33-777

Today

- The Early Universe
- The Cosmic Microwave Background (CMB)
- Big Bang Nucleosynthesis (BBN)

In this lecture we will wrap-up our discussion of classic cosmology topics by reviewing some topics regarding the early universe, $z > z_{eq}$.

- The Early Universe

In the last few lectures we derived the Friedmann equation,

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{Kc^2}{a^2} + \frac{\Lambda c^2}{3}$$

wich governs the expansion history of the Universe, described through the scale factor, $a(t)$. A "cosmology" in this picture can be defined by specifying the "constituents" of the universe, e.g. matter, radiation, and dark energy, and the expansion rate. These quantities are specified by Ω_x and H_0. The evolution of a particular component depends on its equation of state,

$$P = w\rho_x c^2$$

The current expansion rate, the "Hubble constant" has been measured to be,

$$H_0 = H(t_0) = \frac{\dot{a}}{a_0} = 72 \pm 5 \ \frac{\mathrm{km/s}}{\mathrm{Mpc}}$$

Most (all?) cosmologies indicate that the universe has been monotonically expanding up till the present, e.g.

$$H(t < t_0) > 0$$

This suggests that in the past, some finite time ago, the Universe was in a singular state when $a \sim 0$ and the density of our cosmological fluid approaches infinity, $\rho \to \infty$. This point in the past is called the <u>Big Bang</u>.

In the last lecture we showed that at early times that radiation (or more generally relativistic particles) becomes the dominant component. This is easy to see since the density of matter scales as,

$$\rho_M = \rho_{M,0} \left(\frac{a_0}{a}\right)^3$$

while for radiation,

$$\rho_r = \rho_{r,0} \left(\frac{a_0}{a}\right)^4$$

In this high-density, hot, epoch of the Universe, matter and radiation are expected to be in thermal equilibrium. Radiation in thermal equilibrium has a blackbody spectrum with energy density, Stefan's constant

$$\rho_r = \frac{a_B}{c^2} T^4$$

blackbody temperature

At this point matter and radiation are tightly coupled (necessary for thermal equilibrium) primarily via Thomson scattering between free electrons and photons. As the Universe expands, the density and temperature must drop, and eventually the free electrons become bound to neutral atoms. At this point the opacity of the cosmological fluid to photons decreases to the point such that the Universe becomes transparent to photons.

We can conclude that the Universe was full of free-streaming blackbody photons. As the Universe continues to expand, this blackbody radiation field undergoes adiabatic expansion. As a result, the photons maintain a Planck distribution; however, the temperature must decrease. It is straight forward to show how this temperature evolves,

$$\left.\begin{aligned} \rho_\gamma &= \frac{a_B}{c^2} T^4 \\ \rho_r &= \rho_{r,0} \left(\frac{a_0}{a}\right)^4 \end{aligned}\right\} \Rightarrow \frac{T_0}{T} = \frac{a}{a_0}$$

That is, the temperature drops as $T \propto \frac{1}{a}$.

$\Rightarrow$ A predition of a big bang, FRW, cosmology is that the Universe should be bathed in blackbody photons.

In 1965, Penzias and Wilson discovered an isotropic blackbody source with,

$$T \simeq 2.725 \text{ K}$$

This has become known as the Cosmic Microwave Background (CMB).

Since the discovery of the CMB, a series of mostly space-based missions, e.g.

* COBE (1989 – 1993)
* WMAP (2001 – 2010)
* Planck (2009 – 2013)

produced increasingly detailed maps of the CMB. One important discovery was that the CMB was not precisely a uniform blackbody.

The temperature varies from direction-to direction by,

$$\frac{\Delta T}{T} \simeq 10^{-5}$$

For the most part, these variations are intrinsic to the properties at the "surface of last scattering". It should also be noted that photons in the CMB can be affected by various processes between the surface of last scattering and being observed. These effects can be distinguished as,

- primary CMB anisotropies
- secondary CMB anisotropies.

A significant portion of a full semester cosmology course can be dedicated to the interpretation of the CMB. We will just briefly touch on the topic here. Let's focus on temperature anisotropics, but note that other types of measurements are interesting, e.g. polarization.

Consider the temperature of the CMB in some direction ψ and some other direction an angular distance α away, $\psi + \alpha$. The correlation of these temperatures as a function of α can be written as,

$$C(\alpha) = \left\langle \underbrace{\frac{\Delta T}{T}(\psi)} \frac{\Delta T}{T}(\psi+\alpha) \right\rangle$$

$\frac{\Delta T}{T}$ as a function of ψ.

The angle brackets indicate an average over all directions, ψ, and all possible positions around those directions seperated by α.

This angular correlation function, $C(\alpha)$, can be expanded,

coefficient

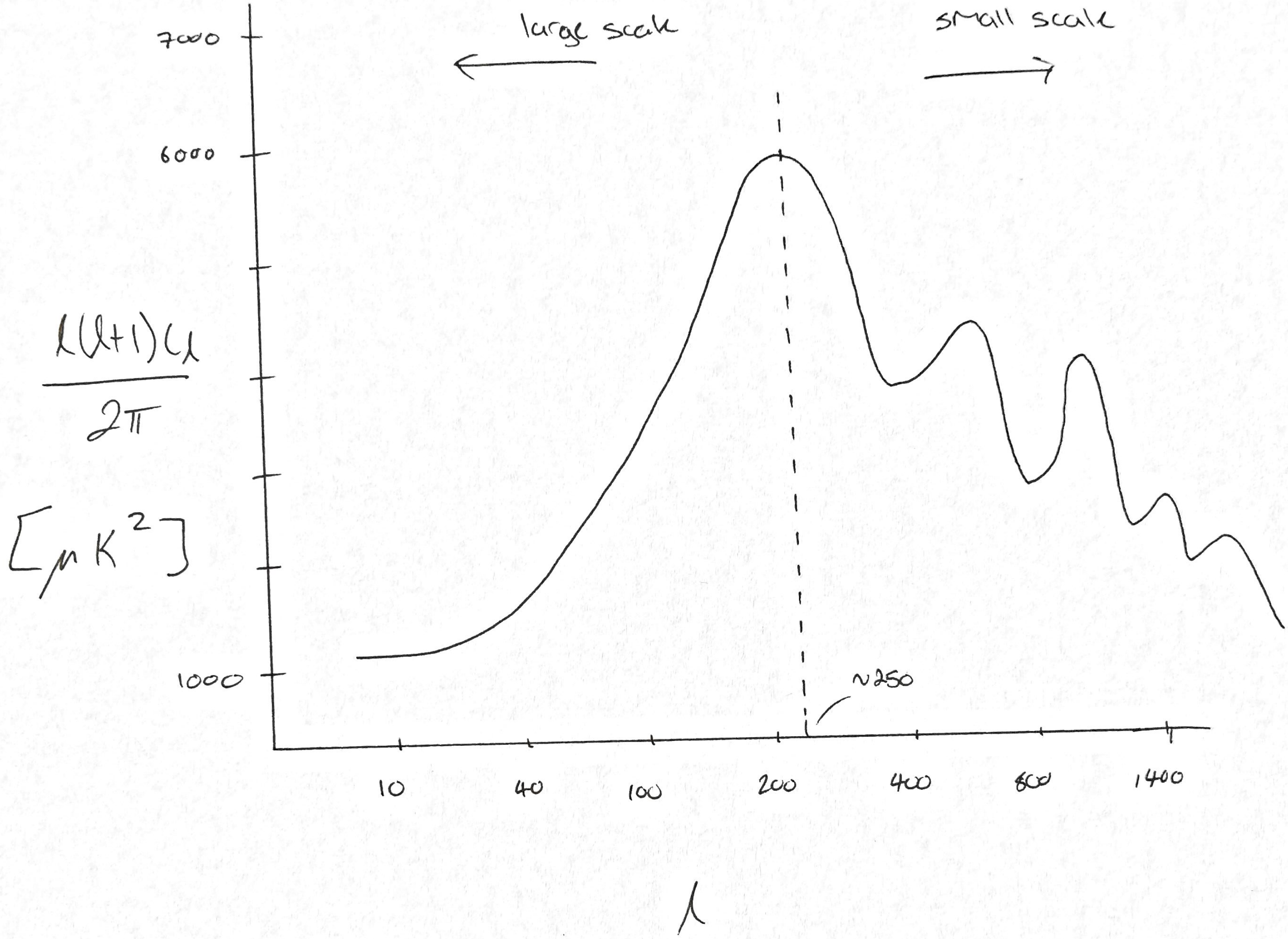
$$C(\alpha) = \sum_{l} \frac{(2l+1)}{4\pi} C_l P_l(\cos\alpha)$$

l^{th} Legendre polynomical

The value of C_l indicates how much "power" is in the temperature fluctuations on a scale,

$$\Delta\alpha \simeq \frac{\pi}{l}$$

A plot of the values of C_l measured for the CMB shows significant structure.



The first (and tallest) peak in the power spectrum occurs at $l \sim 250$. This corresponds to an angular size of,

$$\Delta\alpha \simeq \frac{\pi}{250} = 0.72^\circ \simeq 1^\circ$$

We can relate this to a physical size at the surface of last scattering via the angular diameter distance (see our discussion of distance measures in cosmology). To calculate this quantity we must know the redshift of the surface of last scattering. Also known as decoupling,

$$z_{dec} \simeq 1100$$

By assuming $\Lambda = 0$ (not so bad for $z \gg 1$)

$$\Delta\alpha \simeq (34.4'')\, \Omega_{M,0} h \left(\frac{D}{1\,\mathrm{Mpc}}\right)$$

For $\alpha \sim 1^\circ \Rightarrow D \sim 200\ \mathrm{Mpc}$.

It turns out that during the radiation dominated phase of expansion (to good approximation $z \geq z_{dec}$) only density perturbations larger than,

$$\underbrace{\lambda_J}_{\text{"Jeans wavelength"}} \simeq \sqrt{2}\,\pi c \underbrace{t}_{\text{age of the Universe}}$$

can grow. This is approximatly the Horizon scale at z_{dec}. The physical size of this scale is,

$$\Delta\alpha \simeq (0.87^\circ)\, \Omega_{M,0}^{1/2} \left(\frac{z_{dec}}{1100}\right)^{-1/2}$$

corresponds to the scale of the peak in the $\frac{\Delta T}{T}$ power spectrum.

If we include Λ in the analysis, the location of the first peak constrains:

$$\Omega_{\Lambda,0} + \Omega_{M,0} \simeq 1 \quad (\text{flat})$$

A more detailed analysis shows that the $\frac{\Delta T}{T}$ power spectrum constrains,

* $\Omega_{b,0} \sim 0.044$ (ratio of 1st to 2nd peaks)

* $\Omega_{M,0} \sim 0.26$ (ratio of 2nd to 3rd peaks)

- Relativistic Particles

The distribution function for fermions and bosons may be written as,

$$f(\vec{p}) = \left[\exp\left(\frac{E(\vec{p})-\mu}{K_B T}\right) \pm 1\right]^{-1}$$

+1 for fermions
-1 for bosons

where $E(\vec{p}) = (p^2c^2 + m^2c^4)^{1/2}$

From here we can derive, in the relativistic limit,

Bosons

$$n_B = \frac{\zeta(3)}{\pi^2} g \left(\frac{K_B T}{\hbar c}\right)^3 \quad \text{number density}$$

$$\rho_B = \frac{g}{2}\left(\frac{4\sigma}{c^3} T^4\right) \quad \text{mass-energy density}$$

$$s_B = \frac{2g}{3}\left(\frac{4\sigma}{c} T^3\right) \quad \text{entropy density}$$

Fermions

$$n_F = \frac{3}{4}\left[\frac{\zeta(3)\, g}{\pi^2}\left(\frac{KT}{\hbar c}\right)^3\right] \quad \text{number density}$$

$$\rho_F = \frac{7}{8}\left[\frac{g}{2}\left(\frac{4\sigma}{c^3}T^4\right)\right] \quad \text{mass-energy density}$$

$$s_F = \frac{7}{8}\left[\frac{2g}{3}\left(\frac{4\sigma}{c}T^3\right)\right] \quad \text{entropy density}$$

In the above $\zeta()$ is the Riemann zeta function. $\zeta(3) = 1.202$

g is the degeneracy for a particle in a momentum state $\vec{p}$. For electrons and photons $g=2$ corresponding to the two spin states for electrons and the two degrees of polarization for photons.

- Big Bang Nucleosynthesis (BBN)

In the early Universe we expect matter and radiation to be in equilibrium. Now we want to address how various constituents evolve as the Universe expands.

Consider some reaction,

$$A + B + C + \ldots \longleftrightarrow L + M + N + \ldots$$

can proceed both ways

Under certain circumstances we expect this reaction to reach chemical equilibrium when the backward and forward reaction rates balance. The condition for this equilibrium is,

chemical potential of particle "c".

$$\mu_A + \mu_B + \mu_C + \ldots \longleftrightarrow \mu_L + \mu_M + \mu_N + \ldots$$

$\Rightarrow$ The "chemical potentials" balance.

Under normal circumstances, given enough time this condition will be met. However, in the early Universe, expansion at the rate $H = \dot{a}/a$ causes this condition to continually change as the energy density of the species changes. It is reasonable to expect the condition for equilibrium to be met when,

$$\frac{\Gamma}{H} \gg 1$$

reaction rate

Since Γ is usually a function of ρ, as the Universe expands

$$\Gamma \gg H \longrightarrow \Gamma \ll H$$

universe expands

Note that the reaction rate

$$\Gamma = \sigma n c \propto T^2 T^3$$

Thus the reaction time-scale is,

$$t_r = \frac{1}{\Gamma} = \frac{1}{\sigma n c} \propto T^{-5}$$

We can compare this to the Hubble time,

$$t_H = \frac{1}{H(a)} \propto \frac{1}{T^2}$$

$\Rightarrow$ As the Universe expands and cools, eventually

$$t_r < t_H \longrightarrow t_r > t_H$$

consider the annihilation reaction,

$$\underbrace{X + \bar{X}}_{\text{particle + anti-particle pairs}} \leftrightarrow \underbrace{\gamma + \gamma}_{\text{protons}}$$

when $KT > mc^2$

- particles are relativistic
- reaction occurs in both directions
- number density of particles and photons is approximatly equal

when $KT < mc^2$

- non-relativistic
- forward reaction dominates
- results in a small excess in particles relative to anti-particles
- annihilation energy goes into photons and other relativistic particles

when kT drops below mc^2, the particle species "decouples".

* $e^+ + e^- \leftrightarrow \nu + \bar{\nu}$ $(T \sim 3\text{ MeV})$

* $p^+ + e^- \leftrightarrow n + \nu_e$

  $n + e^+ \leftrightarrow p^+ + \bar{\nu}_e$ $(T \sim 0.8\text{ MeV})$

* $e^+ + e^- \leftrightarrow \gamma + \gamma$ $(T \sim 0.5\text{ MeV})$

Finally, it is interesting to note that the ratio of baryon to photon number densities is constant once reactions that create or destroy baryons and photons cease.

$$\eta \equiv \frac{n_{b,0}}{n_{\gamma,0}} = (2.73\times10^{-8})\Omega_{b,0}h^2$$

Now that we have some of the basics written down, let's briefly consider "when" we expect atomic nuclei to form. Earlier we showed that for

$z > z_{dec}$,

$$T \propto \frac{1}{a}$$

Also recall that during the radiation dominated epoch

$$a \propto t^{1/2}$$

These relations can be combined to show that,

$$T = \left(\frac{3c^2}{32\pi G a_B}\right)^{1/4} t^{-1/2}$$

Filling in for constants,

$$T = (1.52 \times 10^{10}\ K)\left(\frac{t}{1s}\right)^{-1/2}$$

In terms of typical photon energy,

$$T = (1.31\ MeV)\left(\frac{t}{1s}\right)^{-1/2}$$

Recall from our discussion of stellar nucleosynthesis that typical fusion energies are of order ~ 1 MeV. This corresponds to an age of ~ 1 s after the big bang.

Let's consider the synthesis of helium in the early universe. In order to produce helium, there must be neutrons present. In the early universe protons and neutrons participate in reactions like,

$$n \longleftrightarrow p + e + \bar{\nu}$$

$$p + e \longleftrightarrow n + \nu$$

Recall that a free neutron has a half-life of ~ 13 min. The reaction rate for these reactions is given by,

$$\frac{\Gamma}{H} \simeq \left(\frac{T}{0.8\,\text{MeV}}\right)^3$$

Thus, we expect chemical equilibrium between protons and neutrons when,

$T \gg 0.8$ MeV

As the Universe cools and $T \to 0.8$ MeV, the ratio of nuetrons to protons is

$$\frac{n_n}{n_p} = \exp\left[\frac{-(m_n - m_p)c^2}{k_B T}\right]$$

At $T = 0.8$ MeV, the reaction rate drops and neutrons "freeze out" with a ratio given by,

$$\frac{n_n}{n_p} \simeq e^{-1.29/0.8} \simeq 0.2$$

These nuetrons can then be used to synthesize helium as long as they do not decay.

If we assume that all these neutrons are converted into ${}^4_2He$, then the mass fraction of helium would be

$$Y = \frac{2n_n}{n_n + n_p} = \frac{2(n_n/n_p)}{1 + (n_n/n_p)} = 0.33$$

More detailed calculations put $Y = 0.25$, close to the value measured in most stellar atmospheres.

In general, the mass fraction of heavier elements produced in BBN depends on η. Higher η means an increased baryon density and higher reaction rate. For most likely cosmologies, only up to ${}^7_3Li$ is produced (in trace amounts).