circle are equal)

$\therefore \angle AEC = 50^\circ$

Next, $\angle EDC = \angle ADE + \angle ADC$

$= 30^\circ + 50^\circ$

$= 80^\circ$ (1)

and $\angle EOC = 2\angle EDC$ (Angle subtended at the centre of a circle is twice the angle subtended in the same segment)

$= 2 \times 80^\circ$

$\therefore \angle EOC = 160^\circ$ (1)

[NOTE: Reason for atleast one step should be given or else aw -1 (1)]

(iii)

Marks	No. of students	Cf
0-10	6	6
10-20	10	16
20-30	15	31
30-40	13	44
40-50	20	64
50-60	9	73
60-70	7	80

$N = 80$,

(i) Median $= \left(\frac{N}{2}\right)^{th}$ score $= 40^{th}$ score

$\therefore$ Median $= 36.5$ marks (1)

(ii) Lower quartile $= Q_1 = \left(\frac{80}{4}\right)^{th}$ score $= 20^{th}$ score

$\therefore Q_1 = 23$

Upper quartile $\Rightarrow Q_3 = \left(\frac{3 \times 80}{4}\right)^{th}$ score $= 60^{th}$ score

(1) $Q_3 = 48.5$

Inter quartile range $Q_3 - Q_1 = 48.5 - 23$

$= 25.5$ (1)

(iii) No. of students who scored more than 45 marks $= 29$ (1)

Smooth curve with correct scale: (1)

Section-B

Question 4

(i) $AC = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 2-3 & 0+12 \\ 5-7 & 0+28 \end{bmatrix} = \begin{bmatrix} -1 & 12 \\ -2 & 28 \end{bmatrix}$ (1)

$B^2 = \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}\begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix} = \begin{bmatrix} 0-4 & 0+28 \\ 0-7 & -4+49 \end{bmatrix} = \begin{bmatrix} -4 & 28 \\ -7 & 45 \end{bmatrix}$ (1)

$10C = 10\begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 10 & 0 \\ -10 & 40 \end{bmatrix}$

$AC + B^2 - 10C = \begin{bmatrix} -1 & 12 \\ -2 & 28 \end{bmatrix} + \begin{bmatrix} -4 & 28 \\ -7 & 45 \end{bmatrix} - \begin{bmatrix} 10 & 0 \\ -10 & 40 \end{bmatrix}$

$= \begin{bmatrix} -15 & 40 \\ 1 & 33 \end{bmatrix}$ (1)

(ii) $x - \frac{18}{x} = 6$

$x^2 - 18 = 6x$

$x^2 - 6x - 18 = 0$

$a = 1,\ b = -6,\ C = -18$

$$\left.\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-18)}}{2(1)} \end{aligned}\right\} (1)$$

$= \frac{6 \pm \sqrt{36 + 72}}{2}$

$= \frac{6 \pm \sqrt{108}}{2}$

$= \frac{6 \pm \sqrt{36 \times 3}}{2}$

$= \frac{6 \pm 6\sqrt{3}}{2}$

$= \frac{6(1 \pm \sqrt{3})}{2}$

$= 3 \pm 3\sqrt{3}$

$\therefore x = 3 + \sqrt{3}$

$= 3 + \underline{1.732 \times 3}$ (1)

$= 8.196$

$\therefore x = 8.2$

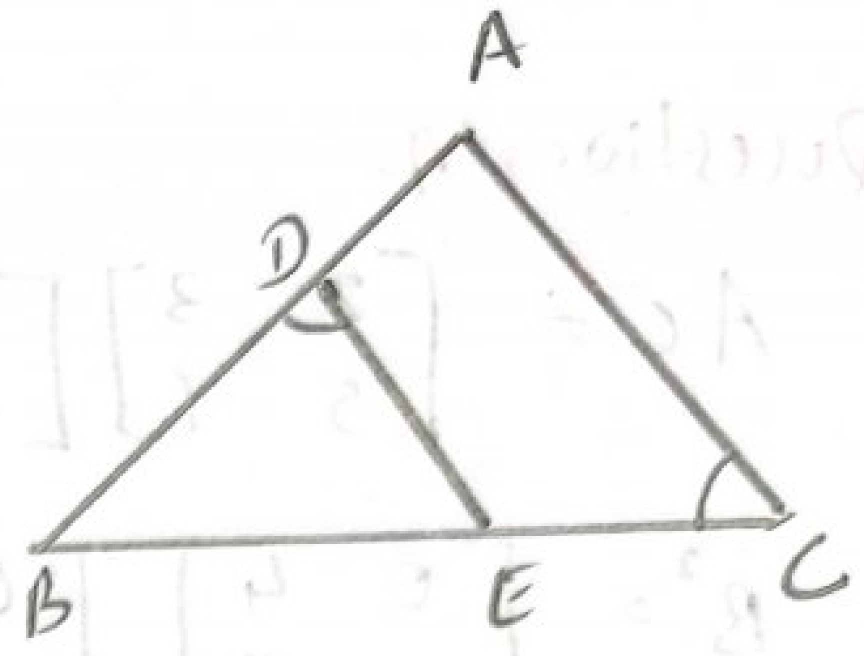
$x = 3 - \sqrt{3}$

$x = 3 - 1.732 \times 3$

$= -2.196$

$x = -2.2$

(both should be correct)



(iii) Given: $\angle EDB = \angle ACB$, $BE = 6cm$

$EC = 4cm$, $BD = 5cm$,

Area of $\triangle BED = 9\,cm^2$

Solution: $\triangle ABC \quad \triangle EBD$

$\angle A = \angle A$ (common)

$\angle ACB = \angle EDB$ (given)

$\therefore \triangle ABC \sim \triangle EBD$ (by AA Similarity) (1)

$\therefore \dfrac{AB}{EB} = \dfrac{BC}{BD} = \dfrac{AC}{ED}$ (Corresponding parts of similar triangles are proportional)

(i) Consider $\dfrac{AB}{EB} = \dfrac{BC}{BD}$

$\dfrac{AB}{6} = \dfrac{10}{5}$

$\therefore AB = 12cm$ (1)

(ii) $\dfrac{Ar.\,\triangle ABC}{Ar.\,\triangle EBD} = \dfrac{AB^2}{EB^2} = \dfrac{12^2}{6^2} = \dfrac{4}{1}$ (1)

[Ratio of areas of two similar triangles is equal to ratio of their corresponding sides]

$\therefore \triangle Ar.\,\triangle ABC = 4 \times 9$

$= 36\,cm^2$ (1)

Question 5

(i) For manufacturer,

$CGST = SGST = \dfrac{14}{100} \times 10000 = ₹1400$

For dealer sells it to a customer at a profit of 12%.

$Sp = 10{,}000 + \dfrac{12}{100} \times 10{,}000 = ₹11{,}200$

$CGST = SGST = \dfrac{14}{100} \times 11{,}200 = ₹1568$

(i) GST paid by dealer to state govt $= 1568 - 1400 = ₹168$ (1)

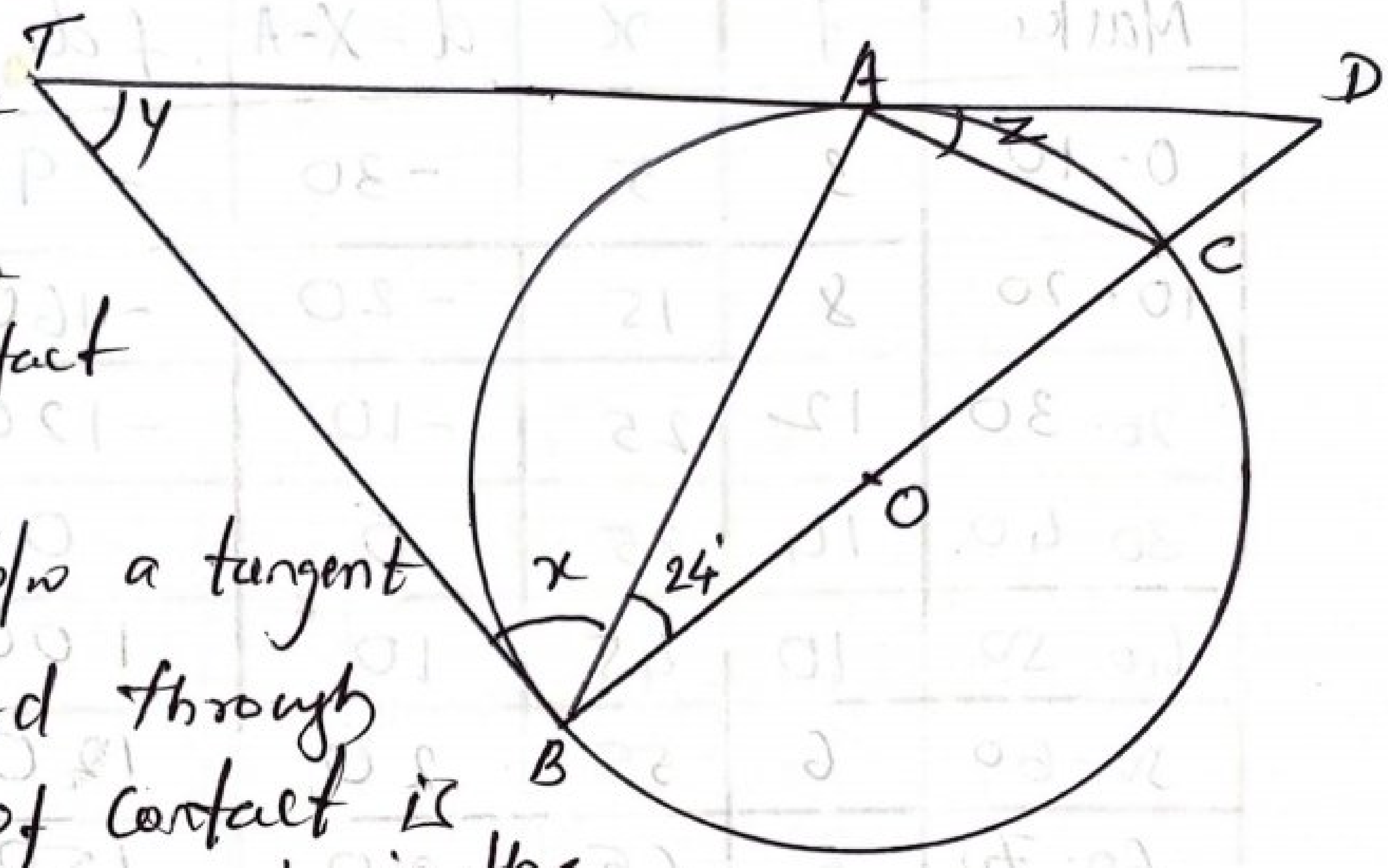
(ii) Total tax received by Central govt $= ₹1400 + (1568 - 1400)$

$= ₹1568$ (1)

(iii) The price paid by customer $= ₹11200 + 1568 + 1568$

$= ₹\,14{,}336$ (1)

(ii) Mode: 65 or 64



(iii) $\angle OBT = 90°$ (Angle b/w the tangent and radii at the point of contact is 90°)

$x + 24° = 90°$

$\boxed{x = 66°}$ (1)

Next,

$\angle CAD = \angle ABC$ (The angle b/w a tangent and a chord through a point of contact is equal to an angle in the alternate segment are equal)

$\therefore \boxed{z = 24°}$ (1)

Next

$AT = BT$ → (Tangent drawn to a circle from an external point are equal in length)

$\Rightarrow \Delta ATB$ is isosceles

$\therefore \angle ATB = 180° - (x + x)$

$= 180 - (66° + 66°)$

$\boxed{y = 48}$ (1)

(NOTE: Atleast one reason to be given or else ow -1)

Question 6

(i) For GP 1, 4, 16 ....

$a = 1, \quad r = \frac{4}{1} = 4, \quad |r| = |4| = 4 > 1$

$\therefore S_n = \frac{a(r^n - 1)}{r - 1}$; $\frac{1(4^n - 1)}{4 - 1} = 341$ (1)

$4^n - 1 = 341 \times 3$ (1)

$4^n = 1024$

$2^{2n} = 2^{10}$

$\therefore 2n = 10$

$n = 5$ (1)

$\therefore$ 5 terms to be taken to sum up to 341

(ii) Mean by short cut method

Marks	f	x	d = X − A	fd
0-10	3	5	−30	−90
10-20	8	15	−20	−160
20-30	12	25	−10	−120
30-40	14	35	0	0
40-50	10	45	10	100
50-60	6	55	20	120
60-70	5	65	30	150
70-80	2	75	40	80
	N=60			

Let $A = 35$ $\qquad \Sigma fd = 80$

$$\left.\begin{aligned}\text{Mean} &= A + \frac{\Sigma fd}{N} \\ &= 35 + \frac{80}{60}\end{aligned}\right\} (1)$$

$= 35 + \frac{4}{3}$

$= \frac{109}{3}$

$= 36.33$ (1)

[Either 4 class marks or 4 deviation are corret –(1)]

(iii) (i) Surface Area of metallic sphere $= 1256 \text{ cm}^2$

$4\pi r^2 = 1256$

$4 \times 3.14 \times r^2 = 1256$ (1)

$r = 10\text{cm}$ (1)

(ii) $n \times$ Vol. of cone = Vol. of sphere

$$\left.\begin{aligned} n \times \frac{1}{3}\pi r_1^2 h &= \frac{4}{3}\pi r_2^3 \\ n \times (2.5)^2 \times h &= 4 r_2^3 \end{aligned}\right\} (1)$$

$2.5 \times 2.5 \times n \times 8 = 4 \times 10 \times 10 \times 10$

$n = 80$ (1)

Question 7

(i) For given line, $2x - 3y = 7$

$y = \frac{2}{3}x - \frac{7}{3}$

Slope $(m_1) = \frac{2}{3}$

∴ Since req. line parall to given line

∴ Slope of req. line $= \frac{2}{3}$ (1)

and y intercept = 4

$\therefore$ Equation of line in slope intercept form,

$y = mx + c$

$y = \frac{2}{3}x + 4$

$\therefore 2x - 3y + 12 = 0$ (1)

This line cuts the x axis at $y = 0$

$\therefore 2x - 3(0) + 12 = 0$

$x = -6$

$\therefore$ The given line cuts the x axis at point $(-6, 0)$ (1)

(ii) $f(x) = 2x^3 + x^2 - 13x + 6$

factors of $6 = \pm 1, \pm 2, \pm 3, \pm 6$

$f(2) = 2(2)^3 + 2^2 - 13(2) + 6$

$f(2) = 0$

$\therefore x = 2$

$x - 2 = 0$

$\therefore (x-2)$ is a factor of $f(x)$ (1) [$\because$ by factor theorem]

$$
\begin{array}{r|l}
x-2 & 2x^3 + x^2 - 13x + 6 \;(2x^2 + 5x - 3 \quad (1) \\
 & \underline{\overset{(-)}{-}2x^3 \overset{(+)}{-} 4x^2} \\
 & \quad 5x^2 - 13x \\
 & \quad \underline{\overset{(-)}{-}5x^2 \overset{(+)}{-} 10x} \\
 & \qquad -3x + 6 \\
 & \qquad \underline{\overset{(+)}{-}3x \overset{(-)}{+} 6} \\
 & \qquad\quad 0
\end{array}
$$

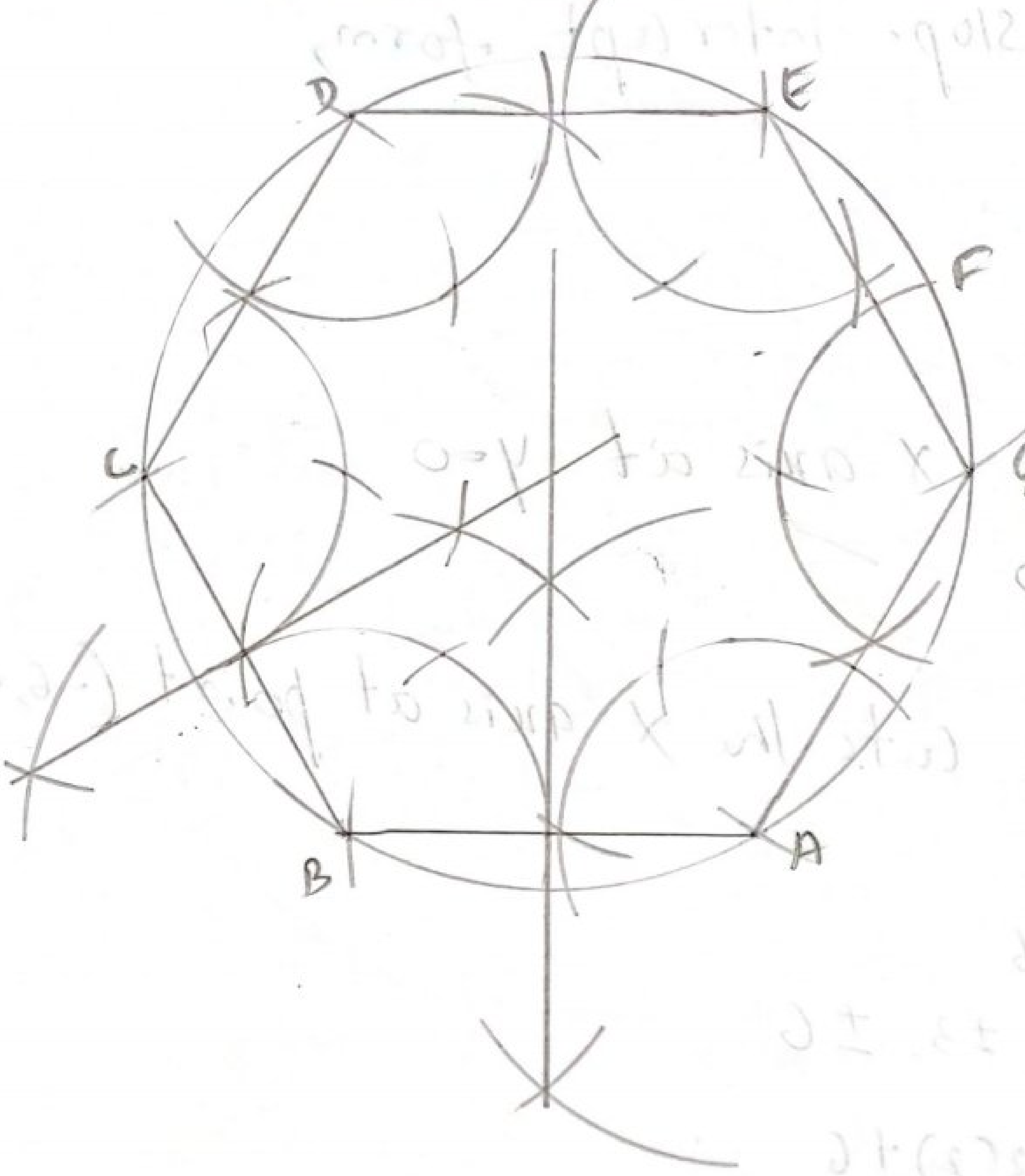
$f(x) = (x-2)(2x^2 + 5x - 3)$

$= (x-2)(2x^2 + 6x - x - 3)$

$= (x-2)[2x(x+3) - 1(x+3)]$

$f(x) = (x-2)(x+3)(2x-1)$ (1)

(iii)



Circum radius = 4cm

Question 8

i) $-3 < -\frac{1}{2} - \frac{2x}{3} \le \frac{5}{6}, x \in R$

$-3 < -\frac{1}{2} - \frac{2x}{3}$ and $-\frac{1}{2} - \frac{2x}{3} \le \frac{5}{6}$

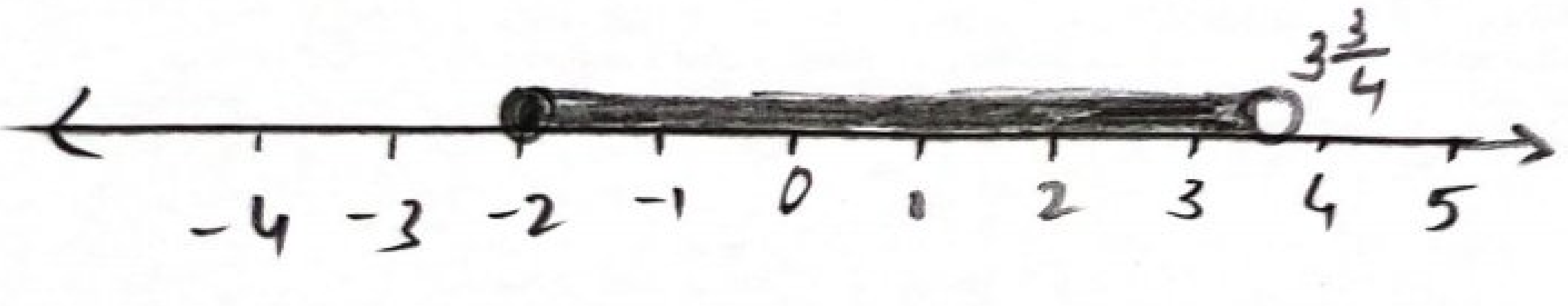
$-3 + \frac{1}{2} < \frac{-2x}{3}$ $\quad$ $\frac{-2x}{3} \le \frac{5}{6} + \frac{1}{2}$

$\frac{-5}{2} < \frac{-2x}{3}$ or $\frac{-2x}{3} \le \frac{8}{6_2}$ (1)

$x < \frac{15}{4}$ $\quad$ $x \geq {}^{-}2$

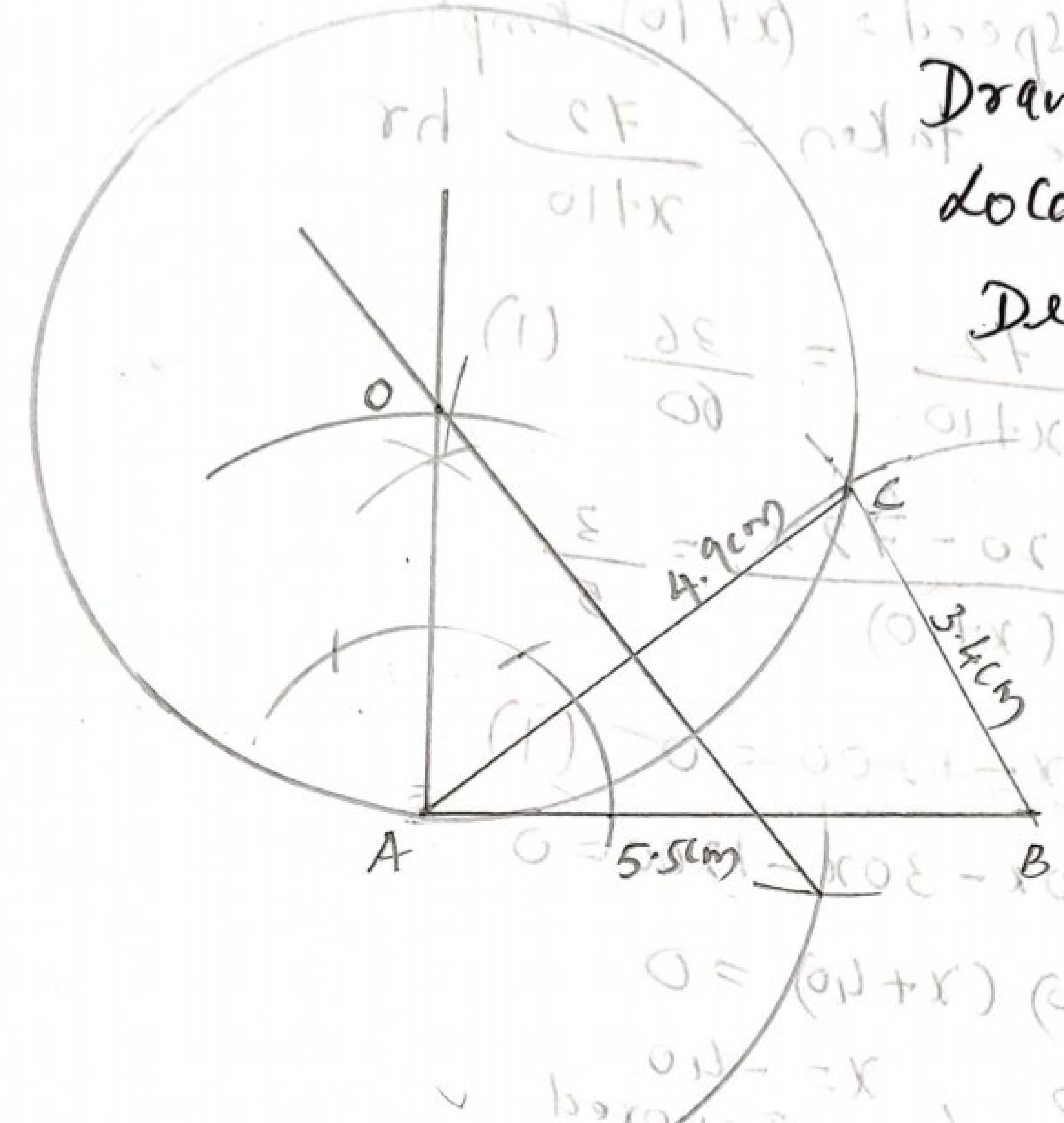
$-2 \le x < 3\frac{3}{4}$

sol. set $= \{x : x \in R, -2 \le x < 3\frac{3}{4}\}$ (1)



(1)

ii)



Drawing Δ = (1)
Locating point O = (1)
Draw circle correctly touching A&C = (1)

iii) a) $M_x P(2,4) = P' = (2,-4)$
$M_x Q(-2,1) = Q' = (-2,-1)$ } (1)

b) Geometrical Name of $PQQ'P'R$ is Pentagon or 5 sided polygon (1)

plotting P, R and Q (1)
plotting P' and Q' (1)

Question 9

i) a) Total dividend paid by Company $= \frac{\text{Dividend\%} \times \text{No. of shares} \times NV}{100}$

$= \frac{15 \times 4000 \times 110}{100}$

$= ₹66,000$ (1)

b) Annual income of Virat $= ₹\frac{15}{100} \times 110 \times 88 = ₹1452$ (1)

c) $NV \times \text{Dividend\%} = MV \times \text{Return\%}$
$110 \times 15 = MV \times 10$
$\therefore MV = ₹165$ (1)

ii) Let the original speed of the Car = x kmph

original time taken = $\frac{72}{x+10}$ hr

Increased speed = $(x+10)$ kmph

New time taken = $\frac{72}{x+10}$ hr

$$\frac{72}{x} - \frac{72}{x+10} = \frac{36}{60} \quad (1)$$

$$\frac{72x+720-72x}{x(x+10)} = \frac{3}{5}$$

$$x^2+10x-1200=0 \quad (1)$$

$$x^2+40x-30x-1200=0$$

$$(x-30)(x+40)=0$$

$x=30$, $x=-40$ Ignored

∴ original speed of Car = 30 kmph (1)

iii) Total no. of Cards = $60-10=50$

$S = \{11, 12, 13, 14, \ldots 60\}$, $n(S) = 50$

a) odd no = $\{11, 13, 15, \ldots 59\} = E_1$

$n(E_1) = 25$

∴ P(number of odd no. cards) = $\frac{\text{No. favourable outcomes}}{\text{Total no. of outcomes}}$

$= \frac{25}{50}$

$= \frac{1}{2}$

b) a perfect square no = $\{16, 25, 36, 49\}$

$n(E_2) = 4$

P(a perfect square card) = $\frac{\text{No. of favourable outcomes}}{\text{Total no. of outcomes}}$

$= \frac{4}{50} = \frac{2}{25}$

c) A no. divisible by 5 = {15, 20, 25, 30, 35, 40, 45, 50, 55, 60}

$n(E_3) = 10$

P(number divisible by 5) = $\frac{\text{No. of favourable outcomes}}{\text{Total no. of outcomes.}}$

$= \frac{10}{50}$

$= \frac{1}{5}$

Question 10

i) AP : PB = 1 : 2

By section formula, the coordinates of the point P which divides the line segment joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$ in the ratio $m_1 : m_2$ is given by

A(2,1) —1— P —2— B(5,-8)

$$p(x,y) = \left(\frac{m_1x_2 + m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$$

$$= \left(\frac{(1)(5)+(2)(2)}{1+2}, \frac{(1)(-8)+(2)(1)}{1+2}\right) \quad (1)$$

$= (3, -2)$ (1)

Since the point P lies on the line $2x - y + k = 0$

$\Rightarrow 2(3) - (-2) + k = 0$

$k = -8$ (1)

(ii) $t_4 + t_8 = 24$

$a + 3d + a + 7d = 24$

$2a + 10d = 24$

$a + 5d = 12$ —①

$t_6 + t_{10} = 44$

$a + 5d + a + 9d = 44$

$2a + 14d = 44$

$a + 7d = 22$ —② } Either of them is correct (1)

Now, on solving (1) & (2)

$d = 5$, $a = -13$ } (1) (if both are correct)

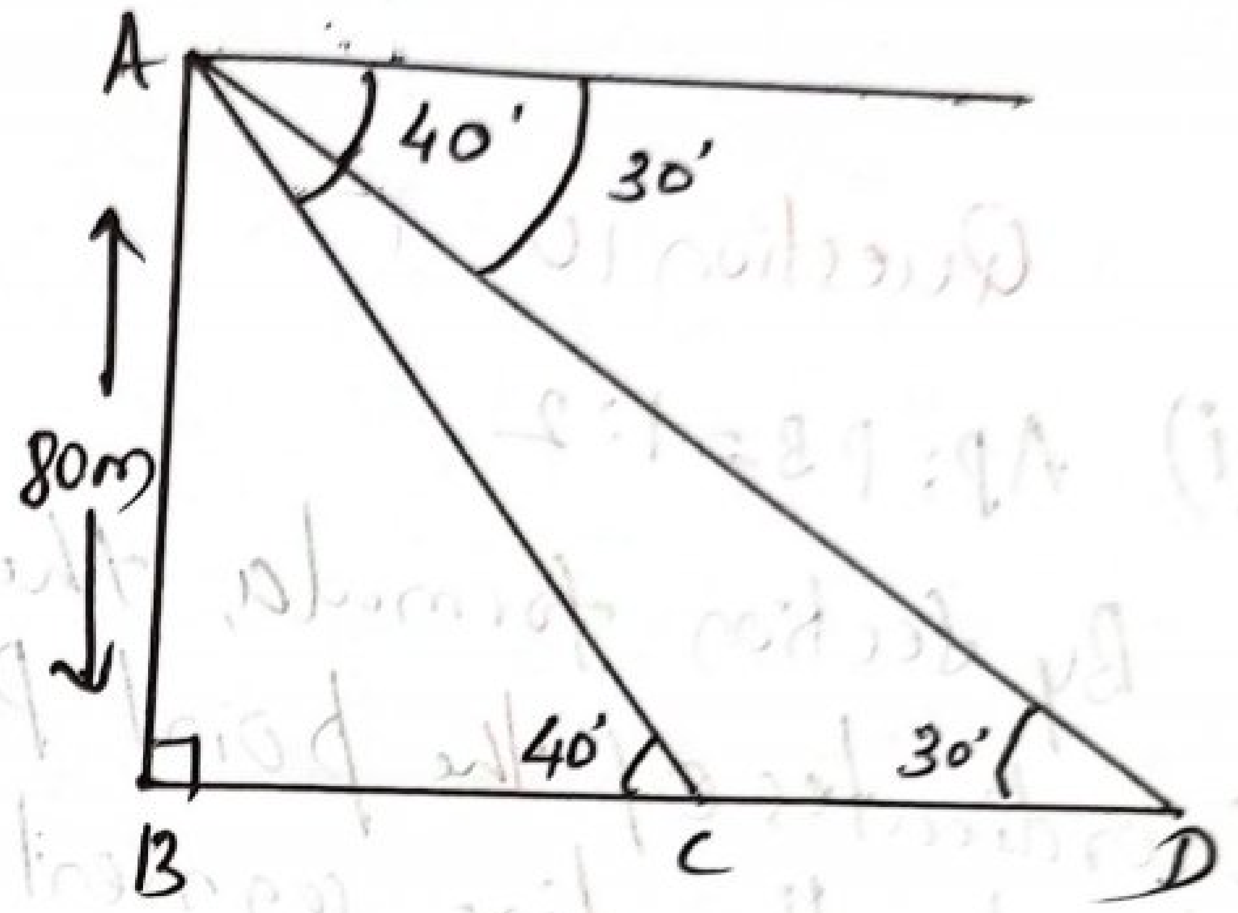
Now, $S_n = \frac{n}{2}[2a+(n-1)d]$

$S_{10} = \frac{10}{2}[2\times(-13)+(10-1)(5)]$

$= 5[-26+45]$

$S_{10} = 95$ (1)

iii) Let AB be light house and C and D be two ships



In $\triangle ABC$,

$\angle BAC = 180° - (90+40)$

$= 50°$

$\tan 50° = \frac{BC}{AB}$

(1)

$1.1918 = \frac{BC}{80}$ [Either for finding tan40°]

$BC = 95.344\ m$ (1)

[This question can also be solved by taking tan40°]

In $\triangle ABD$,

$\angle BAD = 180° - (90+30)$

$= 60°$

$\tan 60° = \frac{BD}{AB}$

$1.7321 = \frac{BD}{80}$

$\therefore BD = 138.568\ m$ (1)

$\therefore CD = BD - BC$

$= 138.568 - 95.344$

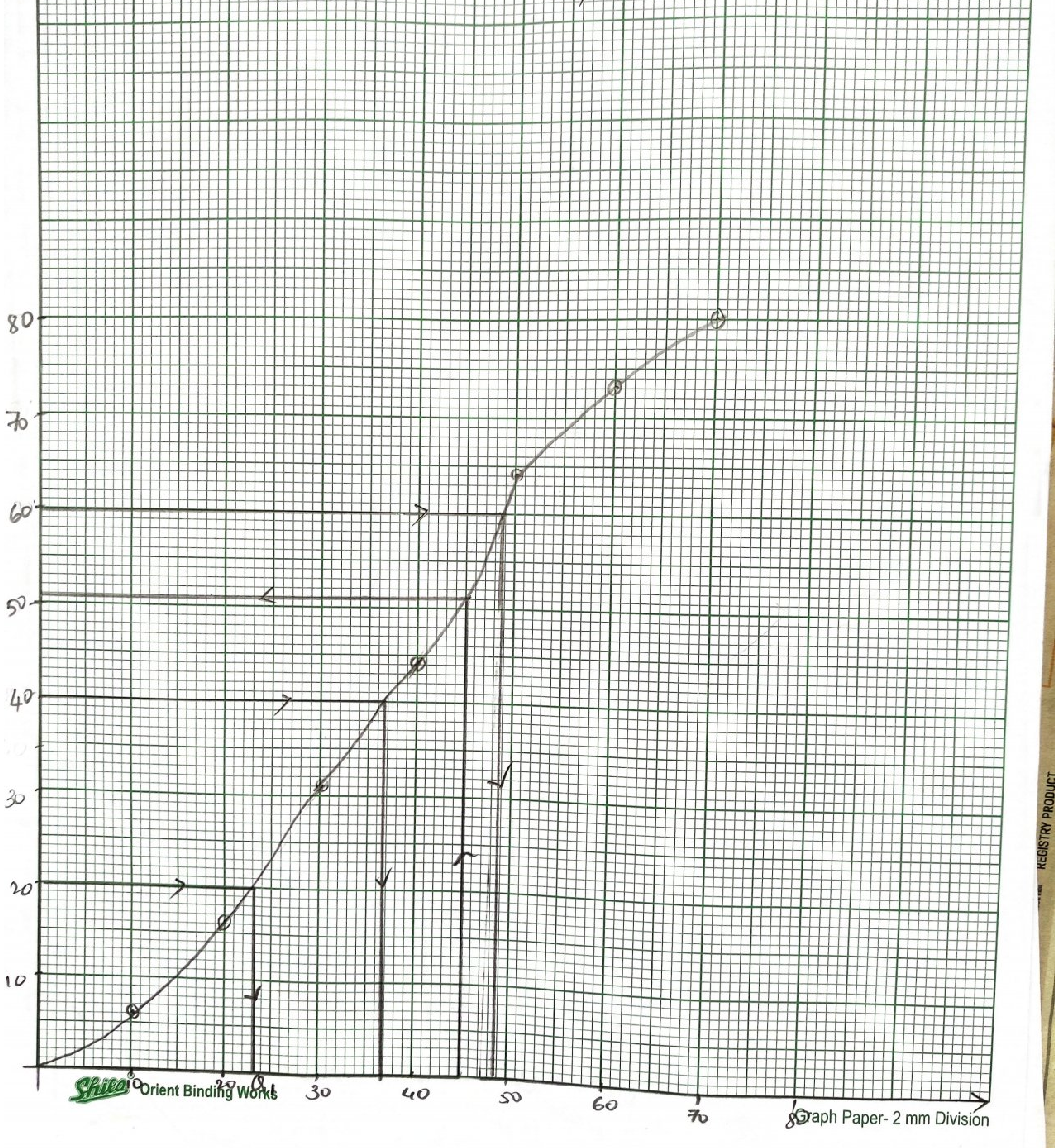
$= 43.224$

$= 43\ m$ (1)

—x—

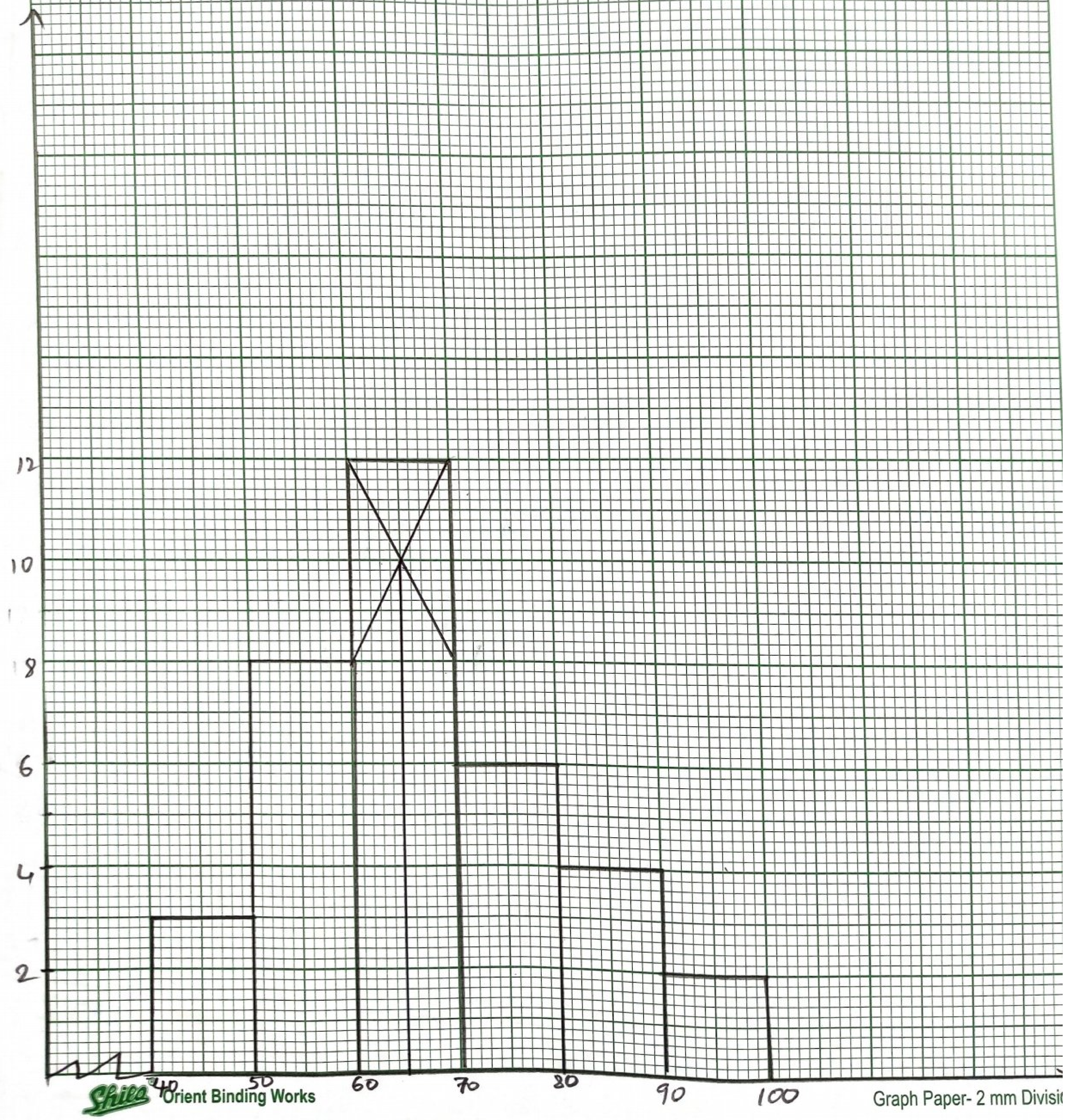
Q.NO: 3 (iii)

Scale: X axis 2cm = 10 marks
Y axis 2cm = 10 students



Q. No: 5, (ii)

Scale X axis: 2cm = ₹10
Y axis: 2cm = 2 workers.



Q. No 8 (iii)

Scale X axis } 2cm = 1 unit
Y axis }