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Today

- The Distance Ladder
- Hubble's Law
- The Galaxy Population

- The Distance Ladder

We have already discussed some techniques for measuring distances in astronomy. Determing distances to astrophysical objects is one of the oldest and persistent problems in the field. However, this problem becomes even more important when we want to study galaxies distributed throughout the Universe, whose scale is truly enormous.

In modern astrophysics and cosmology, we accomplish this using a "ladder" of different techniques where each technique extends out to further distances and is itself callibrated using more nearby objects and shorter range techniques.

From nearby to more distant objects, the distance ladder may be summarized as follows:

① The Solar System

From Kepler's 3rd law,

$$P^2 \propto a^3$$

we can calculate the relative size of solar system bodies' orbits. In order to get an absolute scale, the distance to at least one planet must be directly measured. This was done using trigonometric parallax for Venus in 1761. Distances to solar system bodies has also been measured using <u>radar</u>!

② Trigonometric Parallax

For nearby stars (out to a few kpc now with the Gaia space telescope) the parallax angle is large enough to be measured. Recall that

$$d = \frac{1\ \text{pc}}{p}, \quad p \equiv \text{parallax angle in arcseconds}$$

Note that this equation defines the parsec.

③ Standard Candles

For any object which we may know the intrinsic luminosity a priori or via empirical relation, the observed apparent brightness may be used to calculate a distance. This fact follows from basic geometrical reasoning. In terms of the distance modulus, the relation between apparent magnitude and distance for an object with known absolute magnitude, M, is given by,

$$m - M = 5\log\left(\frac{d}{pc}\right) - 5 + \underset{\substack{\uparrow \\ \text{magnitude of} \\ \text{extinction}}}{A}$$

There are various examples of standard candles in astrophysics. Two of the most important are

- Cepheid variable stars
- Type Ia Supernovae

Cepheid variable stars are stars whose apparent magnitudes are observed to fluctuate with periods between ~ 2-150 days. Empirically, that the period of fluctuation is tightly correlated with their mean luminosity:

$$\underset{\text{absolute magnitude}}{M} = \underbrace{-a - b}_{\text{constants}} \log(\underset{\text{period}}{P})$$

This relation has been extensivly callibrated. However, given the precision requirements for modern cosmology, this is an active area of research. A callibration from Feast + Catchpole (1997) in the V-band gives,

$$M_V = -1.43 - 2.81 \log(P)$$

period in days

Cepheids are some of the most luminous variable stars, typically classified as giants or supergiants. For example, with $P \sim 10$ days

$$M_V \sim -4.24$$

Compare this to the Sun ($M_V = +4.83$). The relative brightness of these objects means they can be observed from large distances. With the HST, cepheids can be observed out to ~ 10 Mpc. Beyond this distance it becomes difficult to resolve individual stars.

The other important standard candle are type Ia supernovae. Type Ia SN are thought to be exploding white dwarf stars that by accretion from a companion reached the Chandrashekhar mass, $\sim 1.4\,M_{\odot}$. Type Ia have well calibrated light-profiles, ~~mass~~ i.e, luminosity as a function of time. Empirically it is observed that all Type Ia SN have very similar peak luminosities, with the variation showing a strong correlation with the luminosity decay rate. Type Ia SN are very luminious, $L \sim 10^{10}\,L_{\odot}$ and as a result these objects can be observed out to ~ 1 Gpc.

It should be noted that there are many other standard candles (that are generally less precise). These include:

- color-magnitud diagram (CMD) features
  - * tip of the RGB
  - * tip of the HB
- other types of variable stars e.g. RR Lyrae
- planetary nebula luminosity function
- empirical galaxy relations, e.g. the Tully-Fisher Relation

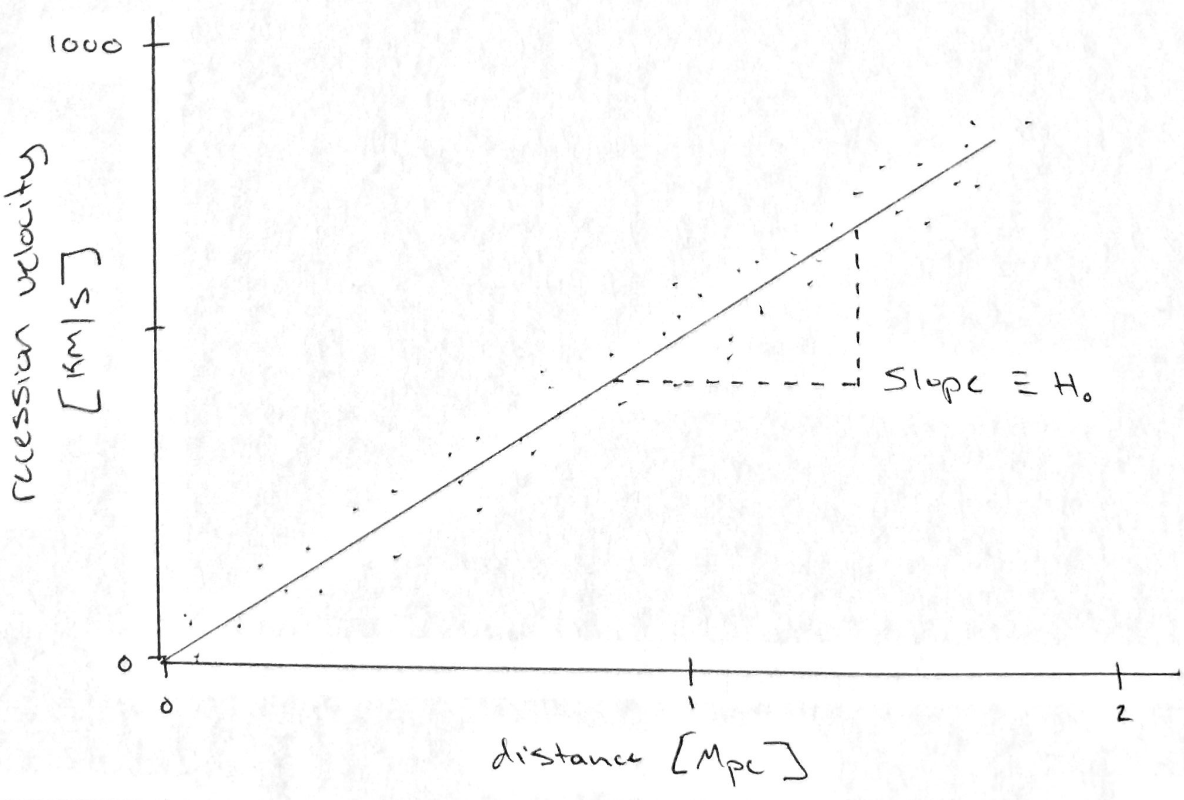
- Hubble's Law

By establishing the distances to galaxies we can start to measure the physical scale of the Universe. A major milestone in establishing our understanding of the extent of the Universe and our understanding of galaxies was advanced by Edwin Hubble. We last encountered Hubble in the "Great Debate" discussion. Hubble resolved the nature of the "spiral nebula" by showing that the distance to M31 (and some other nearby galaxies) that they must be external to the Milky Way and in fact galcxies unto themselves. He did this by observing Cepheid variable stars in these galaxies.

Around the same time, Vesto Slipher began measuring Doppler shift velocities for nearby galaxies.

In addition to discovering that disk galaxies rotate (more on this later) he found that the mean velocity of galaxies was <u>not</u> random with respect to the Sun's velocity. In particular he discovered most galaxies are receeding from the Sun. That is, most galaxies are <u>redshifted</u>.

In 1929 Hubble combined his distance measurements with Slipher's velocity measurements, discovering that our Universe is expanding.



This linear relation between distance and recession velocity is called the Hubble-Lemaître Law:

$$V_r = H_0 d$$

distance

recession velocity

Hubble constant (H-naught)

In our modern cosmological understanding, the distance between two objects co-moving with an expanding background can be written as,

$$r(t) = \frac{a(t)\,r(t')}{a(t')}$$

where $a(t)$ is a time dependent scale factor that is smaller in the past for an expanding universe.

The relative velocity between these two objects is then

$$V_r = \dot{r} = H(t)\, r$$

where $H(t) \equiv \dfrac{\dot{a}(t)}{a(t)}$

If we set $t = t_{now}$ we then recover the Hubble law, $H_0 = H(t_{now})$. Current constraints on H_0 place it at:

$$H_0 \simeq 70 \pm 5 \ \frac{km/s}{Mpc}$$

Because the recession velocity of an object can be measured from its redshift, z, the distance to our co-moving objects is then

$$r = \frac{cz}{H_0}$$

If we assume $V_r \ll c$.

However, it should be noted that objects are rarely perfectly comoving with the expanding background. Typically galaxies have some gravitationally induced peculiar velocity. In this case,

$$V_r = H_0 r + V_{pec,r}$$

Typical $V_{pec} \sim 100$ km/s. Given this, it is only safe to use redshift as a cosmological probe for $cz \gg 1{,}000$ km/s.

Finally, for the aspiring experts, H_0 was not known very precisely. Because of this, it is common to write H_0 as

$$H_0 = 100\, h \, \frac{\text{km/s}}{\text{Mpc}}$$

In this case h parameterizes the uncertainty and is treated kind-of like a unit.

- Galaxy Populations

Over the course of the last 100 years, from the discovery that many "nebula" were in fact galaxies to the present in the era of large all-sky galaxy surveys our understanding of galaxies as a population has grown at a staggering rate.

* Galaxy Morphology

As with so many things in astronomy, we will start with a catagorization scheme. Hubble classified galaxy images by their appearance. Broadly three types of galaxies became apparent:

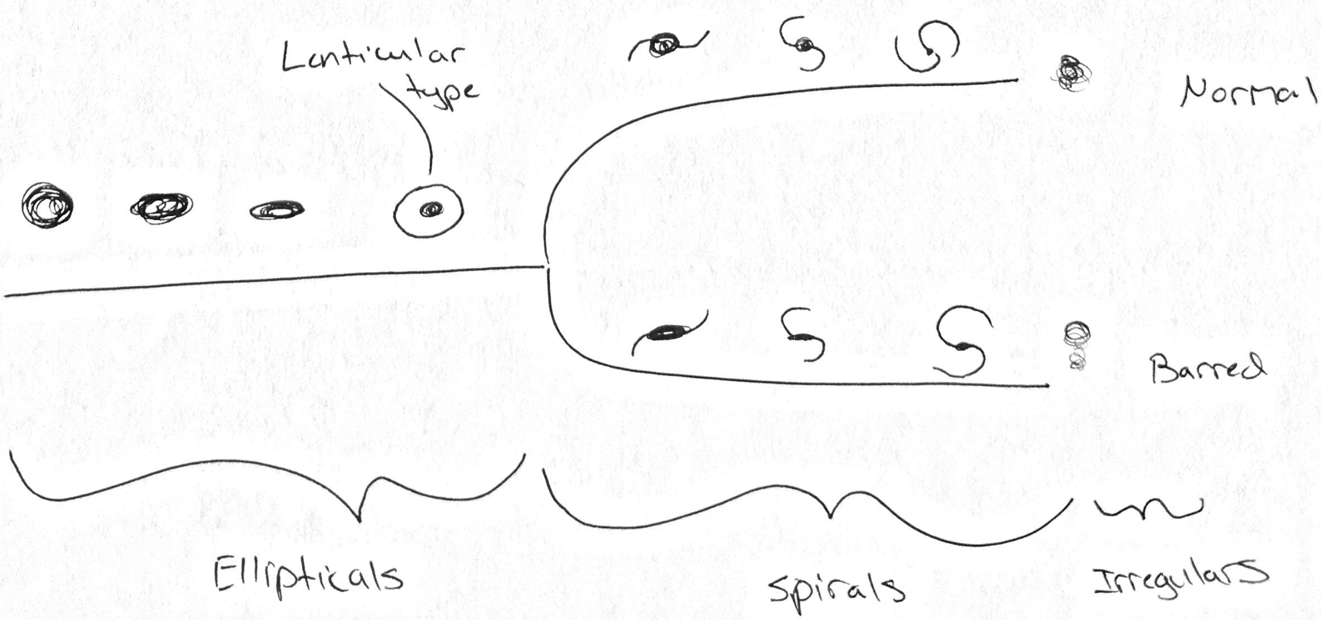
① Elliptical
② Spiral
③ Irregular

Elliptical galaxies were observed to have smooth light profiles with elliptical isophotes, contours of constant surface brightness

Spiral galaxies were observed to have disk like structures with embedded spiral structures. The central regions of spirals are dominated often by more elliptical concentrations of stars called "bulges".

Irregular galaxies are a type of catchall class that are neither clearly elliptical or disky. Often these galaxies are asymmetric with patchy light profiles dominated by a few star forming regions.

Hubble further sub-classed these types according to their isophotal axis ratio (for ellipticals) or by the ratio of the bulge to disk component along with the spiral winding pattern (spiral galaxies). Hubble further distinguished between spiral galaxies with more spherical bulges to those with bar like bulges. He arranged these types into what he thought might be a formation/evolution sequence called the Hubble "tunning fork".



with the addition of lenticular galaxies as an intermediate class between ellipticals and spirals (smooth disk + bulge) one might guess that elliptical galaxies grow spiral arms, evolving from smooth ellipticals to bulge + spiral disk. From this we get the nomenclature "early type" for ellipticals + "late type" for spiral galaxies. Unfortunatly, a modern evolutionary track (if anything) would flip this evolutionary direction. Huzzah!

Finally, note that something of a luminosity class system exists with faint galaxies, $M_B \gtrsim -18$, often called <u>dwarf galaxies</u>.

* Galaxy Light Profiles

It is common to characterize galaxy images with a surface brightness profile, $I(R)$, in units of e.g. $L_\odot/pc^2$, or $\mu(R)$ in units of magnitudes per arcseconds squared.

Be careful with the coordinate R. Depending on the situation it may be the circular or elliptical radial distance.

For elliptical galaxies, the surface brightness profile is often modelled using a <u>Sérsic profile</u>,

$$I(R) = I_o \exp\left[-\beta_n \left(\frac{R}{R_e}\right)^{1/n}\right]$$

$I_0 \equiv$ central surface brightness

$R_e \equiv$ effective radius

$n \equiv$ sersic index

$\beta_n = f(n) \simeq 2n - 0.324$ (for $n > 1$)

The effective radius is defined to enclose half the total light.

Because it is effectively impossible to measure I_0 due to a finite point spread function (PSF) for any observatory, this profile is often written as,

$$I(R) = I_e \exp\left[-\beta_n \left\{\left(\frac{R}{R_e}\right)^{1/n} - 1\right\}\right]$$

where $I_e = I(R_e)$.

In more observationally friendly units,

$$\mu(R) = \mu_e + 1.086\, \beta_n \left[\left(\frac{R}{R_e}\right)^{1/n} - 1\right]$$

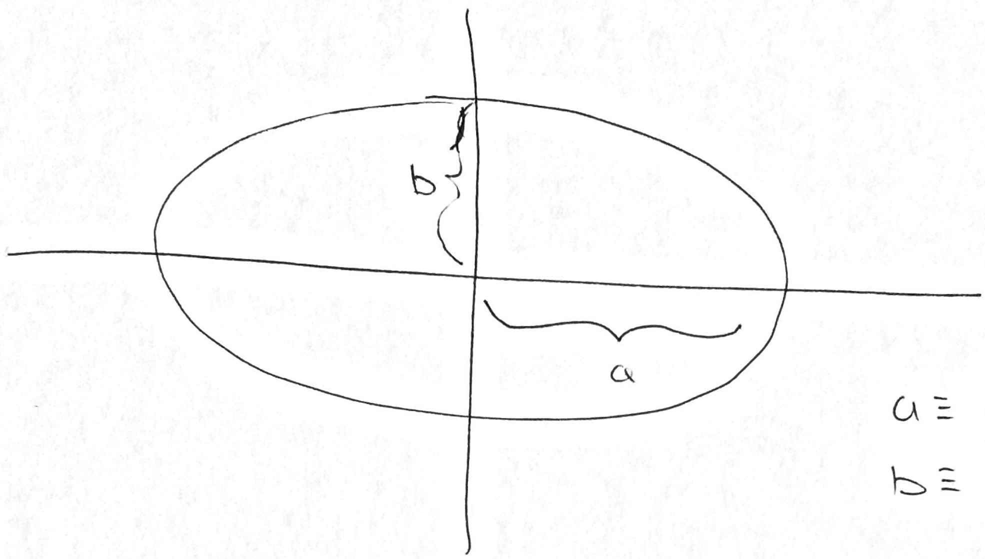
There are two special cases that are important to mention

- $n = 1$ (exponential profile)
- $n = 4$ (de Vaucouleurs profile)

The $n = 4$ is a classic elliptical galaxy profile, although n varies amongst the elliptical population. Note that higher sersic index galaxies are more centrally concentrated.

* Observed Galaxy Shapes

It is common to characterize galaxy shapes by the shapes of their observed isophotes. As you might expect, elliptical galaxies are commonly fit with elliptical isophotes. In addition to the physical size of a given isophote, an elliptical isophote is characterized by an axis ratio, b/a, or ellipticity.



$a \equiv$ major axis
$b \equiv$ minor axis

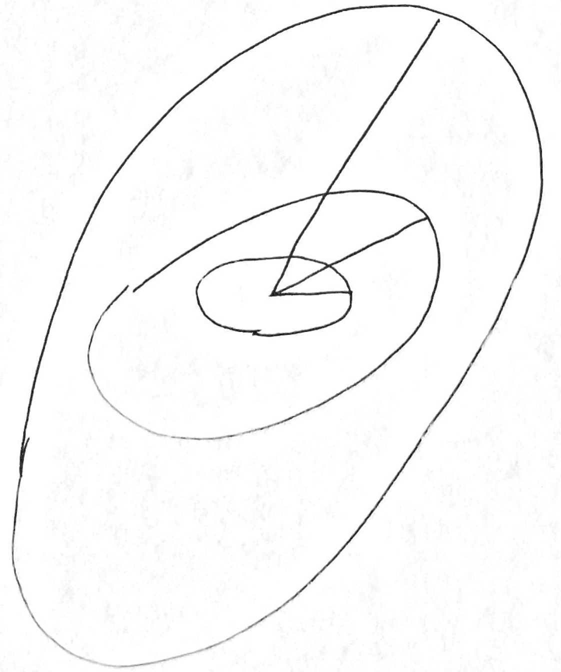
$$\frac{b}{a} \leq 1$$

$$\varepsilon \equiv 1 - \frac{b}{a} \quad (\text{ellipticity})^*$$

* note that their are multiple definitions of ellipticity. For weak lensing studies it is common to use a different definition:

$$\varepsilon' = \left(1 - \frac{b^2}{a^2}\right)^{1/2}$$

In the simplist case, a galaxy could be described by a series of homologous ellipsoids with equivalent ellipticity. For most elliptical galaxies, the ellipticity does not change drastically with 1-2 Re. It should also be noted that most elliptical galaxies show some amount of isophotal twisting, a change in position angle of the elliptical isophote as a function of R.



Isophotal Twisting

Deviations from perfectly elliptical isophotes can also be characterized. This can be done, for example, by comparing the best fit elliptical isophote to the observed isophote.

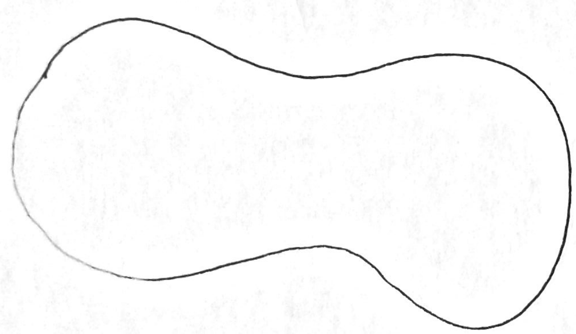
$$\Delta(\phi) \equiv \overset{\text{observed}}{R_{iso}(\phi)} - \underset{\text{best fit}}{R_{ell}(\phi)}$$

These deviations can be quantified by the Fourier coefficients of a function

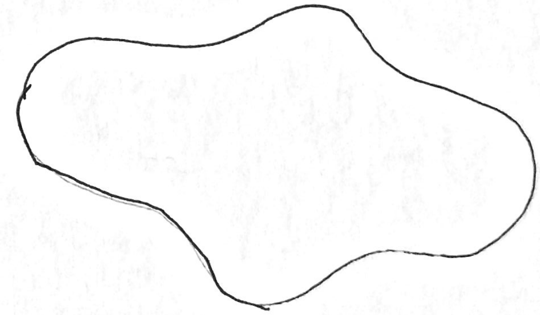
$$\Delta(\phi) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos(n\phi) + b_n \sin(n\phi)\right)$$

$R_{ell}(\phi)$ is taken to be a "best fit" ellipse such that a_0, a_1, a_2 and b_1, b_2 are consistent with zero.

Deviations from elliptical are then characterized by higher order coefficients a_n, b_n for $n \geq 3$. For example, a_4 is often used to characterize disky-ness ($a_4 > 0$) and boxiness ($a_4 < 0$).



Boxy



Disky

* Disk (spiral) galaxies

Unlike elliptical galaxies, spiral galaxies most often clearly have two distinct components

- disk
- bulge

The surface brightness of the disk component is often well fit by a sersic profile with $n=1$,

$$I(R) = I_0 \exp\left(-R/R_d\right)$$

where $I_0 = \dfrac{L}{2\pi R_d^2}$

Note that R is the cylinderical radius and R_d is the exponential scale length, $R_d \simeq \dfrac{R_e}{1.67}$.

If disk galaxies were infinitesimally thin, then this would be sufficient to describe them. However, edge-on observations of disks indicate they have verticle structure. One way to describe a disk galaxy's 3-D structure is then,

coordinate perpendicular to disk.

$$\nu(R, z) = \nu_0 \exp\left(-R/R_d\right) f_n(z)$$

luminosity density

The verticle structure is then described (independant of R) as

$$f_n(z) = \operatorname{sech}^{2/n}\left(\frac{n|z|}{2z_d}\right)$$

Here n controls the structure near $z=0$.
$n=\infty$ corresponds to an exponential profile.