Binary Search Trees

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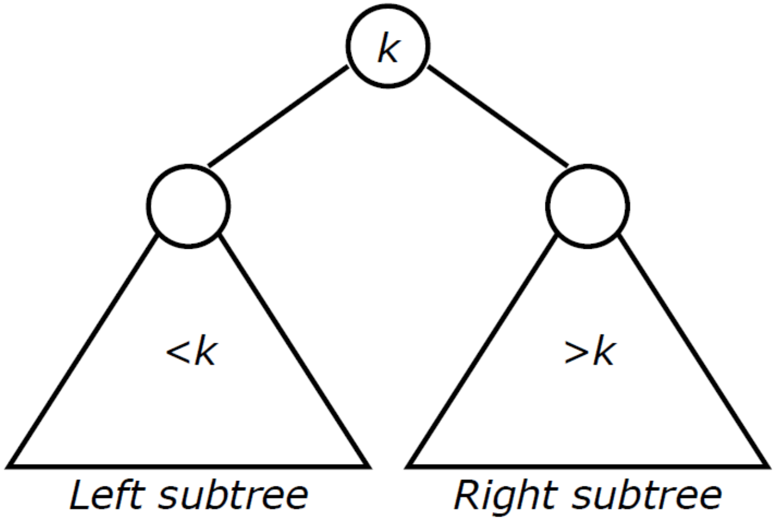
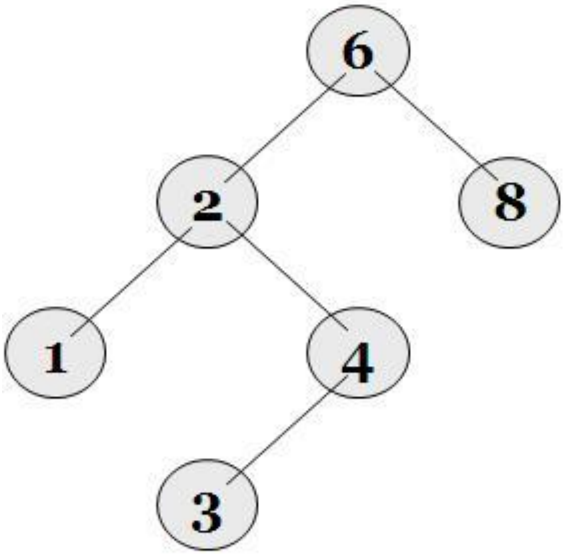
Definition of a Binary Search Tree (BST)

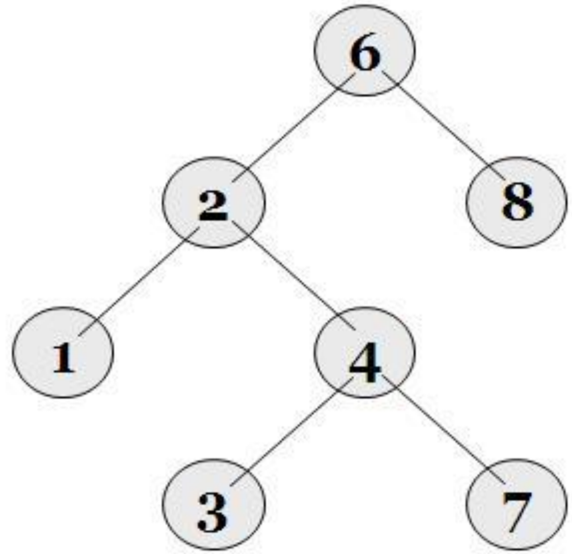
- A binary tree
- Each node has a (*key*, *element*) pair
  - element: value or data
- For every node *x*, all keys in the left subtree of *x* are smaller than that in *x*
- For every node *x*, all keys in the right subtree of *x* are greater than that in *x*
- The left and right subtrees are also binary search trees



Example BST

A *binary search tree*





Not a *binary search tree*, but a *binary tree*

Only keys are shown.

A Dictionary

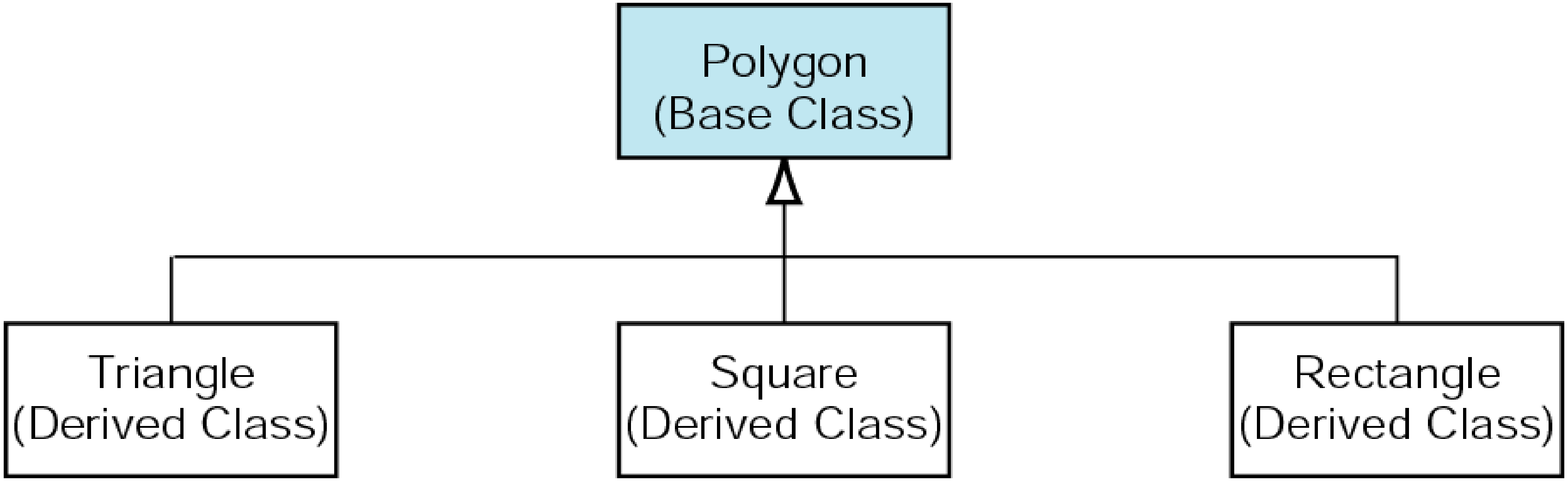
- A *dictionary* is a collection of pairs, each pair has a key and an associated element (or value).
  - It can be implemented using a BST.

Making a member function const means that it cannot call any non-const member functions, nor can it change any member variables.

```
template <class K, class E>
class Dictionary {
public:
        virtual void Ascend(void) const = 0;
          // print the dictionary in ascending order by key

        virtual pair<K, E>*Get(const K&) const = 0;
          // return pointer to the pair with specified key; return NULL if no such pair

        virtual void Insert(const pair<K, E>&) = 0;
          // insert the given pair; if key is a duplicate, update the associated element

        virtual void Delete(const K&) = 0;
          // delete pair with specified key
};
```

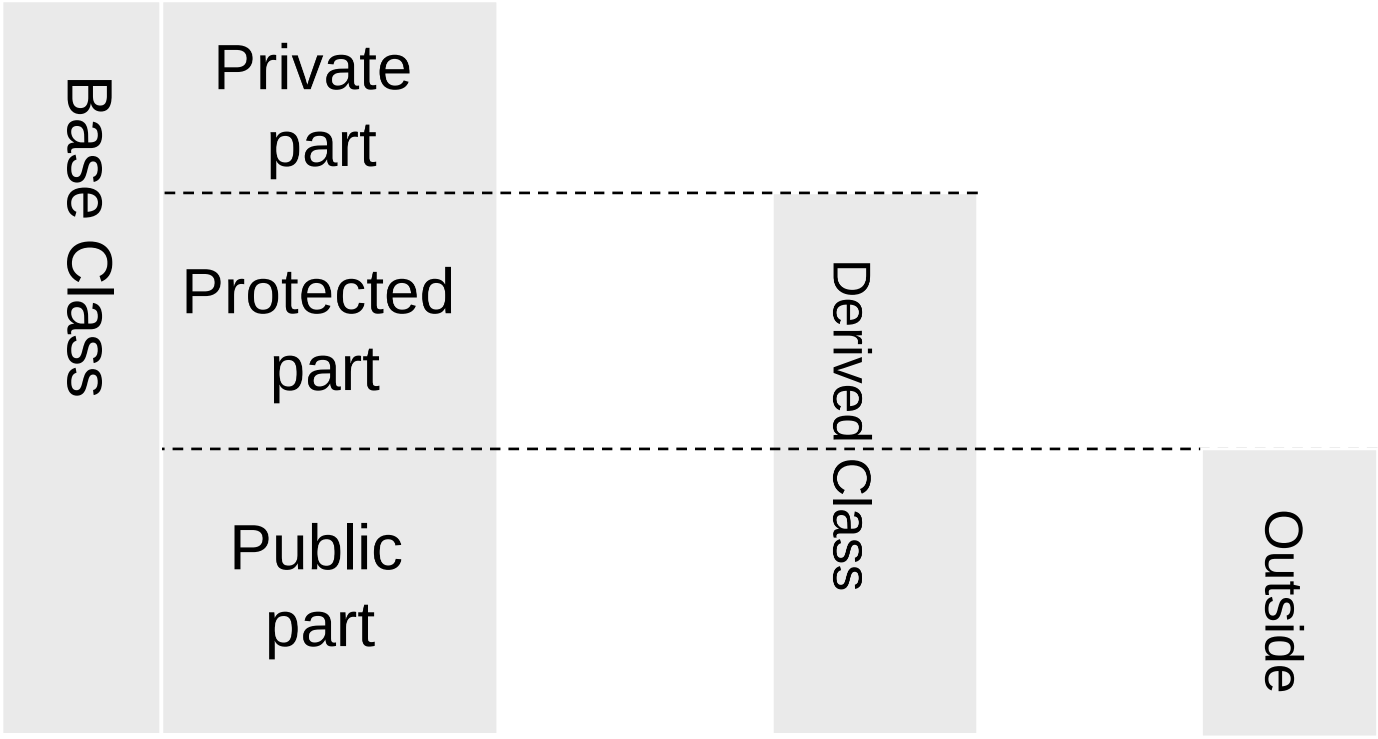
Inheritance

- A class may be derived from a base class by using the inheritance.



Inheritance

- The private data and methods in the base class are inaccessible in the derived class.



Polymorphism

- Polymorphism is the provision of a single interface to entities of different types.

- We use one verb (function) to mean different things. For example, we say “open” meaning to open a door, a jar, or a book; which one is determined by the context.

- Similarly, in C++, we can call `printArea` to print area of a triangle or the area of the rectangle.

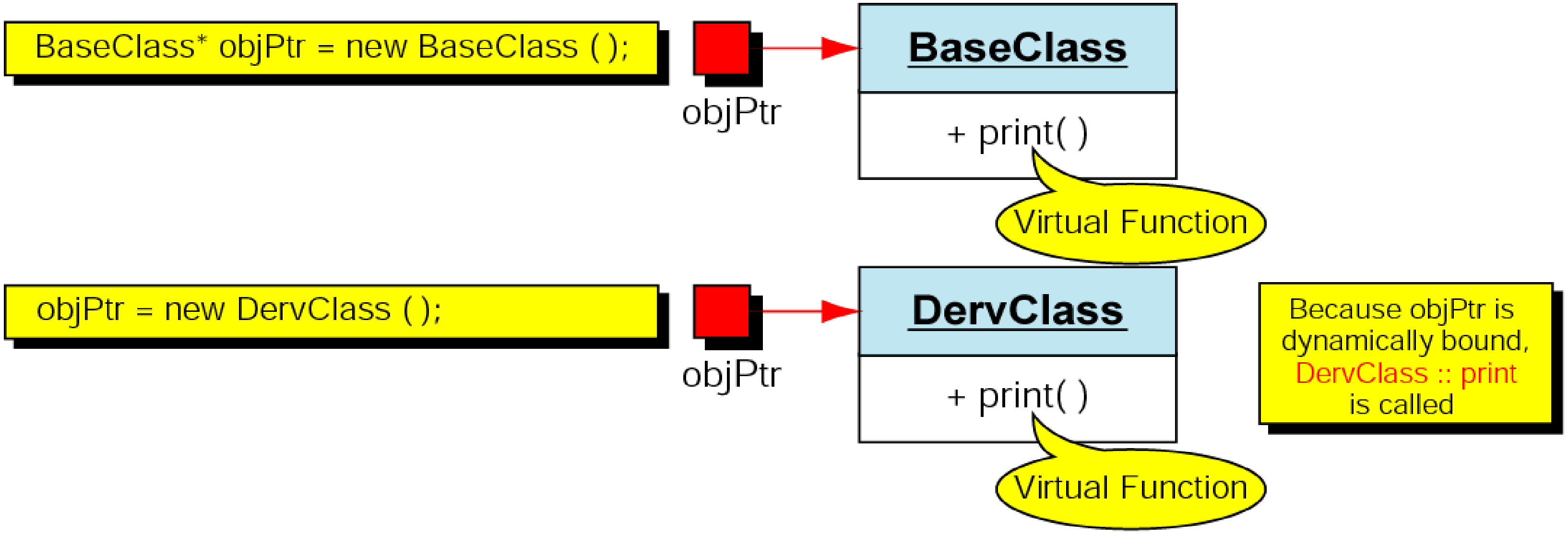
Polymorphism

- Polymorphism is the ability to write several versions of a function, each in a separate class.

- Three conditions for polymorphism to work
  - A hierarchy of inherited classes
  - The function needs to be virtual.
  - We need to use pointers or references to objects.

Virtual Function

- A virtual function tells the compiler to bind a function with an object during the run time, not with the pointer defined during compilation time.



Virtual Function

```
class BaseClass {
public:
    virtual void print(void) const { cout << “Base class\n”; }
};
class DervClass: public BaseClass {
public:
    virtual void print(void) const { cout << “Derived class\n”; }
};
int main(void) {
    BaseClass* objPtr = new BaseClass;
    objPtr->print();    delete objPtr;

    objPtr = new DervClass;
    objPtr->print();    delete objPtr;
}
```

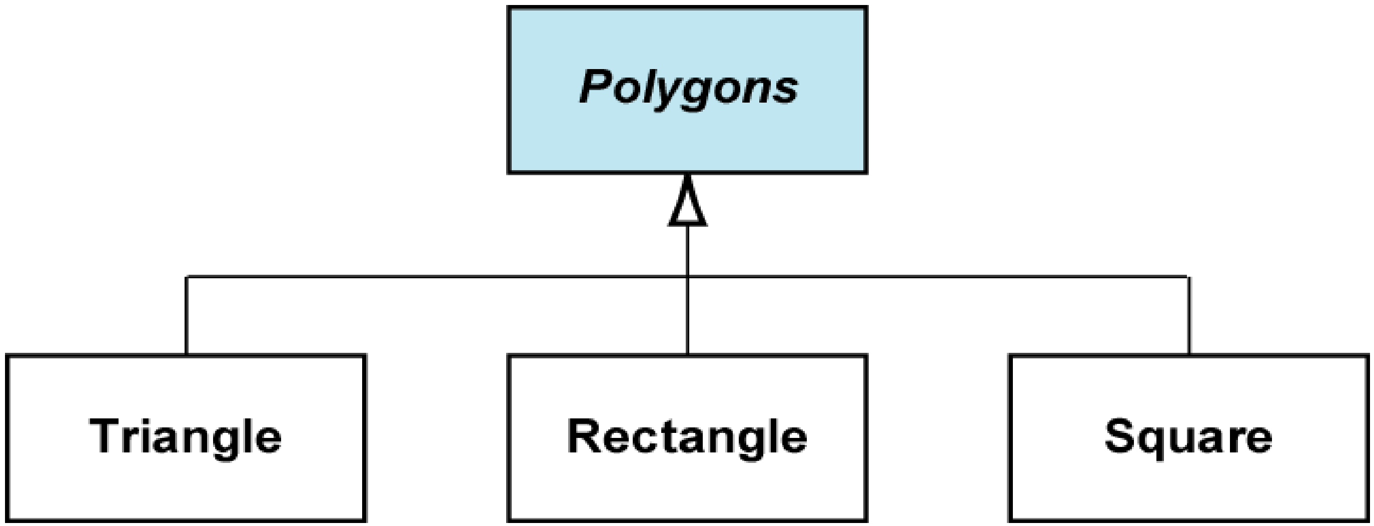
Pure Virtual Function

- In the base class, we can define the minimum number of functions and the format (argument list) that is needed for each derived class to include.
- Whereas a virtual function can have executable code in the base class, a pure virtual function can have no code.
- Pure virtual function is simply a declaration of a function that must be overridden in each derived class.
- Syntax

```
virtual return_type function(parameter list) = 0;
```

Abstract Class

- An abstract class is a class that has at least one pure virtual function.
- It is just a model for all derived classes and cannot be instantiated.
- We cannot have an object of an abstract class because the pure virtual functions cannot be called.



Abstract Class

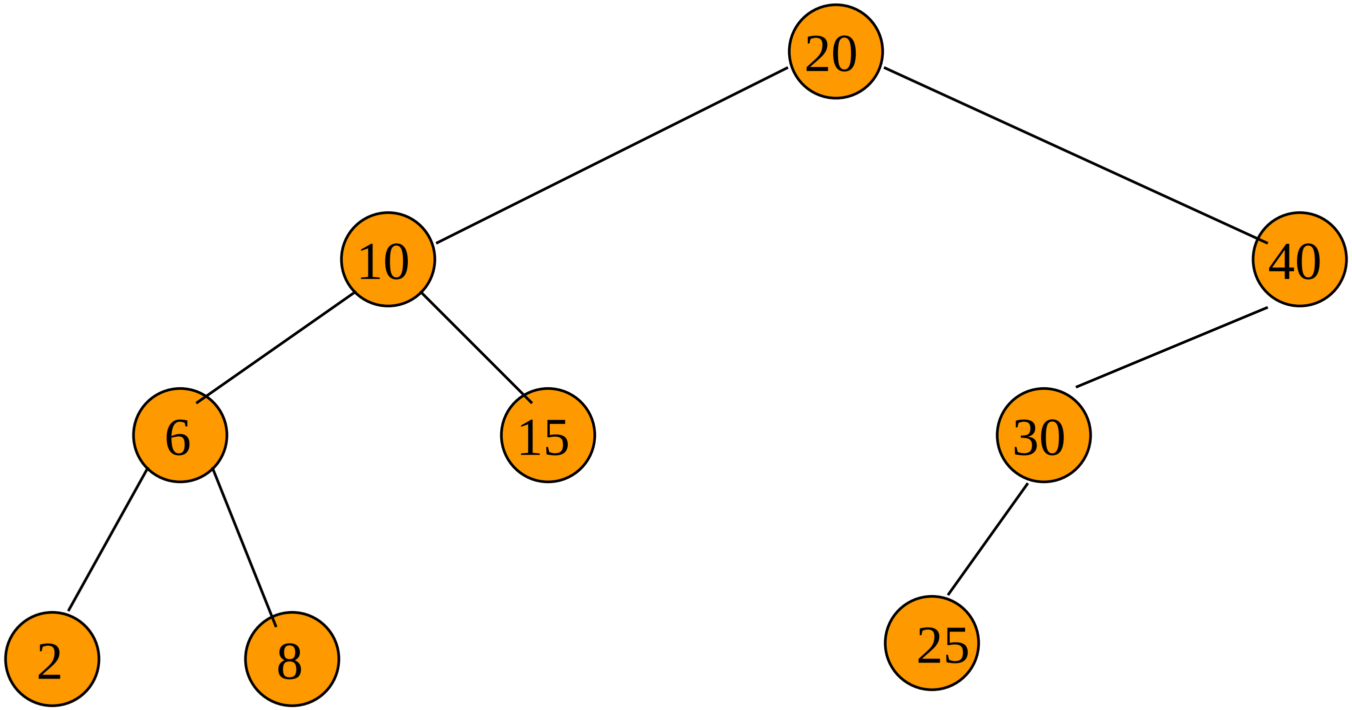
```
class Polygons
{
	protected:
		double area;
		virtual void calcArea()=0;
	public:
		Polygon() {}
		~Polygon() {}
		void printArea() const;
};
```

```
class Triangle : public Polygons
{
	private:
		double sideA;
		double sideB;
		double sideC;
		virtual void calcArea();
	public:
		Triangle(double sideA,
		         double sideB,
		         double sideC);
};
```

Standard Template Library (STL)

- A set of C++ template classes to provide common programming data structures and functions.
- STL components:
  - Containers
    - Data structures: pair, vector, list, queue, priority_queue, stack, set, map, …
  - Iterators
    - Pointer-like objects used to access elements in a container
  - Algorithms
    - Basic algorithms to manipulate the elements of containers (e.g., sorting, searching, …)
  - …
- The pair container is a simple container consisting of two data elements or objects.
  - The first element is referenced as ‘first’ and the second element as ‘second’ and the order is fixed (first, second).

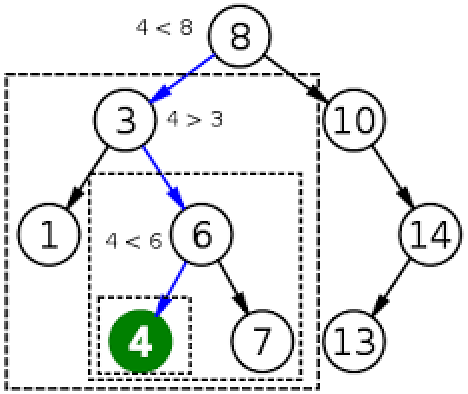
The Operation Ascend()



Do an inorder traversal. O(n) time.

Searching a BST

- Searching for a node with key k
- We begin at the root.
- If the root is NULL, the tree is empty and the search is unsuccessful.
- Otherwise, we compare k with the key k_{root} in the root.
  - If $k < k_{root}$, then only the *left* subtree needs to be searched.
  - If $k > k_{root}$, then only the *right* subtree needs to be searched.
  - Otherwise, $k == k_{root}$ and the search terminates successfully.
- Complexity: O(height)

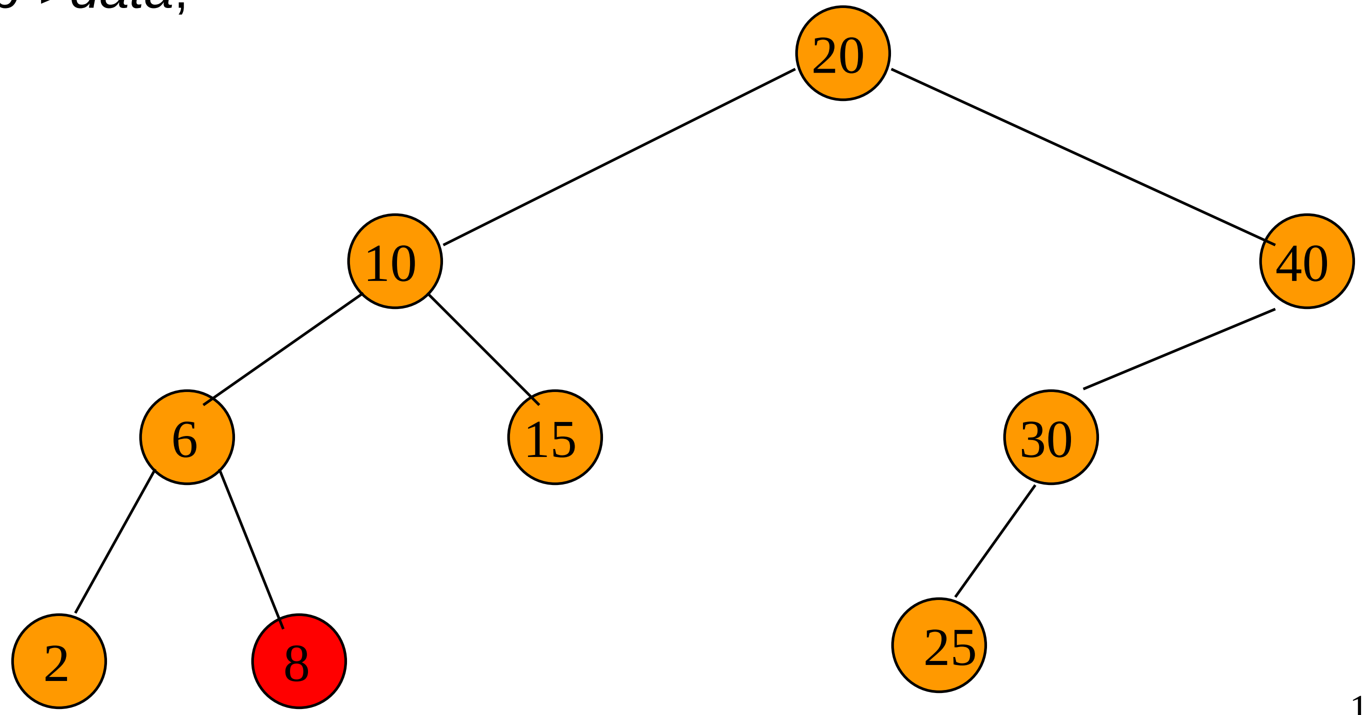


Recursive Search of a BST

```
template <class K, class E> // Driver
pair<K, E>* BST<K, E>::Get(const K& k)
{
// Search the binary search tree (*this) for a pair with key k.
// If such a pair is found, return a pointer to this pair; otherwise, return NULL.
   return Get(root, k);
}

template <class K, class E> // Workhorse
pair<K, E>* BST<K, E>::Get(TreeNode<pair<K, E> >*p, const K& k)
{
   if(p == NULL) return NULL;
   if(k < p->data.first) return Get(p->leftChild, k);
   if(k > p->data.first) return Get(p->rightChild, k);
   return &p->data;
}
```

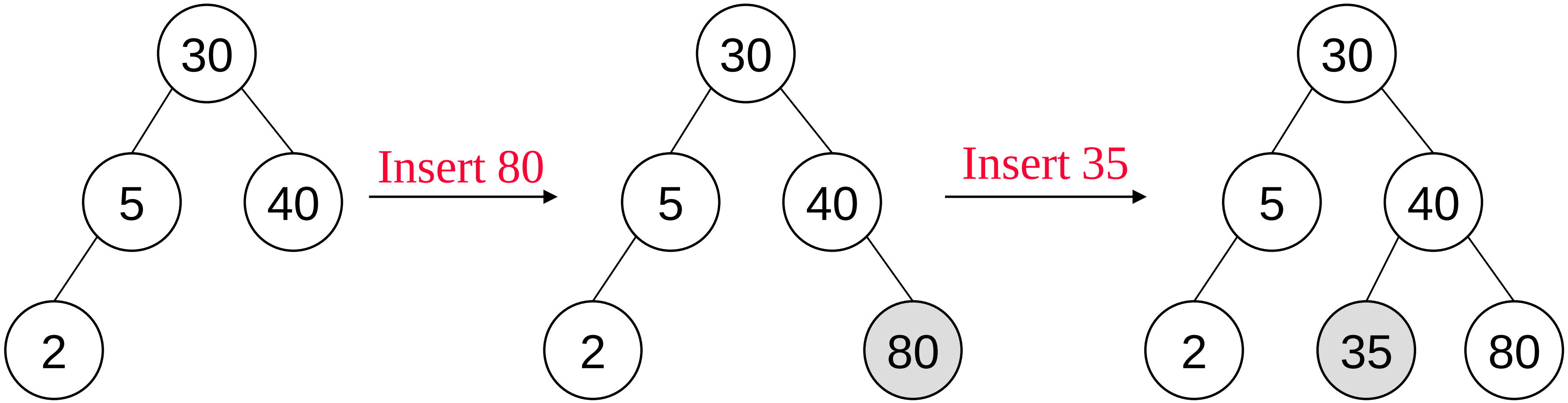
Example (k = 8)

```
template <class K, class E> // Workhorse
pair<K, E>* BST<K, E>::Get(TreeNode<pair<K, E> >*p, const K& k)
{
   if(p == NULL) return NULL;
   if(k < p->data.first) return Get(p->leftChild, k);
   if(k > p->data.first) return Get(p->rightChild, k);
   return &p->data;
}
```



Insertion into a BST

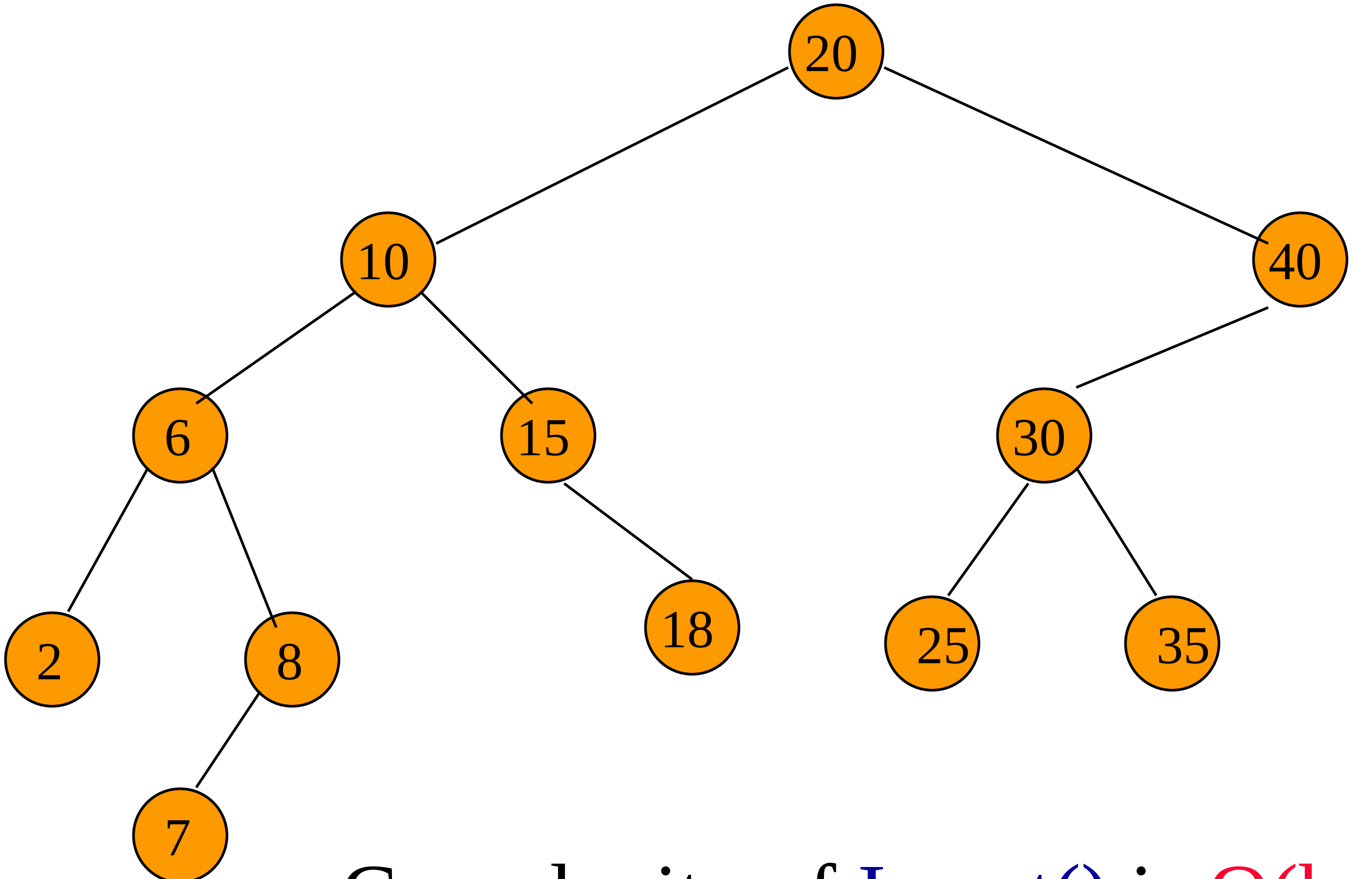
- To insert a pair (*k*, *e*), we first search the tree to verify that its key is different from those of existing nodes.
  - By the definition of BST, no two nodes have the same key.
- If the search is successful (i.e., key is a duplicate), the associated element is updated.
- If the search is unsuccessful, the node is inserted at the point the search terminated.



Insertion into a BST (cont.)

```
================================================================
template <class K, class E>
void BST<K,E>::Insert(const pair<K,E>& thePair)
{// Insert thePair into the binary search tree
 // Search for thePair.first
 // pp is the parent of p
   TreeNode<pair<K,E> > *p = root, *pp = NULL;
   while (p) {
      pp = p;
      if (thePair.first < p->data.first) p = p->leftChild;
      else if (thePair.first > p->data.first) p = p->rightChild;
      else // duplicated, update the associated element
        {p->data.second = thePair.second; return;}
   }

   // Perform insertion
   p = new TreeNode<pair<K,E> > (thePair);
   if (root != NULL)  // tree not empty
      if (thePair.first < pp->data.first) pp->leftChild = p;
      else pp->rightChild = p;
   else root = p;
}
================================================================
```

The Operation Insert()



Complexity of Insert() is O(height).

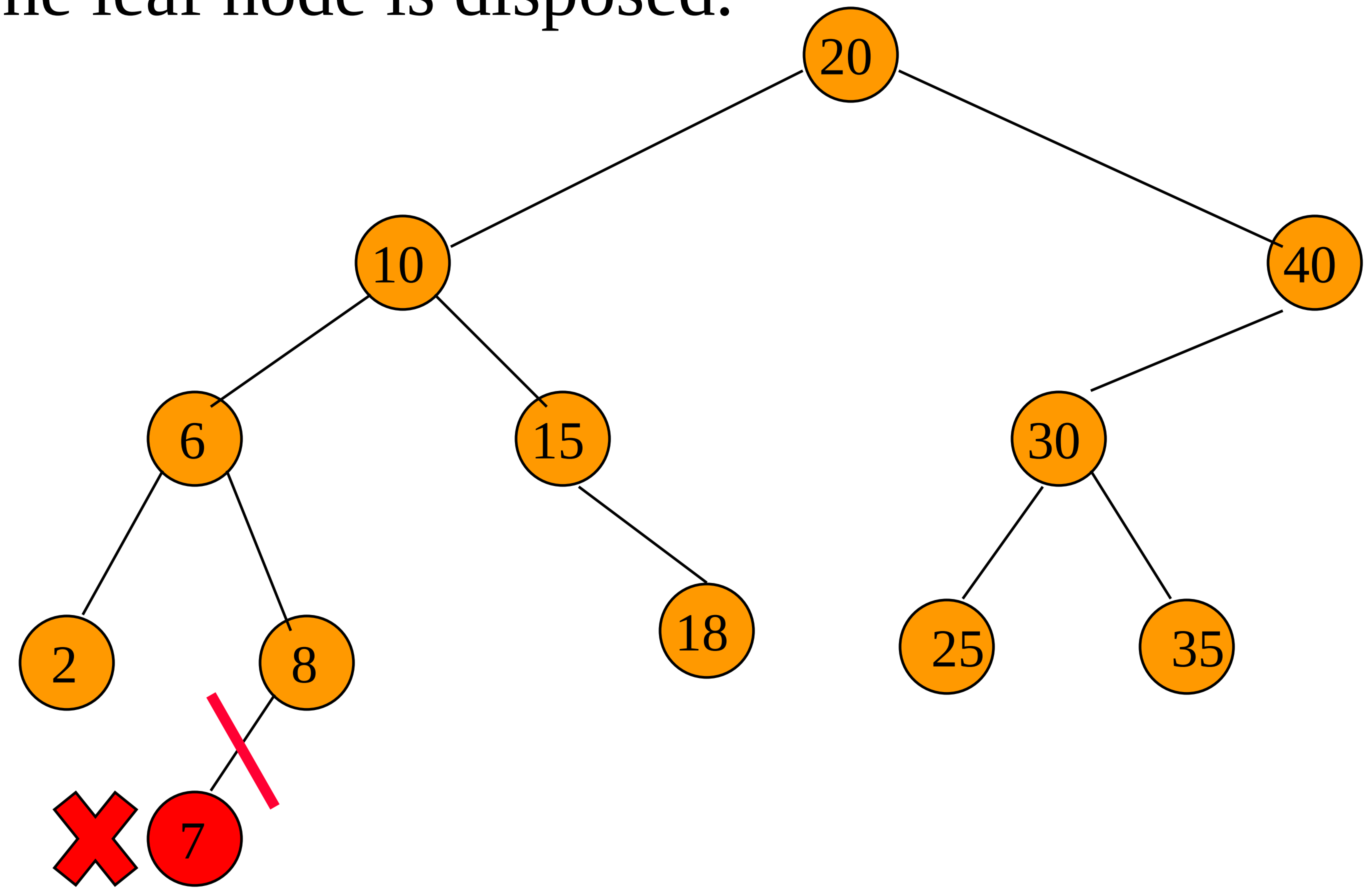
The Operation Delete()

Four cases:

- No node with delete key
- A degree 0 node (leaf node)
- A degree 1 node (internal node)
- A degree 2 node (internal node)

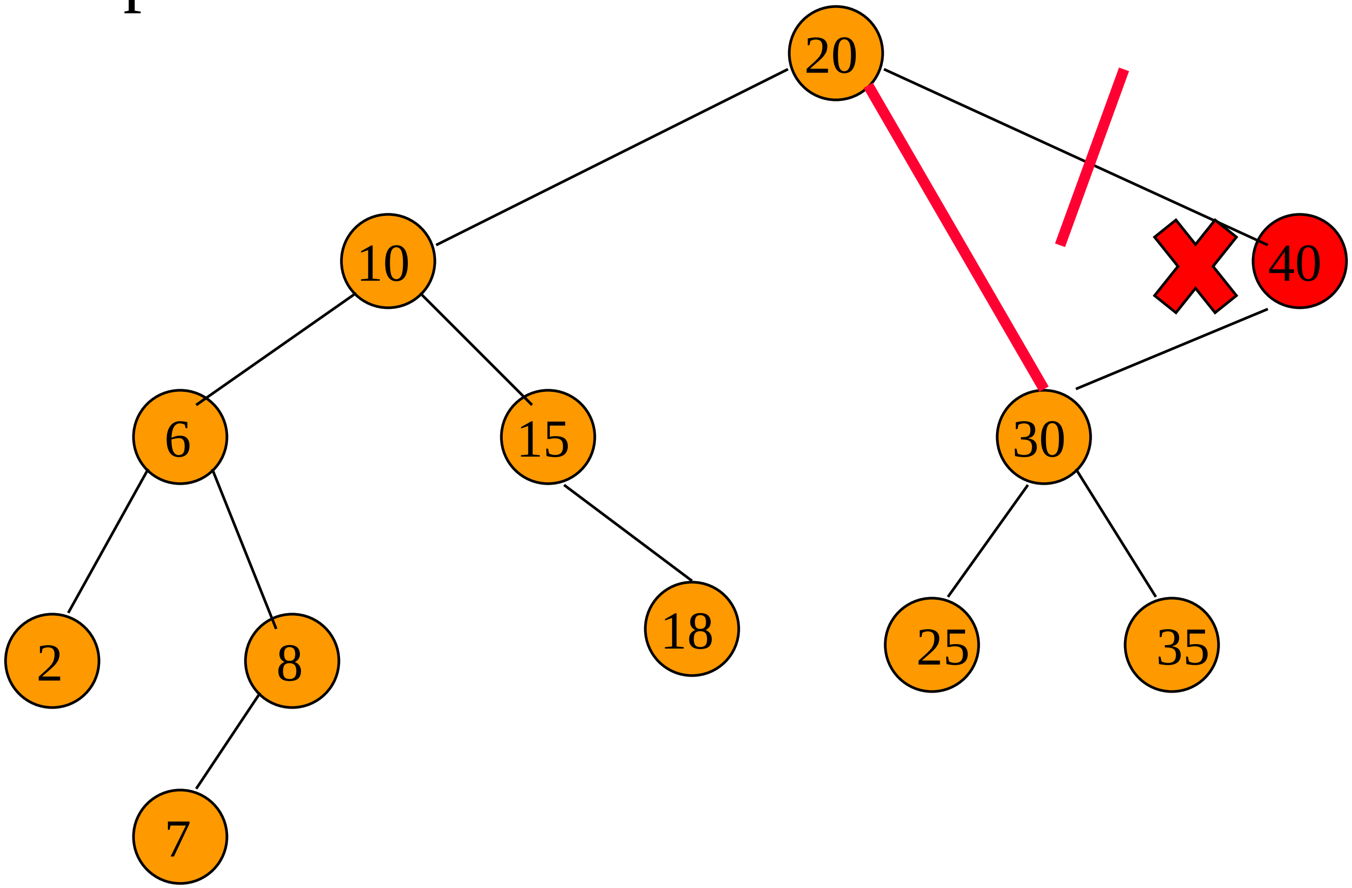
Delete a Leaf Node

- The corresponding child field of its parent is set to NULL.
- The leaf node is disposed.

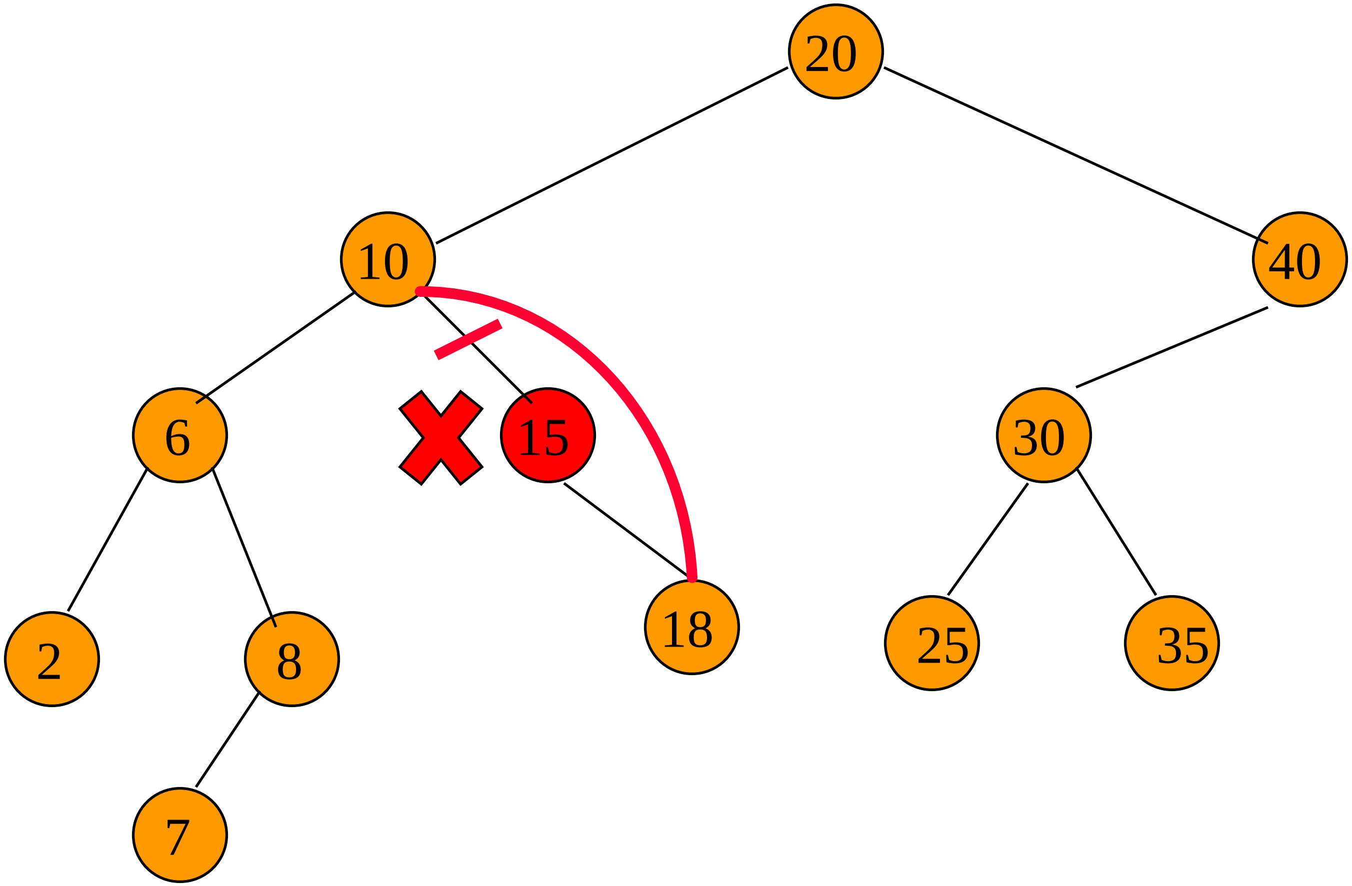


Delete a Degree 1 Node

- The node is disposed
- The single-child of the disposed node takes place of the disposed node



Delete a Degree 1 Node (cont.)

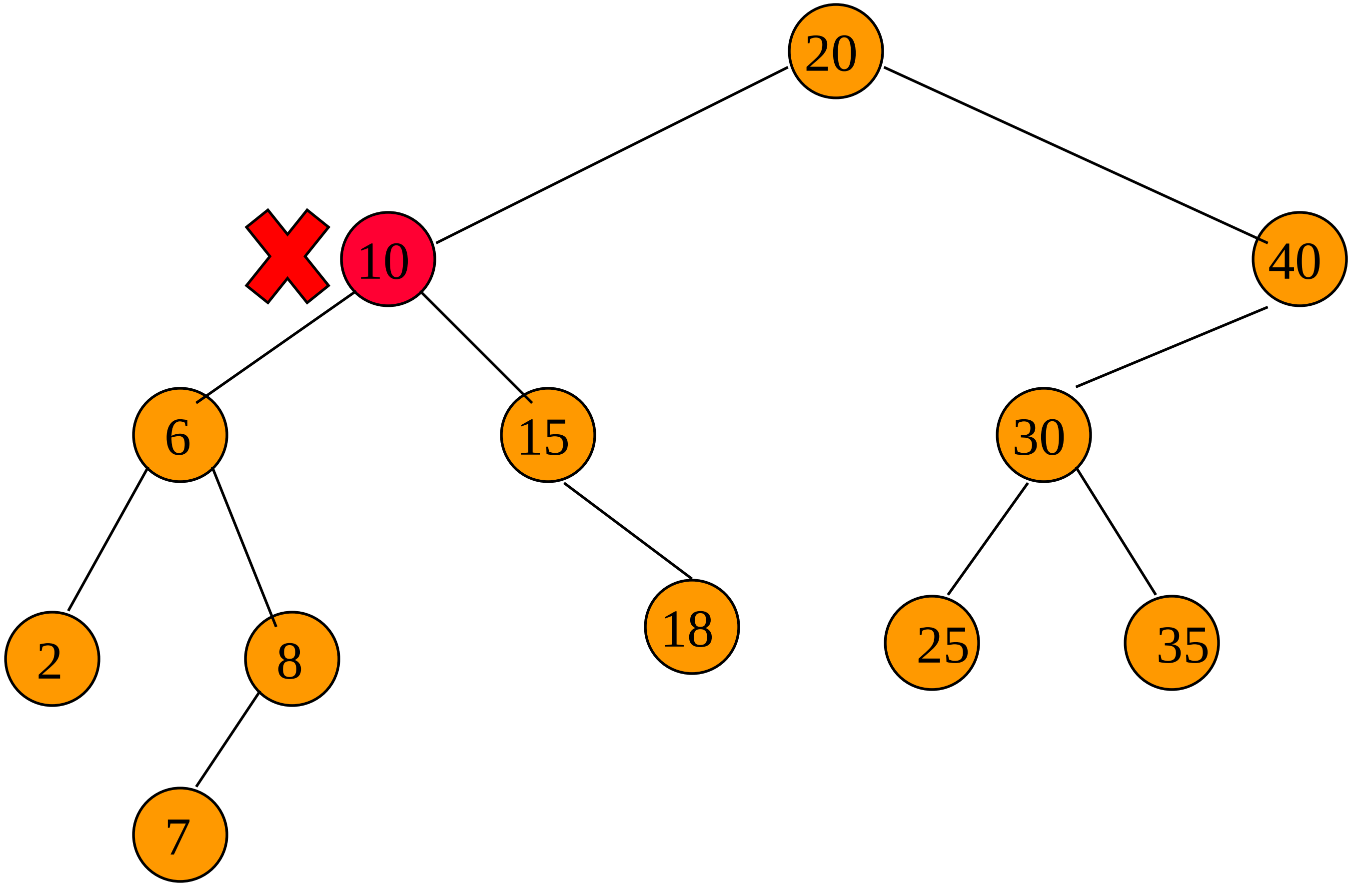


Delete a Degree 2 Node

- The node is replace by either
  - the largest node in its left subtree
  - the smallest node in its right subtree
- Delete this replacing node from the subtree from which it was taken

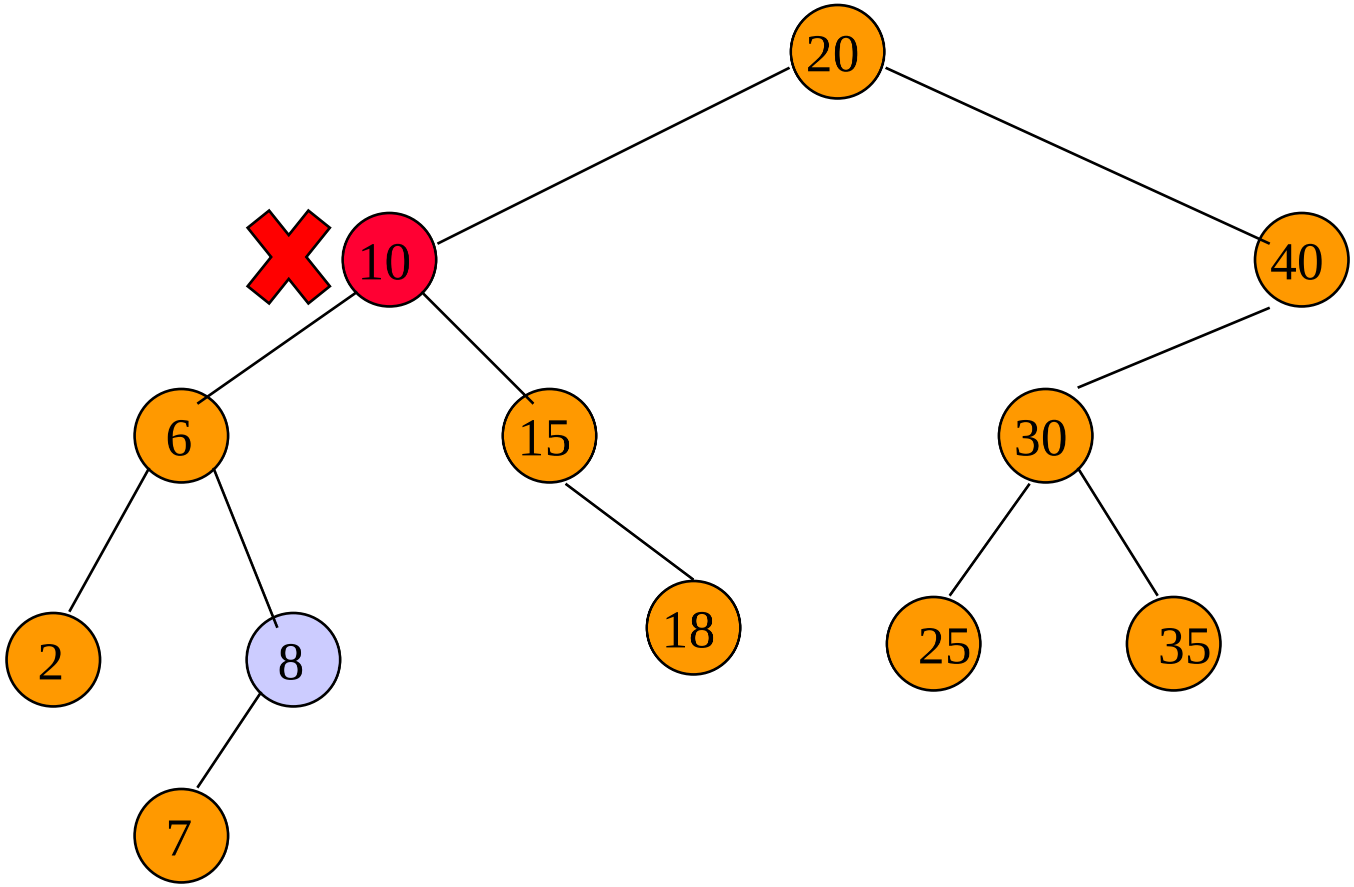
Example

- Delete **10**
- Find the largest key in the left subtree (or the smallest key in the right subtree).



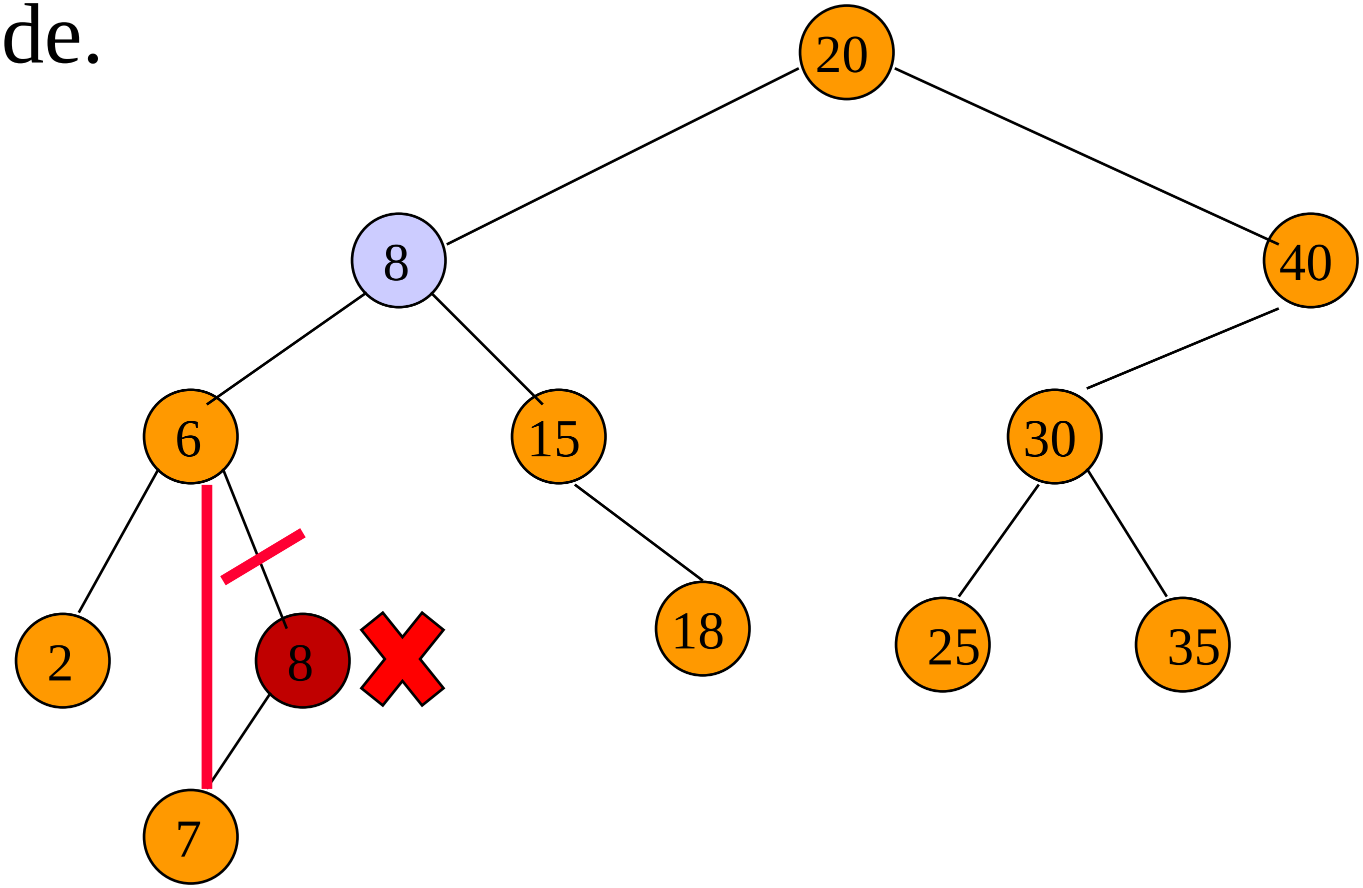
Example (cont.)

- **8** is the largest key in the left subtree
- Replace **10** with **8**

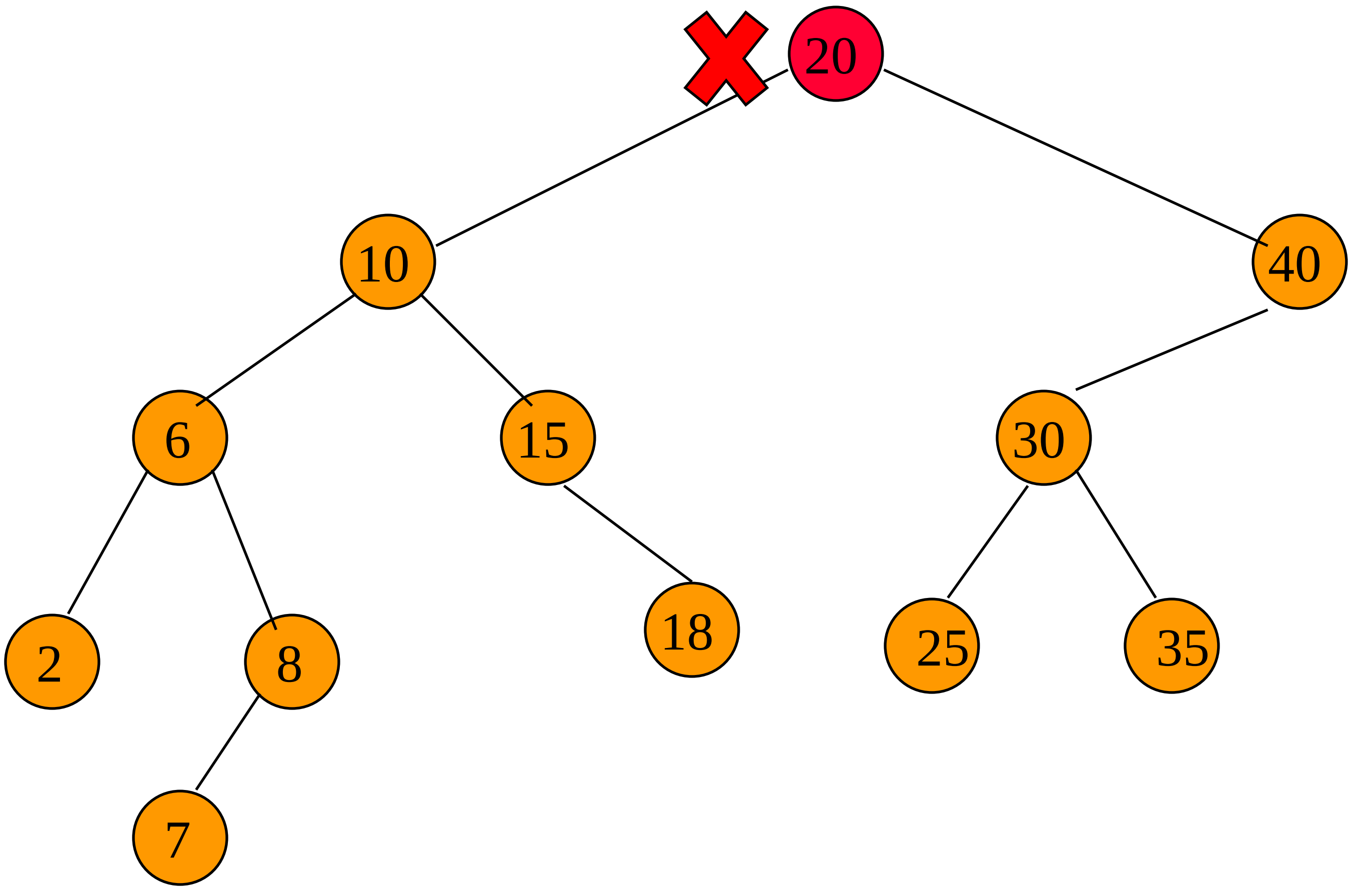


Example (cont.)

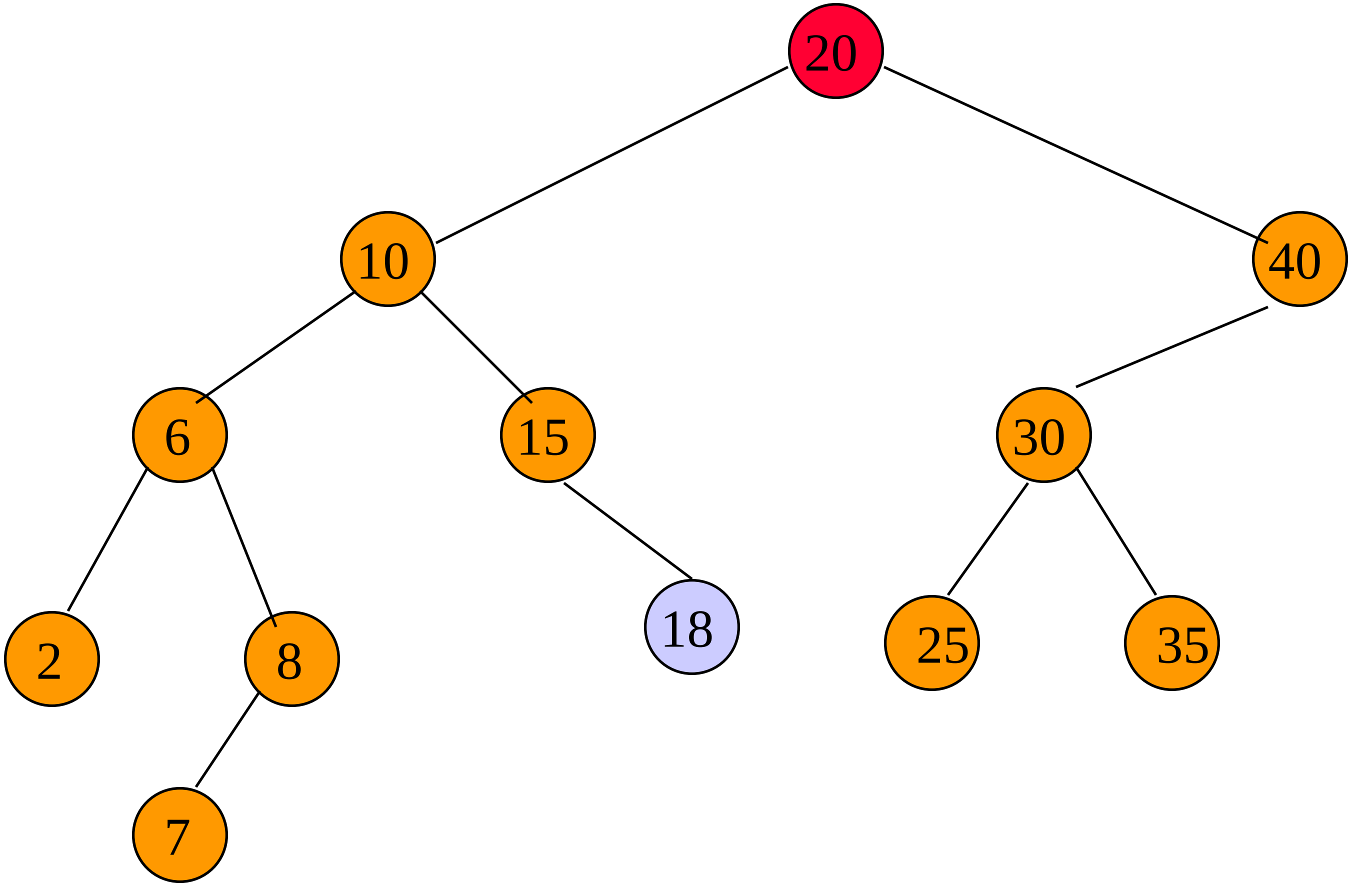
- Delete the replacing node **8**
- The largest key must be in a leaf or degree 1 node.



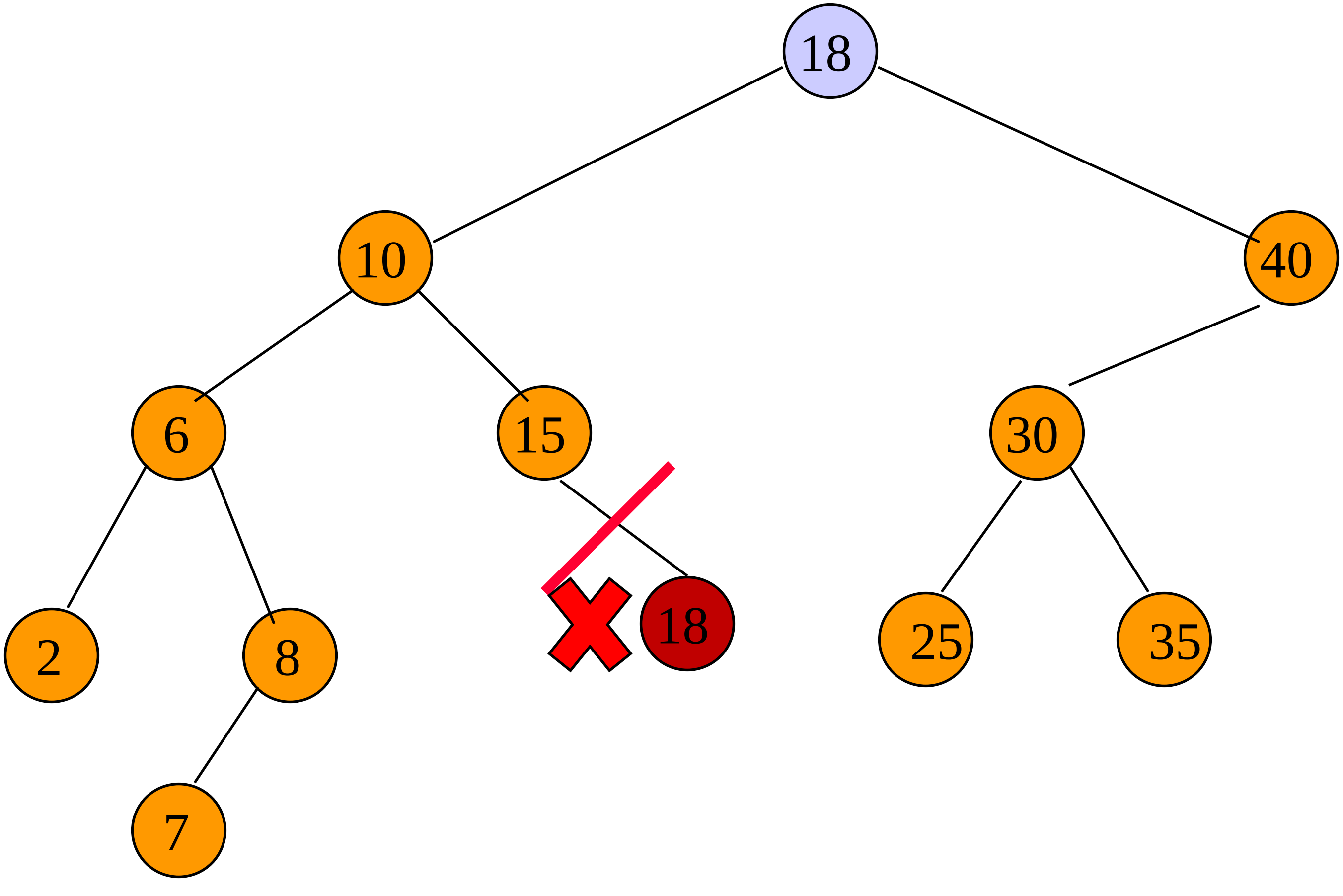
Another Example



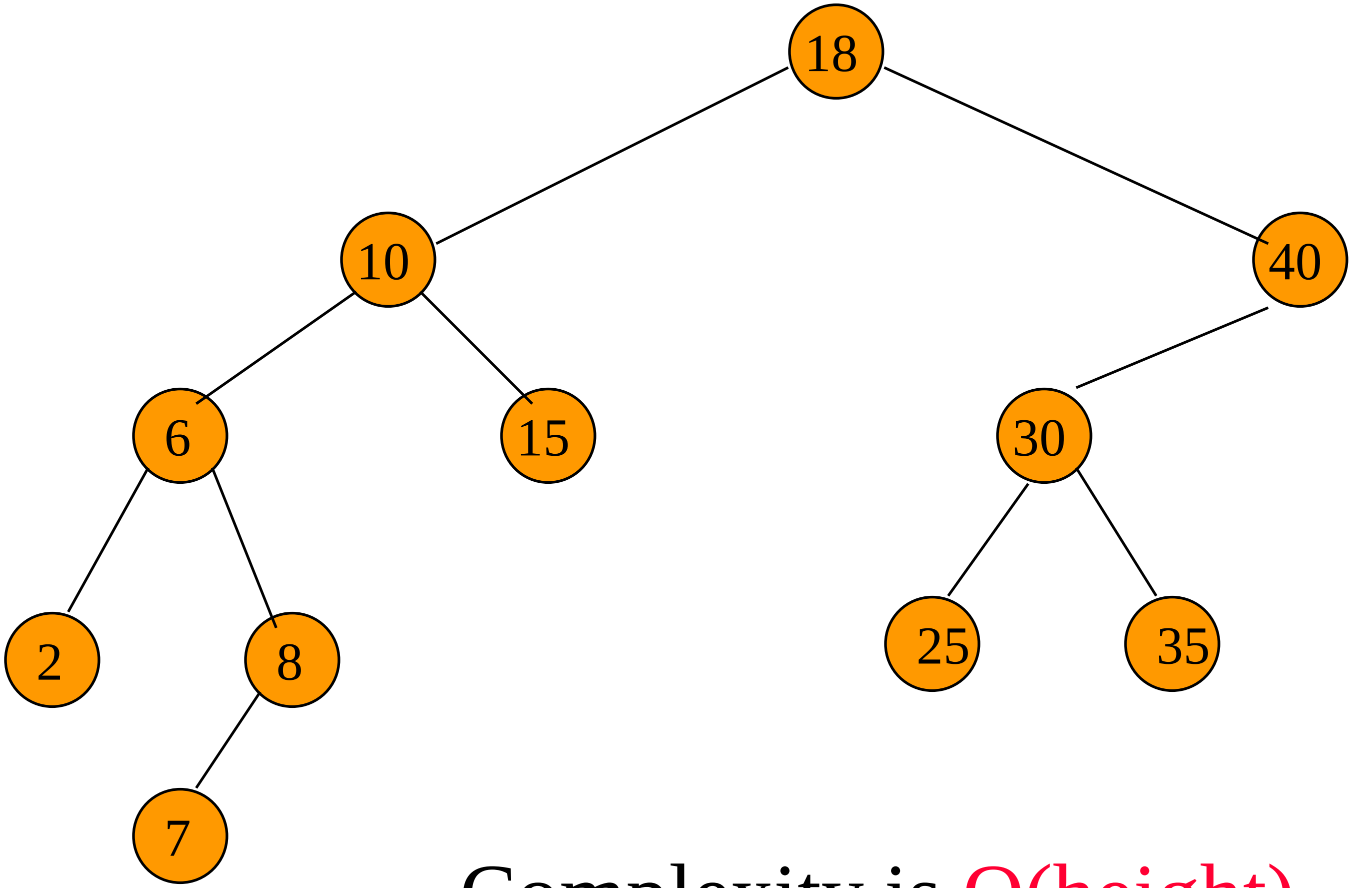
Another Example (cont.)



Another Example (cont.)



Delete a Degree 2 Node



Complexity is O(height).

Implementation

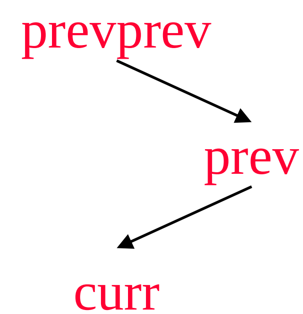
q is the parent of *p*

```
template <class K, class E>
void BST<K,E>::Delete(K k) {
  TreeNode<pair<K,E> > *p = root, *q = 0;
  while (p  && k ≠ p→data.first) {
    q = p;
    if (k < p→data.first) p = p→LeftChild;
    else p = p→RightChild;
  }
  if (p == 0) return; // not found
```

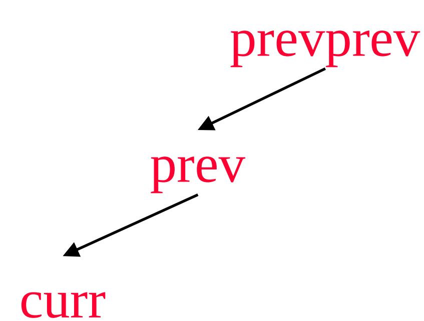
```
if (p→LeftChild == 0 && p→RightChild == 0) // p is leaf
{
  if (q == 0) root = 0;
  else if (q→LeftChild == p) q→LeftChild = 0;
  else q→RightChild = 0;
  delete p;
}

if (p→LeftChild == 0)  // p only has right child
{
  if (q == 0) root = p→RightChild;
  else if (q→LeftChild == p) q→LeftChild = p→RightChild;
  else q→RightChild = p→RightChild;
  delete p;
}
```

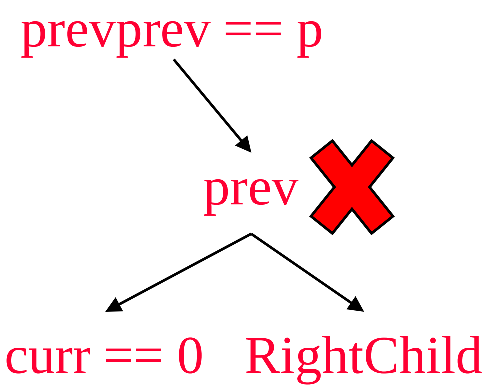
```
if (p→RightChild == 0)  // p only has left child
{
   if (q == 0) root = p→LeftChild;
   else if (q→LeftChild == p) q→LeftChild = p→LeftChild;
   else q→RightChild = p→LeftChild;
   delete p;
}
```

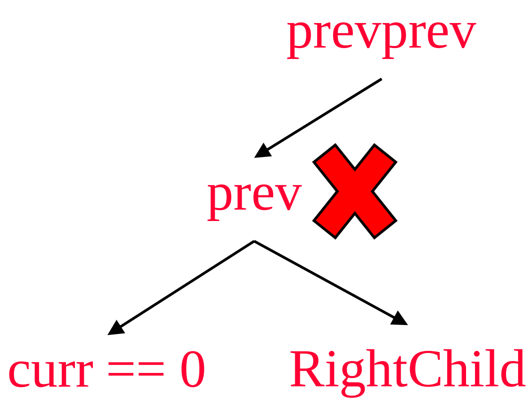
find the smallest node in the right subtree




```
// p has left and right child.
TreeNode<pair<K,E> > *prevprev = p, *prev = p→RightChild,
      *curr = p→RightChild→LeftChild;

while (curr) {
   prevprev = prev;
   prev = curr;
   curr = curr→LeftChild;
}
```







// curr is 0, prev is the node to be deleted, prevprev is prev's
// parent, prev->LeftChild is 0.

p→data = prev→data;
if (prevprev == p) prevprev→RightChild = prev→RightChild;
else prevprev→LeftChild = prev→RightChild;
delete prev;
}

Operations' Efficiency on BST

Operation	Average case	Worst case
Retrieval	O(log n)	O(n)
Insertion	O(log n)	O(n)
Deletion	O(log n)	O(n)
Traversal	O(n)	O(n)

Homework #3

Implement and test

- Programs 5.18, 5.19, 5.21
- Exercise 5.7.1 (the delete function)

Homework을 제출할 필요는 없으나
중간/기말고사에 출제할 계획임