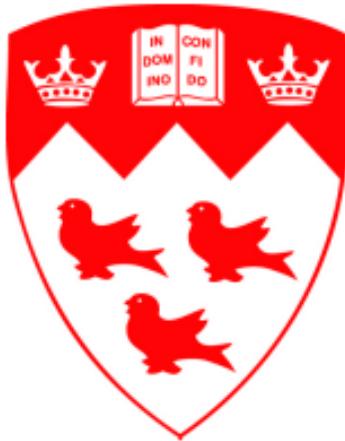


Route Optimization for Bike Rebalancing within the BIXI Bike-Sharing System



Hanna Swail (260746086)

Duncan Wang (260710229)

Tiancheng Zhang (260974250)

Bei Qi Zhou (260742459)

Desautels Faculty of Management

McGill University

Montréal, Québec, Canada

November 30, 2020

Contents

1	Introduction	1
2	Problem Description and Formulation	1
2.1	Problem description and Scope	1
2.2	Variables and Parameters Description	2
2.2.1	Parameters	3
2.2.2	Decision Variables	3
2.3	Main Model Formulation	4
3	Numerical implementation and results	6
3.1	Calculation and Estimation of Parameters	6
3.1.1	Revenue from Ridership	6
3.1.2	Costs Associated with Rebalancing	7
3.2	Dataset Description and Manipulation	7
3.3	Numerical Results	8
4	Problem Extensions	9
4.1	Minimize Operation Time	9
4.2	Routing with Priority	10
5	Conclusion	12
6	Appendix	15

1 Introduction

BIXI is a public bike sharing service located in Montréal, Quebec that started its operations in 2014. Customers are attracted by their mission statement which is centered around the provision of a convenient, environmentally-friendly method of transportation that promotes physical activity and a healthy lifestyle. The organization's main operations involve the supply and maintenance of over 7000 bicycles dispersed across over 600 stations around the Greater Montreal area. The location and capacity of these stations is based on population density, popular tourist attraction locations, and bikes paths. BIXI's operation takes place every year from April 15 to November 15, with trips peaking in the months of July and August (Figure 6). These trips include those made by members, who pay a fixed cost per year for unlimited access to rides within a given time duration, and those made by casual users who pay per trip (Table 2).

To summarize its operations, a customer can pick up a bike from one station, rent it for a certain period of time and cost, and subsequently return to another station. The BIXI system is designed for short-duration trips; thus, the dynamic nature of this system presents the risk of opportunity costs incurred due to bike shortages in locations with high demand and bike surpluses in locations with low demand. The ability to strategically reallocate bikes is a key customer service determinant for BIXI, as the availability of bikes to rent and availability of docks to park bikes between trips leads to more repeat trips and more revenue in the long term. In order to reduce potential imbalances within the bike share system, operators of BIXI known as "drivers" are required to redistribute bicycles from one station to another using trucks, in order to rebalance the system when necessary.

As BIXI is partially funded by the city of Montreal, budget allocation towards system maintenance must be made strategically. The goal of this project will be to meet the fluctuating demands of consumers through a rebalancing system that alleviates bike shortages and surpluses. We will do this by minimizing costs incurred from running the redistribution system, subject to estimated revenues generated from ridership. It is important to note that demand, and consequently revenue, fluctuate greatly within BIXI's months of operations (Figure 8). Thus, we also aim for our model to be applicable to other months to observe how optimal redistribution changes with respect to differing levels of demand.

2 Problem Description and Formulation

2.1 Problem description and Scope

The basic optimization model was built by quantifying the operational costs of the rebalancing system and subtracting these costs from the estimated revenue from ridership in order to minimize

organizational costs, subject to routing and rebalancing constraints. As stated prior, the BIXI system includes 611 stations in the Greater Montreal area; however, the majority of these stations are located in and around the downtown area. To save on computation time as well as achieve a realistic possible route, 18 stations covering 13.65 km^2 within the borough of Verdun were chosen as the region of interest where the rebalancing activities would take place (Figure 1). Verdun was chosen as it is bordered by the Lachine Canal on the West and the St. Lawrence river to the East, hence allowing its borders to be easily delineated for problem scoping purposes. For the same reasons, Nun's Island was excluded from Verdun for the sake of the analysis.

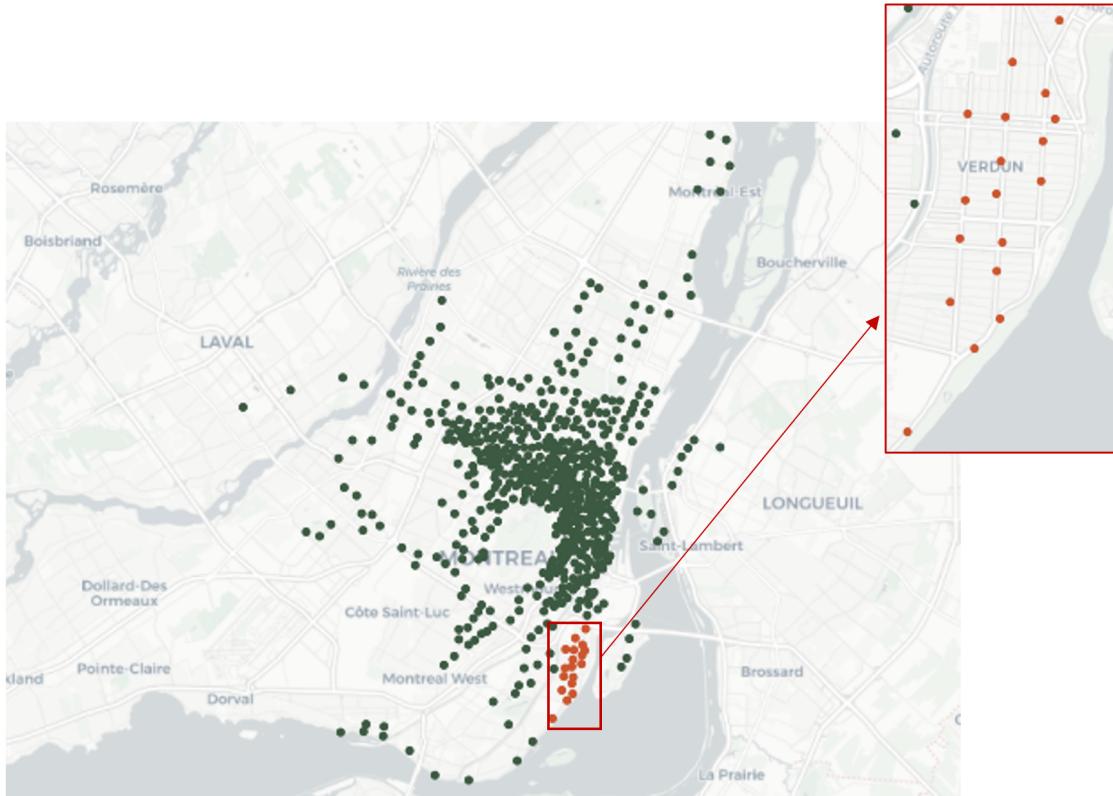


Figure 1: BIXI Stations in the Borough of Verdun

2.2 Variables and Parameters Description

We use capitalized letters for aggregated parameters such as daily revenue and total distance, small letters for decision variables and routing related parameters, and Greek letters for rebalancing related constraints.

2.2.1 Parameters

- C : Capacity of the truck.
- $I_i = [\lambda_i - \omega_i, \lambda_i + \omega_i]$: The ideal threshold of bikes at station i after rebalancing, which is represented by a predetermined idea amount λ_i associated with a symmetric threshold of $2\omega_i$.
- γ_i : Capacity (number of docks) of bike station i .
- e_i : The existing number of bikes at station i before rebalancing.
- $d_{i,j}$: The Manhattan distance between station i and station j .
- n : Number of stations.
- w : Wage for driver, dollars per bike.
- h : Disinfecting cost, dollars per bike.
- g : Price of Gasoline, dollars per kilometer.
- s_i^+ : A binary value that indicates whether there is a bike surplus at station i , i.e. $s_i^+ = 1$ when $e_i > \lambda_i + \omega_i$, and zero otherwise.
- s_i^- : A binary value that indicates whether there is a bike shortage at station i , i.e. $s_i^- = 1$ when $e_i < \lambda_i - \omega_i$, and zero otherwise.

In order to complete rebalancing activities for the region, one truck with a capacity of 40 bikes (Figure 7) was assigned to pick up bikes in stations with surplus and transport them to stations with shortage. Therefore, at each hour, each of the n stations in the chosen sub-region would be evaluated as to whether the difference in number of initial bikes at that hour (e_i) and the ideal range of bikes (I). The ideal number of bikes was set at around 70% of the capacity (number of bike docks) of each station, and this could vary by ω bikes, positively or negatively, creating the ideal range (I).

2.2.2 Decision Variables

- Decision Variables
- $x_{i,j}$: A binary variable that indicates the relocation truck passes through station i to station j if $x_{i,j} = 1$, and zero otherwise.
- c_i : The current number of bikes on the truck before arriving at station i .
- d_i : The number of bikes the truck drops off at station i .
- p_i : The number of bikes the truck picks up at station i .

- Dummy Variables

z_i : Non-decreasing variables to ensure continuity of the route, for $i \in \{1 \dots n\}$.

- Other Notations

B : The total number of bikes that are relocated.

D : The total distance that the truck has traveled.

Thus, each hour, depending on which station needed which service, the truck would make a different sequence of passes through the stations (x_{ij}) yielding a different distance (d_{ij}) and number of bikes relocated. Each time the truck would arrive at a particular station i , the number of bikes being carried would be noted (c_i), as well as the number of bikes being dropped off (d_i) or picked up (p_i).

2.3 Main Model Formulation

Our objective function focuses only on minimizing the operational costs of rebalancing the selected stations, costs of which were subtracted from the revenue derived from the usage of bikes from the selected stations. The revenues used for the calculation of the objective function are "fixed" in the sense that they were derived directly from historical ridership data, and hence not an objective of the optimization problem. We will refer to the following model as Model 1 or the base model.

Objective:

$$\min (hB + gD + w)$$

Subject to:

$$\sum_i x_{ij} \leq 1 \quad i = \{2 \dots n\} \quad (1.1)$$

$$\sum_j x_{ij} \leq 1 \quad j = \{2 \dots n\} \quad (1.2)$$

$$\sum_i x_{ij} = \sum_i x_{ji} \quad (1.3)$$

$$\sum_i x_{ii} = 0 \quad (1.4)$$

$$\sum_i x_{i1} = 1 \quad (1.5)$$

$$\sum_j x_{1j} = 1 \quad (1.6)$$

$$\sum_{i,j} x_{ij} d_{ij} = D \quad (1.7)$$

$$z_j \geq z_i + 1 - 1996(1 - x_{ij}) \quad i \in \{1, \dots, n\}, j \in \{2, \dots, n\} \quad (1.8)$$

$$c_i \leq C \quad (2.1)$$

$$p_i \leq e_i \quad (2.2)$$

$$p_i \geq e_i - (\lambda_i + \omega_i) \quad (2.3)$$

$$d_i \leq \gamma_i - e_i \quad (2.4)$$

$$d_i \geq (\lambda_i - \omega_i) - e_i \quad (2.5)$$

$$\sum_i d_i + p_i = B \quad (2.6)$$

$$d_j + p_j \geq \sum_i x_{ij} \quad (2.7)$$

$$c_j + d_j + p_j + z_j \leq 2C \sum_i x_{ij} \quad (2.8)$$

$$c_j \geq c_i - d_i + p_i - C(1 - x_{ij}) \quad (2.9)$$

$$c_j \leq c_i - d_i + p_i - C(1 - x_{ij}) \quad (2.10)$$

$$x_{ij} \in \{0, 1\} \quad (\text{Binary})$$

$$c_i, d_i, p_i, z_i \in \mathbb{N} \quad (\text{Integrality})$$

In the objective function, we are minimizing the total operational cost, which includes the disinfecting costs, gas costs and a fixed hourly wage for the driver, with respect to routing constraints (1.1) - (1.8) and re-balancing constraints (2.1) - (2.10).

Constraint (1.1) and (1.2) ensure that each station is either visited once or not visited in each hourly operation. Constraint (1.3) ensures that the total number of arcs going out from each station is equal to the number of arcs going in. Constraint (1.4) prohibits self-loops, and Constraint (1.5) makes sure that the route starts and ends at the first station. Constraint (1.7) ensures that D keeps track of the total distance or the route. In most cases, we are searching for a feasible subtour traversing desired stations that minimize the total cost. However, when searching for subtour with the constraints (1.1) - (1.7) we defined, it could return a set of disconnected cycles (Route in red in Figure 2) without violating any of the previous constraints. Therefore, in Constraint (1.8), we use a non-decreasing variable z to ensure the continuity of the route, where if the truck goes from station i to station j then $z_j > z_i$. Notice that in such invalid route, each disconnected cycle would raise a contradiction to constraint (1.8), hence we will always end up with a continuous route.

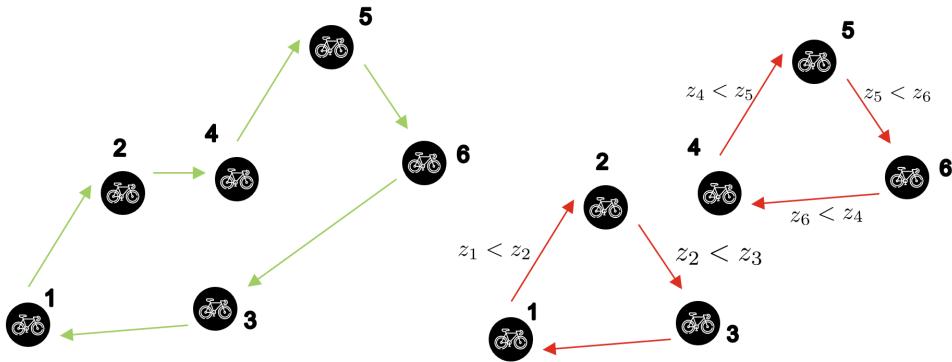


Figure 2: Constraint (1.8) on Avoiding Disconnected Cycles

Constraint (2.1) ensures that the numbers of bikes on the truck cannot exceed its capacity. If a pickup was being performed, the station had to be left within its ideal threshold; less than the existing number of bikes and greater than the difference between the existing bikes and ideal upper limit (Constraint (2.2) - (2.3)). Similarly, the number of bikes dropped off also had to be within the ideal threshold; less than the difference between the capacity of the station and existing number of bikes, but greater than the difference between the ideal lower limit and existing number of bikes (Constraint (2.4) - (2.5)). Constraint (2.6) ensures that B keeps track of the total number bikes re-balanced. Constraint (2.7) - (2.10) are the constraints on the synergy with the route, where (2.7) - (2.8) ensures that a station is visited only necessary and (2.9) - (2.10) ensures the number of bikes on the truck is consistent with the rebalancing along the route.

3 Numerical implementation and results

3.1 Calculation and Estimation of Parameters

In order to replicate real world circumstances, several other parameters relevant to the system were estimated. The parameter inputs utilized for the calculation of the objective function were broken down into two parts: revenue from ridership that was estimated separately, and costs associated with redistribution that were used in the model.

3.1.1 Revenue from Ridership

Total revenue from ridership on a daily and weekly basis for the area of interest was estimated using the number and length of member vs. non-member trips, given the cost per trip provided on the BIXI website (Table 2). Note that members pay a \$97 flat rate per season for access to unlimited rides under 45 minutes. Therefore, while rides under 45 minutes are technically free for members, we broke down the revenue generated through membership into the average cost per trip

for members by calculating the average number of trips taken by members relative to the number of memberships purchased in 2019. Also note that members pay an additional \$3 every 15 minutes beyond 60 minutes, and non-members pay an additional \$3 for every 15 minutes beyond 45 minutes, up to 24 hours, after which the bike is declared as lost. Using the data provided, we found that all rides were less than 120 minutes in length. While BIXI offers additional discounts and packages, we were unable to differentiate these trips from member vs. non member trips, and so all trips were ultimately grouped under these two general cost schemes.

3.1.2 Costs Associated with Rebalancing

We estimated the hourly wage for BIXI truck drivers to be \$20.52/hour using salary data from the job site Indeed for transporters in Quebec. The cost of fuel was calculated based on the average across all months in 2019 in Montreal; at 125.6 c/L. We determined the approximate truck type used for rebalancing based on photos of BIXI trucks found online (Figure 7), and the fuel efficiency was estimated to be 4.25km/L based on a low-end estimate of truck models in comparable groups, given that fuel efficiency will likely be lower than advertised depending on age of the truck and weight of the bike load carried. Accordingly, the estimated price of gas for travel was estimated to be \$0.294/km for the optimization model. Due to COVID-19 and the additional public health safety regulations put in place for businesses, BIXI has ensured that every time a bike is handled it must be sanitized using a disinfectant solution. This was estimated to be \$0.875/relocated bike, based on prices of industrial sized bottles and information obtained from a BIXI representative.

3.2 Dataset Description and Manipulation

The data used to build the model was publicly available from bixi.com. The available information from [Stations_2019.csv](#) consisted of BIXI station identification codes, names and coordinates from 2019. This was used to calculate the distance between each station. The data also includes usage information for each station, including start date and end date, trip duration, trip start station code, trip end station code and type of customer (member or non member) from the years 2014-2019. Due to the large expansion of BIXI's operations since the system's inception, ridership and station configuration vary widely between years; thus, the 2019 dataset was chosen to test the model. In reality, the number of docks at each station is set by BIXI depending on several specific factors. Unfortunately, this data was not available to us, so the capacity for the 18 stations of the region was chosen by looking at the maximum bike difference per hour for each station to assign a reasonable dock capacity based on the demand. For rebalancing operations, we calculated the net difference (number of bikes dropped off - number of bikes picked up) at the beginning of each hour from 8:00 to 20:00, as these were observed as the most active times.

3.3 Numerical Results

The model formulation was done using Gurobi 9.1 and python 3.7, and a computer with a quad core Intel i5 at 2.5 GHz and 16 GB RAM was used to solve the base problem. The model was made to minimize the objective function relating to rebalancing costs in a linear fashion. In the basic model, there were 5 decision variables for each station, pertaining to whether the truck would travel between stations, how many bikes it was carrying before arriving at a station, the number of bikes it was dropping off and number of bikes it was picking up. These generated variables pertaining to total bikes relocated and total distance traveled, which were used directly in the objective function and served the optimization purpose. Another variable was necessary to ensure the continuity of the problem as well as the avoidance of sub-tours. To calculate aggregated level costs and revenues, we also used three variables to keep a record of total bikes picked up (p_sum), dropped off (d_sum) and total stations visited for the hour (V).

In the tested model, ridership and estimated revenue from the week of July 08, 2019 (Jul 08 - Jul 14) was used. For each day in the week and each of the 13 operational hours in a day, the rebalancing process was evaluated by the model. The costs associated with hygiene and worker wage were multiplied by the number of bikes and the total distance traveled according to the route, respectively. The cost was accumulated for the entire week and subtracted from the revenue (Table 5).

The results of the rebalancing operations for Verdun on the week of July 08, 2020 are shown in Table 4. To give an example of what the rebalancing system looks like on an hourly basis, we created a map showcasing the movements of 8am on July 08. At this hour, a total of 5 stations needed to be rebalanced. When the truck arrived at Station (1), we assume it was carrying 20 bikes (half of the capacity). As shown in the map on the right (Figure 3), at the next station (2), it picked up two bikes, meaning the station was over the upper ideal limit according to its dock capacity. Arriving at the next station (3) it was carrying 22 bikes and subsequently dropped off one bike, meaning the station was under its lower ideal limit. At the next station (4) it was carrying 21 bikes and dropped off two bikes. At the next station (5) it was carrying 19 bikes and dropped off one. The truck then returned to station (1), and left for the next region. At this hour a total of 5.56km was traveled, 6 bikes were rebalanced and the cost came to \$27.40.



Figure 3: Rebalancing for Verdun BIXI Stations at 8:00-9:00 July 8th 2019

The most costly hour for the model was from 16:00-17:00, where 10 bikes had to be redistributed. The middle of the day, 12:00-13:00, was the second most costly, however this seemed to relate more to the larger distance traveled rather than the number of bikes redistributed. Logically speaking, these busier times make sense for the time period chosen, as 16:00-17:00 is a common time wherein customers would be using the service to travel home from work, while from 12:00-13:00, customers could be using the service during their lunch break.

4 Problem Extensions

4.1 Minimize Operation Time

One major aspect that was lacking from the base model, was the aspect of time. As basic model focuses on maximizing the profits relative to costs of operating the system, this extension focuses on reframing cost optimization from a time perspective. This is crucial, since the basic model was created on the assumption that each rebalancing operation could be completed within the hour and did not account for factors such as road congestion and time spent handling the bikes.

The following extension of the problem aimed to minimize total operation time by calculating the time spent traveling and rebalancing the bikes at each hour. Specifically, we took into account time necessary move and disinfect the bikes (1.5 minutes per bike), time necessary park the truck (2 minutes per stop) and time spent driving. The time spent driving was computed by estimating the effects of congestion (20 Km/h during rush hours 8:00-10:00 & 12:00-14:00 and 30 Km/h during regular hours). This gives us the following objective function:

$$\min 1.5B + 2V + s_j D,$$

where V is the number of stations visited and s_j is the assumed traveling time per kilometer. We observed that the operations on 12:00-13:00 and 13:00-14:00 exceeded 60 minutes with the rebalancing strategy from the basic model (Table 1 on the left).

In addition, if the anticipated rebalancing requirement in a given hour could not be satisfied within the 60 allotted minutes, proactive measures were set in place by transferring the extra bikes and associated time that could not be rebalanced within the period to another period in the day with the anticipated time capacity to accommodate extra rebalancing. In practice, this was done by adjusting the threshold of bikes to rebalance. For hours where anticipated rebalancing time exceeded 60 minutes, we reduced the number of bikes that required rebalancing (by increasing the threshold and loosening the constraint at which rebalancing was necessary), and for hours where there was extra time capacity available, we increased the number of bikes to be rebalanced (by decreasing the threshold and tightening the constraint at which rebalancing was necessary). In Table 1 on the right, it is seen that the high demands from 12:00 to 14:00 are distributed to less busy hours with a greater time capacity available for extra rebalancing.

Operation Hour	Time (min)	Stations Visited	Threshold
8:00 - 9:00	41.16	5	± 3
9:00 - 10:00	24.3	3	± 3
10:00 - 11:00	20.55	2	± 3
11:00 - 12:00	45.02	7	± 3
12:00 - 13:00	66.06	8	± 3
13:00 - 14:00	68.43	11	± 3
14:00 - 15:00	21.05	3	± 3
15:00 - 16:00	24.94	4	± 3
16:00 - 17:00	35.15	5	± 3
17:00 - 18:00	31.01	5	± 3
18:00 - 19:00	46.64	8	± 3
19:00 - 20:00	27.28	4	± 3
20:00 - 21:00	0	0	± 3

Operation Hour	Time (min)	Stations Visited	Threshold
8:00 - 9:00	41.16	8	± 3
9:00 - 10:00	52.33	3	± 2
10:00 - 11:00	33.86	5	± 2
11:00 - 12:00	59.73	9	± 2
12:00 - 13:00	27.37	4	± 4
13:00 - 14:00	38.13	6	± 4
14:00 - 15:00	21.05	3	± 4
15:00 - 16:00	40.29	7	± 3
16:00 - 17:00	49.64	8	± 3
17:00 - 18:00	47.25	7	± 3
18:00 - 19:00	48.14	8	± 3
19:00 - 20:00	27.28	4	± 3
20:00 - 21:00	0	0	± 3

Table 1: Adjustment of Threshold to Alleviate High Demands

4.2 Routing with Priority

With the recent COVID-19 pandemic, businesses have had to drastically evolve their business models to adapt to changing conditions and newly arising customer demands. Bike sharing systems are no exception. In March of 2019, the New York City bike sharing system, “Citi Bike,” reported an increase in use of stations near hospitals [5]. BIXI also experienced an increase in popularity of their system among public service workers, and specifically essential health care workers. In order

to meet these needs, BIXI began offering free 30-day memberships to essential workers, which was taken advantage of by 1300 people. Based on this information, we suggest that accommodating this specific user profile would add new value to the BIXI system by better serving the Montreal community during the pandemic; specifically, we explored the impacts of prioritizing the availability of bikes near hospitals to ensure access to healthcare workers.

Thus, we modified the original optimization problem to include a required sub-tour for hospitals within a new sub-region of the system. Essentially, the truck would be required to visit hospitals sequentially, to ensure the demands of hospital workers were met preferably compared to the general system. The sub-region chosen was 6.07 km^2 in size, and encompassed 57 stations within the downtown region, including McGill University, the Milton-Parc community, and Quartier des Spectacles. The region also encompassed the Montreal Neurological Hospital (Neuro) on the West end, and is in close proximity to the Centre Hospitalier de l'Université de Montréal (CHUM) on the East end. Three stations around the CHUM and seven stations around the Neuro were chosen to be the predefined subtours. The starting station was also set to be one of the priority stations near the CHUM hospital.

While a recent study [1] proposed a similar idea in a more complex setting, we achieved a routing solution with priority by adding the following two constraints:

$$\sum_{i \in \mathbb{P}} x_{0i} = 1 \quad (3.1)$$

$$\sum_{i \in \mathbb{P}, j \in \mathbb{R}} x_{ij} \leq 1 \quad (3.2)$$

where \mathbb{P} denotes set of priority stations and \mathbb{R} denotes set of regular stations. Constraint (3.1) ensures that the truck first visits a priority station after leaving the starting priority station, and constraint (3.2) guarantees that visits from a priority station to a regular station happen no more than once. This action occurs only when all priority stations have been serviced and there are subsequent regular stations to be serviced afterwards. Note that when all stations that needed rebalancing are priority stations, constraint (3.2) is non-binding. The two maps (Figure 5) below show how the route changed with the addition of the priority constraints.

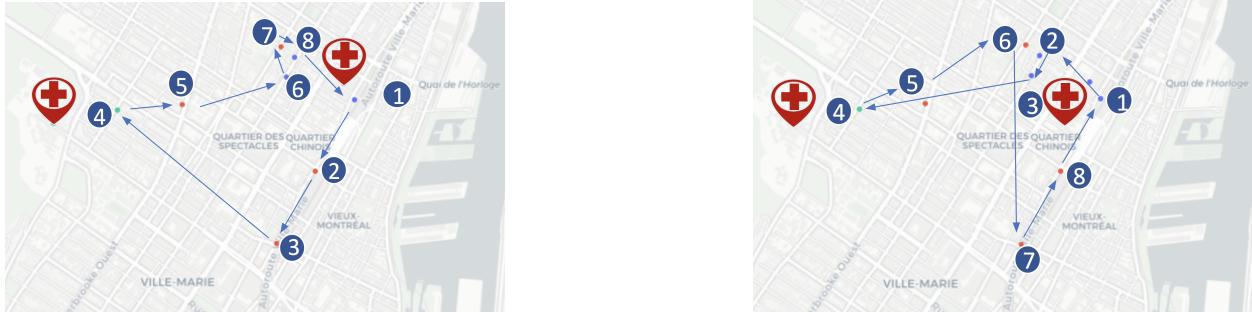


Figure 5: Route with (right) and without (left) priority visitation of hospital stations

By adding this set of constraints, we observe that the truck is still able to satisfy the hourly rebalancing demands, but that as expected, the order of visitation is shifted so that each selected hospital station is visited first. For instance, by running one iteration of rebalancing from 12:00PM-1:00PM on July 8th, 2019, it is observed that the distance increases from 6.97KM to 7.62KM, representing an associated cost increase from \$ 33.94 to \$ 40.14. It is worthy to note that the computational complexity also increases significantly; while the base model was solved in under 1 second, solving for an equal time period of one hour with priority constraints took 36.84 seconds. Given that there are over 10 major health networks in Montreal and many with several branches, such potential increases in hourly rebalancing costs and computational intensity is not trivial when considering whether or not to undertake such operations.

5 Conclusion

In reality, the BIXI rebalancing system is automated such that thresholds for bike rebalancing requirements are generated through complicated algorithms that take into account weather, population density, time of the day, street condition etc. For example, the threshold of one station on a sunny Monday morning during rush hour might be very different compared to the same station on a rainy Sunday afternoon. The assumption we made to choose fixed thresholds for rebalancing evidently do not take these criteria into account and limit the realistic nature of the problem. As well, BIXI normally deploys multiple trucks running through the same region simultaneously, making it easier to handle busy areas such as downtown. While we created fixed boundary regions within which the truck would operate exclusively, trucks may in reality travel flexibly across the island depending on fluctuations in rebalancing demand. Therefore, future iterations of this analysis could explore extending the region of interest or resolving with the addition of multiple trucks within each region.

Although our model can suggest a certain measure of weekly “success” by calculating weekly profit based on estimating rebalancing costs and revenue generated through ridership, it is limited

in the sense that it does not account for the costs of the entire business. In reality, revenue from ridership covers about half the cost of running the entire system [4], as BIXI must also invest a lot into marketing, management, and data collection expenses. Furthermore, we calculated revenue from ridership through historical data, and did not consider how the frequency or effectiveness of rebalancing may in consequence alter customer behaviour or ridership rates. Therefore, our model cannot be interpreted in terms of the success of the business as a whole, rather only in terms of the effectiveness of the rebalancing process.

Following the optimization of the rebalancing process, further issues within BIXI's business model could be tackled. Optimal placement of docks is also a problem that is ongoing and evolving, as road construction and city attractions change from year to year. Boroughs such as Verdun are also transforming rapidly, and certain processes such as gentrification in such historically working-class neighbourhoods may drive forth new efforts to "revitalize" these regions through the introduction of new docks, especially as the popularity of the BIXI system has shown continued growth in the six years since inception. Therefore, it is worthwhile to explore how station allocation can be optimized with respect to factors such as historical and forecasted demand in the region, ease of access, and efficiency of redistribution in the area.

Overall, our analysis aimed to take a deep dive into Montreal's BIXI bike sharing operations by using mixed integer programming techniques to optimize the rebalancing of bikes in a way which meets anticipated demands, alleviates surpluses and shortages in the system, and remains cost efficient. While we focused on a few select regions and time periods, the models created are scalable and can be extended to other boroughs or time periods. By providing key insights into the requirements of maintaining such a dynamic system, we ultimately sought to demonstrate the opportunities which exist to further enhance the value of friendly and efficient transportation shared transportation systems on the island of Montreal.

References

- [1] P. Avila-Torres, N. Arratia-Martinez, and E. Ruiz-y-Ruiz. *The Inventory Routing Problem with Priorities and Fixed Heterogeneous Fleet*, Applied Sciences. 3052. 10, 2020.
- [2] L Flosi. Private Communications.
- [3] J. Schuijbroek, R. Hampshire, and W. Hoeve. *Inventory Rebalancing and Vehicle Routing in Bike Sharing Systems*, European Journal of Operational Research 257(3): 992-1004, 2017.
- [4] BIXI Financial Report (2019). https://ville.montreal.qc.ca/documents/Adi_Public/CE/CE_DA_ORDI_2020-05-20_08h30_Presentation_Depot_des_etats_financier_2019_de_BIXI.pdf
- [5] L. Sabbatini. (2020) Cycling and bike-share take centre stage in the new COVID-reality <https://bixi.com/en/cycling-and-bike-share-take-centre-stage-in-the-new-covid-reality>

6 Appendix

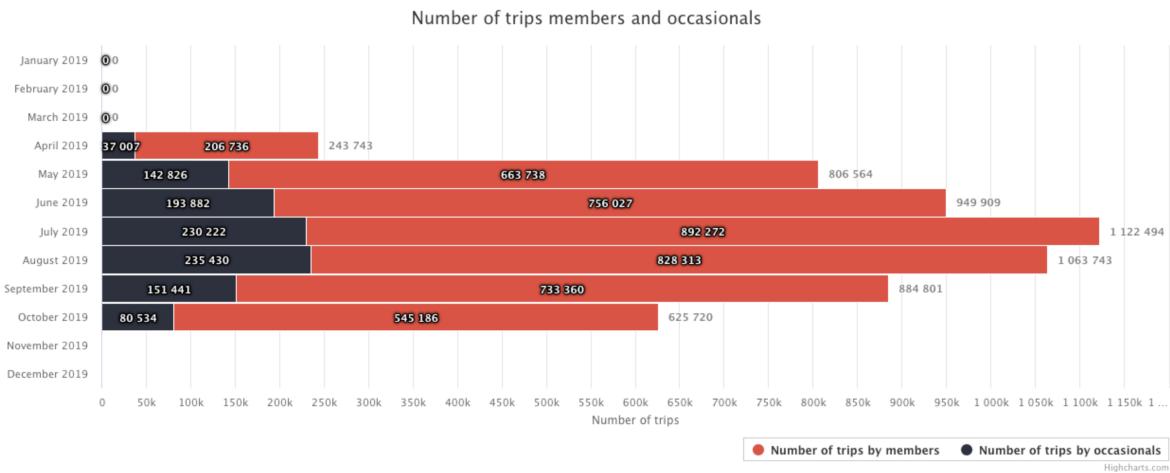


Figure 6: Number of trips made by members vs. non-members in 2019

Trip Length (minutes)	Members (\$)	Non Members (\$)
30	*1.015 (base rate)	2.99 (base rate)
31-45	+0	+1.8
45-60	+1.8	+3
+15	+3	+3

Table 2: Trip fare scheme for members and non-members



Figure 7: BIXI Rebalancing Truck

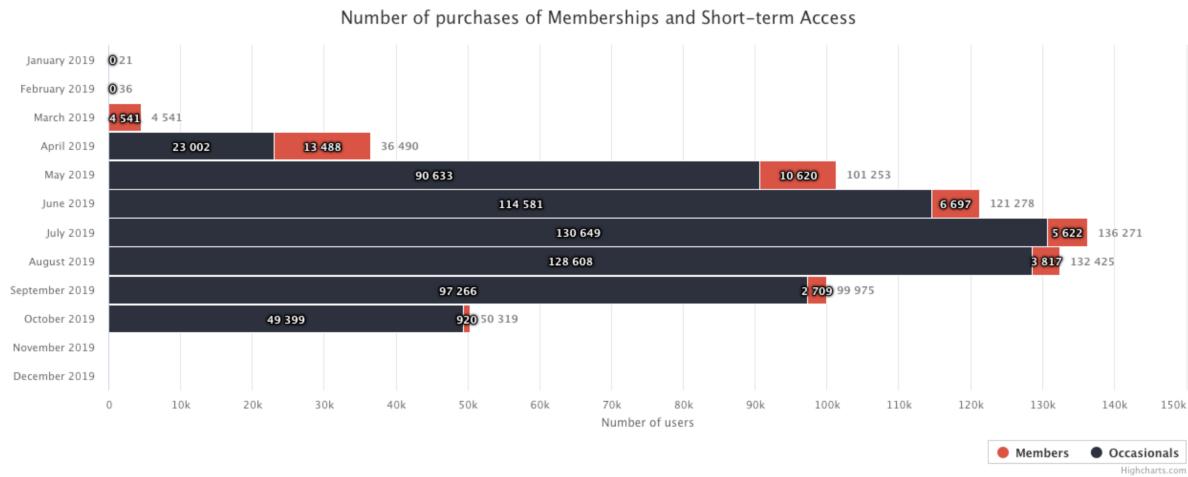


Figure 8: Purchases of memberships vs. short-term access trips in 2019

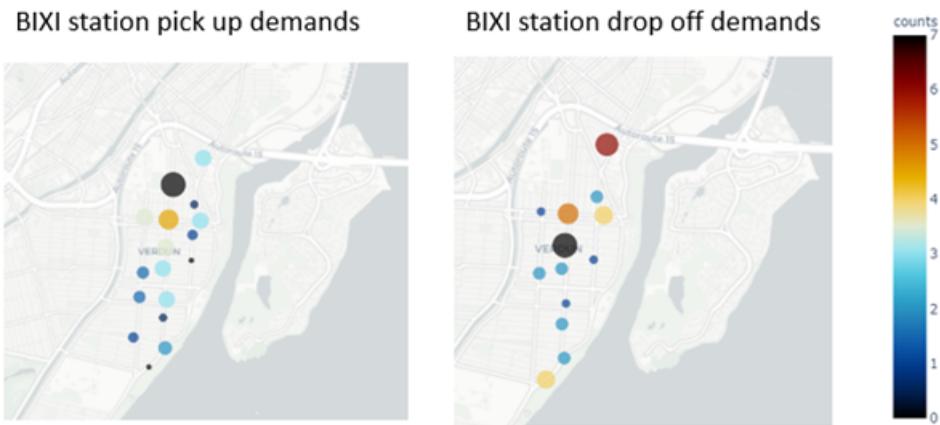


Figure 9: Differences between demand for BIXI bike pick-ups vs. drop-offs for stations in Verdun, between 8:00 - 9:00 on July 8th, 2019

Station Code	Station Name	Dock Capacity	Latitude	Longitude
6309	4e avenue / de Verdun	25	45.45688445	-73.572607
6341	Regina / de Verdun	20	45.467369	-73.570769
6379	de l'Église / Bannantyne	20	45.46325354	-73.575879
6425	Métro de l'Église (Ross/de l'Église)	15	45.46284605	-73.565921
6426	Métro Verdun (Willibrord/de Verdun)	15	45.45948769	-73.572092
6427	Métro Lasalle (de Rushbrooke/Caisse)	15	45.47069643	-73.565422
6705	5e avenue / Bannantyne	15	45.456365	-73.57614
6706	Beatty / de Verdun	15	45.45072998	-73.572574
6712	LaSalle / Crawford	15	45.4379138	-73.582740
6715	Natatorium (LaSalle/Rolland)	15	45.44454701	-73.575090
7056	Bibliothèque de Verdun	15	45.44826205	-73.577856
7057	2e avenue / Wellington	15	45.45789353	-73.567528
7058	Gordon / Wellington	10	45.46107799	-73.567307
7059	Argyle / de Verdun	10	45.45301584	-73.571915
7060	de l'Église / de Verdun	10	45.46300109	-73.571568
7143	LaSalle / Godin	10	45.44693448	-73.572199
7144	Hickson / Wellington	10	45.46489252	-73.566997
7145	Argyle / Bannantyne	10	45.45331877	-73.576775

Table 3: BIXI Stations in Verdun

Time	Total Stations Visited	Bikes Picked Up	Bikes Dropped Off	Bikes Rebalanced	Total Distance (KM)	Operation Cost (\$)
8:00 - 9:00	5	2	4	6	5.56	27.4
9:00 - 10:00	2	1	0	1	2.81	22.24
10:00 - 11:00	3	0	3	3	3.93	24.30
11:00 - 12:00	6	0	7	7	9.48	29.43
12:00 - 13:00	7	1	7	8	9.30	30.25
13:00 - 14:00	5	6	0	6	5.38	27.35
14:00 - 15:00	6	4	1	5	6.56	26.83
15:00 - 16:00	4	6	0	6	4.88	27.20
16:00 - 17:00	6	10	3	13	5.12	33.40
17:00 - 18:00	5	3	1	4	7.96	26.36
18:00 - 19:00	6	1	8	9	5.17	29.16
19:00 - 20:00	5	3	3	6	6.31	27.62
20:00 - 21:00	6	6	3	9	6.50	30.04
Total	61	71 bikes handled			78.96	335.21

Table 4: Summative rebalancing results for stations in Verdun on July 8th, 2019

Month	Total Revenue (\$)	Week	Weekly Revenue (\$)
July	32626.26	1	8671.32
		2	6875.35
		3	6757.71
		4	10321.87

Table 5: Weekly revenue generated through ridership for 18 stations in Verdun, in July 2019