STEM Expo 2025

This worksheet is accompanied by web apps that can be found here (https://dundeemath.github.io/Admissions/).

1 Estimating π

Consider Figure 1. The ratio of the area of the circle to that of the circumscribed square is

$$\frac{\text{Area circle}}{\text{Area square}} = \frac{\pi}{4}.$$

Suppose we uniformly sample points from within the square. The probability of a point landing in the circle is proportional to the relative area of the circle, i.e.

$$\frac{\pi}{4}.$$

Hence we can estimate π by counting the number of uniformly sampled points that fall within the circle.

To sample the points:

1. Hold a small number of rice grains above the circle in Figure 1 and drop onto the page. Vary dropping height to ensure the rice grains land approximately uniformly in the square.
2. (i) Drop rice grains on to the square.
   (ii) Count the total number of rice grains that fall inside the square, N_S, and the circle, N_C.
   (iii) Input the data into Table 1 and compute the quantity

   $$\hat{\pi} = 4\frac{N_C}{N_S}.$$

   (iv) Repeat Steps (i)-(iii).
3. Compute the mean of the samples.
4. Explore computationally using the 'Estimating π' app.

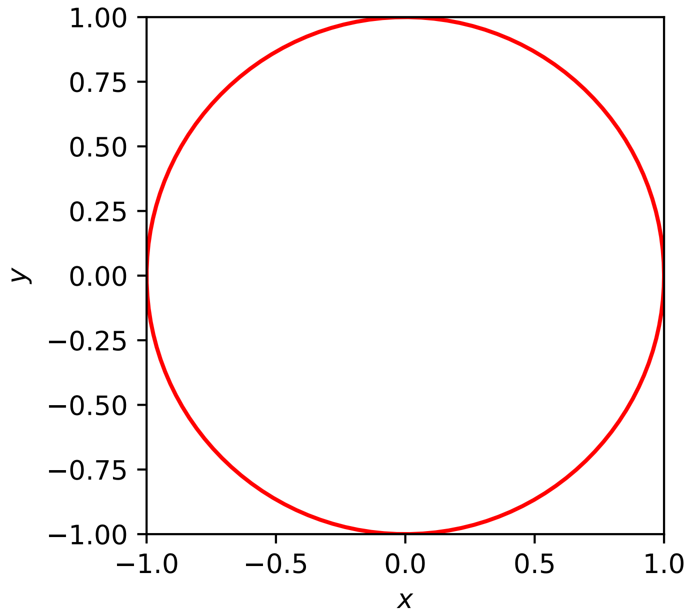

Figure 1

Table 1: A table to record counts.

Iteration	N_S	N_C	$\hat{\pi} = 4N_C/N_S$
1			
2			
3			
4			
5			
6			
—	———	——	——
Mean	N/A	N/A	

2 Estimating an integral

Consider Figure 2. Here we have plotted the function

$$y = x^2, \quad x \in [0, 1].$$

The shaded 'area under the curve' can be represented by the integral

$$I = \int_0^1 x^2 \mathrm{d}x.$$

To estimate the integral we again uniformly sample points from within the circumscribed square. By counting how many points fall inside the shaded region we can estimate the integral to be

$$\hat{I} = \text{Area square} \times \text{Frac points in shaded region} = 1 \times \frac{N_R}{N_S}$$

1. Hold a small number of rice grains above the square in Figure 2 drop onto the page.
2. Count the total number of rice grains that fall inside the square, N_S, and the shaded region, N_R.
3. Input data into Table 2 and compute the quantity

$$\frac{N_R}{N_S}.$$

4. Repeat Steps 1-3.
5. Compute the mean of the samples.
6. Explore computationally using the 'Monte Carlo integration' app.

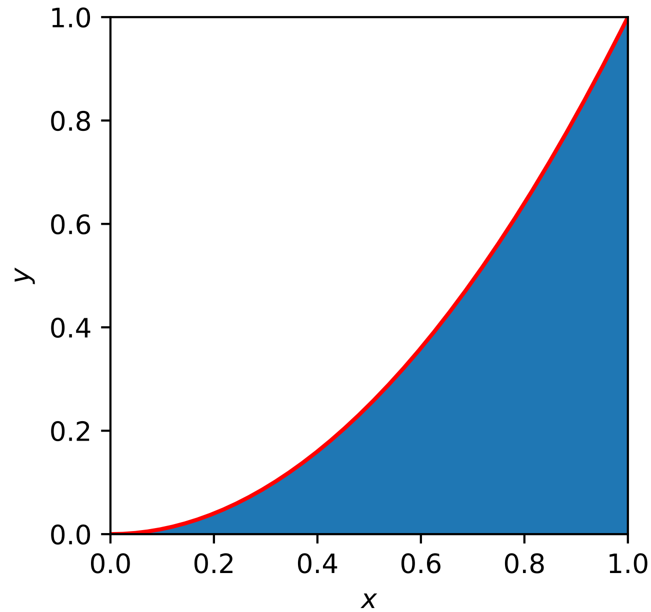

Figure 2

Table 2: A table to record counts.

Iteration	N_S	N_R	$\hat{I} = N_R/N_S$
1			
2			
3			
4			
5			
6			
—	—	—	—
Mean	N/A	N/A	