D'Arcy Thompson and 2D mappings
Worksheet

We can represent a point, P, in the plane using a vector with components representing its x and y coordinates, i.e.

$$\mathbf{v}_P = [x_P, y_P]$$

represent a point with coordinates x_P and y_P.

A transformation is performed by defining a new point, P', with new cooridnates that are some functions of the old coordinates, i.e.

$$\mathbf{v}_{P'} = [x_{P'}, y_{P'}] = [f(x_P, y_P), g(x_P, y_P)].$$

where $f(.,.)$ and $g(.,.)$ are functions that will represent a particular transformation.

Linear transformation

Translation

Suppose that f and g are defined such that

$$f(x, y) = x + t_x$$

and

$$g(x, y) = y + t_y$$

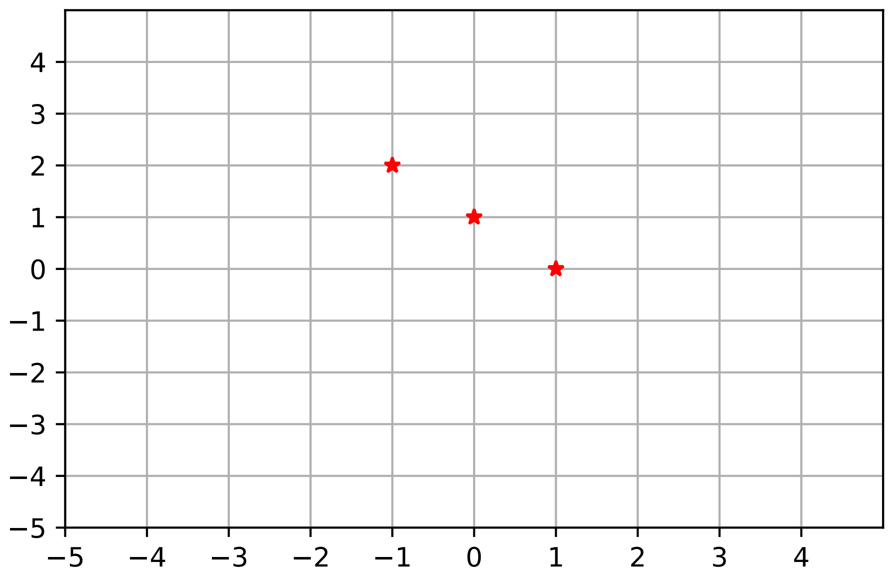
where t_x and t_y are constants.

1. Compute the transformation of the point $P(1, 2)$ in the case $(t_x, t_y) = (3, 0)$.
2. Identify the inverse of the transformation in 1.

```
import matplotlib.pyplot as plt
import numpy as np


fig,ax=plt.subplots()
ax.plot()
ax.set_xlim([-5,5])
ax.set_xticks(np.arange(-5,5))
ax.set_yticks(np.arange(-5,5))
ax.plot([-1,0,1],[2,1,0],'r*')
ax.set_ylim([-5,5])
ax.grid(True)

plt.show()
```



Scalings

Consider a scaling transformation defined such that

$$f(x, y) = a * x$$

and

$$g(x, y) = b * y$$

where b is a constant.

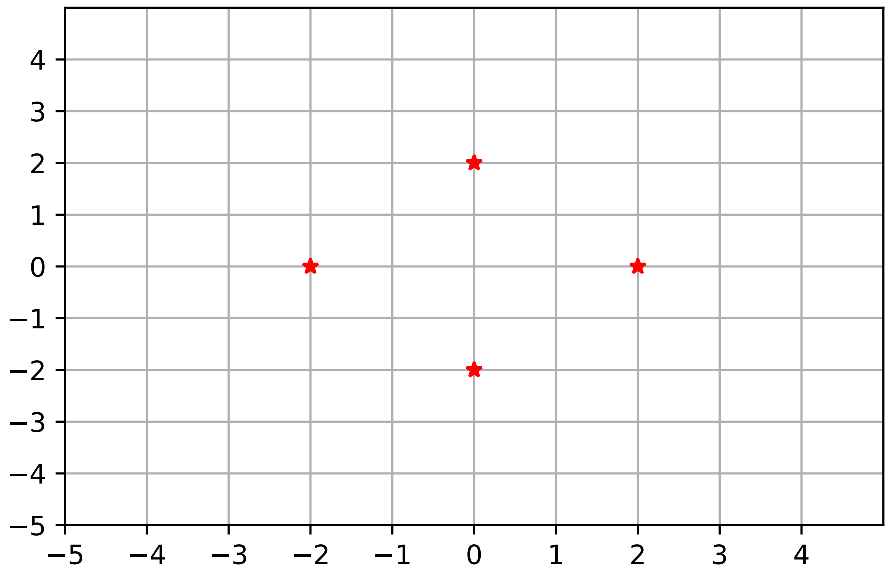
1. Compute the transformation of the point $P(2, 1)$ in the case $b = 2$.
2. Identify the inverse of the transformation in 1. Are there values of b for which an inverse does not exist?

```
import matplotlib.pyplot as plt
import numpy as np

fig,ax=plt.subplots()
ax.plot()
ax.set_xlim([-5,5])
ax.set_xticks(np.arange(-5,5))
ax.set_yticks(np.arange(-5,5))
ax.plot([2,-2,0,0],[0,0,2,-2],'r*')
ax.set_ylim([-5,5])
```

```
ax.grid(True)

plt.show()
```



Rotation

Now consider a transformation

$$f(x, y) = \cos\theta x - \sin\theta y$$

and

$$g(x, y) = \sin\theta x + \cos\theta y$$

where θ is a constant value.

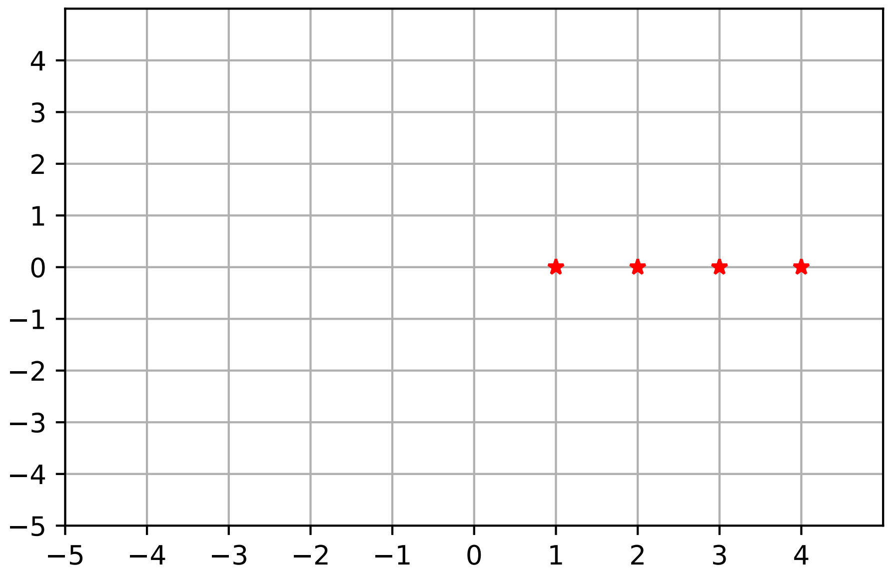
1. Compute the transformation of the point $P(2, 0)$ in the case $\theta = 90\deg(\frac{\pi}{2}rad)$.
2. Identify the inverse of the transformation in 1.

```
import matplotlib.pyplot as plt
import numpy as np

fig,ax=plt.subplots()
ax.plot()
ax.set_xlim([-5,5])
```

```
ax.set_xticks(np.arange(-5,5))
ax.set_yticks(np.arange(-5,5))
ax.plot([1,2,3,4],[0,0,0,0],'r*')
ax.set_ylim([-5,5])
ax.grid(True)

plt.show()
```



Shear

$$f(x, y) = x + ay$$

and

$$g(x, y) = y$$

where a is a positive constant.

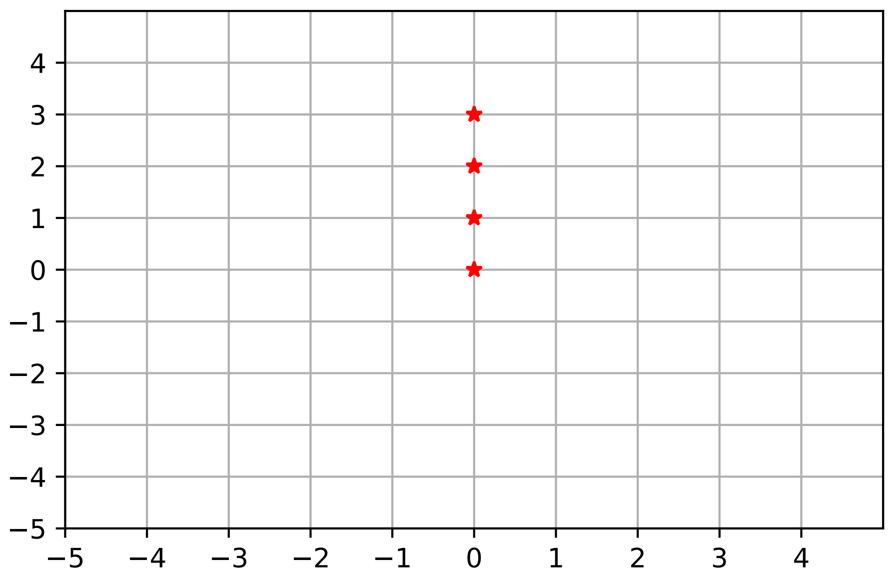
1. Compute the transformation of the point $P(0, 2)$ in the case $a = 2$.

```
import matplotlib.pyplot as plt
import numpy as np

fig,ax=plt.subplots()
ax.plot()
ax.set_xlim([-5,5])
```

```
ax.set_xticks(np.arange(-5,5))
ax.set_yticks(np.arange(-5,5))
ax.plot([0,0,0,0],[0,1,2,3],'r*')
ax.set_ylim([-5,5])
ax.grid(True)

plt.show()
```



A general representation

If you have been introduced to vectors and matrices then you may spot that the above transformations can be represented as a matrix multiplication of a vector.

Consider the matrix

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}.$$

Define

$$\mathbf{v}_{P'} = A\mathbf{v}_P$$

Can you identify values of the parameters a_{11}, a_{12} etc. that descibe the rotation, shear and scaling transformations?