Week 1

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Lecture 1

Conservation equations

$$\begin{pmatrix} \text{rate of change} \\ \text{in the population density} \end{pmatrix} = (\text{spatial movement}) + \begin{pmatrix} \text{birth, growth,} \\ \text{production or deg} \\ \text{due to chemical r} \end{pmatrix}$$

Deriving spatially homogeneous models

$$N(t + \Delta t) = \quad N(t) + \quad KN(t)\Delta t. \tag{1}$$

A model for cell growth under nutrient depletion

$$\begin{aligned} \frac{dN}{dt} &= K(c)N = \kappa c N, \\ \frac{dc}{dt} &= -\alpha \frac{dN}{dt} = -\alpha \kappa c N, \end{aligned} \tag{2}$$

Logistic growth equation

The last equation can be rewritten as

$$\frac{dN}{dt} = \rho N \, (1 - \frac{N}{B}) \qquad N(0) = N_0, \tag{3}$$

Exercise

Consider a well mixed bio reactor.

A biologist cultures an initial cell population of size N_0 in the bioreactor for 72 h.

Cells undergo division with a period of 14 h.

Each cell produces a non-degradable waste product, W, at rate k_1.

When total waste levels exceed a threshold, W^*, cell division stops. Otherwise the cell population grows exponentially.

How many cells are there at the end of the experiment?

Lecture 2

The SIR model

Consider the SIR model equations:

$$\frac{dS}{dt} = -rIS,$$
$$\frac{dI}{dt} = rIS - aI,$$
$$\frac{dR}{dt} = aI.$$

What are the variables? What are the parameters?

Identify an expression for the reproduction number, R_0.

Hence explain why the condition $R_0 < 1$ is necessary to avoid an epidemic?

An activator inhibitor model

Consider the reqction schematic

$$2A + B \to A.$$

Assume that species A is produced at constant rate k_1 and degrades at rate k_2.

Assume that A is linearly degraded.

Hence obtain the ODEs

$$\frac{dA}{dt} = k_1 - k_2 A + k_3 A^2 B,$$
$$\frac{dB}{dt} = k_4 - k_3 A^2 B,$$

Identify the steady state of the ODEs.

Spatiotemporal models - derivation

Deriving a conservation equation

Spatiotemporal models - fluxes