

# **D'Arcy Thompson and 2D mappings Worksheet**

We can represent a point,  $P$ , in the plane using a vector with components representing its  $x$  and  $y$  coordinates, i.e.

$$\mathbf{v}_P = [x_P, y_P]$$

represent a point with coordinates  $x_P$  and  $y_P$ .

A transformation is performed by defining a new point,  $P'$ , with new coordinates that are some functions of the old coordinates, i.e.

$$\mathbf{v}_{P'} = [x_{P'}, y_{P'}] = [f(x_P, y_P), g(x_P, y_P)].$$

where  $f(.,.)$  and  $g(.,.)$  are functions that will represent a particular transformation.

## Linear transformation

### Translation

Suppose that  $f$  and  $g$  are defined such that

$$f(x, y) = x + t_x$$

and

$$g(x, y) = y + t_y$$

where  $t_x$  and  $t_y$  are constants.

1. Compute the transformation of the point  $P(1, 2)$  in the case  $(t_x, t_y) = (3, 0)$ .
2. Identify the inverse of the transformation in 1.



## Scalings

Consider a scaling transformation defined such that

$$f(x, y) = a * x$$

and

$$g(x, y) = b * y$$

where  $b$  is a constant.

1. Compute the transformation of the point  $P(2, 1)$  in the case  $b = 2$ .
2. Identify the inverse of the transformation in 1. Are there values of  $b$  for which an inverse does not exist?



## Rotation

Now consider a transformation

$$f(x, y) = \cos \theta x - \sin \theta y$$

and

$$g(x, y) = \sin \theta x + \cos \theta y$$

where  $\theta$  is a constant value.

1. Compute the transformation of the point  $P(2,0)$  in the case  $\theta = 90 \deg(\frac{\pi}{2} \text{rad})$ .
2. Identify the inverse of the transformation in 1.



### Shear

$$f(x, y) = x + ay$$

and

$$g(x, y) = y$$

where  $a$  is a positive constant.

1. Compute the transformation of the point  $P(0,2)$  in the case  $a = 2$ .



### A general representation

If you have been introduced to vectors and matrices then you may spot that the above transformations can be represented as a matrix multiplication of a vector.

Consider the matrix

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}.$$

Define

$$\mathbf{v}_{P'} = A\mathbf{v}_P$$

Can you identify values of the parameters  $a_{11}$ ,  $a_{12}$  etc. that describe the rotation, shear and scaling transformations?