# STAT 421 Assignment #6

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## 4.8

### a.

For k to be a probability density function it must be equal to 1 so we have

$$\begin{aligned}
1 &= \int_{-\infty}^{\infty} f(y)dy \\
&= \int_0^1 ky(1-y)dy \\
&= k\int_0^1 y(1-y)dy \\
&= k\int_0^1 y\,dy - k\int_0^1 y^2 dy \\
&= \frac{k}{6} \\
k &= \boxed{6}
\end{aligned}$$

### b.

$$\begin{aligned}
P(0.4 \leq Y \leq 1) &= F(1) - F(0.4) \\
F(y) &= 6\int_0^y y(1-y)\,dy \\
&= y^2(3-2y) \\
P(0.4 \leq Y \leq 1) &= 1 - 0.352 = \boxed{0.648}
\end{aligned}$$

### c.

$$\begin{aligned}
P(0.4 \leq Y < 1) &= F(1) - F(0.4) \\
&= \boxed{0.648}
\end{aligned}$$

### d.

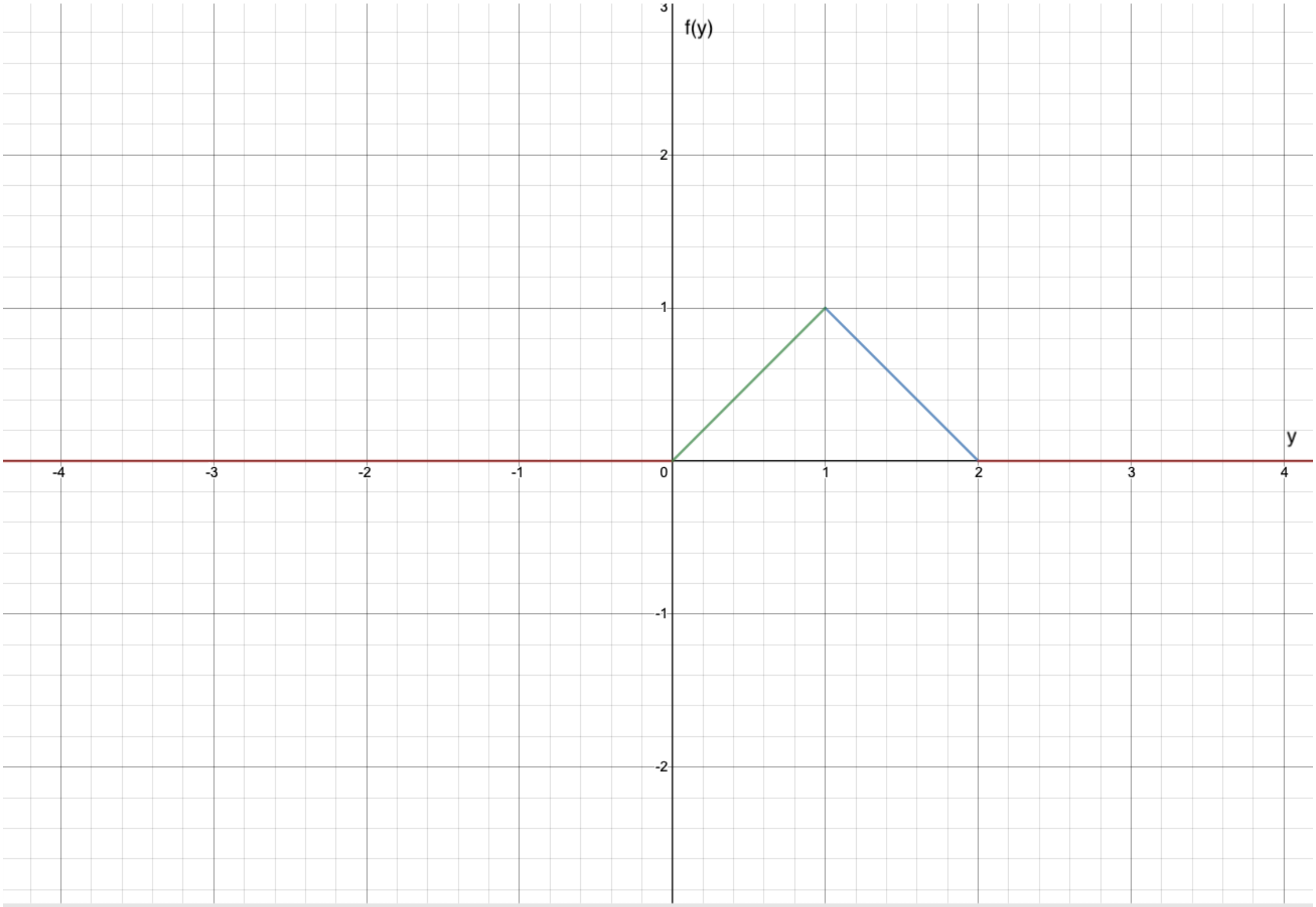
$$
\begin{aligned}
P(Y \leq 0.4 | Y \leq 0.8) &= \frac{P(Y \leq 0.4 \cap Y \leq 0.8)}{P(Y \leq 0.8)} \\
&= \frac{P(Y \leq 0.4)}{P(Y \leq 0.8)} \\
&= \frac{0.352}{F(0.8)} \\
&= \frac{0.352}{0.896} \\
&= \boxed{0.3928}
\end{aligned}
$$

**e.**

$$
\begin{aligned}
P(Y < 0.4 | Y < 0.8) &= \frac{P(Y < 0.4 \cap Y < 0.8)}{P(Y < 0.8)} \\
&= \frac{P(Y < 0.4)}{P(Y < 0.8)} \\
&= \boxed{0.3928}
\end{aligned}
$$

## 4.14

**a.**




Graph of f(y)

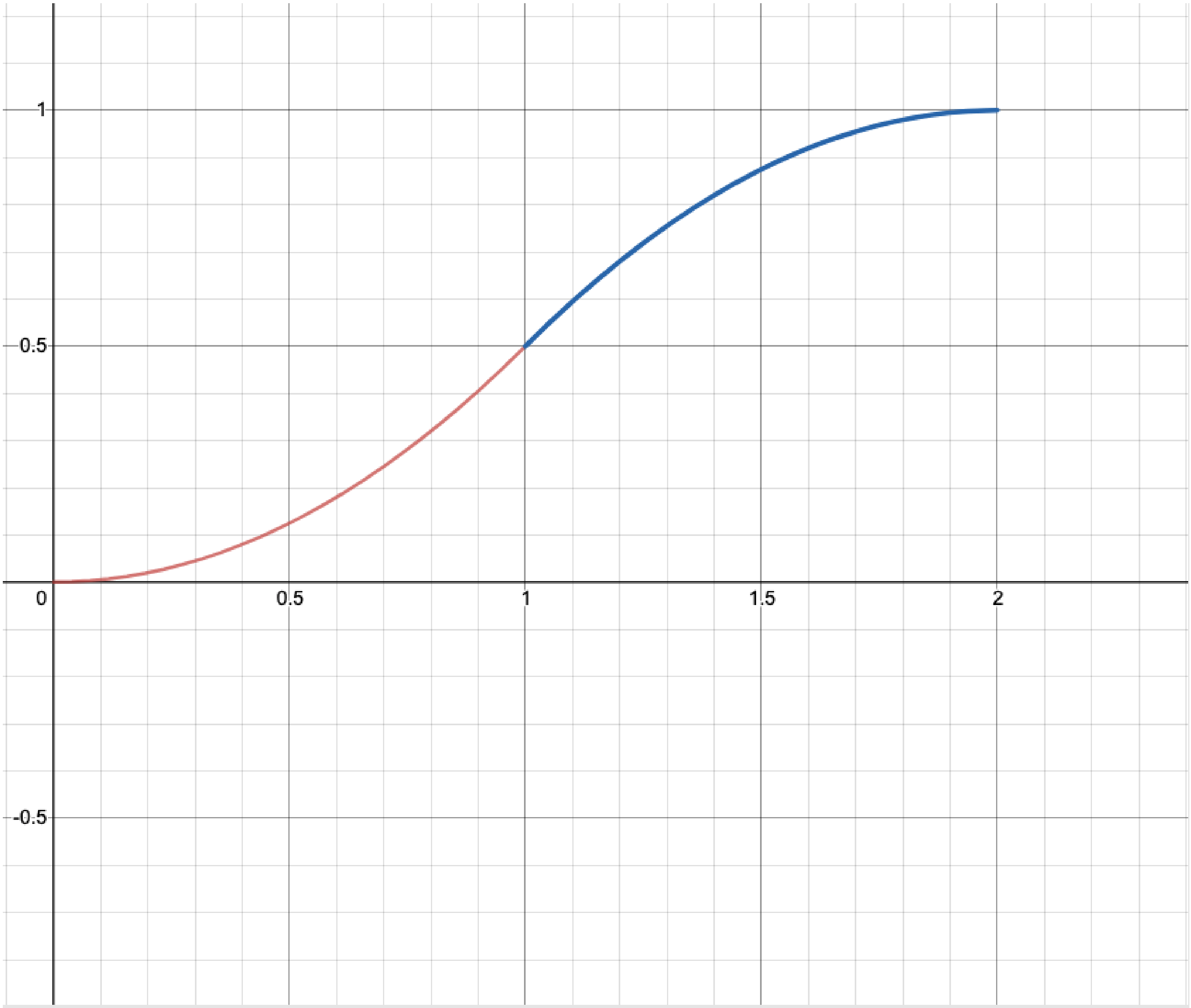
**b.**

For $0 < y < 1$ we have,

$$F(y) = \int_0^y y dy$$
$$= \frac{y^2}{2}$$

For $1 \leq y < 2$ we have,

$$F(y) = 2y - \frac{y^2}{2} - 1$$




Graph

**c.**

$$\begin{aligned} P(0.8 < Y < 1.2) &= P(1.2) - P(0.8) \\ &= \frac{1.2^2}{2} + 2(1.2) - 1 - \frac{0.8^2}{2} \\ &= \boxed{0.36} \end{aligned}$$

**d.**

$$
\begin{aligned}
P(Y > 1.5 | Y > 1) &= \frac{P(Y > 1.5 \cap Y > 1)}{P(Y > 1)} \\
&= \frac{P(Y > 1.5)}{0.5} \\
&= \frac{P(Y < 0.5)}{0.5} && \text{by symmetry we have} \\
&= \frac{0.5^3}{0.5} \\
&= \boxed{0.25}
\end{aligned}
$$

# 4.18

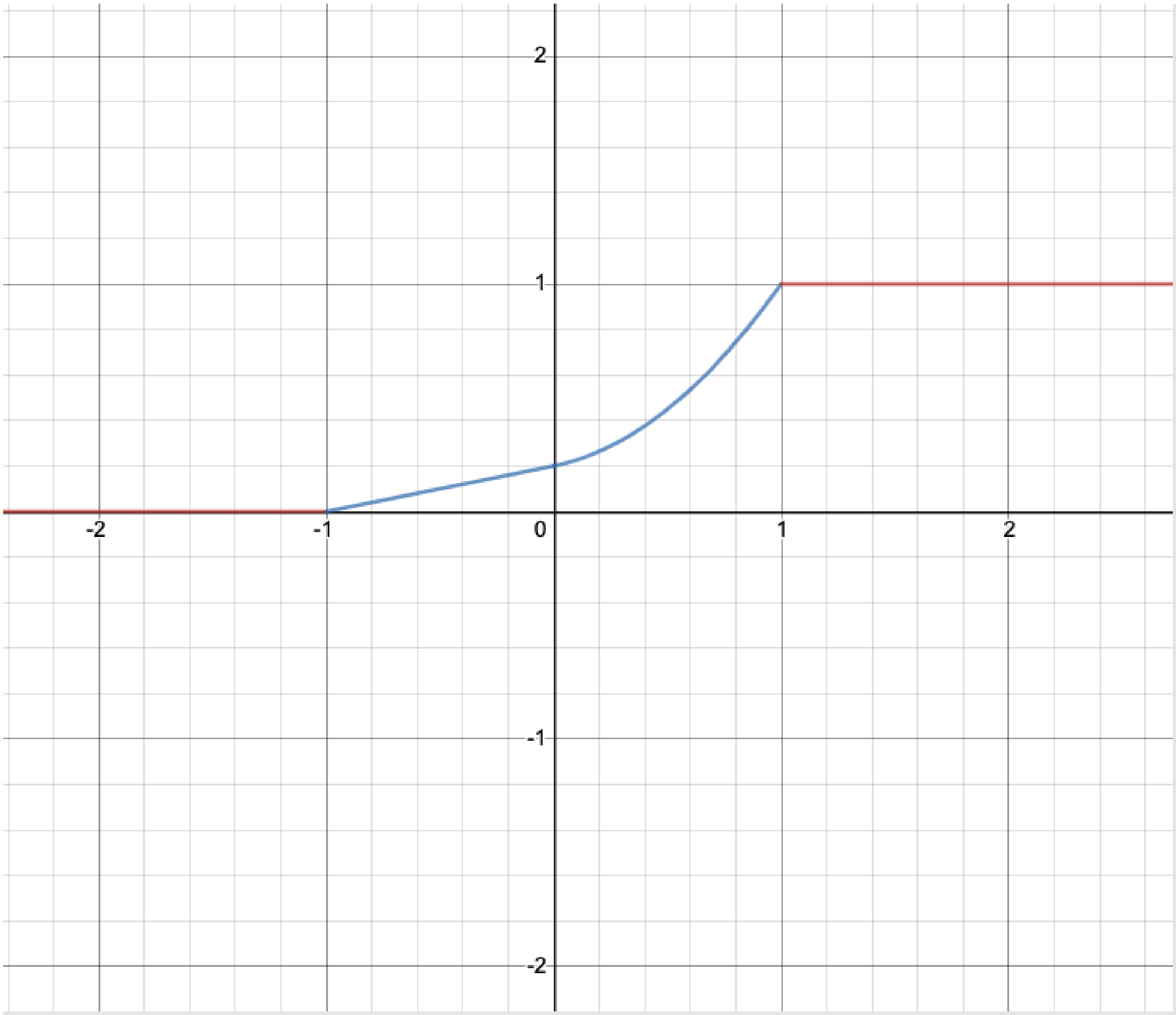
**a.**

$$
\begin{aligned}
1 &= \int_{-1}^{0} 0.2\, dy + \int_{0}^{1} (0.2 + cy) dy + 0 \\
&= 0.4 + 0.5c \\
c &= \boxed{1.2}
\end{aligned}
$$

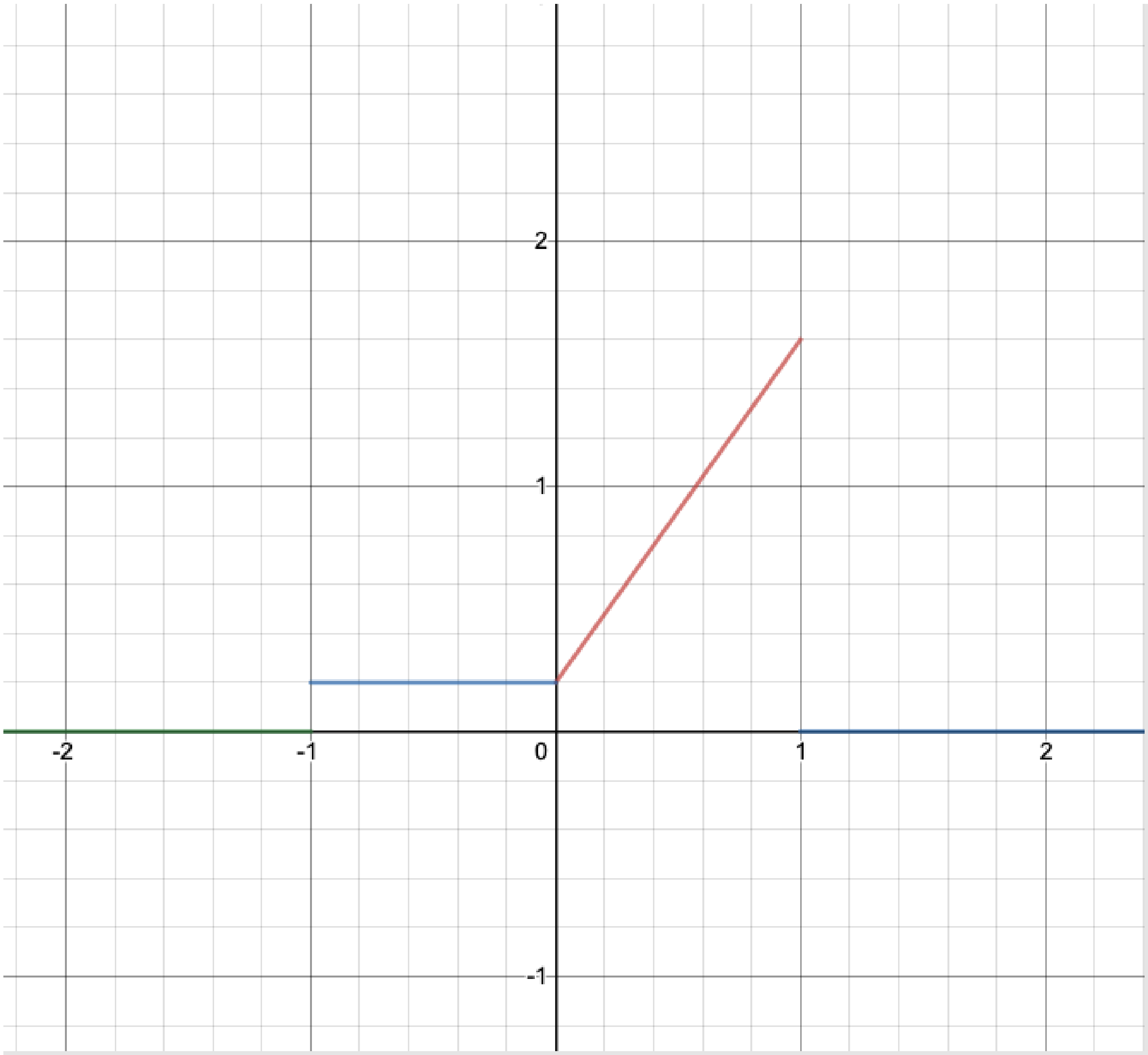
**b.**

$$
F(y) = \begin{cases} 0 & y \leq -1 \\ 0.2(1+y) & -1 \leq y \leq 0 \\ 0.2(1+y+3y^2) & 0 \leq y \leq 1 \\ 1 & y > 1 \end{cases}
$$

**c.**




Graph of F(y)




Graph of f(y)

**d.**

$$F(-1) = 0$$
$$F(0) = 0.2$$
$$F(1) = 1$$

**e.**

$$\begin{aligned} P(0 \le Y \le 0.5) &= 1 - P(Y > 0.5) \\ &= 1 - 0.55 \\ &= \boxed{0.45} \end{aligned}$$

**e.**

$$
\begin{aligned}
P(Y > 0.5|Y > 0.1) &= \frac{P(Y > 0.5 \cap Y > 0.1)}{P(Y > 0.1)} \\
&= \frac{P(Y > 0.5)}{P(Y > 0.1)} \\
&= \frac{0.55}{0.774} \\
&= \boxed{0.71}
\end{aligned}
$$

# 4.28

## a.

$$
\begin{aligned}
1 &= \int_{-\infty}^{\infty} f(y)dy \\
&= \int_{0}^{1} cy^2(1-y)^4 dy \\
&= c\int_{0}^{1} \left(y^2 - 4y^3 + 6y^4 - 4y^5 + y^6\right) dy \\
1 &= \frac{c}{105} \\
c &= \boxed{105}
\end{aligned}
$$

## b.

$$
\begin{aligned}
E[Y] &= \int_{-\infty}^{\infty} yf(y)dy \\
&= \int_{0}^{1} \left(y105y^2(1-y)^4\right) dy \\
&= 105\int_{0}^{1} \left(y^3 - 4y^4 + 6y^5 - 4y^6 + y^7\right) dy \\
&= 105 * \frac{1}{280} \\
&= \boxed{\frac{105}{280}}
\end{aligned}
$$

# 4.32

## a.

$$
\begin{aligned}
E[Y] &= \int_0^4 yf(y)dy \\
&= \int_0^4 y(\frac{3}{64}y^2(4-y))dy \\
&= \frac{3}{64}\int_0^4 (4y^3 - y^4)dy \\
&= \frac{3}{64} * 51.2 \\
&= \boxed{\frac{12}{5}}
\end{aligned}
$$

$$
\begin{aligned}
V(Y) &= E(Y^2) - E(Y)^2 \\
&= E(Y^2) - 2.4^2 \\
E(Y^2) &= \int_0^4 y^2 f(y)dy \\
&= \frac{3}{64}\int_0^4 (4y^4 - y^5)dy \\
&= \frac{3}{64}(\frac{4(4)^5}{5} - \frac{4^6}{6}) \\
&= 6.4 \\
V(Y) &= 6.4 - 2.4^2 \\
&= \boxed{0.64}
\end{aligned}
$$

**b.**

Given that the CPU time costs the firm \$200 per hour we find the expected value as

$$
\begin{aligned}
E(200Y) &= 200E(Y) \\
&= 200 * 2.4 \\
&= \boxed{480}
\end{aligned}
$$

and the expected variance as

$$
\begin{aligned}
V(200Y) &= 200^2 V(Y) \\
&= 200^2(0.64) \\
&= \boxed{25,600}
\end{aligned}
$$

**c.**

We determine if the cost will exceed 600 by testing

$$\begin{aligned} P(200Y > 600) &= P(Y > 3) \\ &= \int_3^4 f(y)dy \\ &= \int_3^4 (\frac{3}{64}y^2(4-y))dy \\ &= \frac{3}{64}(\frac{4}{3}(4^3-3^3) - \frac{1}{4}(4^4-3^4)) \\ &= 0.2617 \end{aligned}$$

So the cost will exceed $600 approximately 26.17% of the time which is not too often.

## 4.40

The probability based on the uniform distribution of landing past the midpoint is $\frac{1}{2}$ in 39. The probability that exactly one of the three lands past the midpoint is given by solving for the probability in a binomial distribution with parameters $n = 3$ and $p = \frac{1}{2}$. Let X be the number of parachutists that land past the midpoint (A,B)

$$\begin{aligned} P(X = 1) &= 3 * \frac{1}{2}^3 \\ &= \boxed{\frac{3}{8}} \end{aligned}$$

## 4.48

**a.**

$$\begin{aligned} P(475 < Y < 500) &= \int_{475}^{500} f(y)dy \\ &= \int_{475}^{500} \frac{1}{500}dy \\ &= \frac{500-475}{500} \\ &= \boxed{0.05} \end{aligned}$$

There is a 5% chance that she selects an area within 25 feet of the end of the line.

**b.**

$$
\begin{aligned}
P(0<y<25) &= \int_0^{25} f(y)dy \\
&= \int_0^{25} \frac{1}{500} dy \\
&= \frac{25-0}{500} \\
&= \boxed{0.05}
\end{aligned}
$$

The probability that the point selected is within 25 feet of the beginning of the line is 5%.

**c.**

$$
\begin{aligned}
P(0<y<250) &= \int_0^{250} f(y)dy \\
&= \int_0^{250} \frac{1}{500} dy \\
&= \frac{250-0}{500} \\
&= \boxed{0.5}
\end{aligned}
$$

There is a 50% chance that the point is closer to the beginning of the line than to the end of the line.

## 4.50

$$
\begin{aligned}
P((0<y<1)\cup(3<y<4)) &= P(0<y<1)+P(3<y<4) \\
&= \int_0^1 f(y)dy + \int_3^4 f(y)dy \\
&= \int_0^1 \frac{1}{5}dy + \int_3^4 \frac{1}{5}dy \\
&= \frac{1}{5}+\frac{1}{5} \\
&= \boxed{\frac{2}{5}}
\end{aligned}
$$

The probability that the center is up when the person calls is 40%.