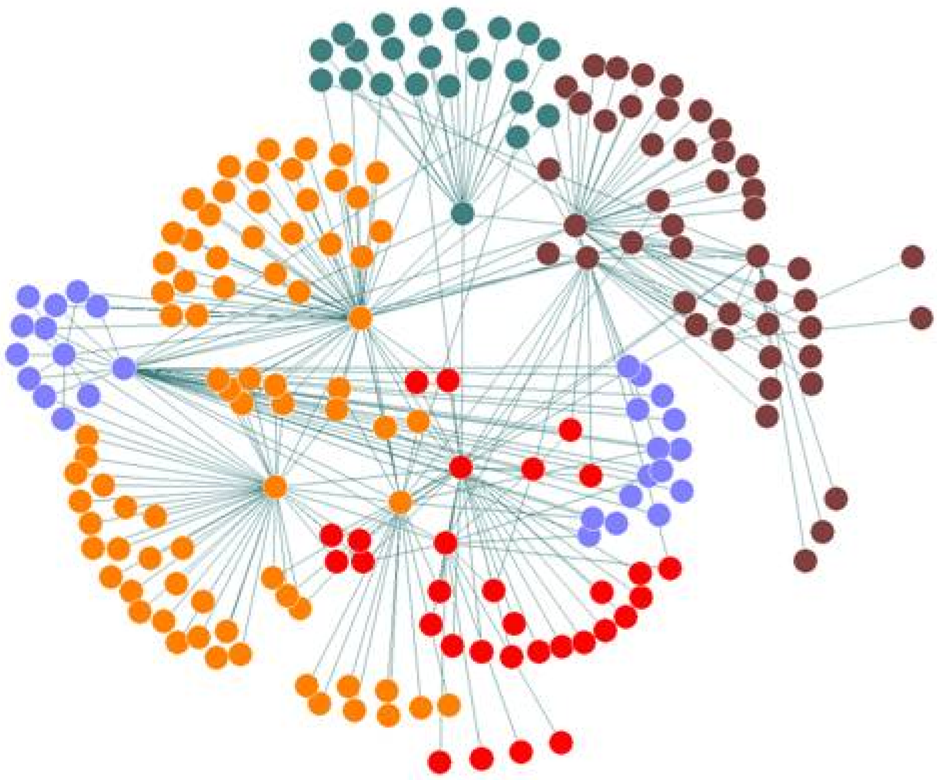
Escola Tècnica Superior
d'**Enginyeria de Telecomunicació de Barcelona**

Laboratory Session 3

Network Performance Analysis and Evaluation

Simulation and Performance Evaluation of Delay Systems. Part 2.



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1. Objectives

The objectives of this session may be summarized in the following points:

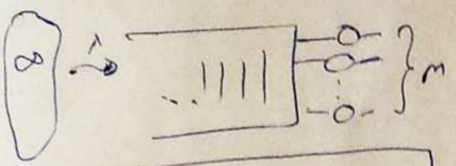
- Consolidate the theoretical knowledge, previously learnt by the students, about transmission systems modeled as M/M/1 and M/M/m.
- Become familiar with the simulation environment provided by Scalev Lite.
    - Single simulations.
    - Result files.
- Analysis of the results with MATLAB.

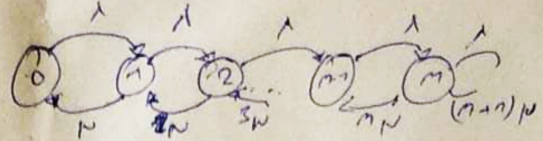
2. Previous work.

Erik & Gokarna — AAX: Sstudi previ 3 — 12/10/2022

(a) obtain, for M/M/m queue system

· State probabilities





$$\boxed{\begin{aligned} P(k) &= P_0 \frac{(m\rho)^k}{k!} & k \le m \\ P(k) &= P_0 \frac{m^m \rho^k}{m!} & k \ge m \end{aligned}}$$

$$\boxed{P_0 = \frac{1}{\sum_{k=0}^{m-1} \frac{(m\rho)^k}{k!} + \frac{m^m}{m!}\frac{\rho^m}{1-\rho}}} \sim \text{Erlang C}$$

· waiting probability

$$\boxed{P_D = \sum_{k=m}^{\infty} P_k = \sum_{k=m}^{\infty} P_0 \frac{m^m \rho^k}{m!} = P_0 \frac{m^m}{m!} \frac{\rho^m}{1-\rho}}$$

· waiting time pdf

$$f_{tw} = (1-P_D)\,\delta(t) + P_D\, m\mu(1-\rho)\, e^{-m\mu(1-\rho)t}$$

(b) · interarrival time: 0,1 · L = exp, 840 bits · Queue size: infinite, · 10 servers · C = 1200 bps

· Transmission time: $T = \frac{1}{\mu} = \frac{L}{C} = \frac{840}{1200} = \boxed{0{,}7\,s}$

· offered traffic: $\lambda = \frac{1}{t_{ic}} = \frac{1}{0{,}1} = \boxed{10\ \text{pag/s}}$

$$\boxed{\mu = \frac{1}{T} = \frac{1}{0{,}7}} \qquad \boxed{A = \frac{\lambda}{\mu} = \frac{10}{1/0{,}7} = 7}$$

· Prob of 0 packets in the system: $m = 10$

$0 < 10 \quad \rho < m \quad P(0) = P_0 \frac{(m\rho)}{m!}$

$$P_0 = \left(\sum \frac{(10\cdot 0{,}7)^0}{0!} + \frac{10^{10}}{10!}\frac{0{,}7^{10}}{1-0{,}7}\right)^{-1} = 854{,}5\cdot 10^{-6}$$

· prob 10 packets

$$P(10) = 854{,}5\cdot 10^{-6}\, \frac{(10\cdot 0{,}7)^{10}}{10!}$$

$$\boxed{P(10) = 66{,}52\cdot 10^{-3}}$$

· Prob. Delay

$$P_D = P_0 \frac{m^m}{m!}\frac{\rho^m}{1-\rho} = 854{,}5\cdot 10^{-6} \cdot \frac{10^{10}}{10!}\frac{0{,}7^{10}}{1-0{,}7} = \boxed{0{,}22}$$

· mean number of waiting packets

$$\boxed{N_w = P_D \frac{\rho}{1-\rho} = 0{,}52}$$

· mean num. of packets in system

$$\boxed{N = P_D \cdot \frac{A}{m-A} + A = 0{,}22 \cdot \frac{7}{10-7} + 7 = 7{,}5}$$

· mean waiting time

$$T_w = \frac{N_w}{\lambda} = \frac{0{,}52}{10} = \boxed{52\,ms}$$

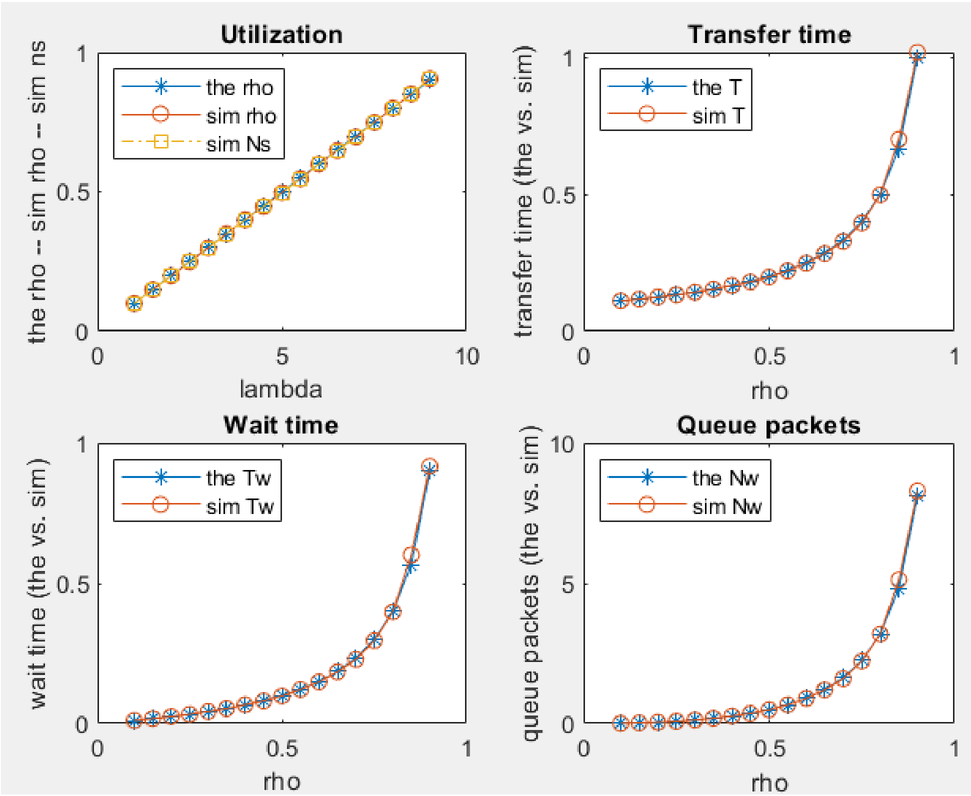
· mean transfer time

$$T = T_S + \frac{T_S}{m(1-\rho)} P_D = 0{,}7 + \frac{0{,}7}{10(1-0{,}7)}\, 0{,}22 = \boxed{0{,}75}$$

3. M/M/1 batch Simulation

3.1 Sweeping the arrival rate

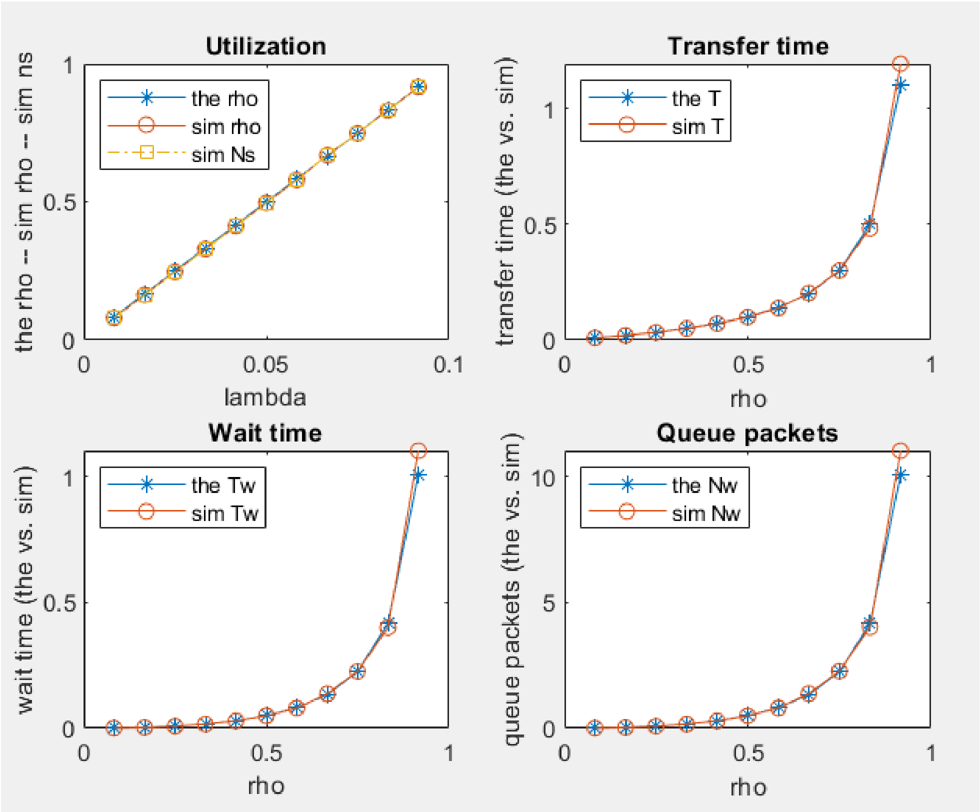
- Name: MM1b
- Length: 200000.
- Intervals: 5
- Type: Packet rate.
- Scheduler: FCFS.
- Traffic sources: 1.
- Categories: 1.
- Traffic 1:
    - Arrival rate: exp, 1:9:0.5 paq/s.
    - Packet length: exp, 120 bits.
    - Category: 1.
- Category 1 queue size: infinite.
- 1 server.
- Channel capacity: 1200 bps.



En la primera gráfica tenemos la ρ teórica y la simulada y también tenemos el número de unidades en el servidor. Los 3 coinciden porque recordemos que en la M/M/1, Ns=ρ. Vemos que la ρ progresa linealmente y está comprendido entre 0 y 1 (condición de estabilidad). En las otras gráficas vemos el tiempo de transferencia, el tiempo de espera y el número de paquetes en cola y observamos que los 3 parámetros hay coincidencia entre el teórico y el práctico y son exponenciales, debido que el sistema es M/M/1.
Con el aumento de ρ, vemos que se aumentan los otros parámetros.

3.2 Sweeping the packet length

- Name: MM1b2
- Length: 200000.
- Intervals: 5
- Type: Packet size.
- Scheduler: FCFS.
- Traffic sources: 1.
- Categories: 1.
- Traffic 1:
  - Arrival rate: exp, 0.1 paq/s.
  - Packet length: exp, 10:110:10 bits.
  - Category: 1.
- Category 1 queue size: infinite.
- 1 server.
- Channel capacity: 1200 bps.



Ahora el parámetro variante es el tamaño de paquete, entonces hemos adaptado el código del primer ejercicio y nos han salido estas gráficas.$\rho = \frac{\lambda}{\mu}$, $\mu = \frac{L}{c}$ =>$\rho = \frac{\lambda * L}{c}$.

El factor de utilización es directamente proporcional al tamaño de paquete, por eso si aumenta el tamaño también aumenta la ρ. En cuanto a las otras gráficas, no hay novedad respecto al ejercicio anterior.

4. M/M/1 batch Simulation

4.1 System analysis

a) Compare the results obtained in the previous work for a M/M/m system with the simulated values.

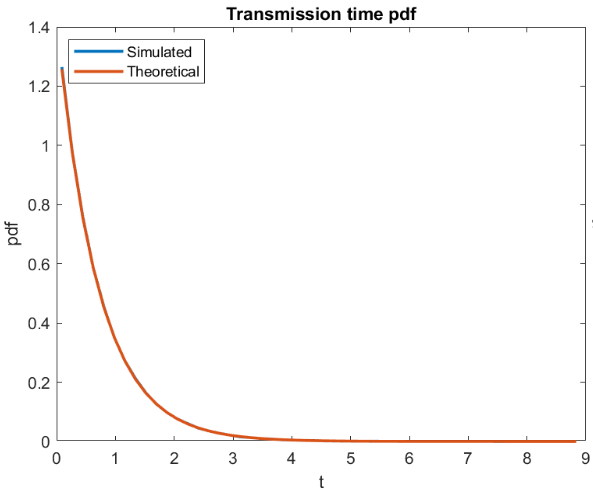
```
TRAFFIC PARAMETERS

 Traffic number: 1
 Arrivals distribution: EXPONENTIAL
 Arrivals average: 0.1
 Length distribution: EXPONENTIAL
 Length average: 840.0
 Category: 1
 Theoric utilization: 7.0

SIMULATION RESULTS

 Category number: 1
 Simulated utilization: 6.991178237290655
 Transmision time: 0.6971593396643356 Var: 0.48657421574054177     CV2:
1.0011173581467954
 Transference time: 0.7498326843577335
 Wait time: 0.052673344693397905
 Packet number in queue: 0.5282131647022085
 Packet number in server: 6.991212597222937
 Packet number in system: 7.519425761925145
 Served packets: 417838
 Lost packets: 0
 Loss probability: 0.0

End time: Tue Oct 18 23:18:58 CEST 2022
```

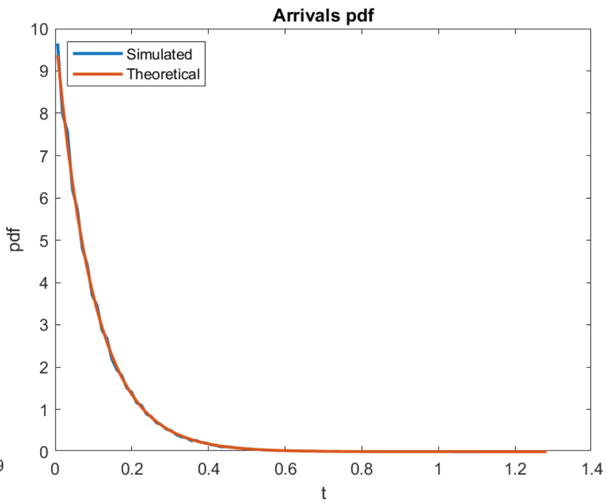
```
 Theoretical delay probability:   0.22173
 Simulated delay probability:  0.22216
```

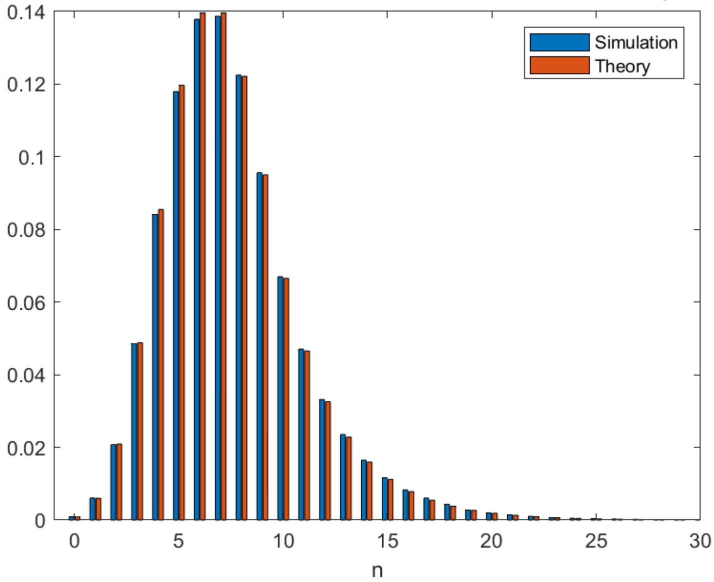
p0 8.5454e-04

	Theoretical	Simulation
Transmission time.	**0.75**	**0.6971593396643356**
Offered traffic (A)	**7**	$A = \frac{\lambda}{\mu}, (\lambda = \frac{1}{Tia=0.1}, \mu = \frac{1}{Ts=0-7})$**=7**
P(0)	**8.545e-04**	**8.545e-04**
P(10)	**$66.52*10^{-3}$**	$\frac{(10*0.7)^{10}}{10!}$ **=66.52e-3**
Pd	**0.22173**	**0.22216**
NW	**0.52**	**0.5282131647022085**
Ns	**7.5**	**7.519425761925145**
Tw	**52 ms**	**0.052673344693397905**
T	**0.75**	**0.7498326843577335**

b) Open and analyze the MATLAB script mmm.m. Next, run it and check the results.





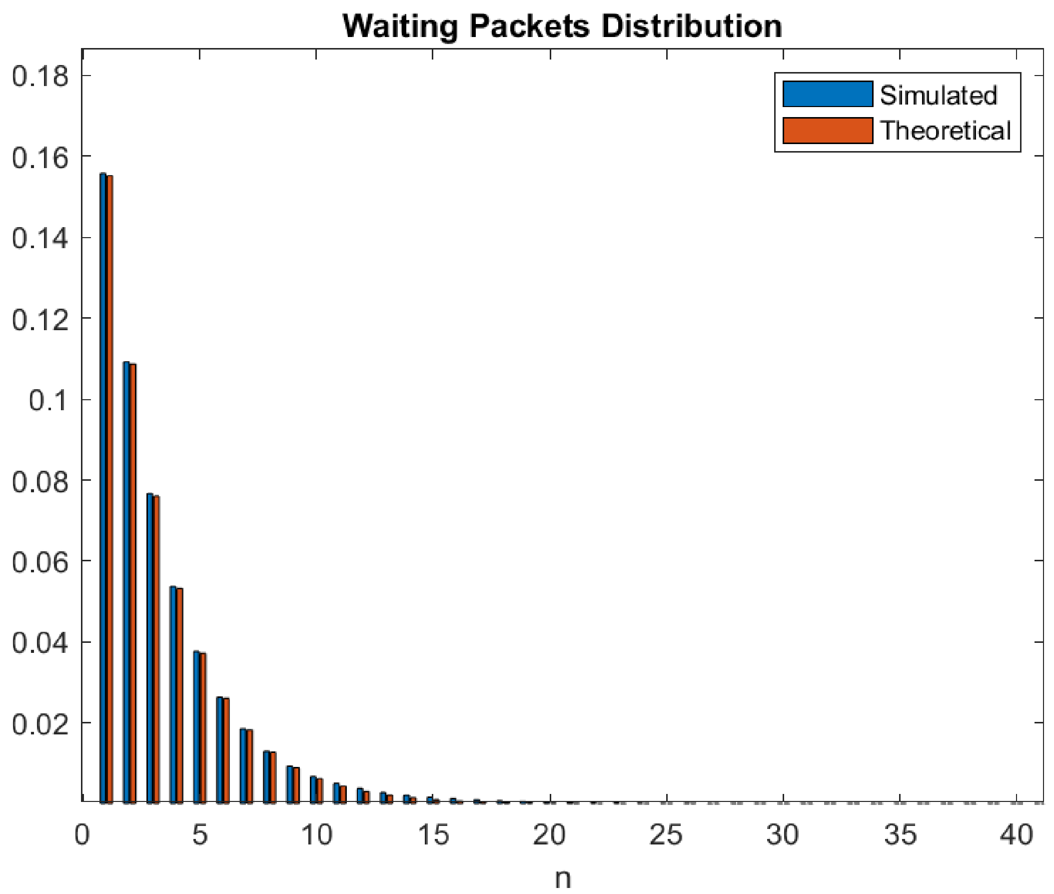


The theoretical and simulated values matched, we obtain identical plots and results.

4.2 Waiting packets

From the script mmm.m write a new one to obtain the simulated tail distribution for the number of packets in the queue and compare it with the theoretical values. Hint: Sum the corresponding states probabilities.

```
% 4.2 waiting packets
%comprobación de valor de Nw
NW=the_p_esp*(the_rho/(1-the_rho))
%NW= 0.5174 which is really close to the theoretical value

% la idea es crear 2 vectores en las que se almacenara las probabilidades
para estados mayores que m
Tail_sim_queue= zeros(1,length(sim_probs_states));
Tail_teo_queue= zeros(1,length(the_probs_states));
for i=1:length(sim_probs_states)-m
   Tail_sim_queue(i)=sum(sim_probs_states(i+1+m:length(sim_probs_states)));
   Tail_teo_queue(i)=sum(the_probs_states(i+1+m:length(the_probs_states)));
end
figure;
bar(1:length(Tail_sim_queue),[Tail_sim_queue' Tail_teo_queue']),colormap([0
0 0; 1 1 1]);
axis([-1 30 0 0.14]);
legend('Simulated','Theoretical','Location','northEast');
title('Waiting Packets Distribution');
xlabel('n');
```



The simulated and theoretical values are identicals

4.3 Waiting time

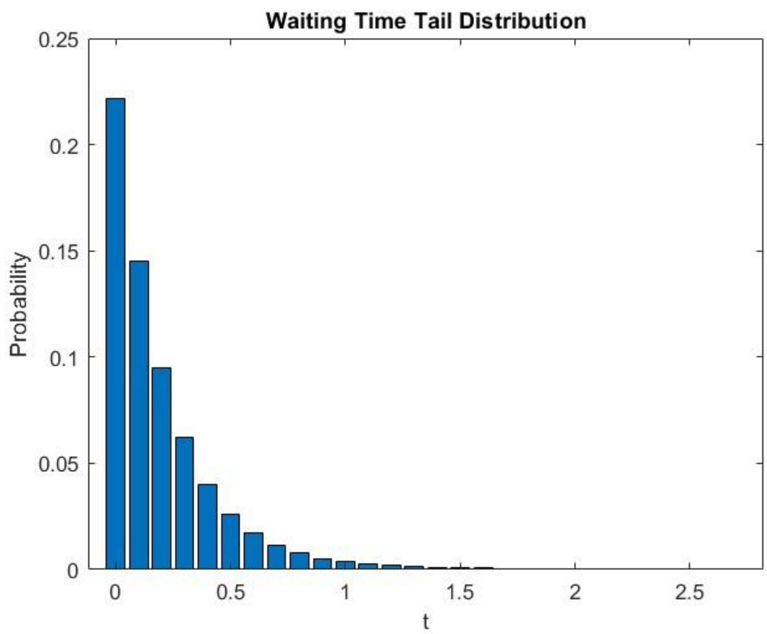
Write a script to obtain:

a) The simulated delay probability pd. Hint: use the simulated waiting time to find how many packets had to wait.

```
%4.3 Waiting time
clc;
clear;
output=load('output_MMm_source_1.txt');
tw=output(:,5); %En la columna 5 tendremos los tw
waited=find(tw);
pd=length(waited)/length(tw)
% 0.2219 probability of Wait, identical to theoretical probability
```

b) The simulated tail distribution function for the waiting time. Hint: Find values in steps of 0.1 s.

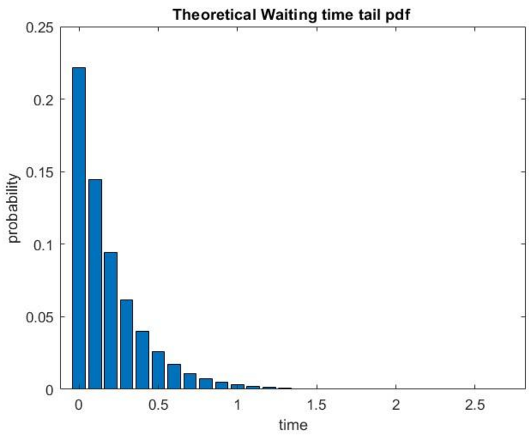
```
tail=[]
suma=1;
for i=0:0.1:max(tw)
   tail(suma)=sum(tw>i)/length(tw);
%para mirar si hay algun paquete con delay mas alto
   suma=suma+1;
end
figure;
bar(0:0.1:max(tw),tail);
title('Waiting Time Distribution');
xlabel('n');
ylabel('Probability')
```



c) The tail distribution function evaluating the theoretical expression with the pd values obtained in a).

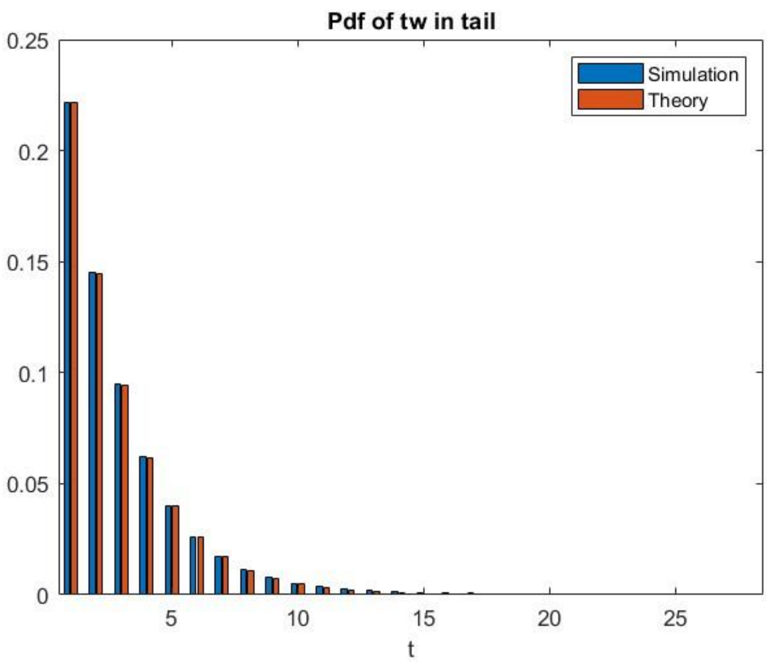
$$p[t_w > t] = 1 - p[t_w \le t] = 1 - F_{t_w}(t) = P_D\, e^{-(1-\rho)m\mu t}$$

```
% 4.3.c
%Theorical pdf
l=840;
c=1200;
m=10;
aux=0:0.1:max(tw)
mu = c / l;
tw_theo                                    =
pd*exp(-m*mu*(1-the_rho).*aux);
figure();
bar(aux, tw_theo);
title("theo pdf of tw tail");
xlabel("time");
ylabel("probability");
```



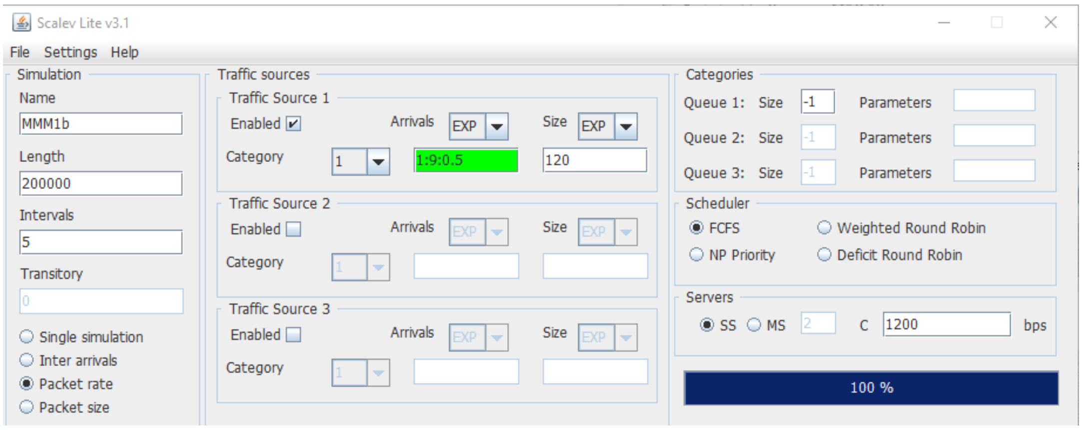
d) Compare the simulated and theoretical values.

```
%4.3.d
figure;
bar(1:length(tw_theo),[tail' tw_theo']),colormap([0 0 0; 1 1 1]);
title("Pdf of tw in tail");
legend('Simulation','Theory','NorthEast');
xlabel('t')
```

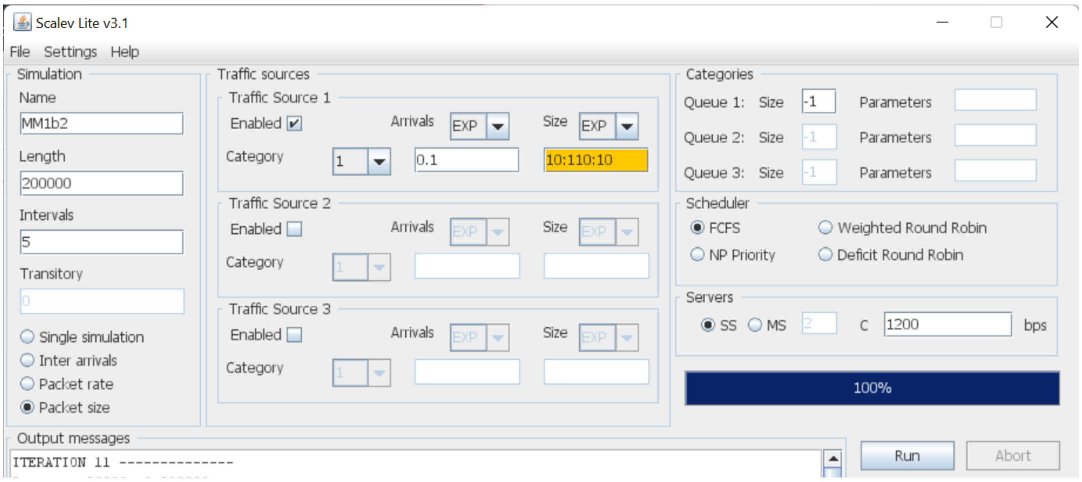


Annex 1: Scalev lite simulations

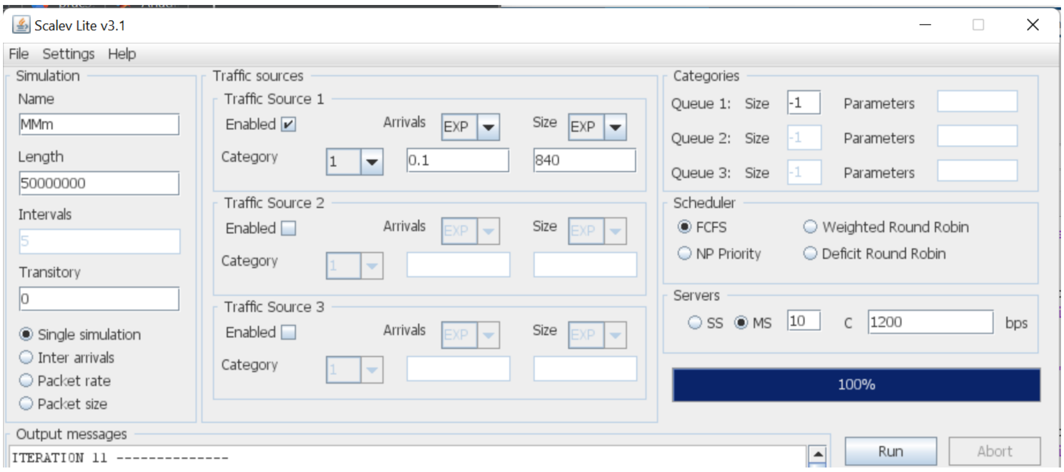
3.1



3.2



4.1



Annex2: matlab script

```
clear all;
lambda=1/0.1;
l=840;
c=1200;
m=10;
disp('Wait please...');
the_ts=l/c;
the_rho=(lambda*the_ts)/m; %rho por servidor
%Llegadas
output=load('output_MMm_source_1.txt');
at=output(:,2);
at=sort(at);
iat=diff(at);
[ht,x]=hist(iat,100);
area=sum(ht)*(x(2)-x(1));
sim_pdf=ht./area;
tmp=lambda*exp(-lambda.*x);
figure;
plot(x,[sim_pdf' tmp'],'LineWidth',1.75)
title('Arrivals pdf')
legend('Simulated','Theoretical','Location','northwest')
xlabel('t')
ylabel('pdf')

%Probabilidades estados
%%Por fichero con todas las transiciones (exacto)
occupancy=load('occupancy_MMm_1.txt');
tmp=[occupancy(:,1),occupancy(:,2)+occupancy(:,3)];
sim_probs_states=state_prob_function(tmp);
%Este sim_probs_states marca el numero total de estados para el resto
de calculos, ya que al venir del fichero donde estan todas las
transiciones es seguro que ha capturado el numero maximo de estados
%%Teorico
tmp5=0;
for i=0:m-1
   tmp5=tmp5+(m*the_rho)^i/factorial(i);
end
p0=1/(tmp5+(m*the_rho)^m/(factorial(m)*(1-the_rho)));
the_probs_states=zeros(1,length(sim_probs_states));
the_probs_states(1)=p0;
maxstate=length(sim_probs_states)-1;
for i=1:m
   the_probs_states(i+1)=p0*(m*the_rho)^i/factorial(i);
end
for i=m+1:maxstate
   the_probs_states(i+1)=p0*m^m*the_rho^i/factorial(m);
end
figure;
bar(0:maxstate,[sim_probs_states'  the_probs_states']),colormap([0  0
0; 1 1 1]);
axis([-1 30 0 0.14]);
legend('Simulation','Theory','Location','northwest')
xlabel('n')
%Tiempo transmision
ts=output(:,3);
[ht,x]=hist(ts,50);
area=sum(ht)*(x(2)-x(1));
sim_pdf=ht./area;
tmp=(1/the_ts)*exp(-(1/the_ts).*x);
figure;
plot(x,[sim_pdf' tmp'],'LineWidth',1.75)
title('Transmission time pdf')
```

```
legend('Simulated','Theoretical','Location','northwest')
xlabel('t')
ylabel('pdf')
%Probabilidad de espera
the_p_esp=p0*((m*the_rho)^m/factorial(m))*(1/(1-the_rho));
sim_p_esp=0;
for i=m+1:length(sim_probs_states)
   sim_p_esp=sim_p_esp+sim_probs_states(i);
end
disp(['Theoretical delay probability:   ' num2str(the_p_esp)])
disp(['Simulated delay probability:  ' num2str(sim_p_esp)])

% 4.2 waiting packets
%comprobacion de valor de Nw
NW=the_p_esp*(the_rho/(1-the_rho))
% la idea es crear 2 vectores en las q se almacenara las probalidades
para
% estados mayores que m
Tail_sim_queue= zeros(1,length(sim_probs_states));
Tail_teo_queue= zeros(1,length(the_probs_states));
for i=1:length(sim_probs_states)-m

Tail_sim_queue(i)=sum(sim_probs_states(i+1+m:length(sim_probs_states)
));

Tail_teo_queue(i)=sum(the_probs_states(i+1+m:length(the_probs_states)
));
end
figure;
bar(1:length(Tail_sim_queue),[Tail_sim_queue'
Tail_teo_queue']),colormap([0 0 0; 1 1 1]);
axis([-1 30 0 0.14]);
legend('Simulated','Theoretical','Location','northEast');
title('Waiting Packets Distribution');
xlabel('n');

%4.3 Waiting time
output=load('output_MMm_source_1.txt');
tw=output(:,5); %En la columna 5 tendremos los tw
waited=find(tw);
pd=length(waited)/length(tw)
tail=[]
suma=1;
for i=0:0.1:max(tw)
   tail(suma)=sum(tw>i)/length(tw);
   suma=suma+1;
end
figure;
bar(0:0.1:max(tw),tail);
title('Waiting Time Distribution');
xlabel('t');
ylabel('Probability')
```

```
% 4.3.c
%Theorical pdf
aux=0:0.1:max(tw)
mu = c / l; %server capacity / packets length
tw_theo = pd*exp(-m*mu*(1-the_rho).*aux);
figure();
bar(aux, tw_theo);
title("theo pdf of tw tail");
xlabel("time");
ylabel("probability");
%4.3.d
figure;
bar(1:length(tw_theo),[tail' tw_theo']),colormap([0 0 0; 1 1 1]);
title("Pdf of tw in tail");
legend('Simulation','Theory','NorthEast');
xlabel('t')


%function de probabilidad de estados de la practica 1
function [output] = state_prob_function(tmp)
time=tmp(:,1);
n=tmp(:,2);
prob=zeros(1,max(n)+1);
Max_time=max(time);
for i=1:length(time)-1
 prob(n(i)+1)=prob(n(i)+1)+(time(i+1)-time(i));
end
prob=prob./Max_time;
output=prob;
end
```