MA32011 Lecture slides

Philip Murray

Lecture 1

Discrete v continuum time

Physics and biology

Autonomous v nonautonomous systems

Linear v nonlinear

Quantitative v qualitative

Phase space

- Phase space
- Phase point
- Trajectory
- Phase portrait

Fixed points and their stability

Fixed points are singular points in the vector/flow field

Uniqueness and existence

Suppose that

$$\dot{x} = f(x), \quad x(0) = x_0. \tag{1}$$

If f is continuously differentiable on an open interval D of the x axis and x_0 is a point in D, Equation 1 possesses a unique solution on some time interval $(-\tau, \tau)$.

Nondimensionalisation

- Rescale variables to obtain system with fewer free parameters

Numerical solutions - Forward Euler

$$x(t + \Delta t) = x(t) + \Delta t f(x(t)).$$

1D flows

Definition

Let $x = x(t)$.

$$\dot{x} = f(x). \tag{2}$$

It is assumed that f is smooth and real valued.

Exact, but not easily helpful

$$\dot{x} = \sin x.$$

$x(0) = \pi/4$, Describe solution behaviour as $t \to \infty$.

Fixed points

Let $x = x^*$ be a fixed point of Equation 2. At $x = x^*$

$$\dot{x} = 0 \implies f(x^*) = 0.$$

Example

Find all the fixed points of

$$\dot{x} = x^2 - 1,$$

Linear stability analysis

How do small perturbation about the fixed point evolve in time?