

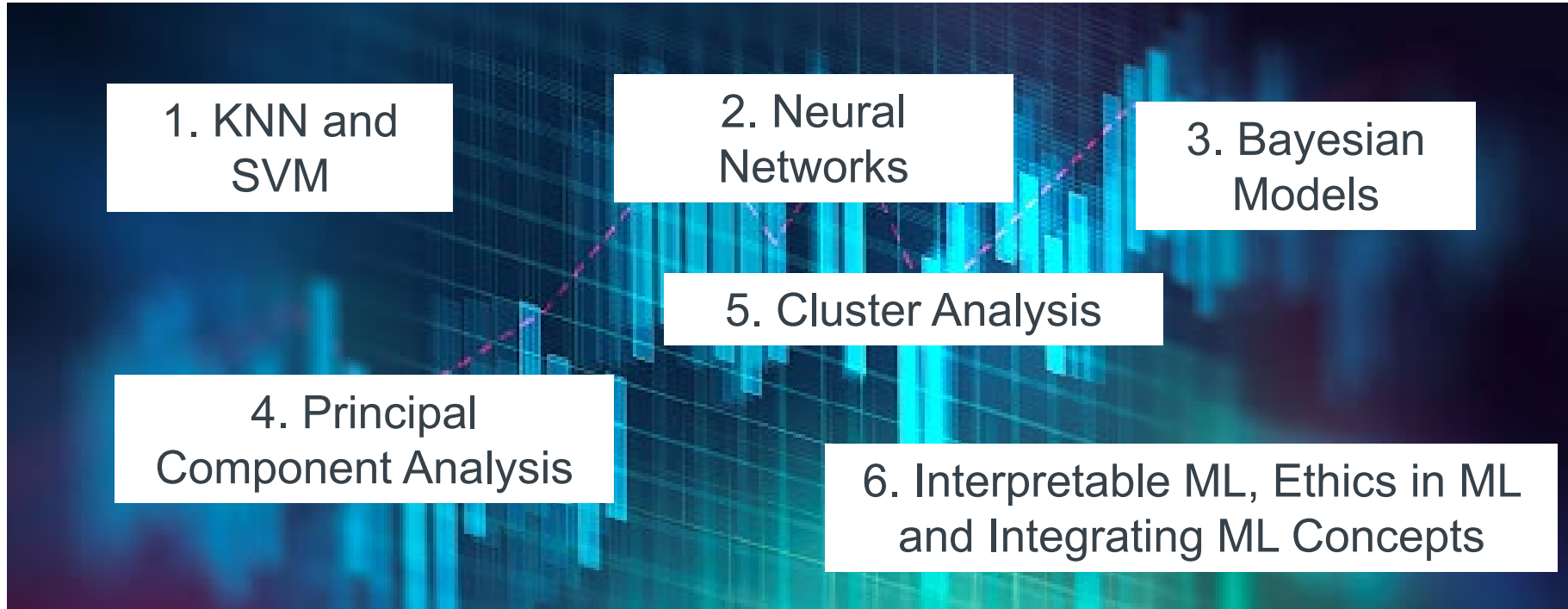
Introduction to Machine Learning

Machine Learning - II

GEORGETOWN UNIVERSITY  McDonough
SCHOOL of BUSINESS



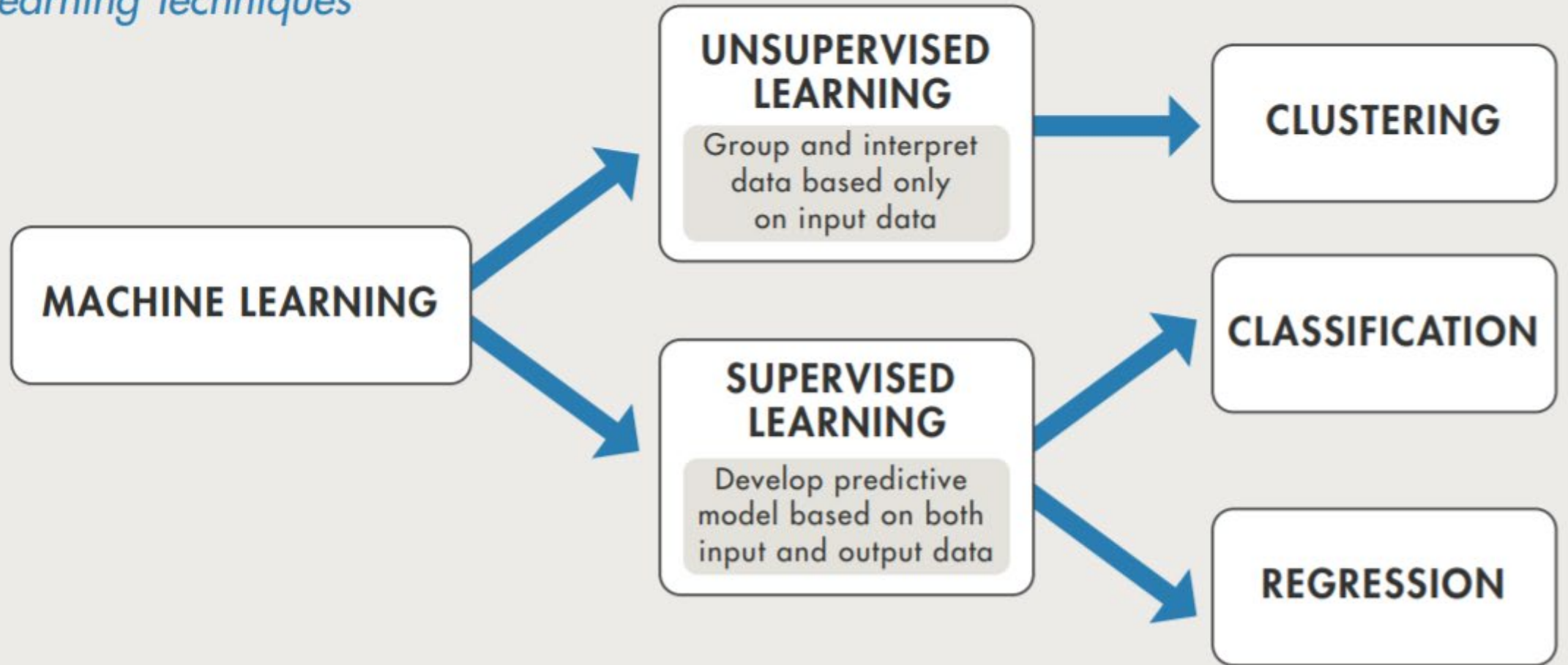
Course Content



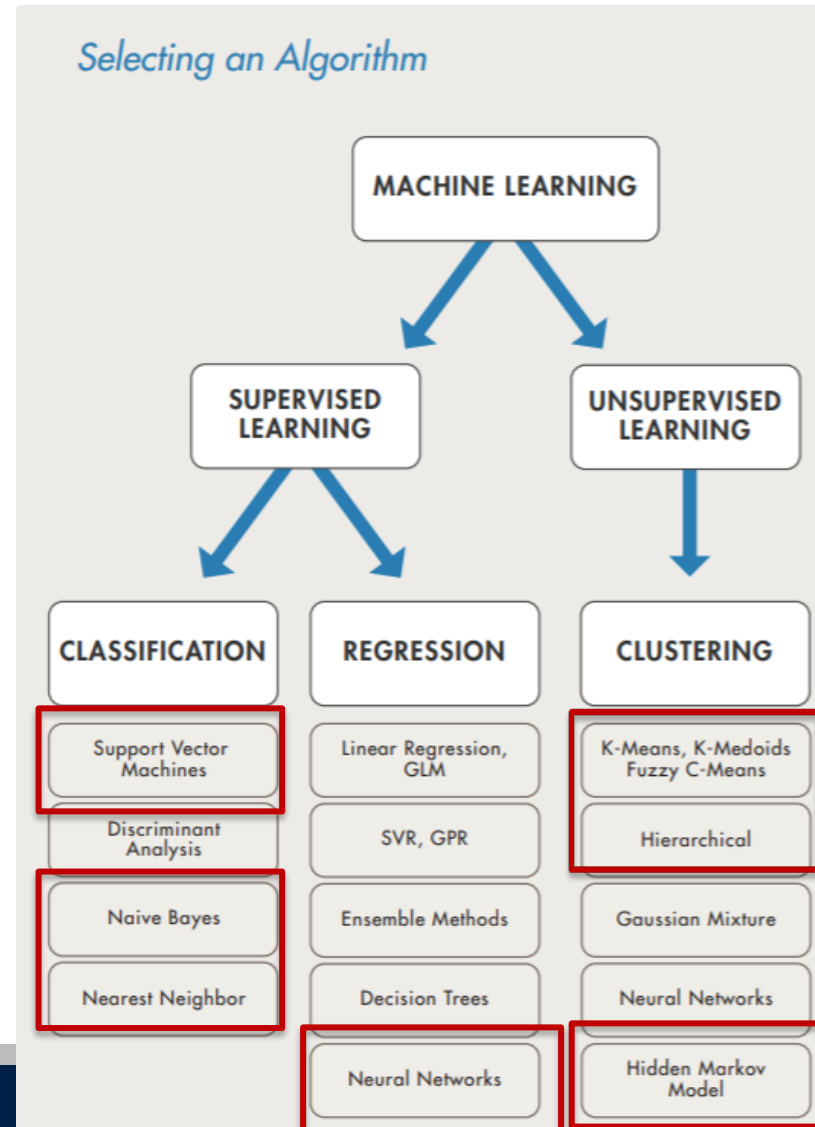
MACHINE LEARNING ALGORITHMS REFRESHER

How Machine Learning Works?

Machine Learning Techniques



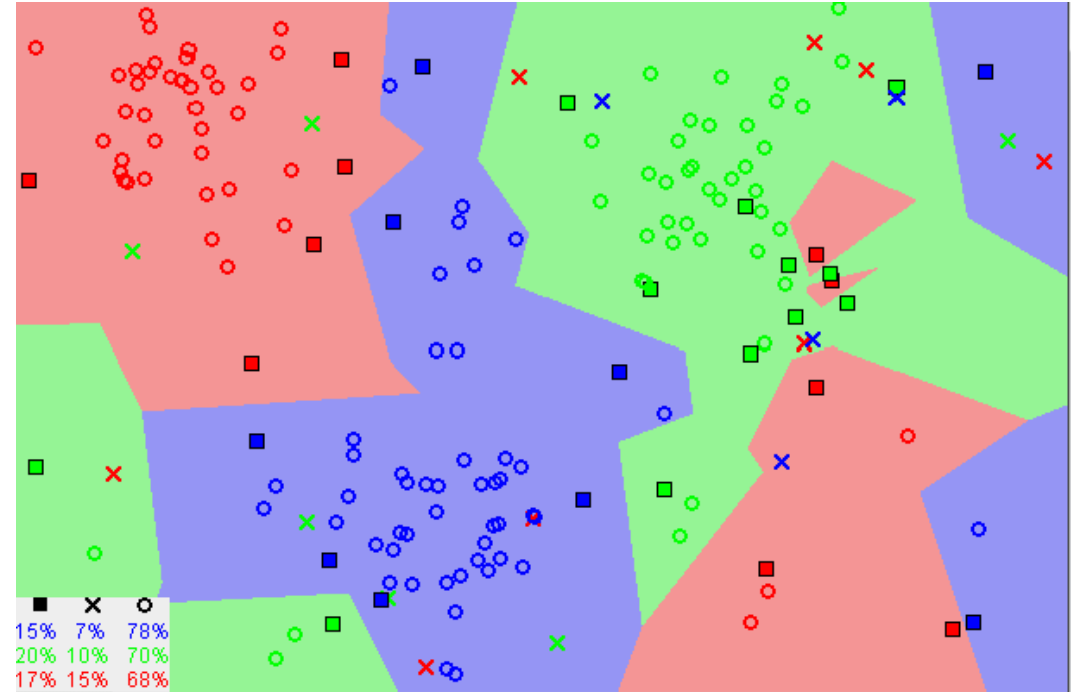
How Do You Decide which Algorithm to Use?



K-NEAREST NEIGHBORS

K-Nearest Neighbors

- Observations close to each other share similar characteristics
- Hinges on the idea of distance between observations
- Distance between observations can be calculated in several ways

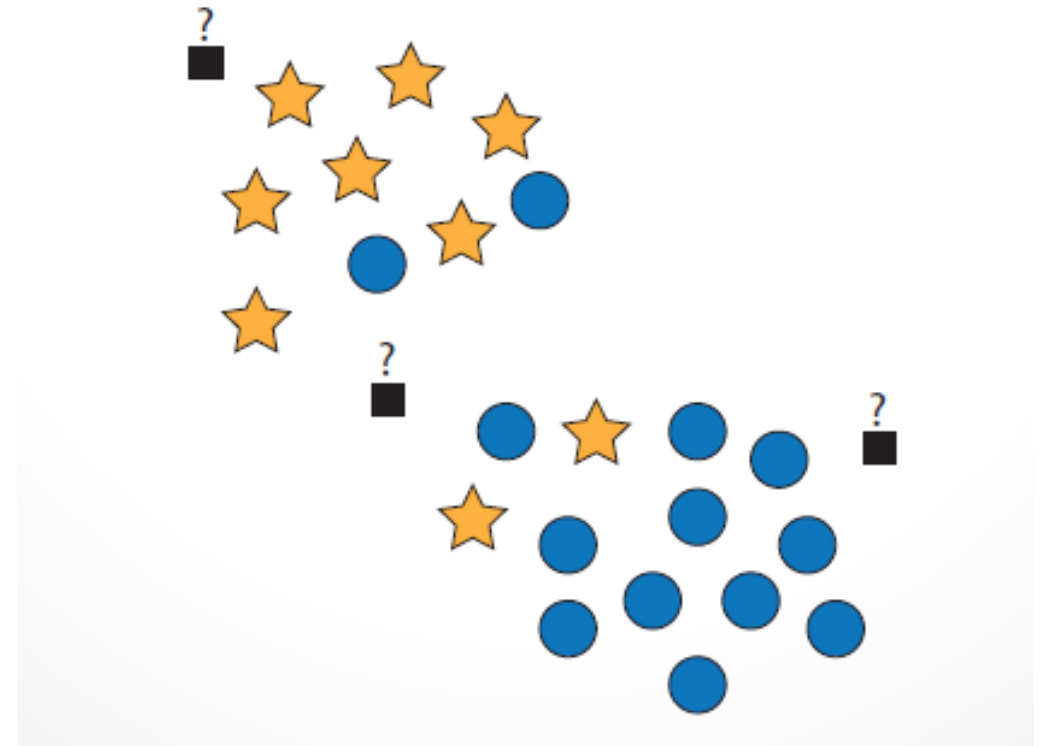


The value of a data point is determined by the data points around it

kNN: Intuition

Identify several cases that are most similar to a given observation

Use the information from those “neighbors” to classify or predict the new observation

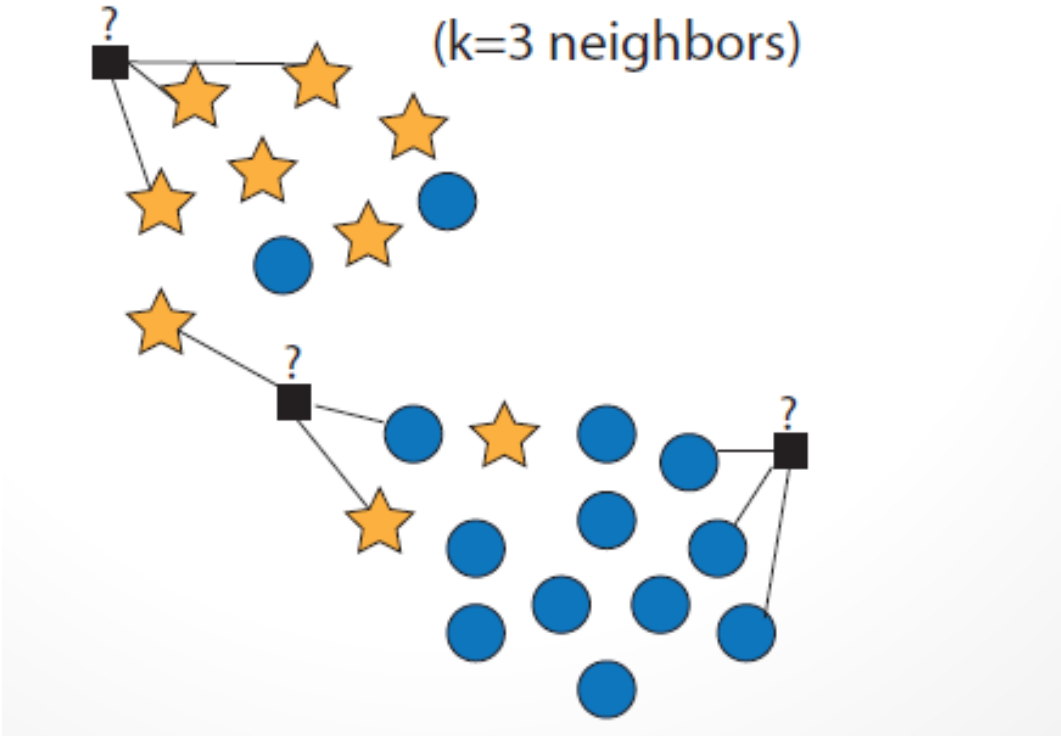


kNN: Intuition

Identify several cases that are most similar to a given observation

Use the information from those “neighbors” to classify or predict the new observation

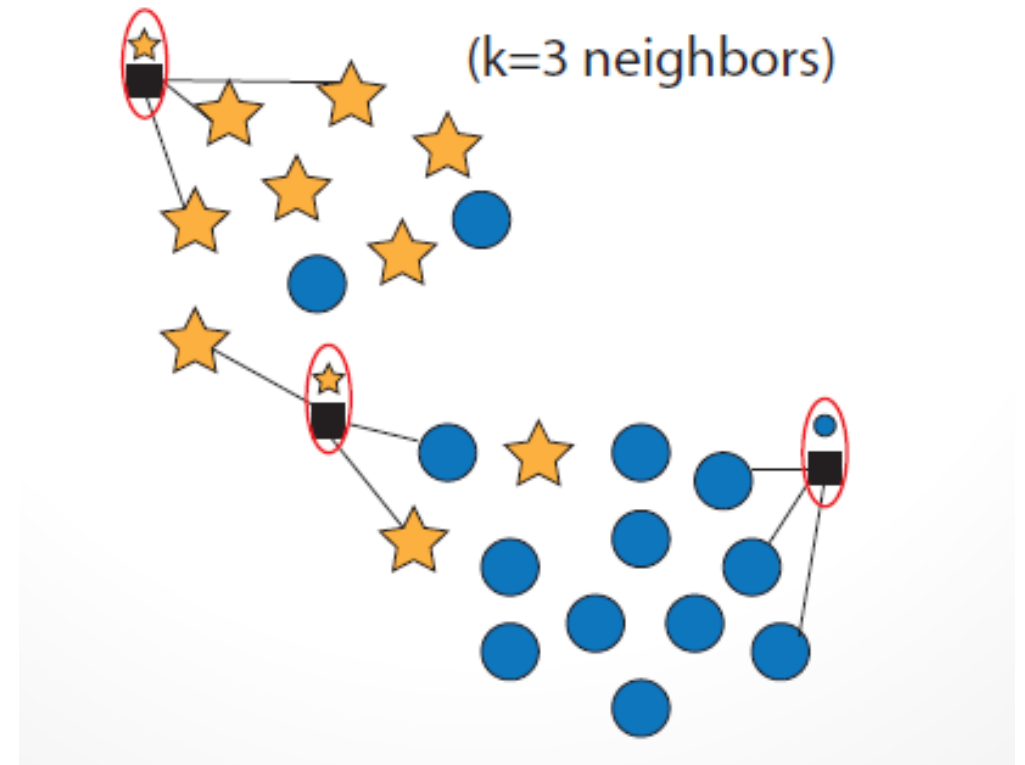
The nearest neighbors are defined by the value of “k”



kNN: Intuition

Identify several cases that are most similar to a given observation

Use the information from those “neighbors” to classify or predict the new observation



Considerations

- How should I measure similarity? Distance measures
 - Numeric attributes
 - Ordinal attributes
 - Categorical attributes
 - How do I combine these?
- How should I combine the results of neighbors?
 - Classification:
 - Majority rules (mode)?
 - Weight votes by nearness?
 - Regression/prediction:
 - Mean?
 - Median?
- How many neighbors should I use (value of k)?

DISTANCE FUNCTIONS

Building Distance Functions

- Numeric variables (includes some ordinal)
 - Some type of normalization or standardization is usually required
 - Standardize the variable before input to the model
 - OR standardize the distance in creating an overall distance matrix
 - Most common type of standardization
 - min-max normalization (feature scaling)
$$\frac{x - x_{min}}{x_{max} - x_{min}}$$
 - z-score standardization

Building Distance Functions

Options for computing distances:

Absolute difference

- No standardization
- Standardize:
 - Divide by max distance (Scales the distance between 0 and 1)
 - Divide by standard deviation for that variable (How many standard deviations apart are the observations?)
 - Can even divide by the standard deviation for the distance!

Name	Income (in Ks)
Sam	50
Pam	65
Tam	75
Original Variable	

$$\begin{array}{c} S \quad P \quad T \\ S \left(\begin{array}{ccc} 0 & 15 & 25 \\ 15 & 0 & 10 \\ 25 & 10 & 0 \end{array} \right) \quad \begin{array}{c} S \quad P \quad T \\ S \left(\begin{array}{ccc} 0 & 0.6 & 1 \\ 0.6 & 0 & 0.4 \\ 1 & 0.4 & 0 \end{array} \right) \\ P \\ T \end{array}$$

Absolute Difference
distance matrix
(Option 1)

Standardized
distance matrix
(Option 2)

Building Distance Functions

- Categorical variables (includes some ordinal):
 - Distance=0 if matching, 1 otherwise

Name	Marital Status
Sam	Single
Pam	Married
Tam	Single

Original Variable

$$\begin{matrix} & S & P & T \\ \begin{matrix} S \\ P \\ T \end{matrix} & \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \end{matrix}$$

Distance Matrix

Building Distance Functions

- Suppose we have a categorical variable with four categories:

Marital Status
Single
Married
Divorced
Widowed

- Software default is usually to create 4 dummy variable (one-hot encoding) and treat them like numeric variables

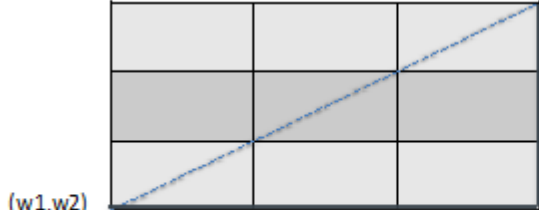
Obs	Single	Married	Divorced	Widowed
i	1	0	0	0
j	0	1	0	0

Building Distance Functions

- You can define anything you find reasonable!
- Example: Zip Codes:
 - $d(i,j)=0$ if zip codes are identical
 - $d(i,j)=0.1$ if the first three digits are identical
 - $d(i,j)=0.5$ if the first digits are identical
 - $d(i,j)=1$ if the first digits are different
 - or use the corresponding geographical distance (more work)

Building Distance Functions

- Used to build distance matrix
 - *Manhattan distance* (L_1 -norm)
 - Measure of similarity in terms of the distance between a pair of objects
 - Distance between two points measured along axis at right angles
- Also used to merge the individual distances together


$$D = \sum_{i=1}^d |x_i - w_i|$$

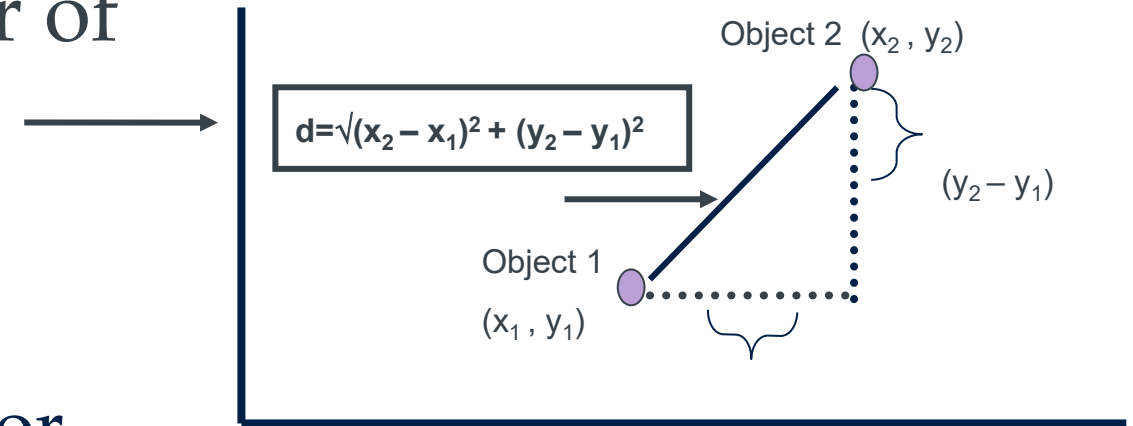
$$d_{x_1}(i, j) + d_{x_2}(i, j) + \dots + d_{x_d}(i, j)$$

where

$d_{x_k}(i, j)$ is the distance between observation i and j on variable x_k

Building Distance Functions

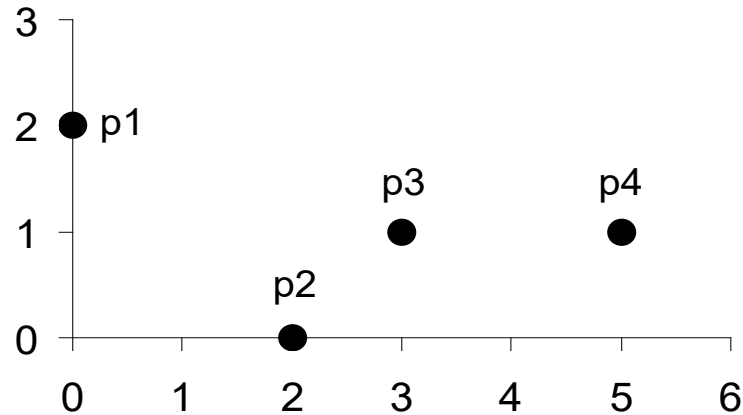
- Compute distance between pair of observations
 - *Euclidean distance (L_2 -norm)*
 - Square root of the sum of the squared differences in values for each variable
- Merge the individual distances together



$$\sqrt{d_{x_1}(i, j)^2 + d_{x_2}(i, j)^2 + \dots + d_{x_k}(i, j)^2}$$

where $d_{x_k}(i, j)$ is the distance between observation i and j on variable x_k

Euclidean Distance



point	x	y
p1	0	2
p2	2	0
p3	3	1
p4	5	1

	p1	p2	p3	p4
p1	0	2.828	3.162	5.099
p2	2.828	0	1.414	3.162
p3	3.162	1.414	0	2
p4	5.099	3.162	2	0

Distance Matrix

Building Distance Functions

- Compute distance between pair of observations (categorical variables only)

- *Jaccard's Coefficient Measure*

$$J = \frac{a}{a + b}$$

- *Jaccard's Distance Measure*

$$d_{ij} = \frac{b}{a + b} \text{ or } 1 - J$$

- *Dice's Coefficient Measure*

$$D = \frac{2a}{2a + b}$$

Gene expression levels under 17 variables (low=0, high=1)

	1	2	3													17
Gene i	0	1	1	0	0	1	0	0	1	0	0	1	1	1	0	0	1
Gene j	0	1	1	1	0	0	0	0	1	1	1	1	1	1	0	1	1

Match of 1s : a = 0 1 1 0 0 0 0 0 1 0 0 1 1 1 0 0 1 = 7

Diff1 (mismatch): b = 0 0 0 1 0 1 0 0 0 1 1 0 0 0 0 1 0 = 5

Match of 0s : c = 1 0 0 0 1 0 1 1 0 0 0 0 0 0 0 1 0 0 = 5

$$J = 7/12 = 0.58$$

$$d_j = 0.42$$

$$D = 0.73$$

Building Distance Functions

- Compute distance between pair of observations
 - *Gower's Measure*
 - Find similarity between observations in a dataset consisting of mixed variables (continuous and categorical)
 - Uses Manhattan distance for continuous variables and Dice Coefficient for categorical variables

For a categorical variable k

$$d_{ijk} = \frac{\sum_k^n w_{ijk} d_{ijk}}{\sum_k^n w_{ijk}}$$

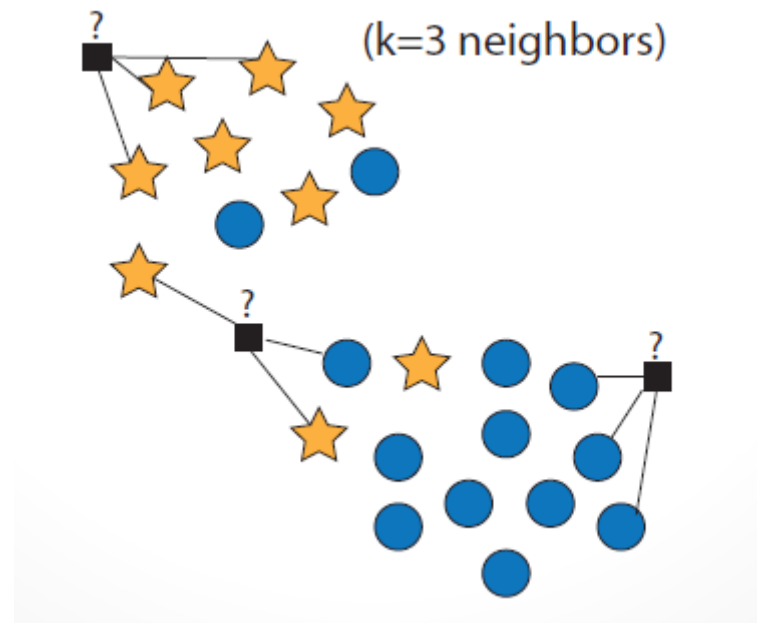
where d_{ij} is the distance between observation i and j ;

d_{ijk} denotes the dice distance between i and j based on the k_{th} variable;

w_{ijk} is usually 1 or 0 depending on if the k_{th} variable is categorical or not.

Combination Functions

- Now we have distances to each k neighbors
- How do I combine that information to make a prediction for the given observation?

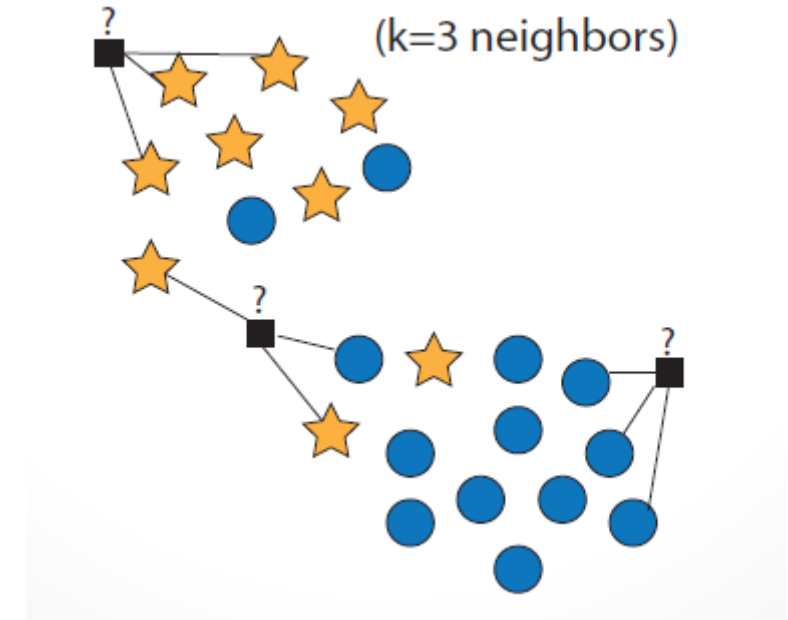


Combination Functions

- Numeric Target
 - Mean or median of the neighbor's target value
- Class Target
 - Basic approach: democracy – majority rules
 - Create probabilities for each class as the proportion of neighbors voting for each class
 - Weighted voting: nearer neighbors have stronger votes
 - This can reduce the sensitivity to the parameter k

$$w_j = \frac{1}{d(i,j)^2}$$

- Add up the weighted votes to see which category has the most



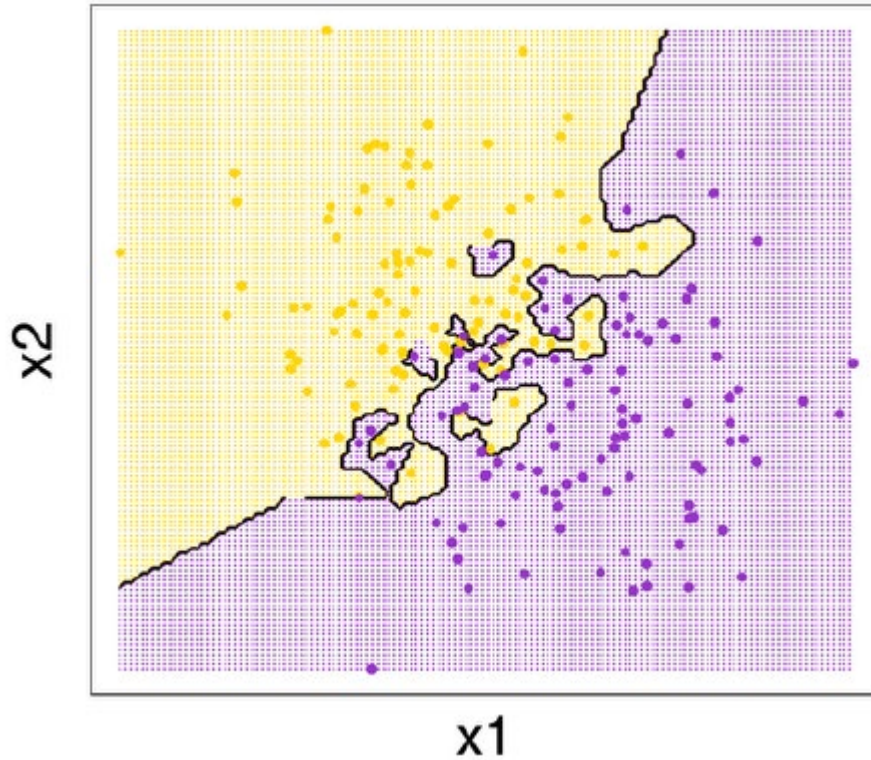
CHOOSING VALUE OF K

Similarity is Measured by Distance between Observations

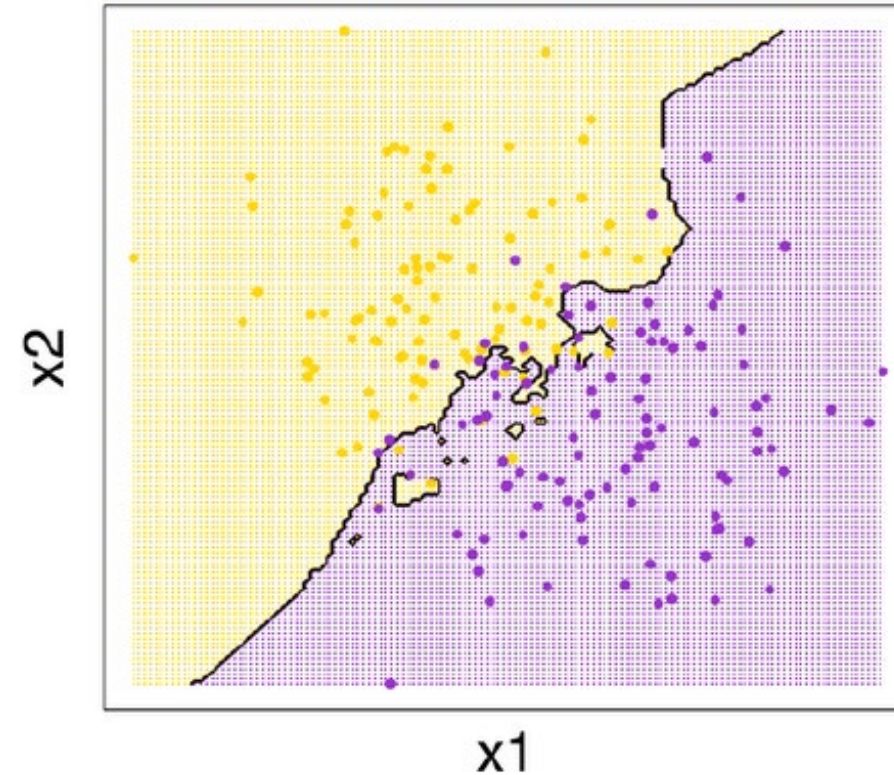
- With just one data point, you will have a kNN with $k=1$
- If you are around a group of 5 data points, each one in the group has an effect on the behavior of the data point and your value = mean of 5 data points = kNN with $k=5$
- kNN classifier determine class of a data point by majority voting principle or mean value of the neighbors for prediction

Choosing k

Binary kNN Classification (k=1)

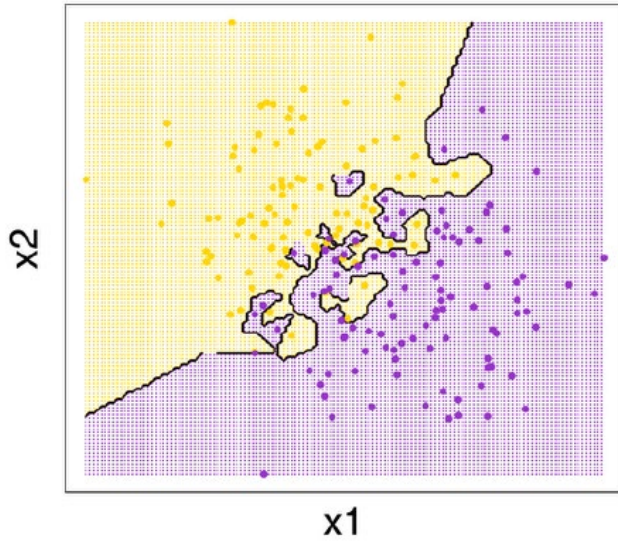


Binary kNN Classification (k=5)

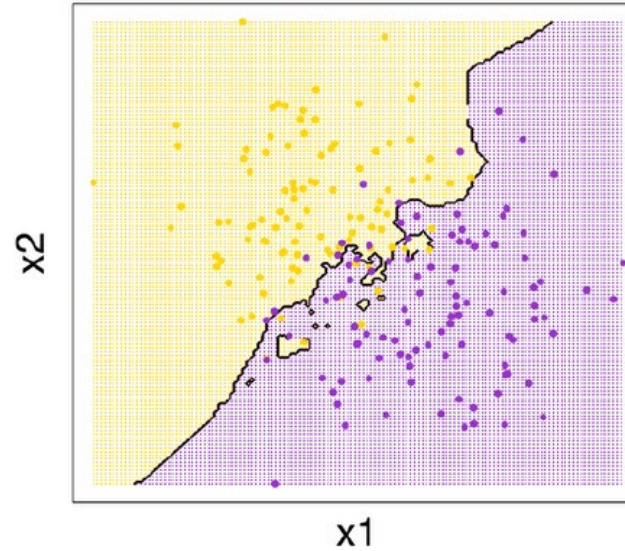


Choosing k

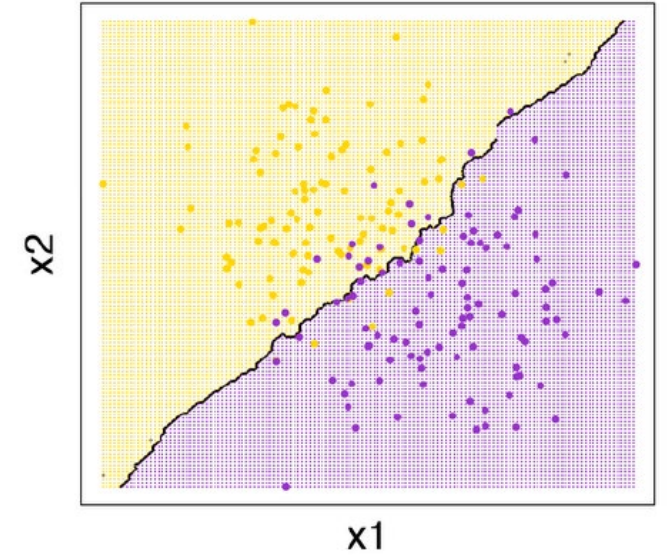
Binary kNN Classification (k=1)



Binary kNN Classification (k=5)



Binary kNN Classification (k=25)

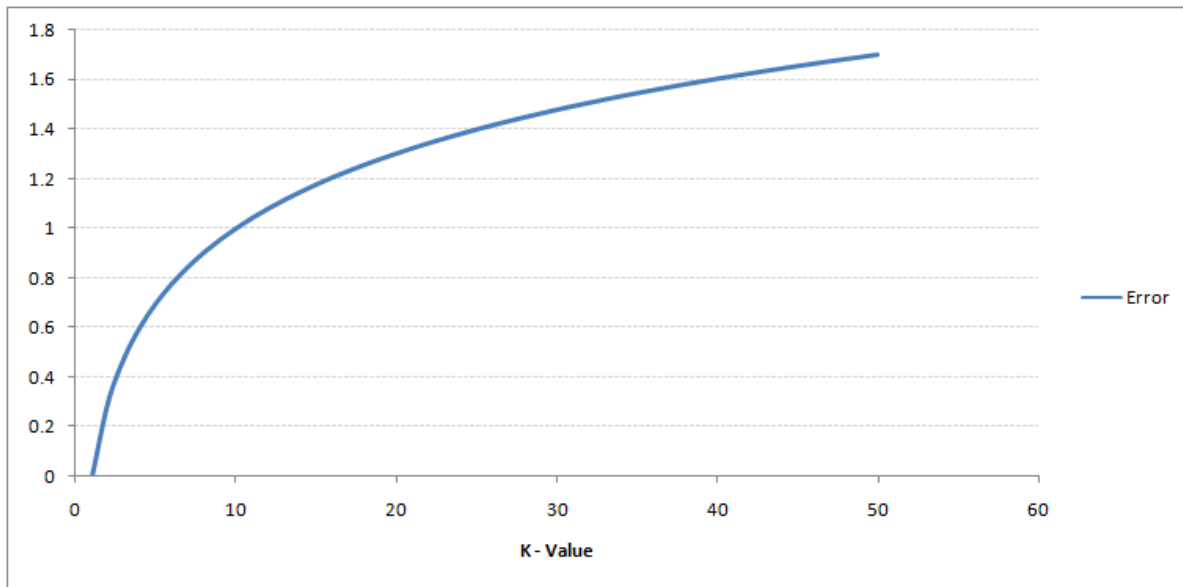


Choosing the Right Value of K

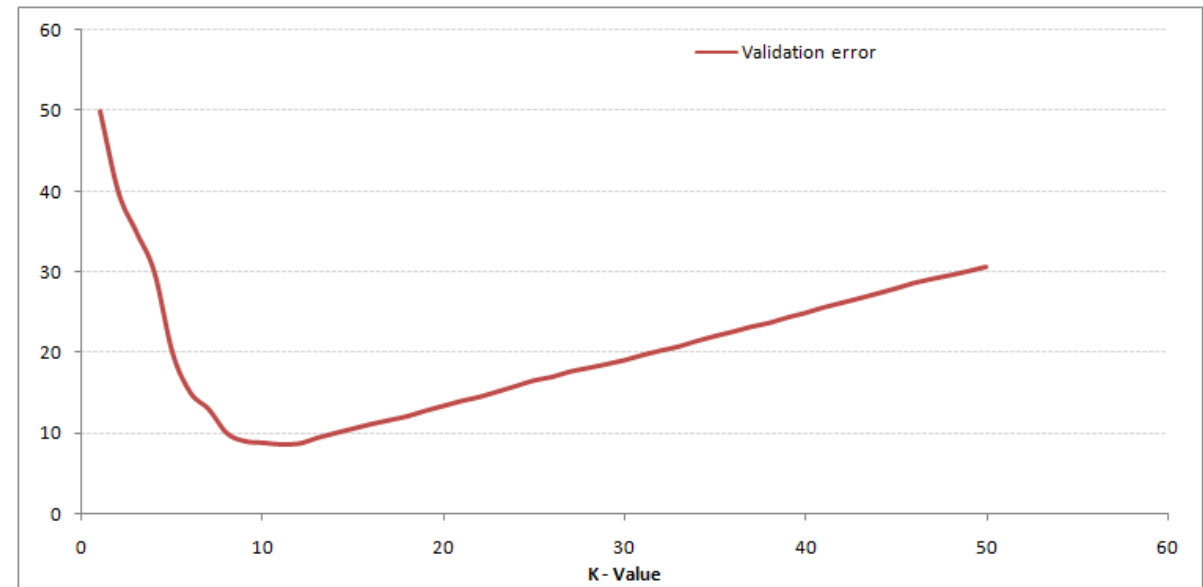
- Smaller values of k -> higher variance model
 - Tends toward overfitting
- Larger values of k -> higher bias model
 - Tends toward underfitting
- Best practice to tune this parameter with a validation set or with cross-validation
 - Run kNN several times with different values of k and choose k that reduces the RMSE in validation data
- Common practice to begin with $k = \sqrt{n}$, n = number of training samples

Choosing k

RMSE (y axis) for Training Set



RMSE (y axis) for Validation Set



kNN Algorithm

- Load the data
- Initialize k to your chosen number of neighbors
- For each example in the data:
 - Calculate the distance between the query example and the current example from the data
 - Add the distance and the index of the example to an ordered collection
- Sort the ordered collection of distances and indices from smallest to largest (in ascending order) by the distances
- Pick the first k entries from the sorted collection
- Get the labels of the selected k entries
 - If regression, return the mean of the k labels
 - If classification, return the mode of the k labels

Advantages and Disadvantages of kNN

Advantage

- Highly unbiased, no prior assumption about underlying data
- The algorithm is simple and easy to implement
- There's no need to build a model, tune several parameters, or make additional assumptions
- The algorithm is versatile - It can be used for classification, regression, and search

Disadvantage

- The algorithm gets significantly slower as the number of examples and/or predictors/independent variables increase
- Results dependent on choice of distance function, no. of neighbors, k
- Does not produce a model: does not help us understand how features are related to classes

KNN in Practice

- Recommendation systems (products): Data consists of customers rating items (Netflix movies, Amazon.com products, etc.)
- Predict a customer's rating on a new product based on the way their nearest neighbors (other customers) rated that product.
- Search for semantically similar documents or observations
- Segmentation/Clustering
- Classification models with class variables (categorical outcome)
- ...

Example: Predicting Red Wine Quality

The red wine dataset includes data on red *vinho-verde* wine samples, from the north of Portugal. The goal is to identify quality class based on physicochemical tests using kNN. Selected variables out of 12:

1. **Fixed acidity:** Most acids involved with wine are fixed or nonvolatile (do not evaporate easily)
4. **Residual sugar:** The amount of sugar remaining after fermentation stops, it is rare to find wines with less than 1 gram/liter; and wines with greater than 45 grams/liter are considered sweet.
5. **Chlorides:** The amount of salt in the wine
9. **pH:** Describes how acidic or basic a wine is on a scale from 0 (very acidic) to 14 (very basic); most wines are between 3-4 on the pH
11. **Alcohol:** The percent alcohol content in the wine
12. **Quality:** Based on sensory data (score between 0- poor and 10 – excellent)



We collapsed the quality metric to two categories:
Low quality (Quality less than 5)
High quality (Quality 5 and over)

kNN Model Results

For the training data (70% of observations, the k value was tuned from 5-43 and ROC computed

Note that the sensitivity and specificity values are given as well

Model accuracy = 71%

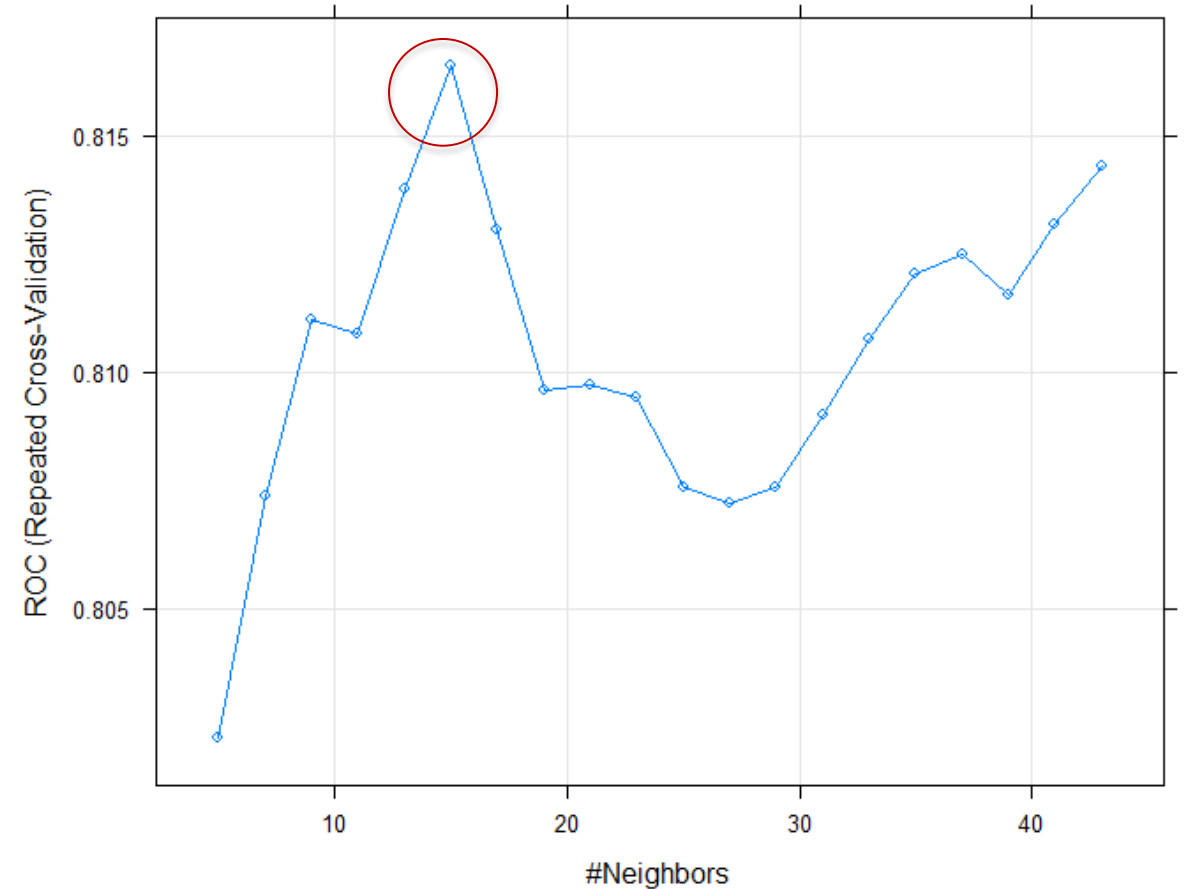
k	ROC	Sens	Spec
5	0.8022560	0.7790772	0.6698355
7	0.8073915	0.7741243	0.6903241
9	0.8111204	0.7752354	0.6890542
11	0.8108145	0.7752448	0.6903120
13	0.8138594	0.7680226	0.7030963
15	0.8164896	0.7686064	0.6999516
17	0.8130138	0.7635876	0.6980164
19	0.8096208	0.7580414	0.7037736
21	0.8097409	0.7630226	0.7114538
23	0.8094651	0.7696987	0.7095670
25	0.8075564	0.7674859	0.6929245
27	0.8072349	0.7619303	0.6942187
29	0.8075579	0.7580320	0.6993348
31	0.8090990	0.7591431	0.6941945
33	0.8107130	0.7574765	0.7006410
35	0.8120695	0.7574670	0.7089260
37	0.8124715	0.7641431	0.7114659
39	0.8116199	0.7608098	0.7165578
41	0.8131362	0.7613653	0.7075955
43	0.8143389	0.7624670	0.7114417

ROC was used to select the optimal model using the largest value. The final value used for the model was k = 15.

kNN Model Results

ROC for 10-fold cross validation

The final value used for the model was $k=15$



kNN Model Results

- Test Data (30% of data)
- The model results show that with 69% accuracy we can predict the quality of red wine using the attributes given

Confusion Matrix and Statistics

	Reference	
Prediction	High	Low
High	184	72
Low	79	144

Accuracy : 0.6848

95% CI : (0.6411, 0.7262)

No Information Rate : 0.5491

P-Value [Acc > NIR] : 9.085e-10

Kappa : 0.3652

Mcnemar's Test P-Value : 0.6254

Sensitivity : 0.6996

Specificity : 0.6667

Pos Pred Value : 0.7188

Neg Pred Value : 0.6457

Prevalence : 0.5491

Detection Rate : 0.3841

Detection Prevalence : 0.5344

Balanced Accuracy : 0.6831

'Positive' Class : High

kNN Summary

- The k-nearest neighbors (kNN) algorithm is a simple, supervised machine learning algorithm that can be used to solve both classification and regression problems
- It's easy to implement and understand but, has a major drawback of becoming significantly slower as the size of that data in-use grows
- kNN works by finding the distances between a query and all the examples in the data, selecting the specified number examples (k) closest to the query, then votes for the most frequent label (in the case of classification) or averages the labels (in the case of regression)
- In the case of classification and regression, we saw that choosing the right k for our data is done by trying several k's and picking the one that works best
- Finally, we looked at an example of how the kNN algorithm could be used in recommender systems, an application of kNN-search