Lecture slides

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Lecture 1

- ▶ Introduction to MA42002
- ▶ Conservation equations
- ▶ Examples of spatially homogeneous models

Conservation equations

$$\begin{pmatrix} \text{rate of change} \\ \text{in the population density} \end{pmatrix} = \begin{pmatrix} \text{spatial movement} \end{pmatrix}$$

$$+ \begin{pmatrix} \text{birth, growth, death,} \\ \text{production or degradation} \\ \text{due to chemical reactions} \end{pmatrix}$$

Spatially homogeneous models (MA32009 revision)

Example problem - bacteria in a dish

$$N(t + \Delta t) = \quad N(t) + \quad KN(t)\Delta t.$$

A model for cell growth under nutrient depletion

$$\frac{dN}{dt} = K(c)N = \kappa c N,$$
$$\frac{dc}{dt} = -\alpha \frac{dN}{dt} = -\alpha \kappa c N, \tag{1}$$

Leading to the logistic growth equation

The last equation can be rewritten as

$$\frac{dN}{dt} = \rho N \left(1 - \frac{N}{B}\right) \qquad N(0) = N_0, \qquad (2)$$

Can also consider other biological processes

Exercise

Consider a well mixed bio reactor.

A biologist cultures an initial cell population of size N_0 in the bioreactor for 72 h.

Cells undergo division with a period of 14 h.

Each cell produces a non-degradable waste product, W, at rate k_1.

When total waste levels exceed a threshold, W^*, cell division stops. Otherwise the cell population grows exponentially.

How many cells are there at the end of the experiment?

Model development

> **i Model checklist**
>
> 1. Variables (dependent, indepedent ?)
> 2. Schematic diagram - what processes are being modelled?
> 3. Governing equations?
> 4. Define model parameters?
> 5. Initial conditions?

Exercise solution

Recap

- Is course layout clear
- Introduction to conservation equation
- Deriving spatially homogeneous models

Lecture 2

The SIR model

Consider the SIR model equations:

$$\frac{dS}{dt} = -rIS,$$
$$\frac{dI}{dt} = rIS - aI,$$
$$\frac{dR}{dt} = aI.$$

What are the variables? What are the parameters?

Identify an expression for the reproduction number, R_0.

Hence explain why the condition $R_0 < 1$ is necessary to avoid an epidemic?

An activator inhibitor model

Consider the reqction schematic

$$2A + B \rightarrow A.$$

Assume that species A is produced at constant rate k_1 and degrades at rate k_2.

Assume that A is linearly degraded.

Hence obtain the ODEs

$$\frac{dA}{dt} = k_1 - k_2 A + k_3 A^2 B,$$
$$\frac{dB}{dt} = k_4 - k_3 A^2 B,$$

Identify the steady state of the ODEs.

Spatiotemporal models - derivation

Deriving a conservation equation

Spatiotemporal models - fluxes

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