Lecture slides

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Lecture 1

- Introduction to MA42002
- Conservation equations
- Examples of spatially homogeneous models

Conservation equations

$$\begin{pmatrix}\text{rate of change}\\ \text{in the population density}\end{pmatrix} = (\text{spatial movement}) + \begin{pmatrix}\text{birth, growth, death,}\\ \text{production or degradation}\\ \text{due to chemical reactions}\end{pmatrix}$$

Spatially homogeneous models (MA32009 revision)

Example problem - bacteria in a dish

$$N(t + \Delta t) = \quad N(t) + \quad KN(t)\Delta t.$$

A model for cell growth under nutrient depletion

$$
\begin{aligned}
\frac{dN}{dt} &= K(c)N = \kappa c N, \\
\frac{dc}{dt} &= -\alpha \frac{dN}{dt} = -\alpha \kappa c N,
\end{aligned}
\tag{1}
$$

Leading to the logistic growth equation

The last equation can be rewritten as

$$\frac{dN}{dt} = \rho N \, (1 - \frac{N}{B}) \qquad\qquad N(0) = N_0, \tag{2}$$

Exercise

Consider a well mixed bio reactor.

A biologist cultures an initial cell population of size N_0 in the bioreactor for 72 h.

Cells undergo division with a period of 14 h.

Each cell produces a non-degradable waste product, W, at rate k_1.

When total waste levels exceed a threshold, W^*, cell division stops. Otherwise the cell population grows exponentially.

How many cells are there at the end of the experiment?

Model development

Model checklist

1. Variables (dependent, indepedent ?)
2. Schematic diagram - what processes are being modelled?
3. Governing equations?
4. Define model parameters?
5. Initial conditions?

Exercise solution

Lecture 2

The SIR model

Consider the SIR model equations:

$$\frac{dS}{dt} = -rIS,$$
$$\frac{dI}{dt} = rIS - aI,$$
$$\frac{dR}{dt} = aI.$$

What are the variables? What are the parameters?

Identify an expression for the reproduction number, R_0.

Hence explain why the condition $R_0 < 1$ is necessary to avoid an epidemic?

An activator inhibitor model

Consider the reqction schematic

$$2A + B \rightarrow A.$$

Assume that species A is produced at constant rate k_1 and degrades at rate k_2.

Assume that A is linearly degraded.

Hence obtain the ODEs

$$\frac{dA}{dt} = k_1 - k_2 A + k_3 A^2 B,$$
$$\frac{dB}{dt} = k_4 - k_3 A^2 B,$$

Identify the steady state of the ODEs.

Spatiotemporal models - derivation

Deriving a conservation equation

Spatiotemporal models - fluxes

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