33-777

Today

- Galaxy Dynamics
- Population Statistics

- Dynamics of Galaxies

Spectroscopic observations of galaxies can provide a wealth of knowledge that is not accessible with imaging data alone. One important example is information about the dynamics of stars and gas within a galaxy. Spectroscopic data for galaxies comes in three flavors:

- long slit observations
- optical fiber-fed spectrographs
- radio observations

Each of these methods can provide varying level of spatially resolved information ranging from

- none (0-D)
  a single optical fiber placed on an object
- 1-D
  long slit observations

- 2-D

radio <u>channel maps</u> or

integral field unit (IFU) spectrographs

Spectroscopy can probe the dynamics of gas or stars by measuring the doppler shifts of known spectroscopic features in the form of:

1. emission lines
2. absorption lines

Different methods, ~~etc~~ and instruments operating in different wavelength regimes acheive different spectral resolutions, quantified as the resolving power,

$$R = \frac{\lambda}{\Delta\lambda}$$

where $\Delta\lambda$ is the smallest difference in wavelength distinguishable in the spectrum.

For dynamical studies, then we are often more interested in the velocity difference that can be measured. In this case,

$$\Delta v = \frac{c}{R}$$

Generally radio telescopes acheive higher resolutions, $\Delta v \sim 1-10$ km/s. Optical and near-infrared (NIR) are often limited to $\Delta v \sim 10$ km/s. Note that these limits can be pushed - for example exoplanet searches in the optical can now reach ~ 10 cm/s. Also note that for faint sources there is a trade-off between resolving power and signal-to-noise.

Let's discuss two applications, the determination of :

1. Galaxy Rotation Curves
2. line-of-sight velocity dispersion

First, let's talk about galaxy rotation curves, mostly applicable to disk galaxies. From 2-D spectra a <u>velocity field</u> can be constructed. Similarly, from a 1-D spectra from a long slit appropiatly orientated on a galaxy, a slice of the velocity field can be measured. For a disk galaxy that is rotating and viewed with an inclination angle, $i \neq 0$, the velocity field will show approaching and receding sides of the galaxy with respect to a systemic velocity.

The observed line-of-sight velocity is then given by,

$$V_{los}(x,y) = V_{sys} + V_{rot}(R)\sin(i)\cos(\theta)$$

observed velocity on the plane of the sky

$$V_{sys} \simeq H_0 r + V_{pec,r}$$

$V_{rot}(R) \equiv$ "rotation curve"

$i \equiv$ inclination angle

$\theta \equiv$ azimuth angle in the disk as measured from the disk

Rotation curves can be used to determine mass distribution within galaxies if we assume that stars and/or gas are/is on circular orbits. Another way to state this is that the stars/gas are kinematically cold

$$\frac{V}{\sigma} \gg 1 \quad , \ \sigma \equiv \text{velocity dispersion}$$

In this case $V_{rot}(R) = V_c(R)$, where $V_c(R)$ is the circular velocity. Recall that the circular velocity is defined such that the centrifugal force balances the gravitational force,

$$V_c^2(R) = R|\vec{F}_R|$$

radial force per unit mass

Because this radial force may be thought of as the sum of forces due to different components in a galaxy, e.g. disk, bulge, halo, etc, we can always write the circular velocity as a sum of components, e.g.

$$V_c^2(R) = V_{c,disk}^2(R) + V_{c,bulge}^2(R) + V_{c,halo}^2(R)$$

Recall that for spherically symmetric mass distributions,

$$V_c^2(r) = r\frac{d\phi}{dr} = \frac{GM(r)}{r}$$

where ϕ is the gravitational potential defined via Poisson's equation

$$\nabla^2\phi = 4\pi G\rho$$

and $M(r)$ is total enclosed mass,

$$M(r) = \int_0^r 4\pi (r')^2 \rho(r')\, dr'$$

It is easy to show how the circular velocity scales for different spherical mass distributions. For example:

* point mass

$$V_c \propto \frac{1}{\sqrt{r}}$$

* singular isothermal sphere (SIS)

$$\rho(r) \propto \frac{1}{r^2}$$

$$\Rightarrow V_c = \text{constant}$$

* uniform density sphere

$$\rho(r) = \text{constant}$$

$$\Rightarrow V_c \propto r$$

Of course, not all (perhaps any) components in galaxies are spherically symmetric. Calculating the circular velocity becomes more difficult. Although for many purposes, assuming spherical symmetry will get you pretty far.

For an axisymmetric disk,

$$V_c^2(R) = R\left(\frac{\partial \Phi}{\partial R}\right)_{z=0} \quad \text{(in the plane of the disk)}$$

Taking, for example, an infinitesimally thin exponential disk, the surface mass density is given by,

$$\Sigma(R) = \Sigma_0 \exp(-R/R_d)$$

the gravitational potential is given by:

$$\Phi(R,z) = -2\pi G \Sigma_0 R_d^2 \int_0^\infty \frac{J_0(KR)\, e^{-K|z|}}{\left[1 + (KR_d)^2\right]^{3/2}}\, dK$$

Here J_0 is the zeroth order cylinderical Bezel function.

The circular velocity is then given by the pretty ugly expression:

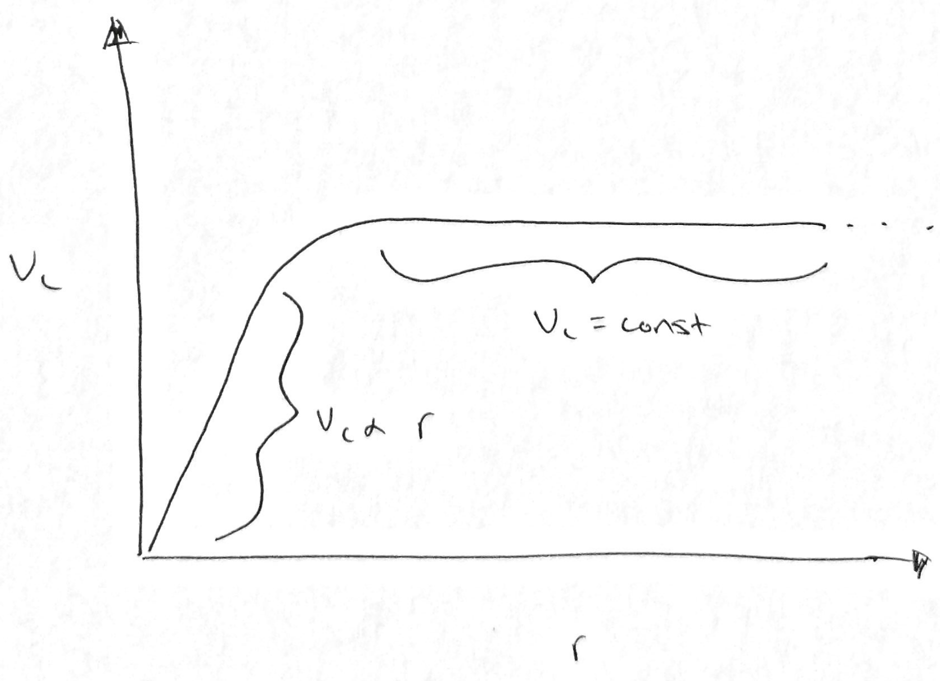
$$V_c^2(R) = -4\pi G \Sigma_0 R_d y^2 \left[I_0(y)K_0(y) - I_1(y)K_1(y) \right]$$

where $y = \dfrac{R}{2R_d}$

and I_n, K_n are modified Bessel functions of the first and second kind.

This rotation curve peaks at $2.16 R_d$ and at large R approaches a Keplerian profile.

For disky galaxies a typical rotation curve may look like,



Compared to disks, the bulge component of spiral galaxies and elliptical galaxies are dynamically hot with

$$\frac{v}{\sigma} \lesssim 1$$

For these types of systems, in addition to any rotation, we are also interested in the velocity dispersion of stars and gas. In order to measure the circular velocity in disk galaxies we were interested in the doppler shifted location of absorption/emission lines. In this case, we are interested in the <u>line profile</u> which contains information on the line-of-sight velocity distribution,

$$F(v_{los}, R)$$

The moments of this distribution contain a wealth of information.

The first moment is the mean velocity, $\bar{v}_{los}$, which tells us about any bulk motion like rotation. The second moment tells us about the velocity dispersion, σ_{los}.

Similar to the rotation velocity field maps we can make maps of the moments of the line-of-sight velocity distribution. The velocity dispersion is often characterized by the central velocity dispersion,

$$\sigma_0^2 = \frac{\int_S \sigma_{los}^2(R) I(R) d^2R}{\int_S I(R) d^2R}$$

which is the luminosity weighted velocity dispersion within a given aperature S. The area associated with a given aperature (circular) is then πR_{ap}^2 where it is typical for

$$R_{ap} \sim R_e$$

It is important to note that the line-of-sight velocity distribution is often not gaussian. In this case there is more information in the higher-order moments, e.g. about the <u>velocity anisotropy</u>.

The velocity dispersion measurements from elliptical galaxies gives us a way to estimate the mass of these galaxies. Given an approximate size and velocity dispersion for a typical elliptical galaxy,

$$R_e \sim 1 \text{ Kpc}$$
$$\sigma_0 \sim 100 \text{ km/s}$$

The crossing time (dynamical time) for a typical star is

$$t_{dyn} \sim \frac{R_e}{\sigma_0} \sim 10^7 \text{ yr} \ll t_{Hubble}$$

Given this, it is not unreasonable to assume the system is in near equilibrium. This allows us to apply the simple form of the scaler virial theorem,

$$2K + W = 0$$

If the system is entirely dispersion supported,

$$K = \frac{1}{2} M_{vir} \sigma_{vir}^2$$

and the potential is

$$W = - \frac{G M_{vir}^2}{r_g}$$

Application of the virial theorem gives

$$M_{vir} = \frac{r_g \sigma_{vir}^2}{G}$$

The gravitational radius, r_g, and the virial velocity dispersion are not directly observable (and perhaps ill-defined). We can relate these to the observables,

$$\sigma_0^2 = a\,\sigma_{vir}^2$$

$$R_e = b\,r_g$$

Then,

$$M_{vir} = \frac{K_{vir} R_e \sigma_0^2}{G}$$

$$K_{vir} \equiv \frac{1}{ab}$$

where K_{vir} depends on a variety of properties of the galaxy. For typical ellipticals,

$$K_{vir} \sim 1.5 - 7.5$$

Using a typical value of $K_{vir} \sim 5$, we can write the dynamical mass for elliptical galaxies as,

$$M_{dyn} \simeq (4.65 \times 10^{10} M_\odot)\left(\frac{R_e}{kpc}\right)\left(\frac{\sigma_0}{200\ km/s}\right)^2$$

Better estimates, taking into account velocity anisotropy and rotation can be acheived with more detailed dynamical modelling using the Mathematics for collisionless systems, e.g. Jeans Modelling.

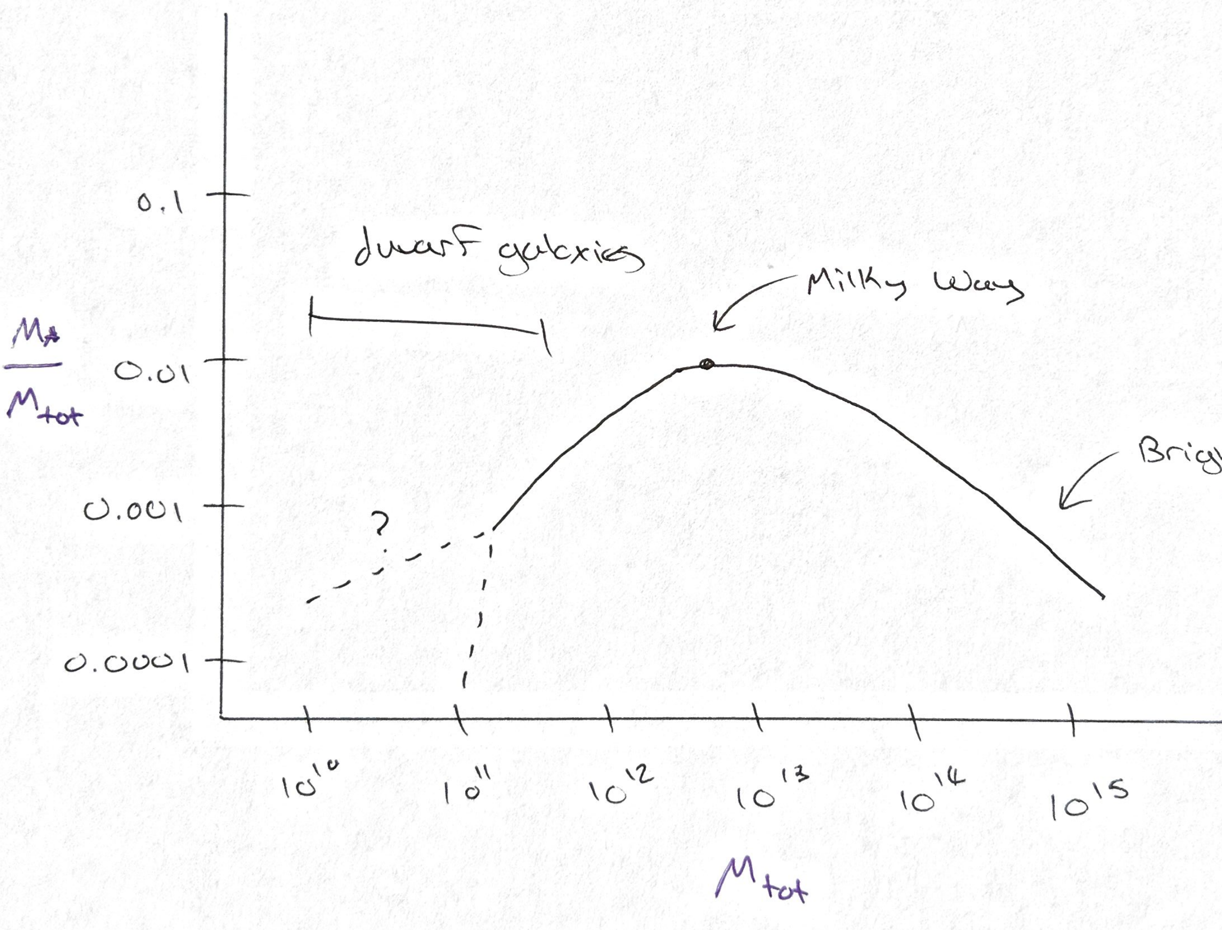
Similar to elliptical galaxies, we can use the dynamics of galaxies in a galaxy cluster to estimate the mass of the entire cluster, again using virial arguments,

$$M_{cluster} = \frac{K_{vir}\, \sigma_{los}^2\, R_{cluster}}{G}$$

Typical masses for the largest clusters are $M_{cluster} \sim 10^{15}\, h^{-1} M_{\odot}$.

From mass estimates for a large range of galaxies and clusters of galaxies we see that there is a characteristic light-to-mass ratio, with low mass and high mass galaxies being the most dominated by dark matter.

By measuring the masses of many galaxies, a relation between total stellar mass, (light) and dynamical mass has emerged.



A quick note on determing stellar mass of a galaxy. Often a stellar mass-to-light ratio is used to convert the observed luminosity of a galaxy in a certain band to an estimate for stellar mass

$$\Upsilon_*^x \equiv \frac{\left(\frac{M_*}{L_x}\right)}{\left(\frac{M_\odot}{L_\odot^x}\right)} \Bigg\} \text{ stellar mass-to-light ratio in X-band}$$

For a given galaxy, or component of a galaxy, the stellar mass is related to the total luminosity

$$M_* = \Upsilon_*^x \left(\frac{L_x}{L_\odot^x}\right)$$

The value of Υ_x^x depends on the stellar population

- Statistics of the galaxy population

The advent of large galaxy surveys has allowed us to learn a lot about the stastical distribution of galaxy properties.

One of the most basic tasks is to count galaxies. Typically this is done as a function of luminosity or stellar mass. The luminosity function is seen to take the form of a Schechter function,

$$\phi(L)\,dL = \phi^* \left(\frac{L}{L_*}\right)^\alpha \exp\left(-\frac{L}{L_*}\right) \frac{dL}{L_*}$$

$\phi(L)\,dL$ is the number density of galaxies with luminosities in the range $L \pm dL/2$.

ϕ^* controls the normalization

L_* is a characteristic luminosity

α controls the faint end power-law slope

While there are a wide variety of stastics to consider, two other important properties include:

- size distribution
- color distribution

Finally, large galaxy surveys have allowed us to map out the location of galaxies of all types throughout the Universe.

On large scales, galaxies appear to be nearly homogenously distributed with a characteristic structure called the cosmic web. Often this structure is characterized via "galaxy clustering". This can be quantified with the two-point correlation function (TPCF).

$$\xi(r) = \frac{DD(r)\Delta r}{RR(r)\Delta r} - 1$$

$DD \equiv$ # of galaxy pairs

$RR \equiv$ # of "random" pairs

To good approximation, in "real space", it is seen that the TPCF of galaxies is well fit by a power law,

$$\xi(r) = \left(\frac{r}{r_0}\right)^{-\gamma}$$

with $r_0 \sim 5\ h^{-1}\mathrm{Mpc}$

$\gamma \sim 1.8$