

STEM Day December 2024

This worksheet is accompanied by web apps that can be found here (<https://dundeemath.github.io/Admissions/>).

1 Estimating π

Consider Figure 1. The ratio of the area of the circle to that of the circumscribed square is

$$\frac{\text{Area circle}}{\text{Area square}} = \frac{\pi}{4}.$$

Suppose we uniformly sample points from within the square. The probability of a point landing in the circle is proportional to the relative area of the circle, i.e.

$$\frac{\pi}{4}.$$

Hence we can estimate π by counting the number of uniformly sampled points that fall within the circle.

To sample the points:

1. Hold a small number of rice grains above the circle in Figure 1 and drop onto the page. Vary dropping height to ensure the rice grains land approximately uniformly in the square.
2.
 - (i) Drop rice grains on to the square.
 - (ii) Count the total number of rice grains that fall inside the square, N_S , and the circle, N_C .
 - (iii) Input the data into Table 1 and compute the quantity

$$\hat{\pi} = 4 \frac{N_C}{N_S}.$$

- (iv) Repeat Steps (i)-(iii).
3. Compute the mean of the samples.
4. Explore computationally using the 'Estimating π ' app.

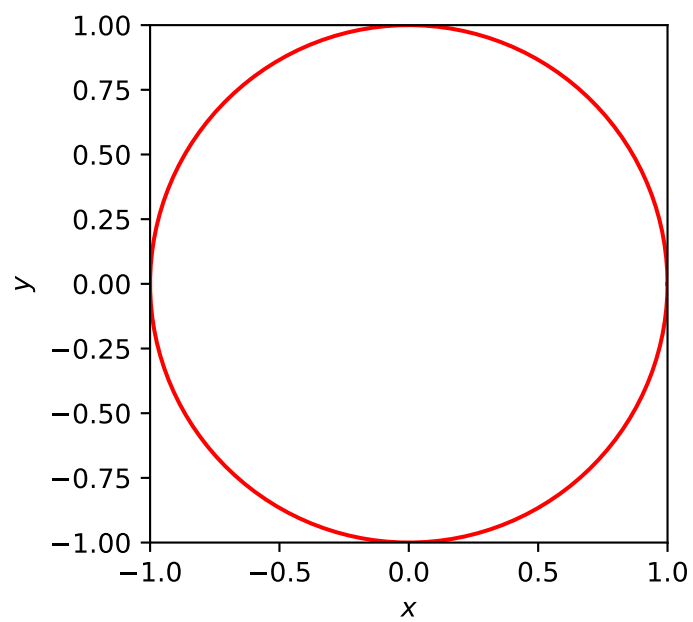


Figure 1

Table 1: A table to record counts.

| Iteration | N_S | N_C | $\hat{\pi} = 4N_C/N_S$ |
|-----------|-------|-------|------------------------|
| 1 | | | |
| 2 | | | |
| 3 | | | |
| 4 | | | |
| 5 | | | |
| 6 | | | |
| Mean | N/A | N/A | |

2 Estimating an integral

Consider Figure 2. Here we have plotted the function

$$y = x^2, \quad x \in [0, 1].$$

The shaded ‘area under the curve’ can be represented by the integral

$$I = \int_0^1 x^2 dx.$$

To estimate the integral we again uniformly sample points from within the circumscribed square. By counting how many points fall inside the shaded region we can estimate the integral to be

$$\hat{I} = \text{Area square} \times \text{Frac points in shaded region} = 1 \times \frac{N_R}{N_S}$$

1. Hold a small number of rice grains above the square in Figure 2 drop onto the page.
2. Count the total number of rice grains that fall inside the square, N_S , and the shaded region, N_R .
3. Input data into Table 2 and compute the quantity

$$\frac{N_R}{N_S}.$$

4. Repeat Steps 1-3.
5. Compute the mean of the samples.
6. Explore computationally using the ‘Monte Carlo integration’ app.

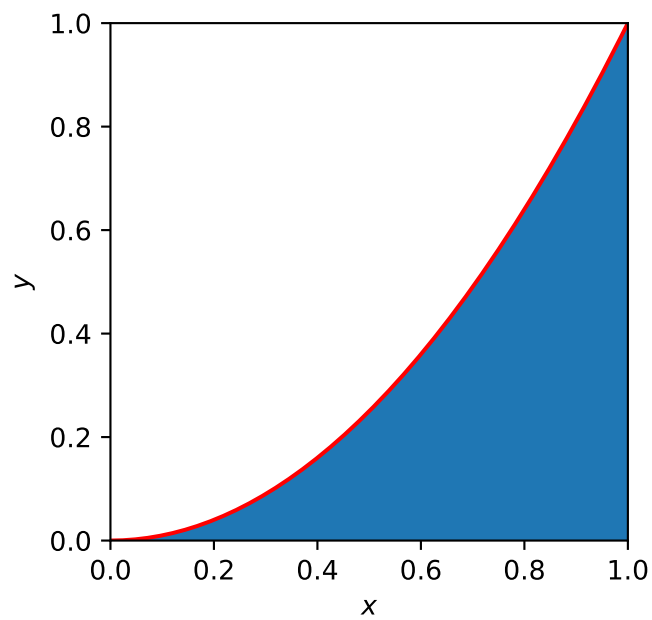


Figure 2

Table 2: A table to record counts.

| Iteration | N_S | N_R | $\hat{I} = N_R/N_S$ |
|-----------|-------|-------|---------------------|
| 1 | | | |
| 2 | | | |
| 3 | | | |
| 4 | | | |
| 5 | | | |
| 6 | | | |
| Mean | N/A | N/A | |