MA22004

Seminar 7

Dr Eric Hall
19/11/2020

Announcements

Reminders

- It is week 7! You should have read the remainder of §5 of the notes on **Perusall**.
- Class test 2 next week.

Comparing many means

We would like to consider k groups/treatment populations

with means $\mu_1, \ldots, \mu_k$.

Based on samples from these k groups, how can we determine whether the means are equal across each of the groups?

> Test procedure based on comparing a measure of difference in variation among the sample means to a measure of variation within each sample.

Variability partitioning

We consider different factors that contribute to variability in our response variable.

> Next week you will all take Class Test 2. What are some factors that might influence one's performance on the test?

Variability partitioning : class test

A number of factors might influence performance:

- Hours
- Completing all components (worksheets, labs, perusall)
- Pre-exposure

If we wanted to consider how strongly completing all components might influence performance, we can partition variability in performance (score) into:

variability due to completing all components and

variability due to all other factors (explanatory variables).

Salary data

Average Salary Data reported from 20 local councils.

Nation	Average salaries ('000 £)	Size (m_i)	Sample Mean $(\overline{x}_i)$	Sample SD (s_i)
England	17, 12, 18, 13, 15, 12	6	14.5	2.588
N Ireland	11, 7, 9, 13	4	10.0	2.582
Scotland	15, 10, 13, 14, 13	5	13.0	1.871
Wales	10, 12, 8, 7, 9	5	9.2	1.924

> What are our groups? What means might we want to compare? What question might we want to ask?

Variability partitioning : salary data

Salary data : hypothesis test

H_0 : the average salary is the same accross all nations

$(\mu_S = \mu_E = \mu_{NI} = \mu_W)$

H_a : the average salaries differ between at least one pair of nations

Nation	Size (m_i)	Sample Mean $(\overline{x}_i)$	Sample SD (s_i)
England	6	14.5	2.588
N Ireland	4	10.0	2.582
Scotland	5	13.0	1.871
Wales	5	9.2	1.924
Total	20	46.7	8.965

Test statistic

$$F = \frac{\text{MSTr}}{\text{MSE}}$$

Computing test statistic

1. Sum of squares total (measures total variation in response variable)

$$\text{SST} = \sum_{i=1}^{m} (x_i - \bar{x})^2$$

2. Sum of squares treatment/group (measures variability between groups)

$$\text{SSTr} = \sum_{j=1}^{k} m_j (\bar{x}_j - \bar{x})^2$$

3. Sum of squares error (measures variability within groups)

$$\text{SSE} = \text{SST} - \text{SSTr}$$

Computing test statistic (degrees of freedom)

To go from sum of squares to mean sum of squares, we need to scale calculations with respect to sample size.

1. total degrees of freedom: $\mathrm{df}_{\mathsf{T}} = m - 1$
2. treatment/group degrees of freedom: $\mathrm{df}_{\mathsf{T}} = k - 1$
3. error degrees of freedom: $\mathrm{df}_{\mathsf{E}} = \mathrm{df}_{\mathsf{T}} - \mathrm{df}_{\mathsf{T}}$

Test statistic and P-value

$$F = \frac{\mathsf{MSTr}}{\mathsf{MSE}} = \frac{\mathsf{SSTr}/\mathrm{df}_{\mathsf{Tr}}}{\mathsf{SSE}/\mathrm{df}_{\mathsf{E}}}$$

P-value area under $\mathsf{F}(\mathrm{df}_{\mathsf{Tr}}, \mathrm{df}_{\mathsf{E}})$ to the right of computed statistic f.

```
pf(obsf, dfTr, dfE, lower.tail = FALSE)
```

- If P-value is small, we reject null hypothesis (i.e. we have sufficient evidence that at least one pair of population means are different from each other at level α)

- If P-value is large, we fail to reject null hypothesis (i.e. the data do not provide convincing evidence that at least one pair of population means are different from each other: the observed difference in the sample means are attributed to sampling variability)

Conditions for ANOVA

1. Independence

- within groups
- between groups (i.e. NOT paired)

1. Approximately normal

2. Equal variance (homoscedasticity)

Summary

Today we discussed single factor anova.

- Idea behind the test statistic (partitioning variability)
- Calculating test statistic
- Concluding hypothesis test
- Conditions for anova

That's a lot to take it!