STAT 421 Assignment #3

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2.172

a.

Given that $P(B) = P(A \cap B) + P(\overline{A} \cap B)$

We can reduce

$$\begin{aligned}
P(A|B) &= \frac{P(A \cap B)}{P(B)} \\
&= \frac{P(B) - P(\overline{A} \cap B)}{P(B)} \\
&= \frac{P(B)}{P(B)} - \frac{P(\overline{A} \cap B)}{P(B)} \\
&= 1 - \frac{P(\overline{A} \cap B)}{P(B)} \\
&= 1 - P(\overline{A}|B)
\end{aligned}$$

So the statement $P(A/B) + P(\overline{A}/\overline{B})$ is $\boxed{\text{False}}$

b.

By the above derivation the statement $P(A/B) + P(A/\overline{B})$ is $\boxed{\text{False}}$

c.

By the derivation in **a** the statement $P(A/B) + P(\overline{A}/B)$ is $\boxed{\text{True}}$

3.14

a.

$$
\begin{aligned}
E(Y) &= \Sigma\, y\, p(y) \\
&= 3*(0.03)+4*(0.05)+5*(0.07)+6*(0.10)+7*(0.14)+8*(0.20)+9*(0.18) \\
&+10*(0.12)+11*(0.07)+12*(0.03)+13*(0.01) \\
&= 7.90
\end{aligned}
$$

Therefore the mean patent life for a drug is $\boxed{7.90}$ years.

b.

$$
\begin{aligned}
SD(y) &= \sqrt{\text{Variance}(Y)} \\
&= \sqrt{E(Y^2)-(E(Y))^2} \\
&= \sqrt{\sum y^2\, p(y) - (7.90)^2} \\
&= \sqrt{3^2*(0.03)+4^2*(0.05)+5^2*(0.07)+6^2*(0.10)+7^2*(0.14)+8^2*(0.20)} \\
&\overline{+9^2*(0.18)+10^2*(0.12)+11^2*(0.07)+12^2*(0.03)+13^2*(0.01)-62.41} \\
&= \sqrt{67.14-62.41} \\
&= 2.17
\end{aligned}
$$

Therefore the standard deviation is $\boxed{2.17}$

c.

$$
\begin{aligned}
P(Y \text{ in interval } \mu \pm 2\sigma) &= P[\mu - 2\sigma < Y < \mu + 2\sigma] \\
&= P[\frac{(\mu-2\sigma)-\mu}{\sigma} < Z < \frac{(\mu+2\sigma)-\mu}{\sigma}] \\
&= P[\frac{(7.9-2*2.17)-7.9}{2.17} < Z < \frac{(7.9+2*2.17)-7.9}{2.17}] \\
&= P[-2 < Z < 2] \\
&= 0.9545
\end{aligned}
$$

Therefore the probability that Y falls in the interval $\mu \pm 2\sigma$ is 95.45%.

3.24

Let P(F) = 0.1 represent the bottle having a serious flaw, and $P(\overline{F}) = 1 - 0.1 = 0.9$ represent not having serious flaws.

The Mean is calculated as

$$
\begin{aligned}
E(Y) &= n * P(F) \\
&= 2 * 0.1 \\
&= \boxed{0.2}
\end{aligned}
$$

The variance is calculated as

$$\begin{aligned} E(Y) &= n * P(F) * P(\overline{F}) \\ &= 2 * 0.1 * 0.9 \\ &= \boxed{0.18} \end{aligned}$$

3.34

Given the cost of the tool is $10 we can find the mean as

$$\begin{aligned} \mu &= E(10Y) \\ &= 10E(Y) \\ &= 10(\Sigma y\, p(y)) \\ &= 10((0 * 0.1) + (1 * 0.5) + 2(*0.4)) \\ \text{Mean cost} &= \boxed{13} \end{aligned}$$

The variance can be calculated as

$$\begin{aligned} V(Y) &= V(10Y) \\ &= 10^2((E(y^2) - \mu^2)) \\ &= 100(E(Y^2) - \mu^2) \\ &= 100(\Sigma y^2 p(y) - (1.3)^2) \\ &= 100(((0^2 * .1) + (1^2 * .5) + (2^2 * .4)) - 1.69) \\ \text{Variance of cost} &= \boxed{41} \end{aligned}$$

3.40

The probability mass function of the binomial distribution is given as $(p)^x(1-p)^{(n-x)}$

a.

Therefore, the probability that 14 will recover is

$$\begin{aligned} P(Y = 14) &= \binom{20}{14}(0.8)^{14}(1-0.8)^{20-14} \\ &= \boxed{0.10909} \end{aligned}$$

b.

The probability that at least 10 recover is

$$P(Y \geq 10) = P(Y=10)+P(Y=11)+P(Y=12)+P(Y=13)+P(Y=14)$$
$$+P(Y=15)+P(Y=16)+P(Y=17)+P(Y=18)+P(Y=19)+P(Y=20)$$

$$P(Y=10) = \binom{20}{10}(0.8)^{10}(1-0.8)^{20-10} = 0.0020314137$$

$$P(Y=11) = \binom{20}{11}(0.8)^{11}(1-0.8)^{20-11} = 0.00738695892$$

$$P(Y=12) = \binom{20}{12}(0.8)^{12}(1-0.8)^{20-12} = 0.02216087676$$

$$P(Y=13) = \binom{20}{13}(0.8)^{13}(1-0.8)^{20-13} = 0.05454985049$$

$$P(Y=14) = \binom{20}{14}(0.8)^{14}(1-0.8)^{20-14} = 0.10909970097$$

$$P(Y=15) = \binom{20}{15}(0.8)^{15}(1-0.8)^{20-15} = 0.17455952156$$

$$P(Y=16) = \binom{20}{16}(0.8)^{16}(1-0.8)^{20-16} = 0.21819940195$$

$$P(Y=17) = \binom{20}{17}(0.8)^{17}(1-0.8)^{20-17} = 0.20536414301$$

$$P(Y=18) = \binom{20}{18}(0.8)^{18}(1-0.8)^{20-18} = 0.13690942867$$

$$P(Y=19) = \binom{20}{19}(0.8)^{19}(1-0.8)^{20-19} = 0.05764607523$$

$$P(Y=20) = \binom{20}{20}(0.8)^{20}(1-0.8)^{20-20} = 0.01152921505$$

$$P(Y \geq 10) = 0.0020314137 + 0.00738695892 + 0.02216087676$$
$$+ 0.05454985049 + 0.10909970097 + 0.17455952156$$
$$+ 0.21819940195 + 0.20536414301 + 0.13690942867$$
$$+ 0.05764607523 + 0.01152921505 = \boxed{\approx 0.999}$$

c.

The probability that at least 14, but no more than 18 recover is

$$P(14 \leq X \leq 18) = P(Y=14)+P(Y=15)+P(Y=16)+P(Y=17)+P(Y=18)$$
$$= 0.10909970097 + 0.17455952156 + 0.21819940195$$
$$+ 0.20536414301 + 0.13690942867 = \boxed{\approx 0.844}$$

d.

The probability that at most 16 recover is

$$\begin{aligned}P(X \leq 16) &= 1 - P(X > 16) \\ &= 1 - (P(17) + P(18) + P(19) + P(20)) \\ &= 1 - 0.20536414301 + 0.13690942867 + 0.05764607523 \\ &+ 0.01152921505 = \boxed{\approx 0.589}\end{aligned}$$

3.56

Given the chance of success is 0.1, and there are 10 explorations, the mean number of successful explorations can be calculated as,

$$\begin{aligned}\mu &= E(X) \\ &= np \\ &= 10 * 0.1 \\ &= \boxed{1}\end{aligned}$$

The variance can be calculated as

$$\begin{aligned}\sigma^2 &= V(X) \\ &= np(1-p) \\ &= 10 * 0.1 * (1 - 0.1) \\ &= \boxed{0.9}\end{aligned}$$

3.60

a.

The probability that exactly 14 fish survive is given by the probability mass function from the binomial distribution

$$\begin{aligned}P(X = 14) &= \binom{20}{14} * (0.8)^{14} * (0.2)^{20-14} \\ &= \boxed{0.109}\end{aligned}$$

b.

The probability that at least 10 fish survive is

$$
\begin{aligned}
P(X \geq 10) &= 1 - P(X > 10) \\
&= 1 - P(X \leq 9) \\
&= 1 - \sum_{x=0}^{9} \binom{20}{x} (0.8)^x (0.2)^{20-x} \\
&= \boxed{\approx 0.999}
\end{aligned}
$$

c.

The probability that at most 16 fish survive is

$$
P(X \leq 16) = \sum_{x=0}^{16} P(X \leq x) = \boxed{\approx 0.589}
$$

d.

The mean number of fish that survive is $20 * 0.8 = \boxed{16}$.

The variance of the number of fish that survive is $20 * (0.8)(1 - 0.8) = \boxed{3.2}$

3.66

a.

$$
\begin{aligned}
\sum_y p(y) &= \sum_{y=1}^{\infty} q^{y-1} p \\
&= q^{1-1} p + q^{2-1} p + q^{3-1} p \ldots \\
&= q^0 p + qp + q^2 p + \ldots \\
&= p(1 + q + q^2 + \ldots) \\
&= p * \frac{1}{1-q} \\
&= p * \frac{1}{p} && \text{as } p + q = 1 \\
&= \boxed{1}
\end{aligned}
$$

b.

Given that a random variable y is said to have a geometric probability distribution iff $p(y) = q^{y-1} p$

$$
\begin{aligned}
\frac{p(y)}{p(y-1)} &= \frac{q^{y-1}p}{q^{y-2}p} \\
&= \frac{q^{y-1}}{q^{y-2}} \\
&= q^{y-1-(y-2)} \\
&= q
\end{aligned}
$$

Therefore the ratio is less than 1 implying that geometric probabilities are a monotonically decreasing function of Y, and that if Y has a geometric distribution the value of Y that is most likely is Y = 0.

That is to say for all possible values of p, $P(Y=1)=p$ is the largest probability that the distribution contains.