ST 421/521 Fall 2020
Assignment #7
Due Date: 11/16/2020 (Mon) by 2:00 pm

4.56 Refer to Example 4.7. Find the conditional probability that a customer arrives during the last 5 minutes of the 30-minute period if it is known that no one arrives during the first 10 minutes of the period.

4.7 Let Y be a binomial random variable with $n = 10$ and $p = .2$.

a Use Table 1, Appendix 3, to obtain $P(2 < Y < 5)$ and $P(2 \leq Y < 5)$. Are the probabilities that Y falls in the intevals $(2, 5)$ and $[2, 5)$ equal? Why or why not?

b Use Table 1, Appendix 3, to obtain $P(2 < Y \leq 5)$ and $P(2 \leq Y \leq 5)$. Are these two probabilities equal? Why or why not?

c Earlier in this section, we argued that if Y is continuous and $a < b$, then $P(a < Y < b) = P(a \leq Y < b)$. Does the result in part (a) contradict this claim? Why?

4.59 If Z is a standard normal random variable, find the value z_0 such that

a $P(Z > z_0) = .5$.
b $P(Z < z_0) = .8643$.
c $P(-z_0 < Z < z_0) = .90$.
d $P(-z_0 < Z < z_0) = .99$.

4.74 Scores on an examination are assumed to be normally distributed with mean 78 and variance 36.

a What is the probability that a person taking the examination scores higher than 72?

b Suppose that students scoring in the top 10% of this distribution are to receive an A grade. What is the minimum score a student must achieve to earn an A grade?

c What must be the cutoff point for passing the examination if the examiner wants only the top 28.1% of all scores to be passing?

d Approximately what proportion of students have scores 5 or more points above the score that cuts off the lowest 25%?

~~**e** **Applet Exercise** Answer parts (a)–(d), using the applet *Normal Tail Areas and Quantiles*.~~

f If it is known that a student's score exceeds 72, what is the probability that his or her score exceeds 84?

4.88 The magnitude of earthquakes recorded in a region of North America can be modeled as having an exponential distribution with mean 2.4, as measured on the Richter scale. Find the probability that an earthquake striking this region will

a exceed 3.0 on the Richter scale.

b fall between 2.0 and 3.0 on the Richter scale.

4.92 The length of time Y necessary to complete a key operation in the construction of houses has an exponential distribution with mean 10 hours. The formula $C = 100 + 40Y + 3Y^2$ relates

the cost C of completing this operation to the square of the time to completion. Find the mean and variance of C.

4.96 Suppose that a random variable Y has a probability density function given by

$$f(y) = \begin{cases} ky^3e^{-y/2}, & y > 0, \\ 0, & \text{elsewhere.} \end{cases}$$

a Find the value of k that makes $f(y)$ a density function.

b Does Y have a χ^2 distribution? If so, how many degrees of freedom?

c What are the mean and standard deviation of Y?

~~**d** **Applet Exercise** What is the probability that Y lies within 2 standard deviations of its mean?~~

4.126 Suppose that a random variable Y has a probability density function given by

$$f(y) = \begin{cases} 6y(1-y), & 0 \le y \le 1, \\ 0, & \text{elsewhere.} \end{cases}$$

a Find $F(y)$.

b Graph $F(y)$ and $f(y)$.

c Find $P(.5 \le Y \le .8)$.

4.134 In the text of this section, we noted the relationship between the distribution function of a beta-distributed random variable and sums of binomial probabilities. Specifically, if Y has a beta distribution with positive integer values for α and β and $0 < y < 1$,

$$F(y) = \int_0^y \frac{t^{\alpha-1}(1-t)^{\beta-1}}{B(\alpha, \beta)}\, dt = \sum_{i=\alpha}^{n} \binom{n}{i} y^i (1-y)^{n-i},$$

where $n = \alpha + \beta - 1$.

a If Y has a beta distribution with $\alpha = 4$ and $\beta = 7$, use the appropriate binomial tables to find $P(Y \le .7) = F(.7)$.

b If Y has a beta distribution with $\alpha = 12$ and $\beta = 14$, use the appropriate binomial tables to find $P(Y \le .6) = F(.6)$.

~~**c** **Applet Exercise** Use the applet *Beta Probabilities and Quantiles* to find the probabilities in parts (a) and (b).~~

4.142 Refer to Exercises 4.141 and 4.137. Suppose that Y is uniformly distributed on the interval $(0, 1)$ and that $a > 0$ is a constant.

a Give the moment-generating function for Y.

b Derive the moment-generating function of $W = aY$. What is the distribution of W? Why?

c Derive the moment-generating function of $X = -aY$. What is the distribution of X? Why?

d If b is a fixed constant, derive the moment-generating function of $V = aY + b$. What is the distribution of V? Why?

4.141 If $\theta_1 < \theta_2$, derive the moment-generating function of a random variable that has a uniform distribution on the interval (θ_1, θ_2).

4.137 Show that the result given in Exercise 3.158 also holds for continuous random variables. That is, show that, if Y is a random variable with moment-generating function $m(t)$ and U is given by $U = aY + b$, the moment-generating function of U is $e^{tb}m(at)$. If Y has mean μ and variance σ^2, use the moment-generating function of U to derive the mean and variance of U.

***4.190** A function sometimes associated with continuous nonnegative random variables is the failure rate (or hazard rate) function, which is defined by

$$r(t) = \frac{f(t)}{1 - F(t)}$$

for a density function $f(t)$ with corresponding distribution function $F(t)$. If we think of the random variable in question as being the length of life of a component, $r(t)$ is proportional to the probability of failure in a small interval after t, given that the component has survived up to time t. Show that,

a for an exponential density function, $r(t)$ is constant.

b for a Weibull density function with $m > 1$, $r(t)$ is an increasing function of t. (See Exercise 4.186.)

***4.186** The random variable Y, with a density function given by

$$f(y) = \frac{my^{m-1}}{\alpha} e^{-y^m/\alpha}, \qquad 0 \le y < \infty,\ \alpha,\ m > 0$$

is said to have a *Weibull* distribution. The Weibull density function provides a good model for the distribution of length of life for many mechanical devices and biological plants and animals. Find the mean and variance for a Weibull distributed random variable with $m = 2$.