Universitat Politècnica De Catalunya

Escuela Técnica Superior de Ingeniería de Telecomunicación de Barcelona

Laboratory Session 4

Network Performance Analysis and Evaluation

Anàlisi i Avaluació de Xarxes

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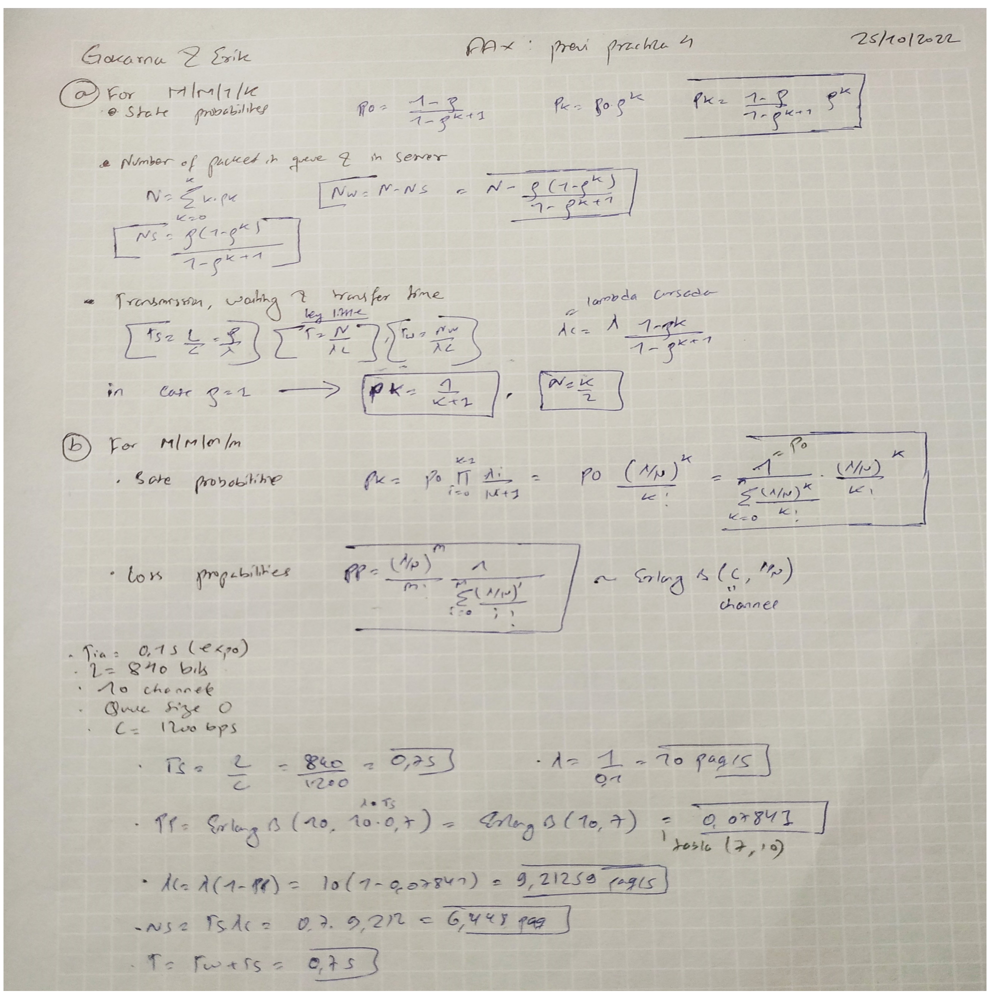
Contents

1 Objectives

The objectives of this session may be summarized in the following points:

- Consolidate the theoretical knowledge, previously learnt by the students, about transmission systems modeled as M/M/1/K and M/M/m/m.
- Become familiar with the simulation environment provided by Scalev Lite.
  - Single simulations.
  - Batch simulations.
  - Result files.
- Analysis of the results with MATLAB.

2 Previous work



Gaizka & Erik   AAX: previ practica 4   25/10/2022

ⓐ For M/M/1/K

- State probabilities $\quad p_0 = \frac{1-\rho}{1-\rho^{K+1}} \quad p_k = p_0 \cdot \rho^k \quad \boxed{p_k = \frac{1-\rho}{1-\rho^{K+1}} \rho^k}$
- Number of packet in queue & in server

$N = \sum_{k=0}^{K} k \cdot p_k \qquad \boxed{N_w = N - N_s = N - \frac{\rho(1-\rho^K)}{1-\rho^{K+1}}}$

$\boxed{N_s = \frac{\rho(1-\rho^K)}{1-\rho^{K+1}}}$

- Transmission, waiting & transfer time

$\boxed{T_s = \frac{L}{C} = \frac{\rho}{\lambda}} \quad \boxed{\underbrace{T = \frac{N}{\lambda_c}}_{\text{key time}}} \quad \boxed{T_w = \frac{N_w}{\lambda_c}} \qquad \lambda_c = \lambda \frac{1-\rho^K}{1-\rho^{K+1}}$ lambda cursada

in case $\rho = 1 \longrightarrow \boxed{p_k = \frac{1}{K+1}}, \boxed{N = \frac{K}{2}}$

ⓑ For M/M/m/m

- State probabilities $\quad p_k = p_0 \prod_{i=0}^{k-1} \frac{\lambda_i}{\mu_{i+1}} = p_0 \frac{(\lambda/\mu)^k}{k!} = \boxed{\underbrace{\frac{1}{\sum_{k=0}^{m} \frac{(\lambda/\mu)^k}{k!}}}_{= p_0} \cdot \frac{(\lambda/\mu)^k}{k!}}$
- Loss probabilities $\quad \boxed{P_P = \frac{(\lambda/\mu)^m}{m!} \frac{1}{\sum_{i=0}^{m} \frac{(\lambda/\mu)^i}{i!}}} \sim \text{Erlang B}(C, \lambda/\mu)$ (C = channel)

- $T_{ia} = 0{,}1$ s (expo)
- $L = 840$ bits
- 10 channels
- Queue size 0
- $C = 1200$ bps

- $T_s = \frac{L}{C} = \frac{840}{1200} = \boxed{0{,}7\text{ s}}$ $\qquad$ - $\lambda = \frac{1}{0{,}1} = \boxed{10 \text{ paq/s}}$
- $P_P = \text{Erlang B}(10, \overbrace{10 \cdot 0{,}7}^{\lambda \cdot T_s}) = \text{Erlang B}(10, 7) = \boxed{0{,}07874}$ taula (7, 10)
- $\lambda_c = \lambda(1 - P_P) = 10(1 - 0{,}07874) = \boxed{9{,}21259 \text{ paq/s}}$
- $N_s = T_s \lambda_c = 0{,}7 \cdot 9{,}212 = \boxed{6{,}448 \text{ paq}}$
- $T = T_w + T_s = \boxed{0{,}7\text{ s}}$

Figure 1: Previous Work.

3 M/M/1/K System Simulation

- Length: 50000000.
- Type: Single simulation.
- Scheduler: FCFS.
- Traffic sources: 1.
- Categories: 1.
- Traffic 1:
  - Interarrival time: exponential, 0.2 s.
  - Packet length: exponential, 120 bits.
- 1 server.
- Channel capacity: 1200 bps. .

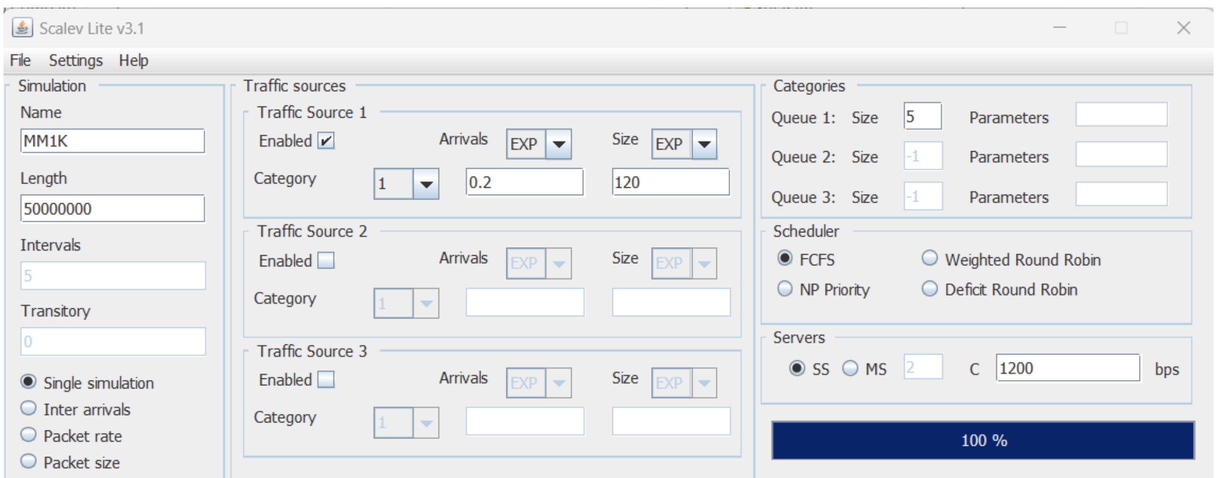


Figure 2: MM1K Scalev Simulation.

3.1 Average values analysis

a) Write a script (MM1K.m) similar to the one used for the study of the M/M/1 queue. The goal is to obtain the theoretical average values (offered and carried traffic; waiting time, service time and transfer time; number of packets in queue, server and system; loss probability) and the state probabilities.

```
close all
lambda=1/0.2;l=120;c= 1200;q=5;
i=0:q+1;
the_ts=l/c; % tiempo de servicio
the_rho=lambda*the_ts; %factor de utilizacion
if the_rho~=1 % si la rho es descansa
  the_prob=[((1-the_rho)/(1-the_rho^(q+2)))*the_rho.^i]'; %la
   probabilidad de estado
  the_n=((q+1)*the_rho^(q+3)-(q+2)*the_rho^(q+2)+the_rho)/(the_rho^(q
   +3)-the_rho^(q+2)-the_rho+1); % numero medio de unidades
  pl=((1-the_rho)/(1-the_rho^(q+2)))*the_rho^(q+1); %probabilidad de
   perdida, probabilidad del ultimo estado
else
  the_prob=zeros(q+2,1); % en caso de que la rho sea 1
  the_prob=(1/(q+2))+the_prob;
  the_n=(q+1)/2;
  pl=1/(q+2);
end
 lambda_c=lambda*(1-pl);
the_rho_c=lambda_c*the_ts;
the_t=the_n/lambda_c;
the_ns=the_rho_c;
the_nw=the_n-the_ns;
the_tw=the_t-the_ts;
disp(['RESULTS M/M/1/K']);
disp(['---------------']);
disp(['Offered traffic:' num2str(the_rho)]);
disp(['Carried traffic:' num2str(the_rho_c)]);
disp(['Waiting time:' num2str(the_tw)]);
disp(['Service time:' num2str(the_ts)]);
disp(['Transfer time:' num2str(the_t)]);
disp(['Number of waiting packets:' num2str(the_nw)]);
disp(['Number of packets in server:' num2str(the_ns)]);
disp(['Number of packets in system:' num2str(the_n)]);
disp(['Loss probability:' num2str(pl)]);
disp(the_prob)
bar(the_prob);
title("Probabilities");
```

b) Run your script MM1K.m to compare the results with the values provided by the simulator.

	Theoretical	Simulated
Offered traffic (A)	0.5	0.5
Carried traffic	0.49606	0.4973065228012163
Waiting time	0.090476	0.09070988917839821
Service time	0.1	0.0997978465001212
Transfer time	0.19048	0.19050773567851942
Number of packets in queue	0.44882	0.44882
Number of packets in server	0.49606	0.49606
Number of packets in system	0.94488	0.94488
Loss probability	0.007874	0.007874

Table 1: Comparison of theoretical & simulated values.

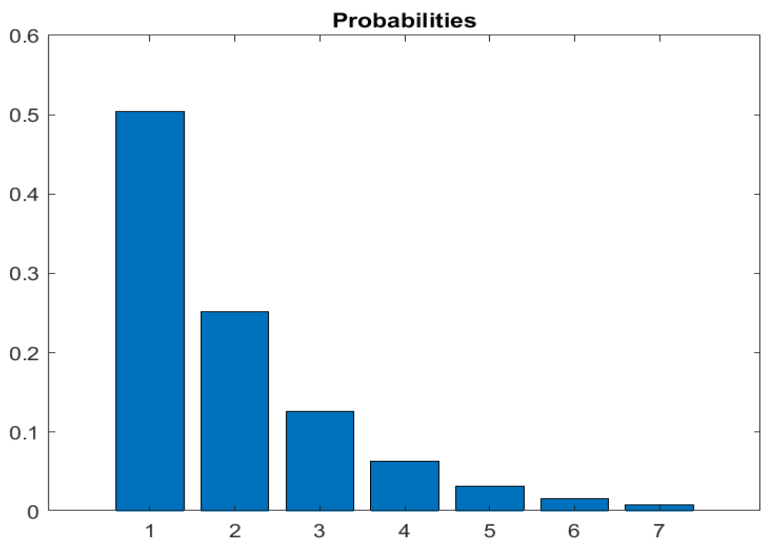


Figure 3: Lost Probabilities.

Como podemos observar, los resultados son idénticos.

c) Now, increase the arrival rate to obtain an offered traffic equal to 2 and compare the results of the script with the simulation.

```
% Solo cambiando el valor de lambda a 20 Obtenemos la  $\rho$ 2.
lambda=20;l=120;c= 1200;q=5;
...
%todo el codigo restante es igual
```

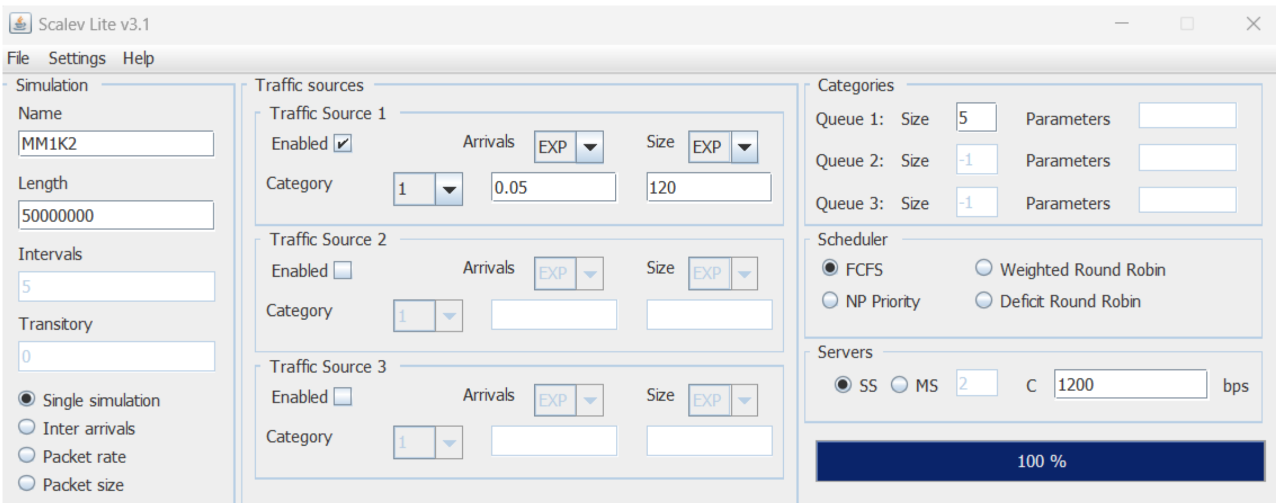


Figure 4: MM1K Scalev Simulation with $\rho = 2$.

	Theoretical	Simulated
Offered traffic (A)	2	2
Carried traffic	0.99213	0.9917839411502483
Waiting time	0.40952	0.4077584756791147
Service time	0.1	0.099588233498493
Transfer time	0.50952	0.5073467091776077
Number of packets in queue	4.063	4.0608715314777454
Number of packets in server	0.99213	0.991786941149823
Number of packets in system	5.0551	5.052658472627277
Loss probability	0.50394	0.5062023400386516

Table 2: Comparison of theoretical & simulated values. with ρ 2.

Aquí también, los resultados son idénticos.

d) Finally, do the same with an offered traffic equal to 1.

```
% Solo cambiando el valor de lambda a 10 Obtenemos la rho 1.
lambda=10;l=120;c= 1200;q=5;
...
%todo el codigo restante es igual
```

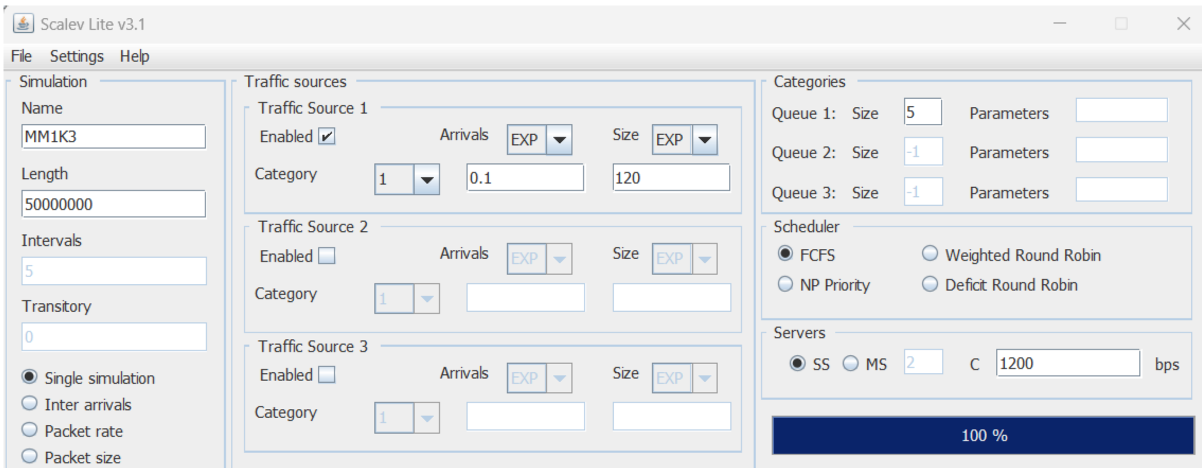


Figure 5: MM1K Scalev Simulation with $\rho = 1$.

	Theoretical	Simulated
Offered traffic (A)	1	1
Carried traffic	0.85714	0.8559820377696996
Waiting time	0.25	0.24893758102827085
Service time	0.1	0.09951044741555855
Transfer time	0.35	0.34844802844438294
Number of packets in queue	2.1429	2.1413493636712917
Number of packets in server	0.85714	0.8559850177680747
Number of packets in system	3	2.9973343814393663
Loss probability	0.14286	0.14379971811471298

Table 3: Comparison of theoretical & simulated values. with ρ 2.

Vemos que los resultados son idénticos.Además como la ρ es 1. La probabilidad de los estados es el mismo, es decir que todos los estados son equiprobables.

3.2 Transfer time analysis

Compare the simulated transfer time pdf with an exponential distribution for the first previous case (ρ=0.5). Repeat with the second case (ρ=2) and give your conclusions.

```
load output_MM1K_source_1.txt;
transfer_time_rho05=output_MM1K_source_1(:,4);
load output_MM1K2_source_1.txt;
transfer_time_rho2=output_MM1K2_source_1(:,4);
[ht,x]=hist(transfer_time_rho05,50);
area=(x(2)-x(1))*sum(ht);
pdf=ht./area;
figure
bar(x,pdf);
title('Ts pdf(rho = 0.5)');
[ht,x]=hist(transfer_time_rho2,50);
area=(x(2)-x(1))*sum(ht);
pdf=ht./area;
figure
bar(x,pdf);
title('Ts pdf(rho = 2)');
```

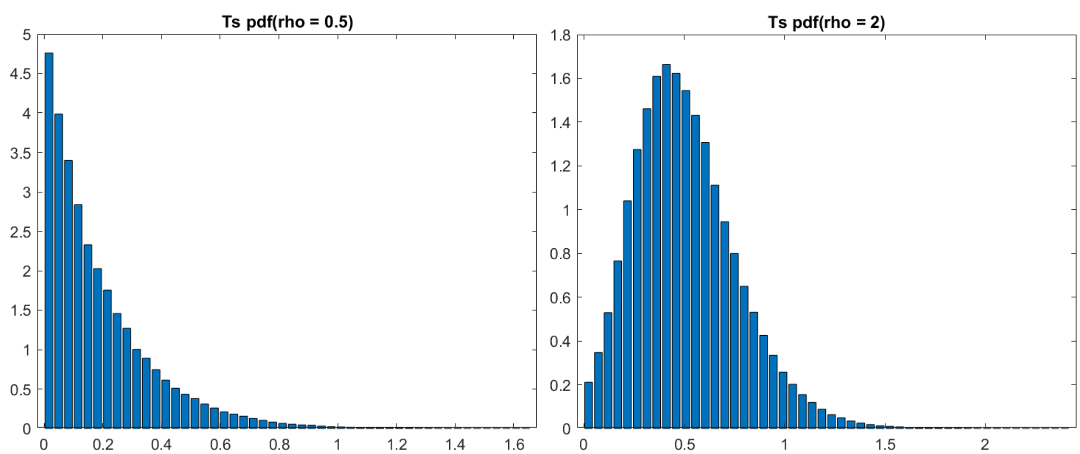


Figure 6: Ts con ρ 0,5 y 2.

En la primera gráfica, sabemos que es un sistema estable, ya que el factor de utilización es menor que 1 y si miramos en la 1era tabla, la probabilidad de pérdida es muy pequeña. También vemos que la pdf del tiempo de servicio sigue una distribución exponencial, como debe ser porque se trata de un sistema M/M/1.

En cambio en la segunda gráfica, el factor de utilización es mayor que 1, por tanto, se trata de un sistema inestable. Si observamos la segunda tabla, vemos que ahora la probabilidad de pérdida es bastante significativa. En cuanto a la pdf, vemos que la pdf del tiempo de servicio sigue una distribución guassiana.

3.3 Loss probability analysis sweeping the packet arrival rate

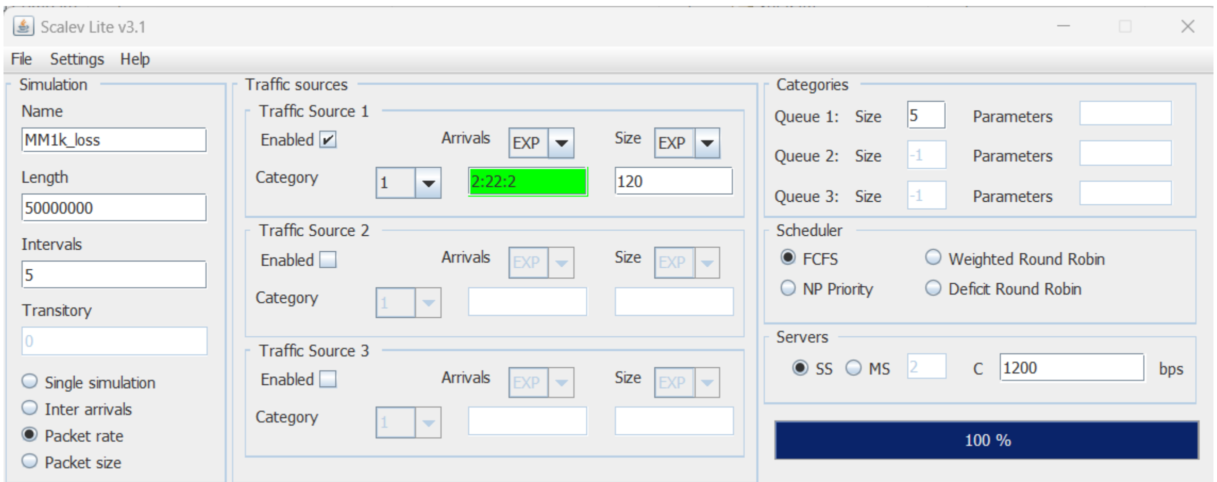


Figure 7: MM1K loss probability Scalev Simulation.

```
load report_MM1k_loss.txt;
rho_offered=report_MM1k_loss(:,1)./10;
loss_prob=report_MM1k_loss(:,10);
rho_carried=rho_offered.*(1-loss_prob);
figure;
plot((0.2:0.2:2.2)', rho_offered, (0.2:0.2:2.2)', rho_carried,
   (0.2:0.2:2.2)', loss_prob);
legend('Offered Traffic','Carried Traffic', 'Loss Probability' ,'
   Northwest')
```

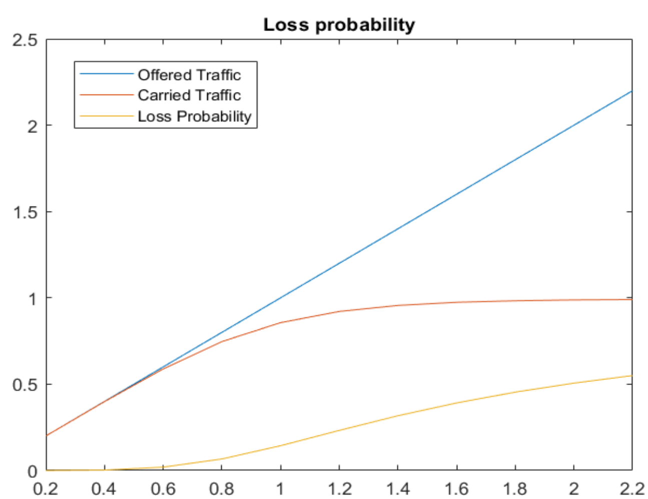


Figure 8: Lost Probability.

La probabilidad de pérdida aumenta a medida que se aumenta el tráfico. Por eso, el tráfico cursado va disminuyendo.

3.4 Additional exercise

Repeat sections 3.1 (only the case ρ=0.5) and 3.2 (ρ=0.5 and ρ=2) with queue size equal to 0. Give your conclusions.

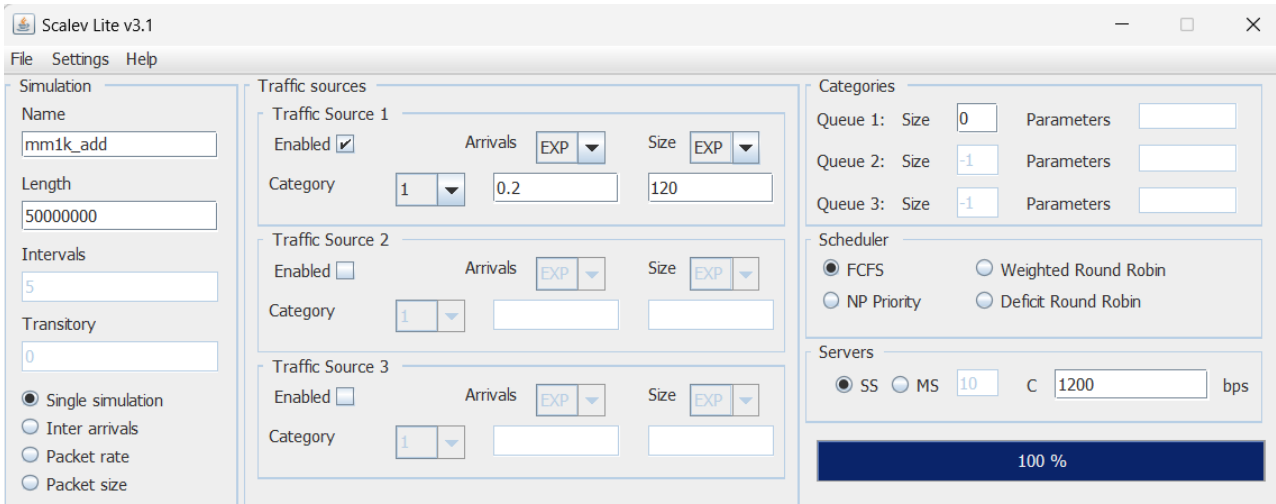


Figure 9: Scalev simulation with ρ=2 & queue=0.

	Theoretical	Simulated
Offered traffic (A)	0.5	0.5
Carried traffic	0.33333	0.3331529134343685
Waiting time	-2.7756e-17	0.0
Service time	0.1	0.10062486451333189
Transfer time	0.1	0.10062486451333189
Number of packets in queue	-1.1102e-16	0.0
Number of packets in server	0.33333	0.33315291343438286
Number of packets in system	0.33333	0.33315291343438286
Loss probability	0.33333	0.33356445728112166

Table 4: Comparison of theoretical & simulated values. with ρ=2 & queue=0.

Al no disponer de la cola, aun teniendo poco trafico, la probabilidad de perdida aun es mucho mayor.

4 M/M/1/K System Simulation

4.1 General study

a) Run a simulation for an M/M/m/m system with the same parameters used in part (b) of the previous work and check the theoretical and simulated results.

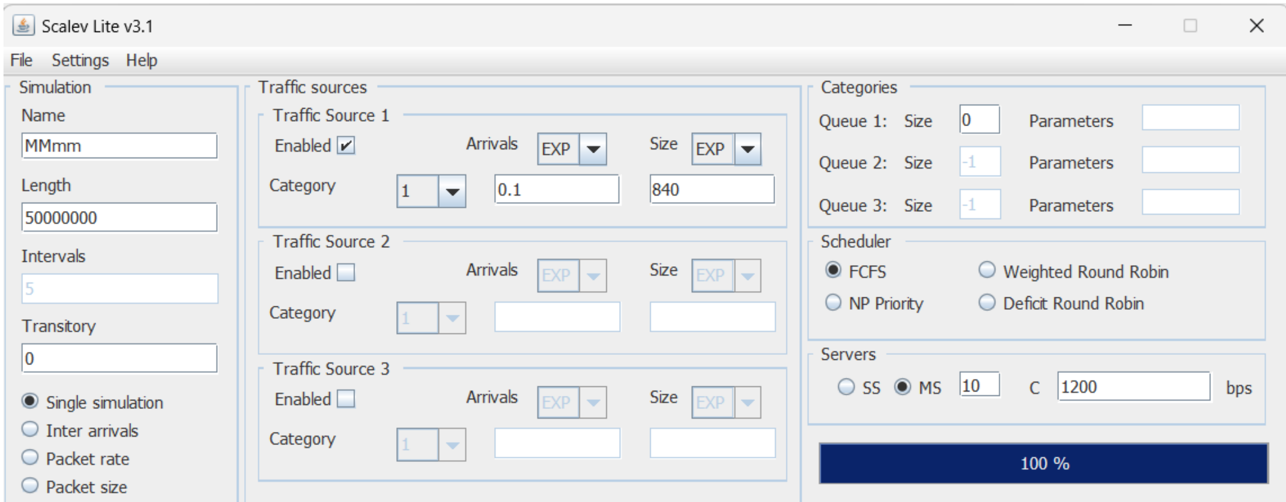


Figure 10: MMmm Scalev simulation.

Los valores de la simulacion coincidieran con los valores del estudio previo.

b) Write a MATLAB script MMmm.m

```
close all;
lambda=1/0.1;
l=840;
c=1200;
ns=10;
the_ts=l/c;
the_rho=lambda*the_ts;
the_mu=1/the_ts;
output=load('output_MMmm_source_1.txt');
%%%Llegadas
at=output(:,2);
at=sort(at);
iat=diff(at);
[ht,x]=hist(iat,100);
area=sum(ht)*(x(2)-x(1));
sim_ia=ht./area;
%iat pdf teorico
teo_ia=lambda*exp(-lambda.*x);
figure;
plot(x,[sim_ia' teo_ia'],'LineWidth',2)
title('Interarrivals Pdf')
legend('Simulated','Theoretical','Location', 'northwest');
xlabel('t')
ylabel('pdf')
```

```
%%%Tiempo transmision
ts=output(:,3);
[ht,x]=hist(ts,50);
area=sum(ht)*(x(2)-x(1));
sim_ts=ht./area;
%ts pdf teorico
teo_ts=(1/the_ts)*exp(-(1/the_ts).*x);
figure;
plot(x,[sim_ts' teo_ts'],'LineWidth',2)
title('Service time pdf')
legend('Simulated','Theoretical','Location', 'northwest');
xlabel('t')
ylabel('pdf')


%%%Probabilidades estados
occupancy=load('occupancy_MMmm_1.txt');
tmp=[occupancy(:,1),occupancy(:,3)];
sim_probs_states=state_prob_function(tmp);
maxstate=length(sim_probs_states)-1;
the_probs_states = zeros(1, length(sim_probs_states));
% prob de p0
aux = 0;
for i = 0:maxstate
   aux = aux + ((the_rho)^i)/(factorial(i));
end
p0 = 1/aux;
%prob de pk
for i = 0:maxstate
   the_probs_states(i+1) = p0*(lambda/the_mu)^i/(factorial(i));%state
    prob
end
figure;
bar(0:maxstate,[sim_probs_states' the_probs_states']),colormap([0 0 0;
    1 1 1]);
legend('Simulation','Theory','Location','Northeast');xlabel('n')
title('The state Probabilities')
%prob de perdida
loss_prob = zeros(1, length(sim_probs_states));
for i = 0:maxstate
   loss_prob(i+1) = p0*(lambda/the_mu)^i/(factorial(i));%state prob
end
figure;
bar(0:maxstate,sim_probs_states);
xlabel('n')
title('The Loss Probabilities')

%%funcion de la practica anterior
function [output] = state_prob_function(tmp)
%UNTITLED3 Summary of this function goes here
%   Detailed explanation goes here
time=tmp(:,1);
```

```
n=tmp(:,2);
prob=zeros(1,max(n)+1);
Max_time=max(time);
for i=1:length(time)-1
 prob(n(i)+1)=prob(n(i)+1)+(time(i+1)-time(i));
end
prob=prob./Max_time;
output=prob;
end
```

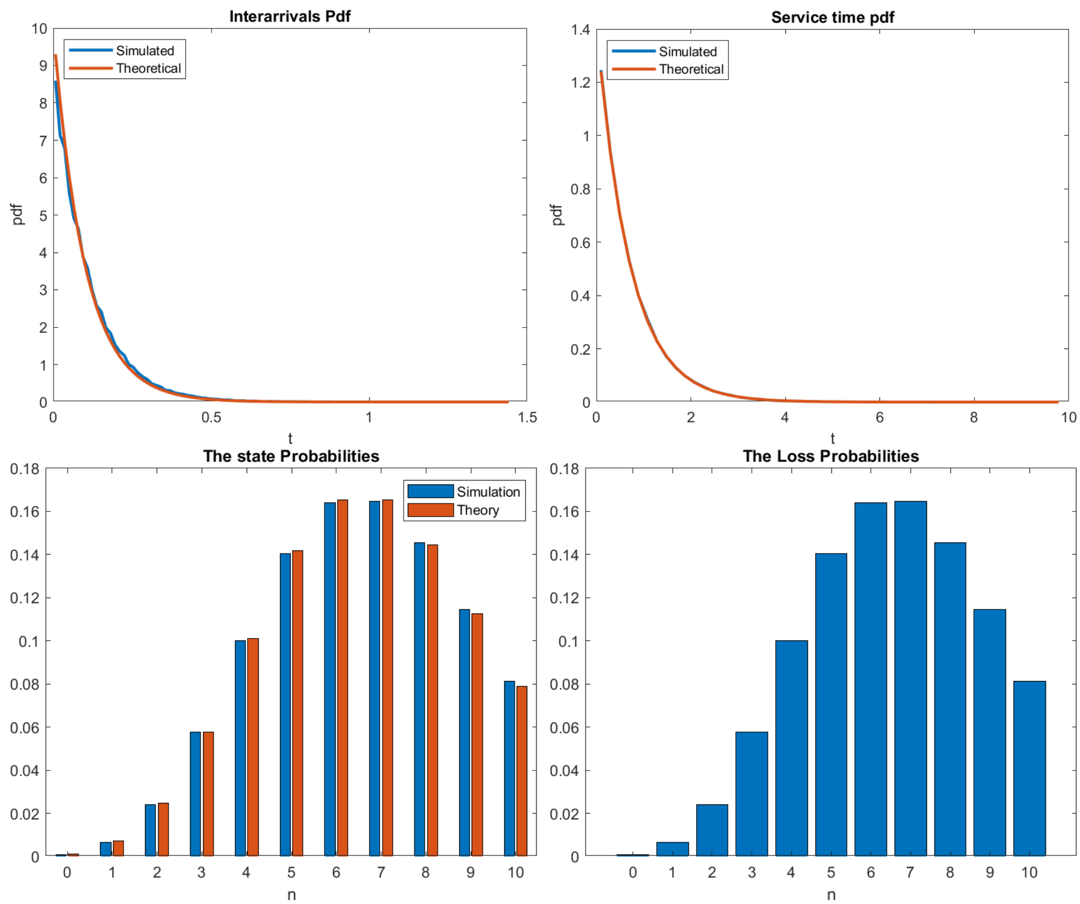


Figure 11: General Study MMmm.

4.2 Increasing the loss probability

Repeat the same simulation with 2 servers. Give your conclusions.

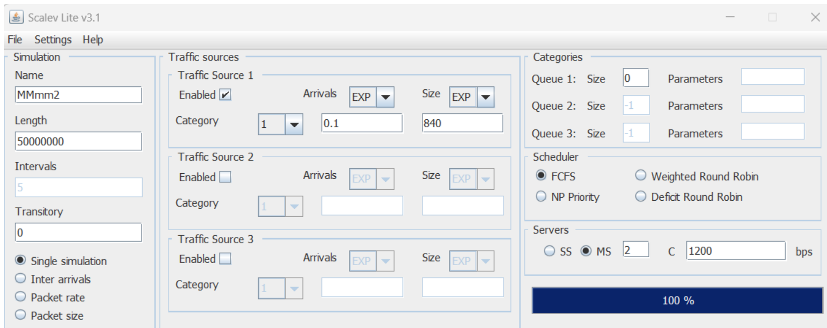


Figure 12: MMmm Scalev simulation.

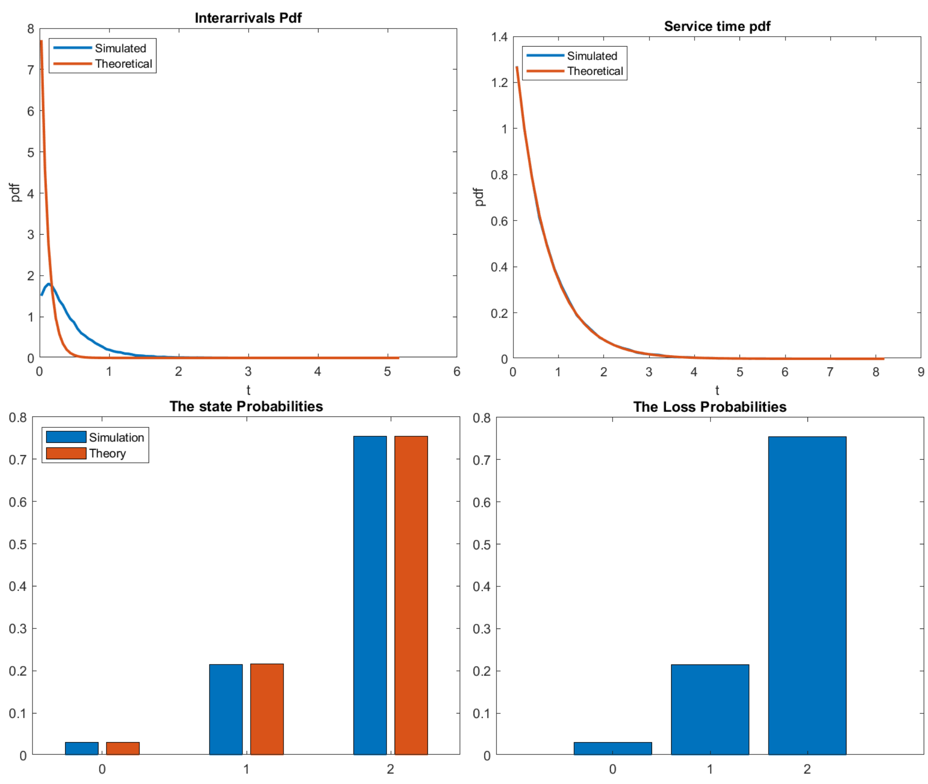


Figure 13: General Study MMmm with 2 servers.

Aquí sucede como en el ejercicio anterior, al disminuir el número de servidores, se aumenta la probabilidad de pérdida, en cuanto a la pdf del tiempo entre llegadas vemos que no coincide la teórica con la simulada, eso es debido a que en la simulación solo se tiene en cuenta el tráfico ofrecido.

5 Anexo

Codigo MATLAB completo apartado 3

```
close all
lambda=input('Arrival rate (packets/s): '); %5,10,20
l=input('Packet length (bits): ');  %120, 840
c=input('Channel capacity (bps): '); %1200
q=input('Queue size (packets): '); %10 ,0
i=0:q+1;
the_ts=l/c; % tiempo de servicio
the_rho=lambda*the_ts; %factor de utilizacion
if the_rho~=1 % si la rho es descansa
   the_prob=[((1-the_rho)/(1-the_rho^(q+2)))*the_rho.^i]'; %la
   probabilidad de estado
   the_n=((q+1)*the_rho^(q+3)-(q+2)*the_rho^(q+2)+the_rho)/(the_rho^(q
   +3)-the_rho^(q+2)-the_rho+1); % numero medio de unidades
   pl=((1-the_rho)/(1-the_rho^(q+2)))*the_rho^(q+1); %probabilidad de
   perdida, probabilidad del ultimo estado
else
   the_prob=zeros(q+2,1); % en caso de que la rho sea 1
   the_prob=(1/(q+2))+the_prob;
   the_n=(q+1)/2;
   pl=1/(q+2);
end

lambda_c=lambda*(1-pl);
the_rho_c=lambda_c*the_ts;
the_t=the_n/lambda_c;
the_ns=the_rho_c;
the_nw=the_n-the_ns;
the_tw=the_t-the_ts;
disp([' ']);
disp(['RESULTS M/M/1/K']);
disp(['---------------']);
disp(['Offered traffic:' num2str(the_rho)]);
disp(['Carried traffic:' num2str(the_rho_c)]);
disp(['Waiting time:' num2str(the_tw)]);
disp(['Service time:' num2str(the_ts)]);
disp(['Transfer time:' num2str(the_t)]);
disp(['Number of waiting packets:' num2str(the_nw)]);
disp(['Number of packets in server:' num2str(the_ns)]);
disp(['Number of packets in system:' num2str(the_n)]);
disp(['Loss probability:' num2str(pl)]);
disp(the_prob)
bar(the_prob);
title("Probabilities")
%

% %%%%%3.2 Transfer time analysis
% load output_MM1K_source_1.txt;
% transfer_time_rho05=output_MM1K_source_1(:,4);
% load output_MM1K2_source_1.txt;
% transfer_time_rho2=output_MM1K2_source_1(:,4);
```

```
% [ht,x]=hist(transfer_time_rho05,50);
% area=(x(2)-x(1))*sum(ht);
% pdf=ht./area;
% figure
% bar(x,pdf);
% title('Ts pdf(rho = 0.5)');
% [ht,x]=hist(transfer_time_rho2,50);
% area=(x(2)-x(1))*sum(ht);
% pdf=ht./area;
% figure
% bar(x,pdf);
% title('Ts pdf(rho = 2)');

%%%%3.3 loss probability analysis
load report_MM1k_loss.txt;
rho_offered=report_MM1k_loss(:,1)./10;
loss_prob=report_MM1k_loss(:,10);
rho_carried=rho_offered.*(1-loss_prob);
figure;
plot((0.2:0.2:2.2)', rho_offered, (0.2:0.2:2.2)', rho_carried,
   (0.2:0.2:2.2)', loss_prob);
legend('Offered Traffic','Carried Traffic', 'Loss Probability' ,'
   Northwest')
title('Loss probability')
```