# Assignment 1

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9/28/2020

## Question 1.11

$$\sum_{i=1}^{n}(y_i - \overline{y})^2$$

$$\sum_{i=1}^{n}(y_i^2 - 2y_i\overline{y} + \overline{y^2})$$

$$\sum_{i=1}^{n}(y_i^2 - 2y_i\overline{y}) + n\overline{y^2}$$ (by a)

$$\sum_{i=1}^{n} y_i^2 - \sum_{i=1}^{n} 2y_i\overline{y} + n\overline{y^2}$$

$$\sum_{i=1}^{n} y_i^2 - 2\overline{y}\sum_{i=1}^{n} y_i + n\overline{y^2}$$ (by b)

$$\sum_{i=1}^{n} y_i^2 - 2\overline{y} * n\overline{y} + n\overline{y^2}$$ (by the definition of the sample mean)

$$\sum_{i=1}^{n} y_i^2 - n\overline{y^2}$$ (combine like terms)

$$\sum_{i=1}^{n} y_i^2 - \frac{1}{n} * n^2\overline{y^2}$$

$\sum_{i=1}^{n} y_i^2 - \frac{1}{n}(\sum_{i=1}^{n} y_i)^2$ (once again by the definition of the sample mean)

Finally we can multiply $\frac{1}{n-1}$ by the original and final equations to prove 1.11.

# Question 1.17

### Exercise 1.2

Exercise 1.2 has a minimum value of 5.7 and a maximum value of 35.1, so the range is $35.1 - 5.7 = 29.4$ and dividing by 4 we have $\frac{29.4}{4} = 7.35$

The standard deviation of exercise 1.2 is calculated with $s = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (x_i - \bar{x})^2}$.

So we have s = 4.14, which is significantly different than the $\frac{range}{4}$ metric for calculating standard deviation because of the effect of the outlier of 35.1.

### Exercise 1.3

Exercise 1.3 has a minumum value of 0.32 and a maximum value of 12.48, so the range is $12.48 - 0.32 = 12.16$ and dividing by 4 we have $\frac{12.16}{4} = 3.04$

The standard deviation of exercise 1.3 is calculated as 3.17, which is very close to the $\frac{range}{4}$ metric.

### Exercise 1.4

Exercise 1.4 has a minumum value of 2.61 and a maximum value of 11.88, so the range is $11.88 - 2.61 = 9.27$ and dividing by 4 we have $\frac{9.27}{4} = 2.3175$

The standard deviation of exercise 1.4 is calculated as 1.87 which is fairly similar to the $\frac{range}{4}$ metric.

# Question 2.2

**a.** Both events occur.

$A \cap B$

**b.** At least one occurs.

$A \cup B$

**c.** Neither occurs.

$\overline{A} \cap \overline{B}$

**d.** Exactly one occurs.

$(A \cap \overline{B}) \cup (B \cap \overline{A})$

## Question 2.6

**a.** Undergraduates, were living off campus or both.

We have 36 undergraduate students, and 9 students living off campus, 3 of whom are undergraduates.

$36 + 9 - 3 = \boxed{42}$

**b.** Undergraduates living on campus.

We have 36 undergraduates and 3 living off campus.

$36 - 3 = \boxed{33}$

**c.** Graduate students living on campus.

We have 60 total students and 36 undergraduate students, 9 students live off campus, but 3 of those are undergraduates.

$60 - 36 - 9 + 3 = \boxed{18}$

## Question 2.14

Let the proportion of adults needing glasses for reading be $p_{11} = 0.44$

Let the proportion of adults who need glasses for reading but don't use them be $p_{12} = 0.14$

Let the proportion of adults who don't need glasses for reading but do use them be $p_{21} = 0.02$

Let the proportion of adults who don't need glasses for reading and don't use them be $p_{22} = 0.40$

**a.** Needs glasses.

$$P(Adult\ needs\ glasses) = p_{11} + p_{12}$$

$$= 0.44 + 0.14 = 0.58$$

Therefore, the probability than an adult needs glasses is 0.58.

**b.** Needs glasses but does not use them.

$$P(Adult\ needs\ glasses\ but\ does\ not\ use) = p_{12}$$

$$= 0.14$$

**c.** Uses glasses whether the glasses are needed or not.

$$P(Adult\ uses\ glasses) = p_{11} + p_{21}$$

$$= 0.44 + 0.02$$

$$= 0.46$$

## Question 2.18

**a.** The sample points of this experiment given that the two balanced coins are tossed and the upper faces are observed are:

Let H = Head of the coin, and T = Tail of the coin

The sample space is $S = \{(H,H),(H,T),(T,H),(T,T)\}$

**b.**

$$P(H,H) = \frac{Number\ of\ sample\ points\ choosing\ (H{,}H)}{Total\ number\ of\ sample\ points}$$

$$= \frac{1}{4}$$

$$P(H,T) = \frac{Number\ of\ sample\ points\ choosing\ (H{,}T)}{Total\ number\ of\ sample\ points}$$

$$= \frac{1}{4}$$

$$P(T,H) = \frac{Number\ of\ sample\ points\ choosing\ (T{,}H)}{Total\ number\ of\ sample\ points}$$

$$= \frac{1}{4}$$

$P(T,T) = \frac{Number\ of\ sample\ points\ choosing\ (T,T)}{Total\ number\ of\ sample\ points}$

$= \frac{1}{4}$

Therefore all sample points are equally likely.

**c.** Let A denote the event that exactly one head is observed and B the event that at least one head is observed.

$A = \{(H,T),(T,H)\}$

$B = \{(H,H),(H,T),(T,H)\}$

**d.**

Given that

$A = \{(H,T),(T,H)\}$

$B = \{(H,H),(H,T),(T,H)\}$

$P(A) = \frac{Number\ of\ sample\ points\ in\ A}{Total\ number\ of\ sample\ points}$

$= \frac{2}{4}$

Therefore $\boxed{P(A) = 0.5}$

$P(B) = \frac{Number\ of\ sample\ points\ in\ B}{Total\ number\ of\ sample\ points}$

$= \frac{3}{4}$

Therefore $\boxed{P(B) = 0.75}$.

$(A \cap B) = \{(H,T),(T,H)\} \cap \{(H,H),(H,T),(T,H)\}$

$= \{(H,T),(T,H)\}$

$P(A \cap B) = P(\{(H,T),(T,H)\})$

Which has previously been proved to be $\frac{2}{4}$

Therefore $\boxed{P(A \cap B) = 0.5}$.

$(A \cup B) = \{(H,T),(T,H)\} \cup \{(H,H),(H,T),(T,H)\}$

$$= \{(H,H),(H,T),(T,H)\}$$

There are three sample points so the probability will be $\frac{3}{4}$

Therefore $P(A \cup B) = 0.75$.

$(\overline{A} \cup B) = \{(H,H),(H,T),(T,H),(T,T)\}$

There are four sample points so the probability will be $\frac{4}{4}$

Therefore $P(\overline{A} \cup B) = 1$.

## Question 2.40

**a.**

$\binom{5}{1} * \binom{4}{1} * \binom{2}{1} = 40$

Therefore the number of autos the dealer would have to stock would be $\boxed{40}$.

**b.**

$\binom{5}{1} * \binom{4}{1} * \binom{2}{1} * \binom{8}{1} = 320$

Therefore the number of autos the dealer would have to stock would be $\boxed{320}$.

## Question 2.58

The total number of ways to draw 5 cards from a deck is $\binom{52}{5} = \frac{52!}{5!(52-5)!} = 2,598,960$

**a.**

The number of possible ways to draw 3 aces from all 4 aces in a standard deck is $\binom{4}{3}$.

The number of possible ways to draw 2 kings from all 4 kings in a standard deck is $\binom{4}{2}$.

Therefore, the total number of favorable cases is $\binom{4}{3} * \binom{4}{2} = 24$

So we have $\frac{24}{2{,}598{,}960} = \boxed{0.0000092}$.

The probability that the five drawn cards contain 3 aces and 2 kings is 0.0000092.

**b.**

There are 4 suits, and 13 cards in each suit.

The number of possible ways to select 2 kinds of 13 cards and order them is $2 * \binom{13}{2}$ ways.

The number of possible ways to select 3 cards of a particular kind and 2 cards of the other kind is $\binom{4}{3} * \binom{4}{2}$ ways.

Therefore, the total number of favorable cases is $2 * \binom{13}{2} * \binom{4}{3} * \binom{4}{2} = 3,744$

So we have $\frac{3{,}744}{2{,}598{,}960} = \boxed{0.0014}$

The probability that a full house is drawn 0.0014.