21st Century Power System Dynamics

EECS 290

Spring 2019

March 18 2019 Lecture notes.

Schedule

Next week: Spring Break

	Book	Group
April 1	Finish Machine Models	Ramasubramanian
8	Y & I	Lin
15	↓	Curi
22		Ramasubramanian,
29		Markovic, Lin

Question:

Final presentations

During reading week?

Deliverable: Machine Modeling code.

Description of additions

Simulation tests.

Introduction

Today's objective: Sketch basic process of building a 6^{th} order generator model.

State variables: ω, δ and d-q voltages for fast and slow voltage dynamics.

- we'll deal w/ voltage and frequency regulation control dynamics next time
- Also next time - generator shaft dynamics.

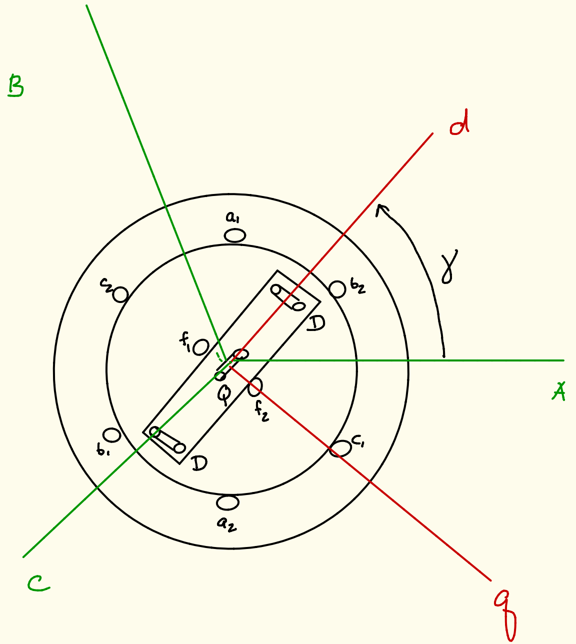
Basic mechanics are pretty simple. we need.

1. "Flux linkage" equations, plus
2. "Voltage equations" from applying KVL to gen circuits - including damper and field winding.
3. Integrate these formulas. Faraday's law from voltage eq'ns has time derivative → ultimately gives us ODEs on generator voltages.

A note about transient and subtransient reactances

- I have struggled a lot to understand the significance of the different flux paths and how they decay
- This chapter uses a bit of the logic of earlier ones to explain.
- However, refreshingly, it ultimately takes a fresh approach that is more rigorous in my opinion
  - Key idea: model voltage transients due to winding currents induced during disturbance.
  - Stay tuned.

Assumptions



- 3ϕ winding symmetrical
- Ignore winding capacitance
- Distributed windings can be modeled as single concentrated
- Stator winding inductance indep. of rotor position
- Ignore harmonics
- Neglect hysteresis loss
- Rotor speed changes negligible
- Magnetic ckts are not saturated.

Flux Linkage equations – Stator perspective (11.1.2)

- First, basic form for flux linkage: $\boxed{\psi = L i}$
- Now, when you have two magnetic circuits in proximity,

$$\begin{bmatrix} \psi_1 \\ \psi_2 \end{bmatrix} = \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

The off-diagonals are mutual inductances.

- MBB first present

$$\begin{bmatrix} \psi_A \\ \psi_B \\ \psi_C \\ \psi_f \\ \psi_D \\ \psi_Q \end{bmatrix} = L \begin{bmatrix} i_A \\ i_B \\ i_C \\ i_f \\ i_D \\ i_Q \end{bmatrix}$$

i_A, i_B, i_C: phase A, B, C armature windings

$i_f \rightarrow$ rotor field winding

$i_D \rightarrow$ Damper windings – d-axis

$i_Q \rightarrow$ Damper windings – q-axis

L ....

- is 6x6
- has mostly time-varying entries, since rotor is spinning

Flux Linkage Equations : Rotor / dq perspective (11.1.3)

$$\begin{bmatrix} \psi_0 \\ \psi_d \\ \psi_q \\ \psi_f \\ \psi_D \\ \psi_Q \end{bmatrix} = \begin{bmatrix} & & \\ & L_{dq} & \\ & & \end{bmatrix} \begin{bmatrix} i_0 \\ i_d \\ i_q \\ \psi_f \\ \psi_D \\ \psi_Q \end{bmatrix} \begin{array}{l} \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} \text{armature variables in d-q} \\ \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} \text{as before} \end{array}$$

All entries of the L_{dq} matrix are constant!

There are a lot of zero entries in L_{dq}...

$$\psi_0 = L_0 i_0 \leftarrow$$

(can neglect if
1. Balanced, or
2. Neutral point is not grounded)

$\Rightarrow$

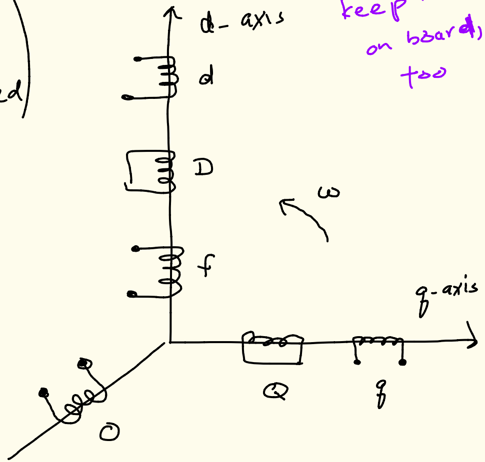
$$\begin{bmatrix} \psi_d \\ \psi_f \\ \psi_D \end{bmatrix} = L_{dfD} \begin{bmatrix} i_d \\ i_f \\ i_D \end{bmatrix}$$

7

These eqn import. ...keep on board

$$\begin{bmatrix} \psi_q \\ \psi_Q \end{bmatrix} = L_{qQ} \begin{bmatrix} i_q \\ i_Q \end{bmatrix}$$

keep this on board, too



o, d and q windings are "fictitious," representing armature circuits in d-q.

One more note on Flux Linkage in d-q ...

- Have a look at the discussion on p.439 regarding d-q transformation coefficients.
- This book uses $\sqrt{2/3}$, as Jonny & Seth discussed, too.
- But there are versions with a coefficient of $2/3$ - be careful as you read the literature!

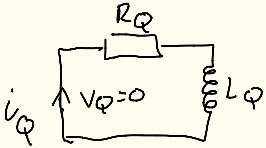
A note on power (discussed a few lectures ago...)

P_g = generator power ... transformation-invariant

$$= v_A i_A + v_B i_B + v_C i_C$$

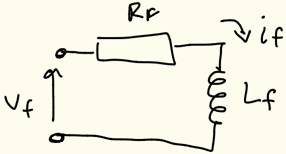
$$= v_d i_d + v_q i_q + v_0 i_0$$

Voltage equations - ABC (11.1.4)

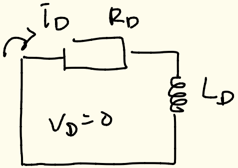


$$\boxed{v = -Ri - \dot{\psi}}$$ ← basic form, derived from KVL.

$$\Downarrow$$



$$\begin{bmatrix} v_A \\ v_B \\ v_C \\ -v_f \\ 0 \\ 0 \end{bmatrix} = \underbrace{- \begin{bmatrix} R_A & & & & & \\ & R_B & & & & \\ & & R_3 & & & \\ & & & R_f & & \\ & & & & R_D & \\ & & & & & R_Q \end{bmatrix} \begin{bmatrix} i_A \\ i_B \\ i_C \\ i_f \\ i_D \\ i_Q \end{bmatrix}}_{\text{ohm!}} \underbrace{- \frac{d}{dt} \begin{bmatrix} \psi_A \\ \psi_B \\ \psi_C \\ \psi_f \\ \psi_D \\ \psi_Q \end{bmatrix}}_{\text{faraday!}}$$



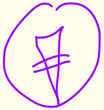
Voltage Equations : dq

$$v_0 = -R i_0 - \dot{\psi}_0$$

$$v_d = -R i_d - \dot{\psi}_d - \omega \psi_q$$

$$v_q = -R i_q - \dot{\psi}_q + \omega \psi_d$$

"rotational emf" come from same mechanism Jonny taught us about two weeks ago.



These eqn import. ...keep on board

$$v_f = R_f i_f + \dot{\psi}_f$$

$$0 = R_D i_D + \dot{\psi}_D$$

$$0 = R_Q i_Q + \dot{\psi}_Q$$

"transformer emfs" due to changing currents on armature

Simplifying voltage equations ...

- Transformer emfs typically small relative to iR and rotational terms

- If you include transformer emfs then currents on tx lines are subject to ODEs $\Rightarrow$ Power transmission equations are ODEs
  - → That adds significant complexity
  - → Textbook recommends ignoring these
  - → Benefit is that d, q voltage equations become algebraic $\Rightarrow$ easier to compute
  - → This also makes it possible to model tx eq'ns as algebraic.

- Separate note: these equations will hold for all states of the generator (s.s., trans, subtrans...)

Generator Reactances (s'n 11.1.5)

As we discussed before, the steady state, transient and subtrans. flux conditions each have different effects on how the armature reactance appears.

Note
At this point in the text it appears that the circuit used to represent d & q axes changes - such that the sign of terms in ckt equations differs from what we had before

→ see fig. 11.5 and eq'n 11.34, which are consistently labeled.
(not 11.4)

Generator Reactances, ctd (s'n 11.1.5)

In this section of the book the authors re-derive expressions for transient and subtransient "effective" reactances, and the time constants governing their decay.

However the following section (11.1.6) derives generator voltage equations in a way I find more comfortable.

- The expressions derived for reactances and time constants in this section are used to simplify the representations in 11.1.6, but we don't need any of the conceptual tools from this section to move forward.

Transient and subtransient mental gymnastics - (why I prefer not to use the book's logic in 11.1.5)

The original justification for trans. & subtrans. states

- came from 3ϕ short circuit modeling
- assumed no electrical torque during fault → armature flux perfectly opposes rotor.
- This gave "alternate path" for armature flux, necessary b/c currents induced in field & damper windings generate their own fluxes, and flux in rotor can't change instantaneously.
- That in turn makes the armature reactance appear different.
- I find it somewhat challenging to see why these trans + subtrans states would appear under all kinds of disturbances - not only 3ϕ.
- But - see last slide - this logic isn't actually required in subsequent derivations (11.1.6)

Synchronous Generator Equations (11.1.6)

The next steps involve substituting $\forall$ equations into $\nexists$ eq'ns with appropriate simplifications

Steady State: ignore ψ_f, ψ_D, ψ_Q resulting from transients. Assume $i_D = i_Q = 0$

$\forall \Rightarrow$

$$\psi_d = L_d i_d + kM_f i_f$$

$$\psi_q = L_q i_q$$

$\leftarrow \nexists \Rightarrow$

$$v_d = -R i_d - X_q i_q$$

$$v_q = -R i_q + X_d i_d + \omega k M_f i_f$$

$$= -R i_q + X_d i_d + e_q$$

$$e_q = \omega k M_f i_f = \frac{\omega k M_f}{L_f} \psi_f (i_d = 0)$$: "open circuit armature voltage induced by field current"

Sync. Gen equations

"Transient" $\Rightarrow$ $i_D = i_Q = 0$ $\Rightarrow$ ignore ψ_D & ψ_Q.

But don't ignore ψ_f ... i.e. model effect of field winding current dynamics.

Voltage equations + flux linkage

$$v_q = -Ri_q + X_d' i_d + \frac{\omega k M_f}{L_f} \psi_f$$

$$\boxed{= -Ri_q + X_d' i_d + e_q'}$$

emf introduced into q-axis by rotor and flux transient

this is a function of d-axis mutual and self inductances. Does not require earlier logic on "screening" flux etc.

$$\boxed{v_d = -Ri_d - X_q' i_q}$$

Sync. Gen equations. Here is the cool part

We can bring the time derivative arising in the rotor voltage equation in to give us dynamics!

$\forall + \exists \quad \Rightarrow v_f = \dot{\psi}_f - R_f \psi_f - R_f \frac{k M_f}{L_f} i_d$

defined on last slide

$$\Rightarrow \dot{e}'_q = \frac{e_f - e'_q + i_d (X_d - X'_d)}{T'_{do}}$$

time constant for decay of "transient state"... formula derived earlier, but falls out naturally here.

$$\dot{e}'_d = \frac{-i_d (X_q - X'_q) - e'_d}{T'_{qo}}$$

Subtransient dynamics

Follow similar logic to identify odes for conditions when $i_D, i_Q \neq 0$

("subtransient" phase... but you don't need to worry about flux paths inside the rotor, just assume all currents are nonzero.)

subtransient emf

$$\Rightarrow \quad \dot{e}_q'' = \frac{e_q' + (X_d' - X_d'')\, i_d - e_d''}{T_{do}''}$$

time constant for decay of "subtransient state"... formula derived earlier, but falls out naturally here.

$$\dot{e}_d'' = \frac{e_d' - (X_q' - X_q'')\, i_q - e_q''}{T_{qo}''}$$

X_d'' this is a function of d-axis mutual and self inductances. Does not require earlier logic on "screening" flux etc.

We have ODEs on generator emfs. Now what?

- You can think of the subtransient ODEs as the most detailed description of generator voltage dynamics

- $e''_{(\cdot)}$ values ....
  - equal $e'_{(\cdot)}$ if you assume $\dot{e}''_{(\cdot)} = 0$ (sustransient gone, but not transient)
  - equal e_f if you assume $\dot{e}'_{(\cdot)} = 0$ (transient is gone)

- with these, we have a system of equations to describe generator voltage and current dynamics in general conditions

  $\Rightarrow$ we will put everything together in a moment

- Keep in mind the generator emf still needs to pass through the remainder of the gen. ckt... (see upcoming slides and s'n 11.1.7)

Phasor interpretations... (s'n 11.1.6.4)

All d-q values are directly related to instantaneous ABC values in prior slides.

Simple transformation...

$$v_d = \sqrt{3}\, V_d \quad , \quad v_q = \sqrt{3}\, V_q$$

$\Leftarrow$ capital letters are consistent w/ rms phasor interpretations in A-B-C frame.

This gives intuitive way to write power

$$P_e = 3\left[V_d I_d + V_q I_q + \left(I_d^2 + I_q^2\right) R\right]$$

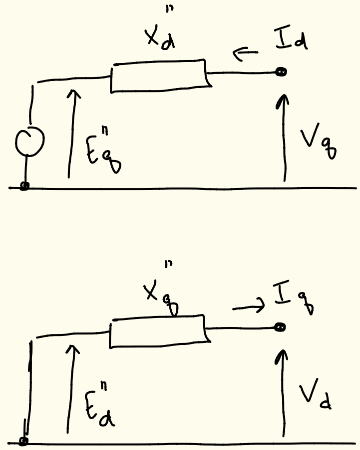
$\Leftarrow$ "power equation"

Finally... the Full Model (11.1.7).

First, basic armature voltage equations

$$\begin{bmatrix} V_d \\ V_q \end{bmatrix} = \begin{bmatrix} E''_d \\ E''_q \end{bmatrix} - \begin{bmatrix} R & X''_q \\ -X''_d & R \end{bmatrix} \begin{bmatrix} I_d \\ I_q \end{bmatrix}$$

Using subtransient. Note there are dynamics behind this (transient) and a steady-state voltage excitation



- Equivalent ckts ignoring generator resistances
- For resistance, we'd need an ad'l line showing parallel currents times resistances... e.g., fig 4.14.

Now the 6^{th} order model

$$M \Delta\dot{\omega} = P_m - P_e$$ ← not typically linear

$$\dot{\delta} = \Delta\omega$$

$$T_{do}' \dot{E}_q' = E_f - E_q' + I_d (X_d - X_d')$$

$$T_{qo}' \dot{E}_d' = - E_d' - I_d (X_q - X_q')$$

$$T_{do}'' \dot{E}_q'' = E_q' - E_q'' + I_d (X_d' - X_d'')$$

$$T_{qo}'' \dot{E}_d'' = E_d' - E_d'' - I_q (X_q' - X_q'')$$

all linear.

$$P_e = E_d'' I_d + E_q'' I_q + \underbrace{(X_d'' - X_q'') I_d I_q}$$

↳ a bit counterintuitive; comes from substituting ~~ϕ~~ into power equation to replace V_q & V_d.

Some follow up thoughts - comparison to earlier models

- 6^{TH} order model includes damper winding influence
  - → remaining swing equation damping is only mechanical ... small ⇒ neglect
- The swing model displayed here assumes $\omega \approx \omega_s$

  ω ↑ actual speed; ω_s ↑ synchronous speed

  → relaxing this assumption would make power $(= \omega\tau)$ vary with speed ⇒ nonlinear model.
- Relax const freq assump. by putting a factor ω/ω_s in front of all reactances

- "Transformer emfs" in ‡ ("voltage equations") ignored
  - Put them back in if you need transients immediately after fault - e.g, short circuit currents.
  - This might be important if you want to study saturation limits in C.I.G.

Multimachine models - stitching things together.

- This model gets "plugged in" to a network model by equating the electrical power on the last slide with power transferred to system. For a single connection,

- Each I_d, I_q can be replaced using

$$I = \frac{\mathbb{V}}{\mathbb{Z}} \rightsquigarrow$$

$\mathbb{V}$ is difference between generator voltage and (remote) bus voltage

$\mathbb{Z}$ is impedance between remote bus and generator.

- Important to get d-q transformations right!

Summary

- we built up a 6^{th} order model describing generator voltage and rotor dynamics
- Strongly nonlinear when connected to a network
- weakly nonlinear when frequency changes influence ① Power ($= \omega \tau$) and ② impedances ($X = \omega L$)
- Next step: Model ① voltage, frequency regulation
  ② Generator shaft dynamics.