Control of Low-Inertia Power Grids: A Model Reduction Approach

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Introduction

Paper objectives and stated contributions

- Develops a reduced-order model for analysis and control design of low-inertia power systems
- Uses singular perturbation theory to build a tractable model for control design
- Bridges gap with device level control of converters and system level control objectives
- Develops a nonlinear droop control that stabilizes the power system (frequency and devices)

Models

Notation

g for generators; c for AC side of converters; l for loads and t for tx. lines.

DC states x:

θ_g	Rotor angle of synchronous generators
ω_g	Rotor angle velocity of synchronous generators
v_{dc}	DC link capacitor voltage of converters.

AC states z:

i_g	Stator current of synchronous generators
i_c	Output AC current of converters
i_l	Load current
i_t	Transmission line current
v	Bus voltages.

Control inputs u:

τ_m	Mechanical torque of synchronous machines
i_f	DC field current in rotor of synchronous generators
i_{dc}	Current source that supply the DC capacitor of converters
m	Modulation signal that control the average AC voltage of converters.

Notation

Electrical parameters:

$\mathcal{E}$	Incidence matrix for lines of the transmission network
$\mathcal{I}_\mu$	Indicator matrices that indicates in which bus the component μ is connected for $\mu \in \{g, c, l\}$
L_γ	Inductance of component γ for $\gamma \in \{g, t, c\}$
R_γ	Resistance of component γ for $\gamma \in \{g, t, c\}$
Z_γ	Impedance of component γ, $Z_\gamma = R_\gamma + j\omega_0 L_\gamma$, for $\gamma \in \{g, t\}$
C_ψ	Capacitance of component ψ, for $\psi \in \{\mathrm{dc}, v\}$
G_ψ	Conductance of component ψ, for $\psi \in \{\mathrm{dc}, v\}$
Y_ψ	Admittance of component ψ, $Y_\psi = G_\psi + j\omega_0 C_\psi$, for $\psi \in \{\mathrm{dc}, v\}$
l_m	Mutual inductance between stator and rotor of synchronous generators.

Other parameters:

ω_0	Nominal frequency of the AC signals
M	Inertia constant of synchronous machines
D	Damping coefficient of synchronous generators.

We denote j as the rotation matrix in 90 degrees in the $\alpha - \beta$ plane and $\mathrm{r}(\theta) := [\cos\theta,\ \sin\theta]^\top$ as the vector in the unit circle with angle θ.

Network

Equations in $\alpha - \beta$ coordinates in a rotating frame at nominal frequency ω_0

Transmission network modeled using dynamic Π-model:

- Lines as series RL circuits.
- Buses as parallel RC circuits.

For a particular transmission line k:

$$L_{t,k}\dot{i}_{t,k} = -Z_{t,k} i_{t,k} + \mathcal{E}_{t,k}^\top v \tag{1}$$

For a particular bus k, its charge RC dynamics are:

$$C_k \dot{v}_k = -Y_{v,k} v_k + \mathcal{E}_{v,k}^\top i_t + i_{\mathsf{in},k} \tag{2}$$

where $i_{\mathsf{in},k}$ includes all the currents of loads, synchronous generators and DC/AC converters connected to the voltage bus k.

Synchronous generators

Equations in $\alpha - \beta$ coordinates in a rotating frame at nominal frequency ω_0

The complete model of a synchronous generator k is given by:

$$\dot{\theta}_{g,k} = \omega_{g,k} - \omega_0 \tag{3}$$

$$M_k \dot{\omega}_{g,k} = -D_k \omega_{g,k} + \tau_{m,k} - \tau_{e,k} \tag{4}$$

$$L_{g,k} \dot{i}_{g,k} = -Z_{g,k} i_{g,k} + \mathcal{I}_{g,k}^\top v - v_{\mathsf{ind},k} \tag{5}$$

$$\tau_{e,k} = -l_{m,k} i_{f,k} i_{g,k}^\top j \mathsf{r}(\theta_{g,k}) \tag{6}$$

$$v_{\mathsf{ind},k} = l_{m,k} i_{f,k} \omega_{g,k} j \mathsf{r}(\theta_{g,k}) \tag{7}$$

where equations (3) and (4) are the swing equations, (5) is the KVL equation, (6) is the electrical torque relationship and (7) is the induced voltage relationship.

DC/AC converters

Equations in $\alpha - \beta$ coordinates in a rotating frame at nominal frequency ω_0

The complete model of a AC/DC converter k use an average dynamics over one switching period as:

$$C_{\mathsf{dc},k}\dot{v}_{\mathsf{dc},k} = -G_{\mathsf{dc},k}v_{\mathsf{dc},k} + i_{\mathsf{dc},k} - i_{\mathsf{sw},k} \tag{8}$$

$$L_{c,k}\dot{i}_{c,k} = -Z_{c,k}i_{c,k} + \mathcal{I}_{c,k}^{\top}v - v_{\mathsf{sw},k} \tag{9}$$

$$i_{\mathsf{sw},k} = -\frac{1}{2}i_{c,k}^{\top}m_k \tag{10}$$

$$v_{\mathsf{sw},k} = \frac{1}{2}v_{\mathsf{dc},k}m_k \tag{11}$$

where:

- equation (8) is the KCL on the DC side of the converter
- (9) is the KVL on the AC side of the converter (including an RL filter)
- (10) is the average current across the switches
- (11) is the average voltage across the switches.

Loads

Equations in $\alpha - \beta$ coordinates in a rotating frame at nominal frequency ω_0

Modeled as RL circuits with constant impedance:

$$L_{l,k}\dot{i}_{l,k} = -Z_{l,k}i_{l,k} + \mathcal{I}_{l,k}^{\top}v \tag{12}$$

Summary

By setting:

- $x = (\theta_g, \omega_g, v_{\mathsf{dc}})$ as the DC states
- $z = (i_g, i_c, i_l, i_t, v)$ as the AC states
- $u = (\tau_m, i_f, i_{\mathsf{dc}}, m)$ as the control inputs.

Our dynamics can be written as:

$$\dot{x} = f_{\mathsf{dc}}(x, z, u) \qquad \dot{z} = f_{\mathsf{ac}}(x, z, u)$$

where

$$f_{\mathsf{ac}}(x, z, u) = M_z^{-1}(A_z z + B_z v_{\mathsf{in}}(x, u))$$

on which M_z, A_z and B_z are appropriated matrices that depends on the parameters, and $v_{\mathsf{in}} = (v_{\mathsf{ind}}, v_{\mathsf{sw}})$, and:

$$f_{\mathsf{dc}}(x, z, u) = \begin{bmatrix} \omega_g - \omega_0 \\ M^{-1}\left(-D\omega_g + \tau_m - \tau_e(\theta_g, i_g, i_f)\right) \\ C_{\mathsf{dc}}^{-1}\left(-G_{\mathsf{dc}} v_{\mathsf{dc}} + i_{\mathsf{dc}} - i_{\mathsf{sw}}(i_c, m)\right) \end{bmatrix}$$

Stability criteria

Device and System Stability

System Stability

- Frequency: AC variables are stable with respect to the desired synchronous frequency. (unclear if they mean asymptotically or bounded stability)
- Angle: Phase angles of the induced and modulated voltages (i.e. generator and converter output) converge to a stable equilibrium, and the equilibrium matches the setpoints.
- Voltage: All AC voltage magnitudes converge

Device Stability:

- Rotor Frequency: Generator rotor speeds stable with respect to synchronous frequency
- DC Voltage: Converter DC voltages stable with respect to setpoints.

System stability implies device stability; device stability does not imply system stability.

Model Reduction

In a nutshell...

Because the model of the AC dynamics is exponentially stable, we can neglect them in stability analysis and only concern ourselves with the DC dynamics.

We can define an approximate system neglecting the AC dynamics and place a bound on the approximation error related to the time constants of the RLC circuits in the AC system.

Argument for neglecting AC dynamics

"DC" dynamics does not mean on the DC side of converters. It means DC *quantities*; i.e. the DC states are bus angle, frequency of each generator and converter, and the (physical) DC converter voltage.

Proof sketch:

- Lyapunov argument for exponential stability of AC dynamics
- Intuition: All the network elements are conservative (L, C) or dissipative (R), so the whole system dissipates energy and there are no active components.
- Uses a quadratic energy function and because AC dynamics are linear can reduce time derivative of $V_z(z)$ to a quadratic and it turns out the matrix is positive definite diagonal, which gives necessary and sufficient conditions for exponential stability.

Form of reduced order model

The reduced order model with $\hat{x}$ being the approximate DC states, and $\hat{z}$ being the approximate AC states is:

$$\dot{\hat{x}} = f_{\mathsf{red}}(\hat{x}, u)$$

$$\hat{z} = h(\hat{x}, u)$$

Note that $\hat{z}$ has no dynamics. The dynamics of $\hat{x}$ are:

$$f_{\mathsf{red}} := f_{\mathsf{dc}}(x, h(x, u), u)$$

so the DC dynamics still depend on the AC states. Recall that the full dynamics were:

$$\dot{x} = f_{\mathsf{dc}}(x, z, u)$$

$$\dot{z} = f_{\mathsf{ac}}(x, z, u)$$

Error bounds

Theorem 2 (simplified): For a **smooth state feedback law** $u = \kappa(\hat{x})$ and initial error conditions $x_0 - \hat{x_0} = O(\epsilon)$ and $z_0 - h(\hat{x_0}, \kappa(\hat{x_0})) = O(\epsilon)$, the solutions of the full and reduced order models satisfy:

$$x(t) - \hat{x}(t) = O(\epsilon)$$

$$z(t) - h(\hat{x}(t), \kappa(\hat{x})) = O(\epsilon)$$

Where ϵ is related to the maximum time constant of the inductor and capacitor dynamics of the AC system (some technicalities about how that is precisely defined).

The theorem implies (I think) that if the reduced order system is stable, then the full order system is stable.

Proof relies on Tikhonov's theorem (Thm. 11.1 in Khalil's *Nonlinear Systems*).

Key assumptions / caveats for model reduction

- Smooth state feedback law places some restrictions on control and strong condition on having full knowledge of the states, though (I think) this is partially addressed in their decentralized controller. Does the smoothness requirement present any issues for digital controllers?
- Electrical network is not actually linear and passive (e.g. constant current or power loads, series power electronics).
- Instantaneous controllability of generator excitation current: read against Lin, et al.
- Time-scale separation: "we exploit the time-scale separation between the DC and AC variables and define the reduced-order model". It's not explicit to me to what extent they are assuming *a priori* time scale separation, what that precisely means, and where that step comes into play. Could time-scale separation break down; e.g. in the switching, filter, excitation, prime mover dynamics?
- Balanced three-phase system: (1) electrical components (resistance, inductance, capacitance) of each device have identical values for each phase, and (2) all three-phase signals are balanced. Thus, a three-phase voltage can be transformed into (α, β) stationary coordinates.

Non-linear droop controller for frequency stabilization

Non-linear droop controller for frequency

To control the frequency the paper proposes the following controller for each synchronous machine and DC/AC converter:

$$\tau_{m,k} = \tau^\star_{m,k} + \omega^{-1}_{g,k} P_{\mathsf{ind},k} - \omega_0^{-1} P^\star_{\mathsf{ind},k} \tag{13}$$

$$i_{\mathsf{dc},k} = i^\star_{\mathsf{dc},k} + v_{\mathsf{dc},k} P_{\mathsf{sw},k} - (v^\star_{\mathsf{dc},k})^{-1} P^*_{\mathsf{sw},k} \tag{14}$$

where:

- $P_{\mathsf{ind},k} = -\hat{i}_{g,k} v_{\mathsf{ind,k}}$ is the inst. act. power flow out the generator
- $P_{\mathsf{sw},k} = -\hat{i}_{c,k} v_{\mathsf{sw,k}}$ is the inst. act. power flow out the converter

The main idea behind these controllers is that it cancels the variations in the electrical torque since:

$$\tau_{e,k} = -\omega^{-1}_{g,k} \hat{i}_{g,k} v_{\mathsf{ind,k}} = \omega^{-1}_{g,k} P_{\mathsf{ind},k}$$

that allow us to prove Lyapunov stability by picking convenient a Lyapunov function. Observe that these are decentralized controllers that only use local measurements.

Conclusions

Conclusions and future directions

Conclusions:

- Paper proposes a reduced-order model for control design of low-inertia power grids that preserves system structure.
- Paper proposes internal controllers for AC/DC converters on the reduced-order models.

Possible extensions:

- Explore classic controllers in simulation and sensing dynamics of frequency for AC/DC converters.
- Explore unbalanced three-phase systems.
- Consider a network that is not linear and passive.
- Avoid using a current source on the DC side of converters and excitation of synchronous machines.