STAT 421 Assignment #7

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4.56

In 4.7 the 30 minute arrival time has a unform distribution therefore we must find,

$$\begin{aligned} P(25 < Y < 30 | Y > 10) &= \frac{\frac{1}{6}}{\frac{2}{3}} \\ &= \boxed{\frac{1}{4}} \end{aligned}$$

4.59

I just typed all of these into my TI-83 hope that's alright.

a.

$$\begin{aligned} P(Z > z_0) &= 0.5 \\ z_0 &= \boxed{0.5} \end{aligned}$$

b.

$$\begin{aligned} P(Z < z_0) &= 0.8643 \\ z_0 &= \boxed{1.1} \end{aligned}$$

c.

$$\begin{aligned} P(-z_0 < Z < z_0) &= 0.9 \\ z_0 &= \boxed{1.645} \end{aligned}$$

d.

$$\begin{aligned} P(-z_0 < Z < z_0) &= 0.99 \\ z_0 &= \boxed{2.576} \end{aligned}$$

4.74

a.

$$\begin{aligned} P(Y > 72) &= P(\frac{y-\mu}{\sigma}) \\ &= P(\frac{72-78}{6}) \\ &= P(z \geq -1) \\ &= \boxed{0.8413} \end{aligned}$$

b.

$$\begin{aligned} 0.9 &= P(z \geq \frac{x-78}{6}) \\ 1.28 &= \frac{x-78}{6} \\ x &= \boxed{85.68} \end{aligned}$$

c.

$$\begin{aligned} 0.281 &= P(z \geq \frac{x-78}{6}) \\ 0.58 &= \frac{x-78}{6} \\ x &= \boxed{81.48} \end{aligned}$$

d.

$$\begin{aligned} 0.25 &= P(z \geq \frac{x-78}{6}) \\ 0.67 &= \frac{x-78}{6} \\ x &= 73.98 \\ P(Y > 78.98) &= P(z \geq \frac{78.98-78}{6}) \\ &= \boxed{0.4369} \end{aligned}$$

f.

$$\begin{aligned} P(Y > 84 | Y > 72) &= \frac{P(Y > 84)}{P(Y > 72)} \\ &= \frac{P(z > 1)}{P(z > -1)} \\ &= \frac{0.1587}{0.8413} \\ &= \boxed{0.1886} \end{aligned}$$

4.88

a.

$$\begin{aligned} P(Y > 3) &= 1 - P(Y \leq 3) \\ &= 1 - \int_0^3 f(y)\,dy \\ &= 1 - \int_0^3 (\frac{1}{2.4})e^{\frac{-y}{2.4}}\ dy \\ &= 1 - \frac{2.4}{2.4}\Bigg[- e^{-3/2.4} - (-e^{-0/2.4})\Bigg] \\ &= \boxed{e^{-1.25}} \end{aligned}$$

b.

$$\begin{aligned} P(2 < y < 3) &= \int_2^3 (\frac{1}{2.4})e^{\frac{-y}{2.4}}\ dy \\ &= \frac{2.4}{2.4}\Bigg[- e^{-3/2.4} - (-e^{-2/2.4})\Bigg] \\ &= e^{-1.25} - (-e^{-.83}) \\ &= \boxed{\approx 0.1481} \end{aligned}$$

4.92

Given $C = 100 + 40Y + 3Y^2$

We find $E(Y)$ as

$$\begin{aligned} E(Y) &= \frac{1}{\mu} \\ &= \frac{1}{(1/10)} \\ &= 10 \end{aligned}$$

and $E(Y^2)$

$$
\begin{aligned}
E(Y^2) &= \int_0^\infty \frac{y^2}{10} e^{\frac{-y}{10}}\, dy \\
&= \frac{1}{10}\left(\frac{\tau(3)}{(1/10)^3}\right) \\
&= 200 \\
E(Y^3) &= \frac{1}{10}\left(\frac{\tau(4)}{(1/10)^4}\right) \\
&= 6000 \\
E(Y^4) &= \frac{1}{10}\left(\frac{\tau(3)}{(1/10)^5}\right) \\
&= 240000
\end{aligned}
$$

$$
\begin{aligned}
E(C) &= E(100 + 40Y + 3Y^2) \\
&= E(100) + 40 * E(Y) + 3 * E(Y^2) \\
&= 100 + 40 * 10 + 3 * 200 \\
&= \boxed{1100}
\end{aligned}
$$

$$
\begin{aligned}
V(C) &= E(C^2) - E(C)^2 \\
&= E(100 + 40Y + 3Y^2)^2 - (1100)^2 \\
&= E((100)^2 + (40Y)^2 + (3Y^2)^2 + 2(3Y^2 * 40Y) + 2(40Y * 100) + 2(3Y^2 * 100)) - (1100 \\
&= 4,130,000 - (1,100)^2 \\
&= \boxed{2,920,000}
\end{aligned}
$$

4.96

a.

We have $\alpha = 4$ and $\beta = 2$ therefore,

$$
\begin{aligned}
1 &= k \int_0^\infty y^3 e^{-y/2}\, dy \\
k &= \frac{1}{\tau(4)2^4} \\
&= \boxed{\frac{1}{96}}
\end{aligned}
$$

b.

$$
\begin{aligned}
f(y) &= \frac{y^3 e^{-y/2}}{96} \\
&= \frac{y^{4-1} e^{-y/2}}{(2^4)(4-1)!} \\
&= \frac{y^{\frac{8}{2}-1} e^{-y/2}}{(2^{\frac{8}{2}})\tau(\frac{8}{2})} \\
\text{and} f(\chi^2) &= \frac{y^{\frac{n}{2}-1} e^{-y/2}}{(2^{\frac{n}{2}})\tau(\frac{n}{2})}
\end{aligned}
$$

$$
\begin{aligned}
n &= 2\beta \\
n &= 8
\end{aligned}
$$

So yes with 8 degrees of freedom.

c.

The mean can be calculated as

$$
\begin{aligned}
E(Y) &= \alpha\beta \\
&= \boxed{8}
\end{aligned}
$$

The variance is

$$
\begin{aligned}
V(Y) &= \alpha\beta^2 \\
&= \boxed{16}
\end{aligned}
$$

d.

$$
\begin{aligned}
\sigma &= \sqrt{V(X)} \\
&= \sqrt{16} \\
P(|Y-8| < 2(4)) &= P(0 < Y < 16) \\
&= 0.95762
\end{aligned}
$$

4.126

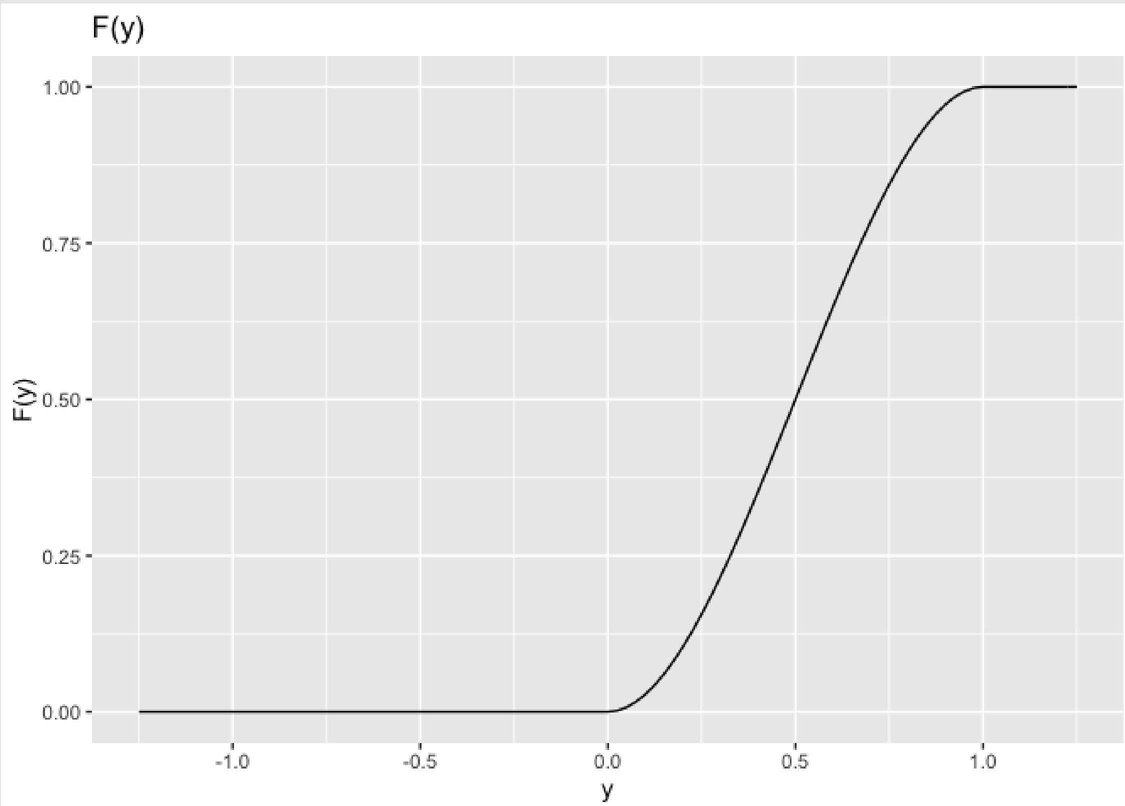
a.

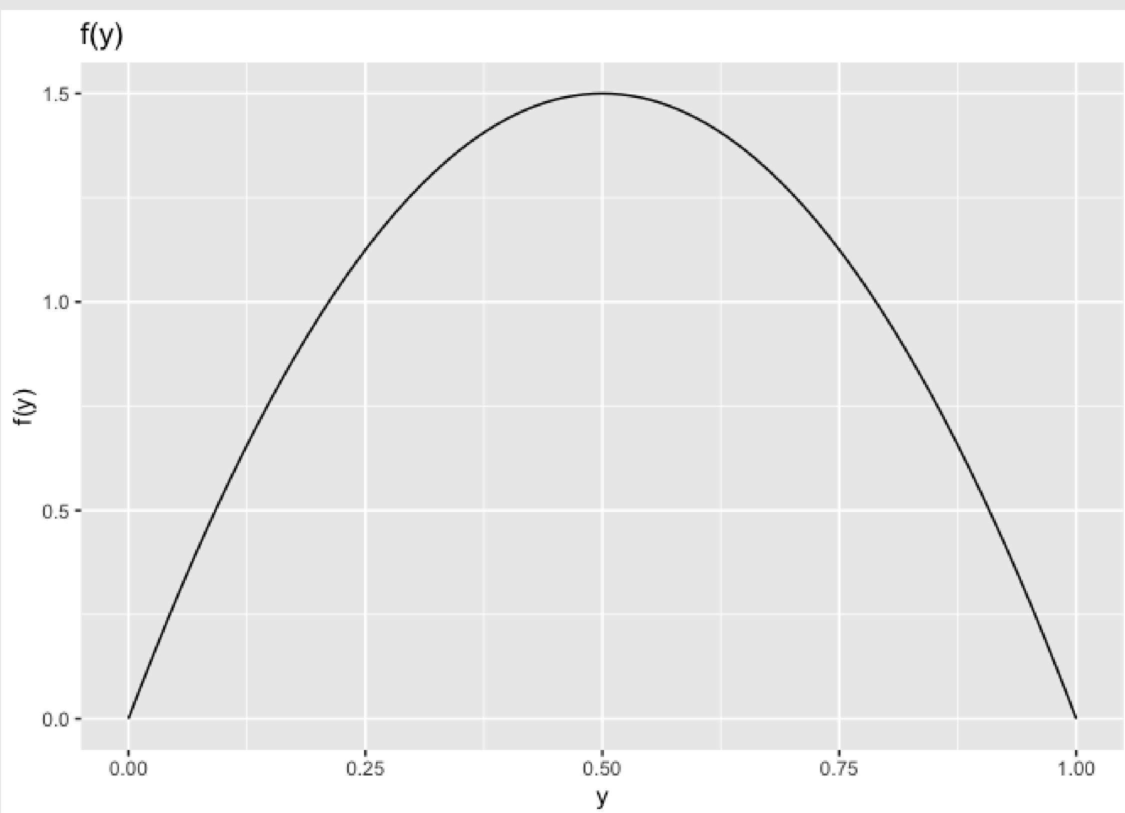
$$
\begin{aligned}
F(y) &= P(Y \leq y) \\
&= \int_0^y (6t - 6t^2)\, dt \\
&= 3y^2 - 2y^3
\end{aligned}
$$

So we have the cumulative probability function

$$F(y) = \begin{cases} 0 & y < 0 \\ 3y^2 - 2y^3 & 0 \le y \le 1 \\ 1 & y > 1 \end{cases}$$

b.





c.

$$\begin{aligned} P(0.5 \leq Y \leq 0.8) &= F(0.8) - F(0.5) \\ &= 1.92 - 1.092 - 0.75 + 0.25 \\ &= \boxed{0.396} \end{aligned}$$

4.134

a.

Given $\alpha = 4$ and $\beta = 7$,

$$\begin{aligned} P(Y \leq 0.7) &= F(0.7) \\ &= \sum_{i=4}^{10} \binom{10}{i} (0.7)^i (0.3)^{10-i} \\ &= P(4 \leq X \leq 10) && \text{distributed binomially n} = 10 \text{ and p} = 0.7 \\ &= \boxed{0.989} \end{aligned}$$

b.

$$\begin{aligned} P(Y \leq 0.6) &= F(0.6) \\ &= \sum_{i=4}^{10} \binom{10}{i} (0.6)^i (0.4)^{10-i} \\ &= P(12 \leq X \leq 25) && \text{distributed binomially n} = 25 \text{ and p} = 0.6 \\ &= \boxed{0.922} \end{aligned}$$

4.142

a.

$$\begin{aligned} m_Y(t) &= E(e^{ty}) \\ &= \int_0^1 e^{ty}\, dy \\ &= \boxed{\frac{e^t - 1}{t}} \end{aligned}$$

b.

Given $W = aY$

$$
\begin{aligned}
m_W(t) &= E(e^{tw}) \\
&= m_Y(at) \\
&= \boxed{\frac{e^{at}-1}{at}}
\end{aligned}
$$

c.

Given $W = -aY$,

$$
\begin{aligned}
m_W(t) &= E(e^{tw}) \\
&= m_Y(-at) \\
&= \boxed{\frac{1-e^{-at}}{at}}
\end{aligned}
$$

d.

Given $V = aY + b$

$$
\begin{aligned}
m_V(t) &= E(e^{tv}) \\
&= E(e^{t(ay+b)}) \\
&= e^{bt}E(e^{aty}) \\
&= e^{bt} * m_Y(at) \\
&= \boxed{\frac{e^{bt}-e^{(a+b)t}}{at}}
\end{aligned}
$$

4.190

a.

$$
\begin{aligned}
r(t) &= \frac{f(t)}{1-F(t)} \\
&= \frac{\lambda e^{-\lambda t}}{1-1+e^{-\lambda t}} \\
&= \lambda
\end{aligned}
$$

b.

For a Weibull function with $m > 1$,

$$
\begin{aligned}
r(t) &= \frac{\frac{my^{m-1}}{\alpha}e^{-y^m/\alpha}}{1-1+e^{-y^m/\alpha}} \\
&= \frac{mt^{m-1}}{\alpha}
\end{aligned}
$$

Therefore $r(t)$ is an increasing function of t when $m > 1$