Hypothesis Testing

Question 1: What is a null hypothesis (H_0) and why is it important in hypothesis testing?

Answer:
The null hypothesis (H_0) is a statement that assumes no effect, no difference, or no change in the population. It is the default assumption in hypothesis testing.

Importance:
• It provides a starting point for analysis.
• It helps decide whether sample evidence is strong enough to support the alternative hypothesis.
• Hypothesis testing is mainly done to determine whether to reject H_0 or not reject H_0 based on data.

Question 2: What does the significance level (α) represent in hypothesis testing?

Answer:
The significance level (α) represents the probability of making a Type I error, i.e., rejecting the null hypothesis when it is actually true. It is the maximum risk level allowed by the researcher.

Common significance levels are:
• $\alpha = 0.05$
• $\alpha = 0.01$

Question 3: Differentiate between Type I and Type II errors.

Answer:
Type I Error (α):
• Rejecting H_0 when H_0 is true
• False positive
Example: A medical test indicates disease when the person does not have it.

Type II Error (β):
• Not rejecting H_0 when H_0 is false
• False negative
Example: A medical test indicates no disease when the person actually has it.

Question 4: Explain the difference between a one-tailed and two-tailed test. Give an example of each.

Answer:
One-tailed test:
A one-tailed test checks for an effect in only one direction (either greater than or less than).
Example:
H_0: $\mu = 60$
H_1: $\mu > 60$

Two-tailed test:
A two-tailed test checks for an effect in both directions (different from).
Example:
H_0: $\mu = 10$
H_1: $\mu \neq 10$

Question 5: A company claims that the average time to resolve a customer complaint is 10 minutes. A sample of 9 complaints gives mean 12 minutes and standard deviation 3 minutes. At $\alpha = 0.05$, test the claim.

Given:
$\mu = 10$, $\bar{x} = 12$, $s = 3$, $n = 9$, $\alpha = 0.05$

Step 1: Hypotheses
H_0: $\mu = 10$
H_1: $\mu \neq 10$ (two-tailed test)

Step 2: Test statistic (t-test)
$t = (\bar{x} - \mu) / (s/\sqrt{n})$
$t = (12 - 10) / (3/\sqrt{9})$
$t = 2 / 1$
$t = 2$

Step 3: Critical value
$df = n - 1 = 8$
tcritical = ± 2.306 ($\alpha = 0.05$, two-tailed)

Decision:
$|t| = 2 < 2.306 \rightarrow$ Fail to reject H_0

Conclusion:

At $\alpha = 0.05$, there is not enough evidence to reject the company's claim. The claim $\mu = 10$ minutes is not rejected.

Question 6: When should you use a Z-test instead of a t-test?

Answer:
Use a Z-test when:
• The population standard deviation (σ) is known, and
• The sample size is large ($n \geq 30$).

Use a t-test when:
• The population standard deviation is unknown (especially when $n < 30$).

Question 7: The productivity of 6 employees was measured before and after a training program. At $\alpha = 0.05$, test if the training improved productivity.

Employee	Before	After
1	50	55
2	60	65
3	58	59
4	55	58
5	62	63
6	56	59

Step 1: Hypotheses (paired)
Let d = After − Before
H_0: $\mu d = 0$
H_1: $\mu d > 0$ (right-tailed)

Step 2: Differences
d = 5, 5, 1, 3, 1, 3
Mean difference: $\bar{d} = 3$
Standard deviation of differences: $sd \approx 1.789$

Step 3: Test statistic
$t = \bar{d} / (sd/\sqrt{n})$
$t = 3 / (1.789/\sqrt{6})$
$t \approx 4.108$

df = 5
Critical value at α=0.05 (right-tailed): tcritical = 2.015

Decision:
$4.108 > 2.015 \rightarrow$ Reject H_0

Conclusion:
At $\alpha = 0.05$, the training significantly improved productivity.

Question 8: A company wants to test if product preference is independent of gender. At $\alpha = 0.05$, test independence.

Gender	Product A	Product B	Total
Male	30	20	50
Female	10	40	50
Total	40	60	100

Hypotheses
H_0: Product preference is independent of gender
H_1: Product preference is dependent on gender

Expected frequencies (E)
E = (Row total × Column total) / Grand total
Male-A = 20, Male-B = 30
Female-A = 20, Female-B = 30

Chi-square statistic
$\chi^2 = \Sigma\, (O-E)^2/E$
$\chi^2 = 16.67$

Step 4: Critical value
$df = (2-1)(2-1) = 1$
χ^2critical (α=0.05, df=1) = 3.841

Decision:
$16.67 > 3.841 \rightarrow$ Reject H_0

Conclusion:
At $\alpha = 0.05$, product preference is NOT independent of gender.