# The Particle Swarm Optimization Algorithm


**Decision Support**
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# Summary

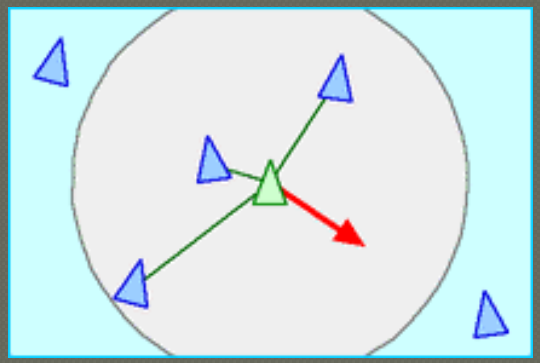
- Introduction to Particle Swarm Optimization (PSO)
  - Origins
  - Concept
  - PSO Algorithm
- PSO for the Bin Packing Problem (BPP)
  - Problem Formulation
  - Algorithm
  - Simulation Results

# Introduction to the PSO: **Origins**

- Inspired from the nature social behavior and dynamic movements with communications of insects, birds and fish

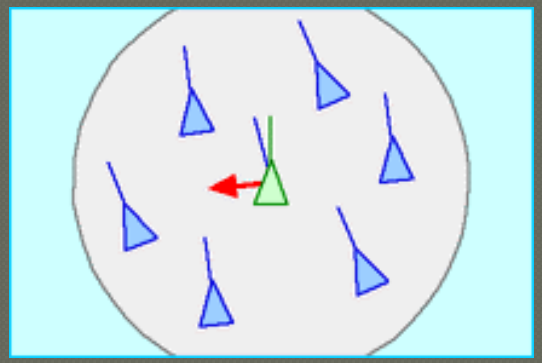
# Introduction to the PSO: **Origins**

- In 1986, Craig Reynolds described this process in 3 simple behaviors:



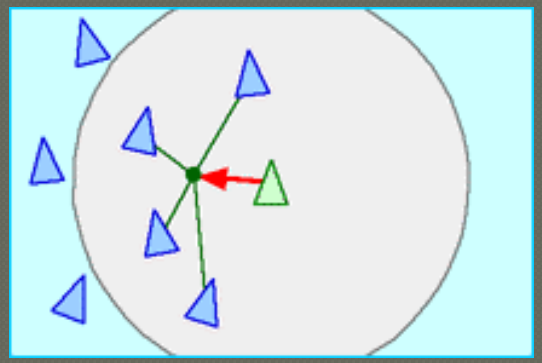
**Separation**

avoid crowding local flockmates



**Alignment**

move towards the average heading of local flockmates



**Cohesion**

move toward the average position of local flockmates

# Introduction to the PSO: **Origins**

- Application to optimization: Particle Swarm Optimization
- Proposed by James Kennedy & Russell Eberhart (1995)
- Combines self-experiences with social experiences

# Introduction to the PSO: **Concept**

- Uses a number of agents (**particles**) that constitute a swarm moving around in the search space looking for the best solution

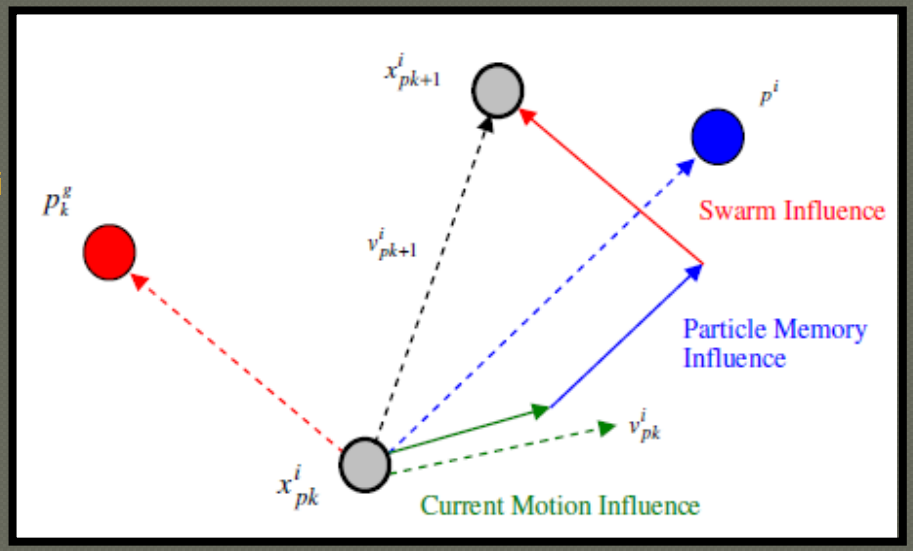
- Each particle in search space adjusts its "flying" according to its own flying experience as well as the flying experience of other particles

# Introduction to the PSO: **Concept**

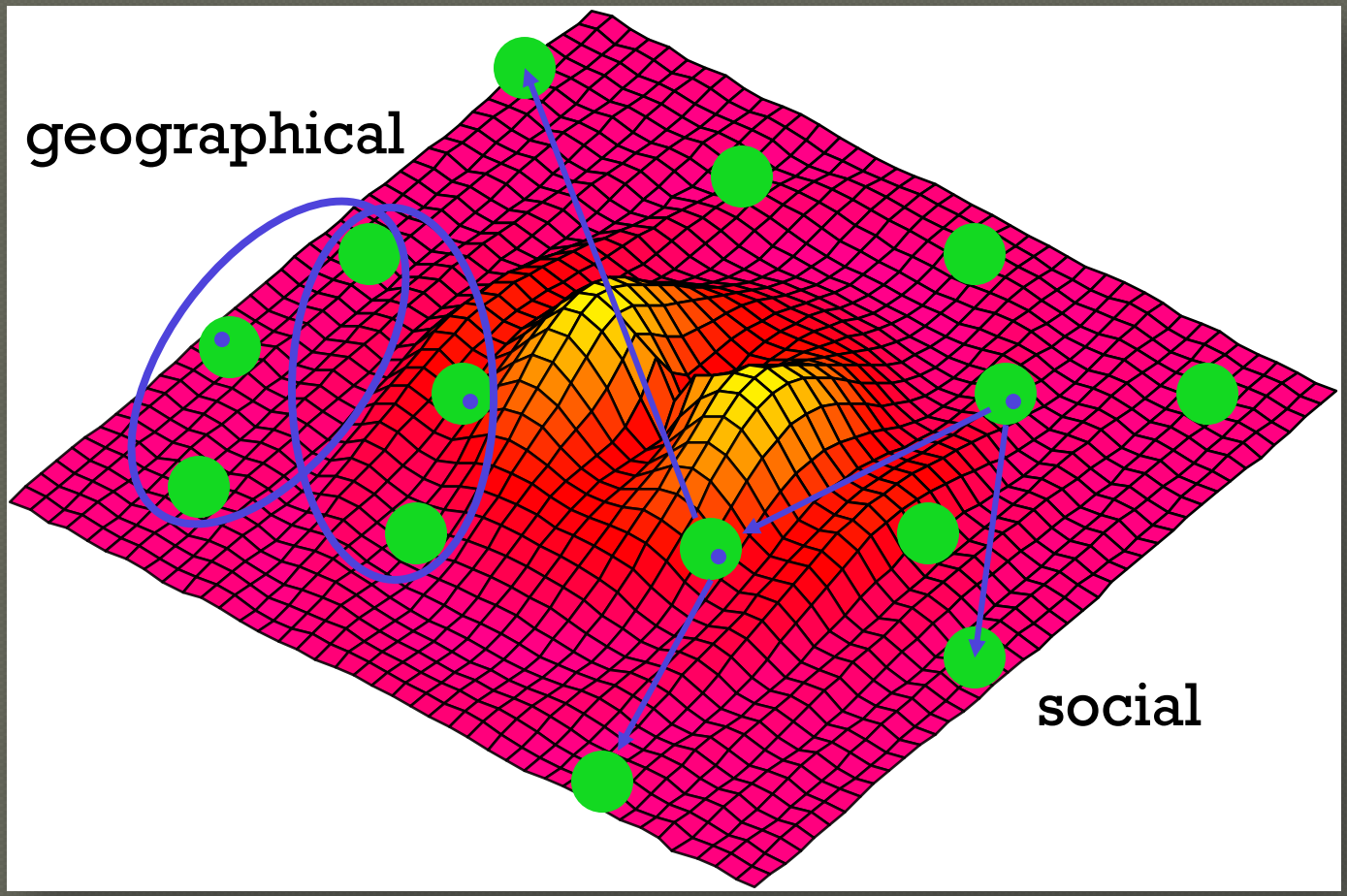
- Collection of flying particles (swarm) - Changing solutions
- Search area - Possible solutions
- Movement towards a promising area to get the global optimum
- Each particle keeps track:
  - its best solution, personal best, *pbest*
  - the best value of any particle, global best, *gbest*

# Introduction to the PSO: **Concept**

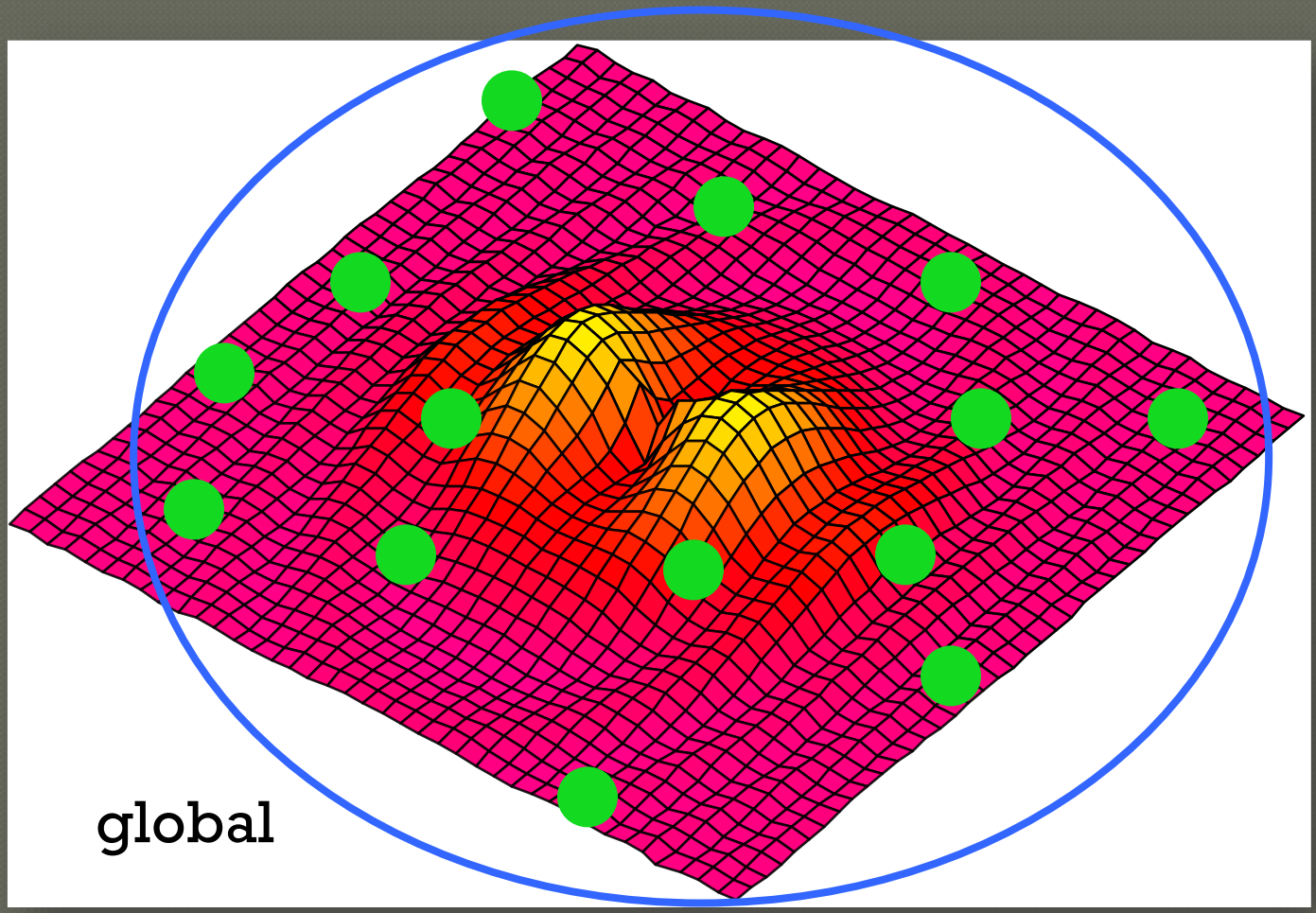
- **Each particle adjusts its travelling speed dynamically corresponding to the flying experiences of itself and its colleagues**
- Each particle modifies its position according to:
  - its current position vị trí hiện tại
  - its current velocity vận tốc hiện tại
  - the distance between its current position and *pbest* (khoảng cách giữa vị trí hiện tại với vị trí tốt nhất của cá thể)
  - the distance between its current position and *gbest* (khoảng cách giữa vị trí hiện tại với vị trí tốt nhất của bầy đàn)



# Introduction to the PSO:
# Algorithm - Neighborhood



# Introduction to the PSO:
# Algorithm - Neighborhood



# Introduction to the PSO: Algorithm - Parameterss

- Algorithm parameters
  - $A$ : Population of agents — Quần thể (tập các cá thể a1, a2, a3, .... aN)
  - $p_i$ : Position of agent $a_i$ in the solution space — Vị trí của cá thể ai trong không gian tìm kiếm
  - $f$ : Objective function — Hàm mục tiêu tính fitness cho từng cá thể
  - $v_i$ : Velocity of agent's $a_i$ — Vector vận tốc hiện tại của cá thể ai
  - $V(a_i)$ : Neighborhood of agent $a_i$ (fixed) — Tập các hàng xóm của cá thể ai (hàng xóm trong PSO được hiểu là N-1 cá thể còn lại)
- The neighborhood concept in PSO is not the same as the one used in other meta-heuristics search, since in PSO each particle's neighborhood never changes (is fixed)

Khái niệm hàng xóm trong PSO khác với khái niệm hàng xóm trong các giải thuật meta-heuristic search

# Introduction to the PSO: Algorithm

[x*] = PSO() ??? Chưa hiểu làm gì?

*P* = Particle_Initialization(); Khởi tạo quần thể (gồm N cá thể ngẫu nhiên)

For *i*=1 to *it_max* Thực hiện lặp qua it_max thế hệ

  For each particle *p* in *P* do

    *fp* = f(*p*);

    If *fp* is better than f(*pBest*)

      *pBest* = *p*;

    end

  end

Tính toán lại fitness cho từng cá thể p[i] trong quần thể

Nếu vị trí mới tốt hơn vị trí của kỷ lục hiện tại thì cập nhật lại kỷ lục (gán kỷ lục bằng vị trí tốt hơn đó). Nghĩa là 1 cá thể luôn lưu 2 giá trị : p[i] (vị trí hiện tại) và p_best[i] (vị trí tốt nhất của cá thể hiện tại)

  *gBest* = best *p* in *P*; Chọn ra cá thể tốt nhất trong quần thể (tức vị trị tốt nhất), chú ý chọn gBest từ tập p trong P chứ không phải chọn từ tập các pBest[]

  For each particle *p* in *P* do

    *v* = *v* + *c1**rand**(*pBest* – *p*) + *c2**rand**(*gBest* – *p*);

    *p* = *p* + *v*; Các công thức cập nhật vị trí cho từng cá thể trong quần thể

  end

end

# Introduction to the PSO: **Algorithm**

- Particle update rule

$$p = p + v$$

  - with

$$v = v + c_1 * rand * (pBest - p) + c_2 * rand * (gBest - p)$$

  - where

- $p$: particle's position
- $v$: path direction
- $c_1$: weight of local information
- $c_2$: weight of global information
- *pBest*: best position of the particle
- *gBest*: best position of the swarm
- *rand*: random variable

# Introduction to the PSO: **Algorithm - Parameters**

- Number of particles usually between 10 and 50
- $C_1$ is the importance of personal best value
- $C_2$ is the importance of neighborhood best value
- Usually $C_1 + C_2 = 4$ (empirically chosen value)
- If velocity is too low → algorithm too slow
- If velocity is too high → algorithm too unstable

# Introduction to the PSO: **Algorithm**

1. Create a 'population' of agents (particles) uniformly distributed over X
2. Evaluate each particle's position according to the objective function
3. If a particle's current position is better than its previous best position, update it
4. Determine the best particle (according to the particle's previous best positions)

# Introduction to the PSO: **Algorithm**

5. Update particles' velocities:

$$\mathbf{v}_i^{t+1} = \underbrace{\mathbf{v}_i^t}_{\textit{inertia}} + \underbrace{c_1 \mathbf{U}_1^t (\mathbf{pb}_i^t - \mathbf{p}_i^t)}_{\textit{personal influence}} + \underbrace{c_2 \mathbf{U}_2^t (\mathbf{gb}^t - \mathbf{p}_i^t)}_{\textit{social influence}}$$

6. Move particles to their new positions:

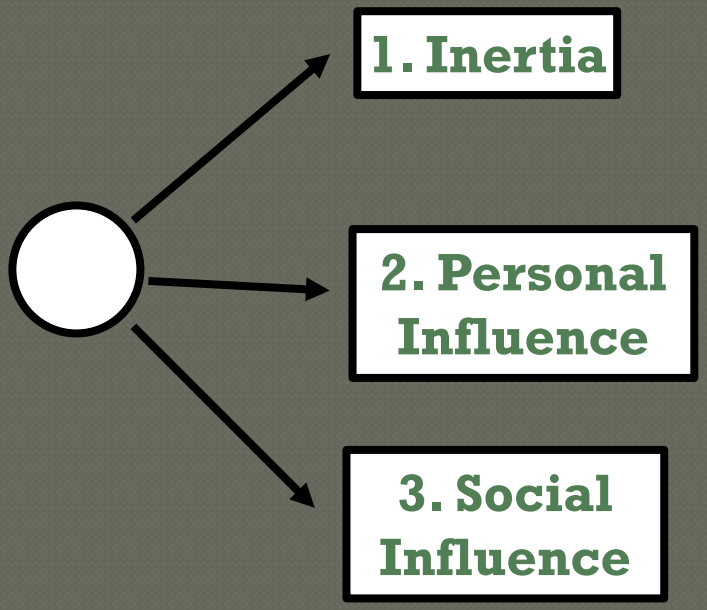
$$\mathbf{p}_i^{t+1} = \mathbf{p}_i^t + \mathbf{v}_i^{t+1}$$

7. Go to step 2 until stopping criteria are satisfied

# Introduction to the PSO: **Algorithm**

Particle's velocity:

$$\mathbf{v}_i^{t+1} = \underbrace{\mathbf{v}_i^t}_{\text{inertia}} + \underbrace{c_1 \mathbf{U}_1^t (\mathbf{pb}_i^t - \mathbf{p}_i^t)}_{\text{personal influence}} + \underbrace{c_2 \mathbf{U}_2^t (\mathbf{gb}^t - \mathbf{p}_i^t)}_{\text{social influence}}$$



**1. Inertia**

- Makes the particle move in the same direction and with the same velocity

**2. Personal Influence**

- Improves the individual
- Makes the particle return to a previous position, better than the current
- Conservative

**3. Social Influence**

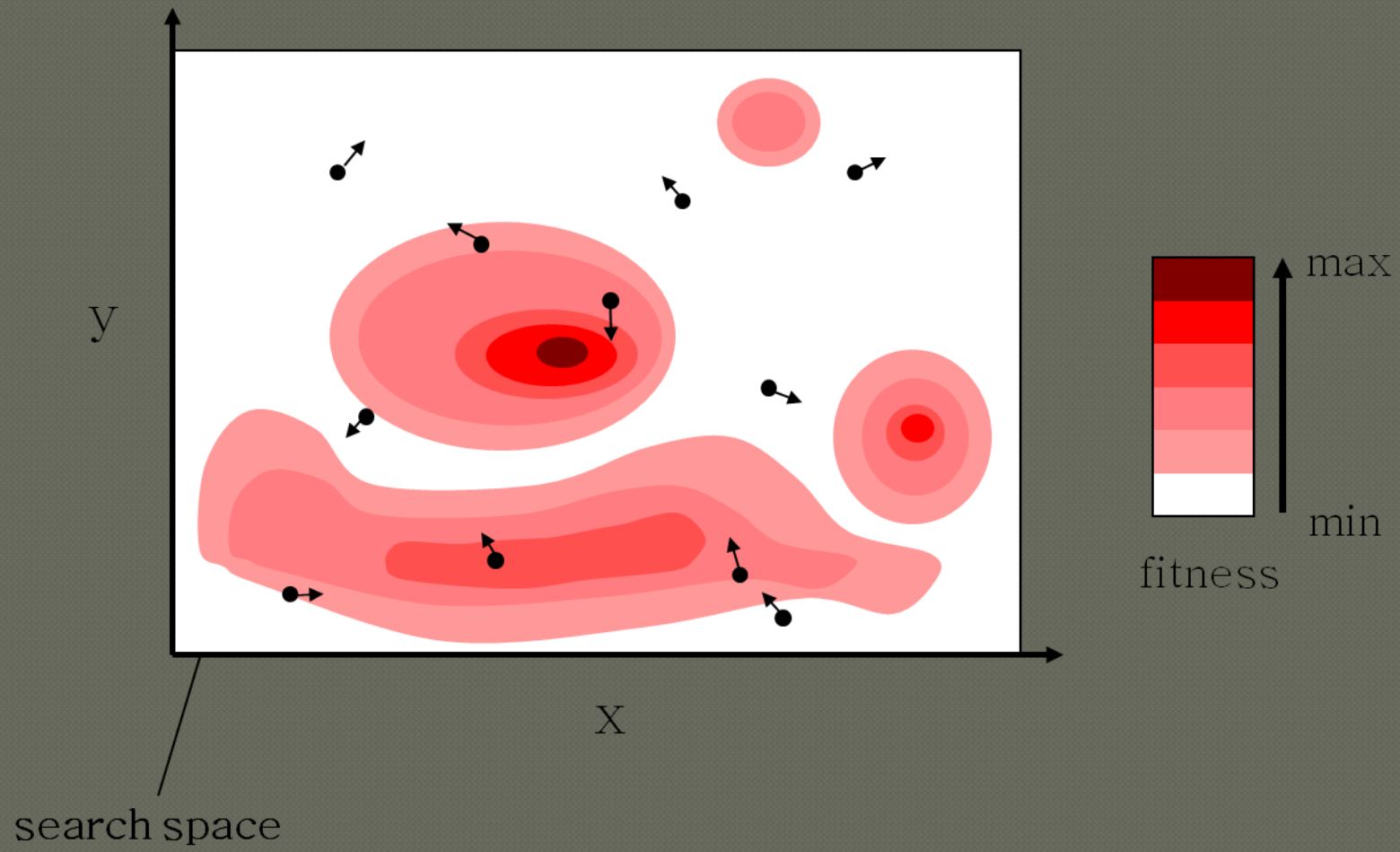
- Makes the particle follow the best neighbors direction

# Introduction to the PSO: **Algorithm**

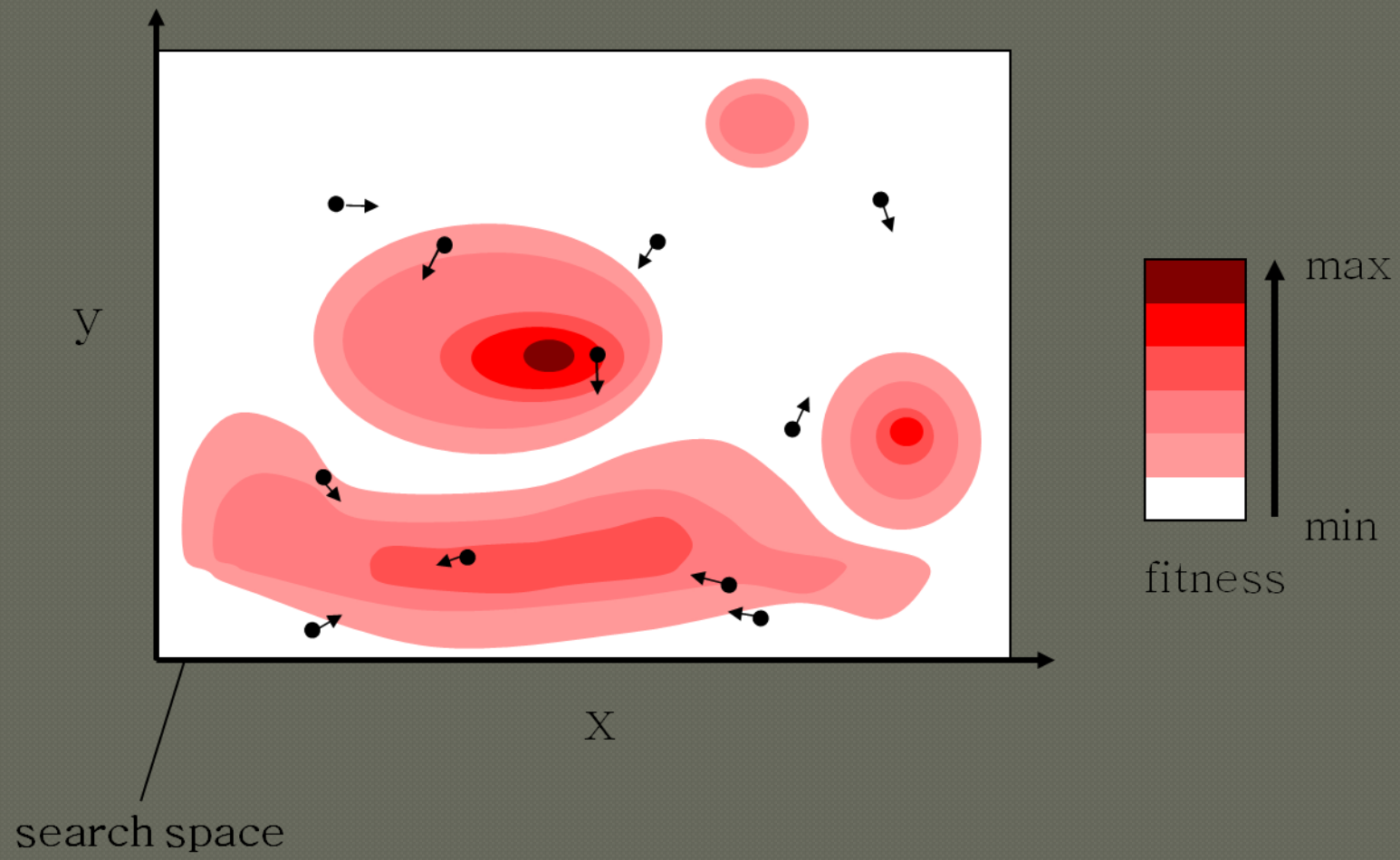
- Intensification: explores the previous solutions, finds the best solution of a given region
- Diversification: searches new solutions, finds the regions with potentially the best solutions
- In PSO:

$$\mathbf{v}_i^{t+1} = \underbrace{\mathbf{v}_i^t}_{\text{Diversification}} + \underbrace{c_1 \mathbf{U}_1^t(\mathbf{pb}_i^t - \mathbf{p}_i^t) + c_2 \mathbf{U}_2^t(\mathbf{gb}^t - \mathbf{p}_i^t)}_{\text{Intensification}}$$

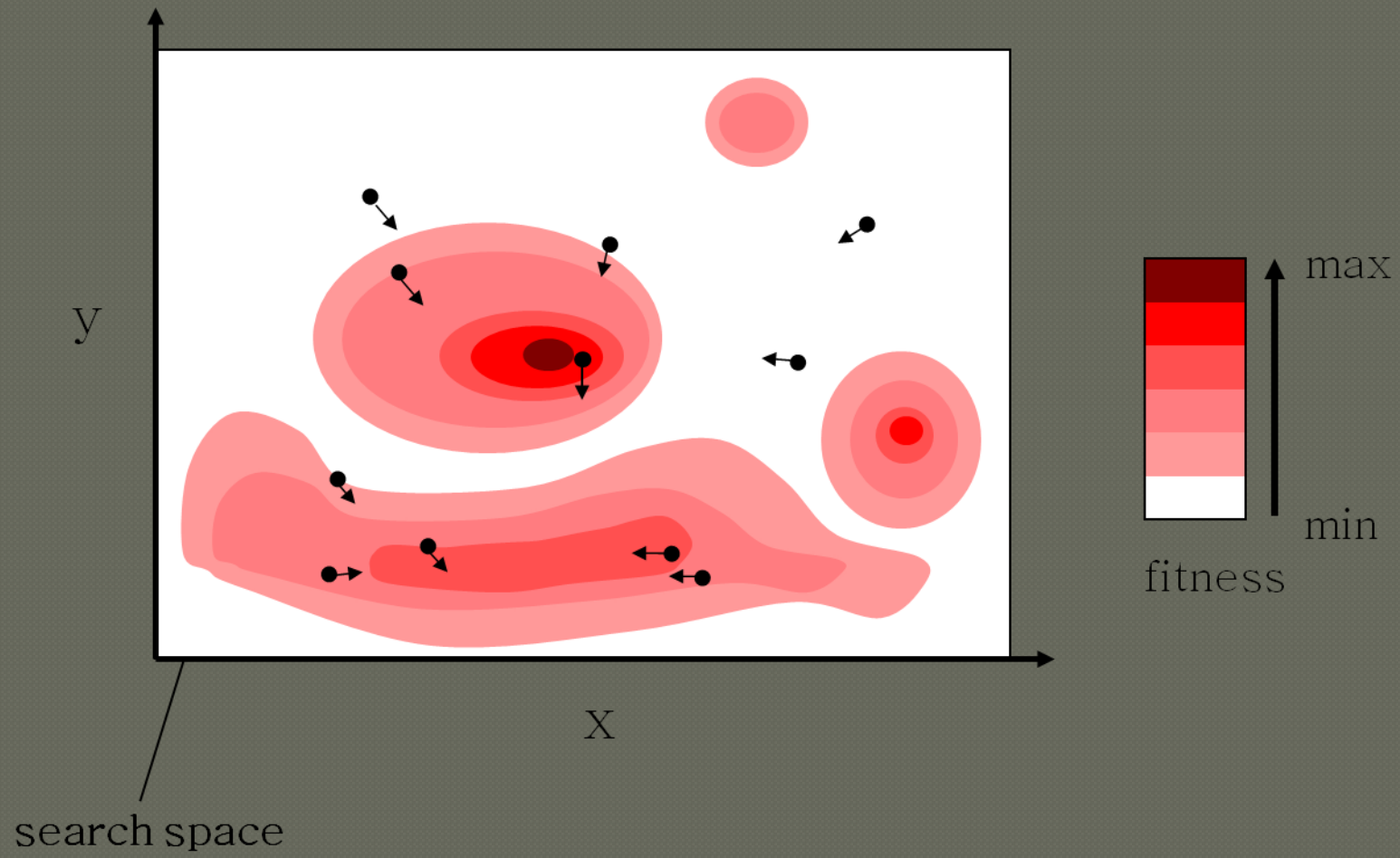
# Introduction to the PSO: **Algorithm - Example**



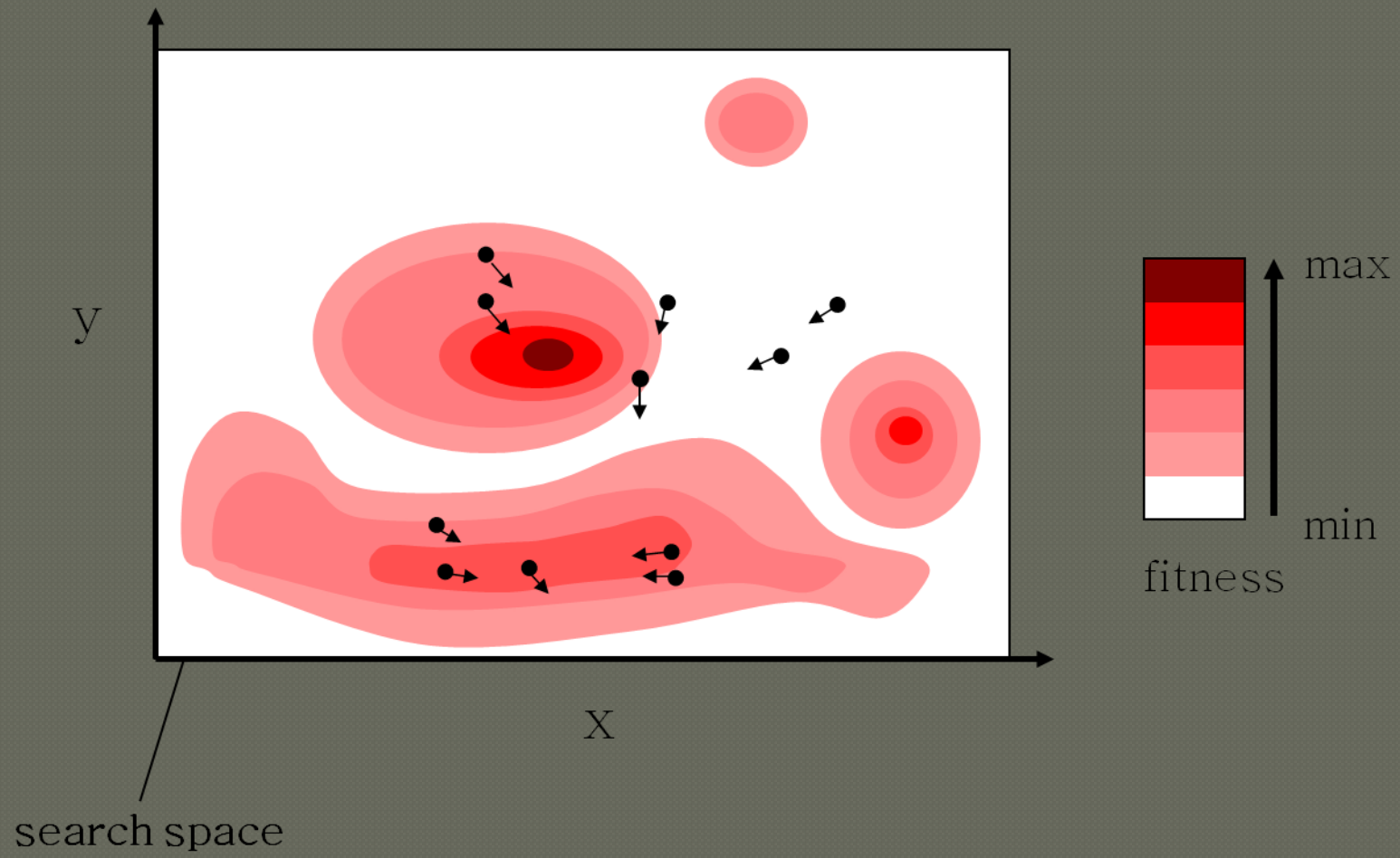
# Introduction to the PSO: **Algorithm - Example**



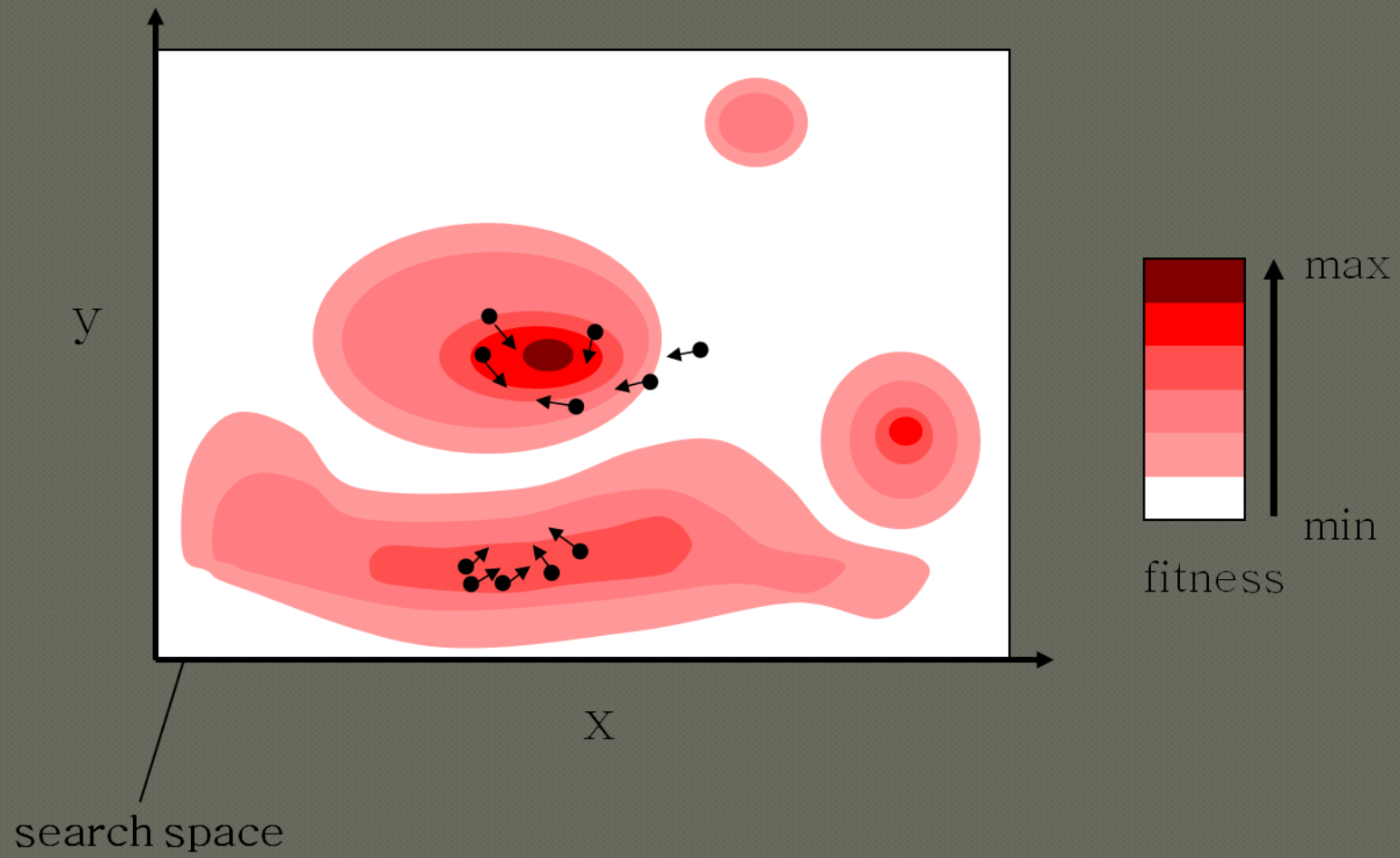
# Introduction to the PSO: **Algorithm - Example**



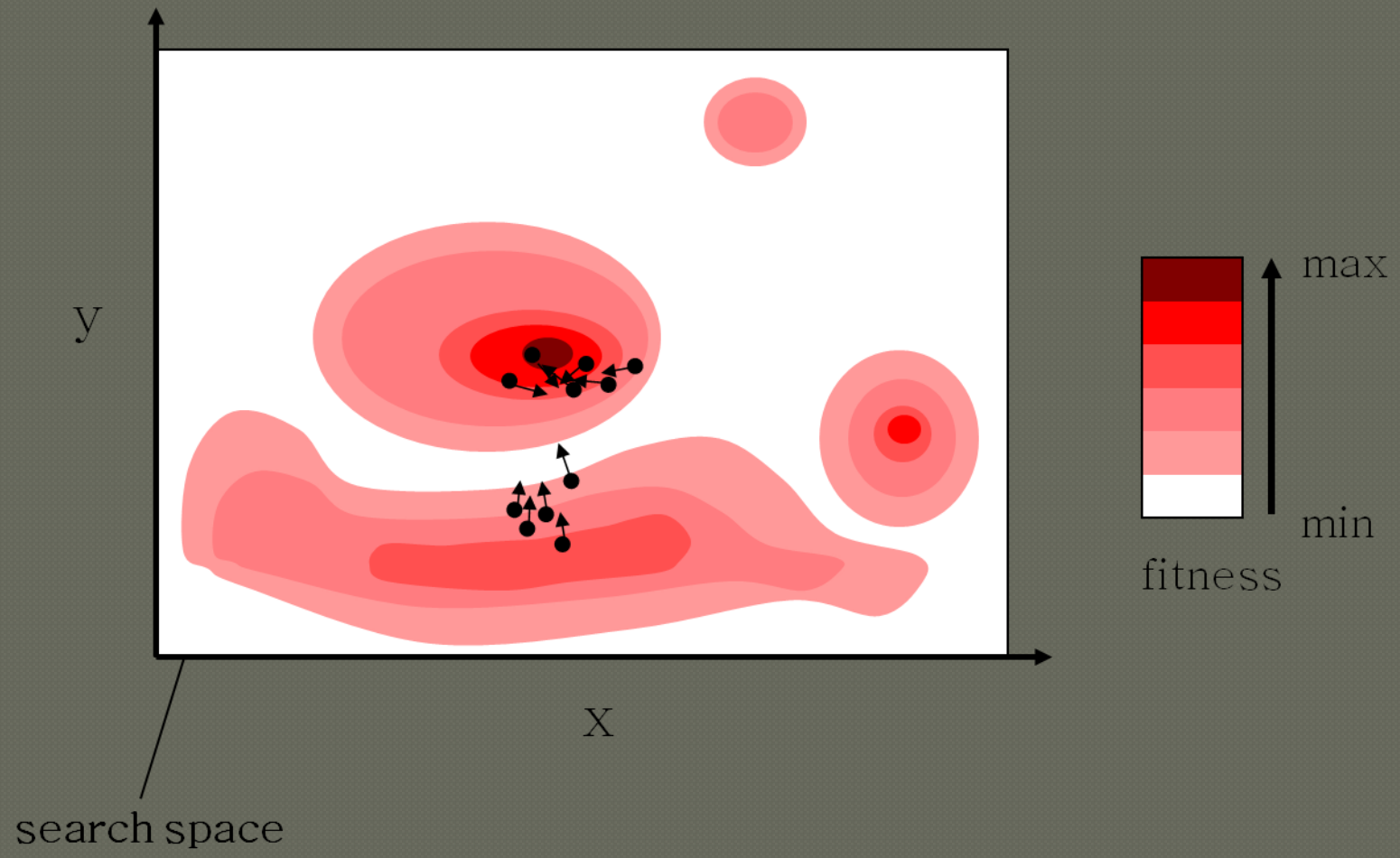
# Introduction to the PSO: **Algorithm - Example**



# Introduction to the PSO: **Algorithm - Example**



# Introduction to the PSO: **Algorithm - Example**



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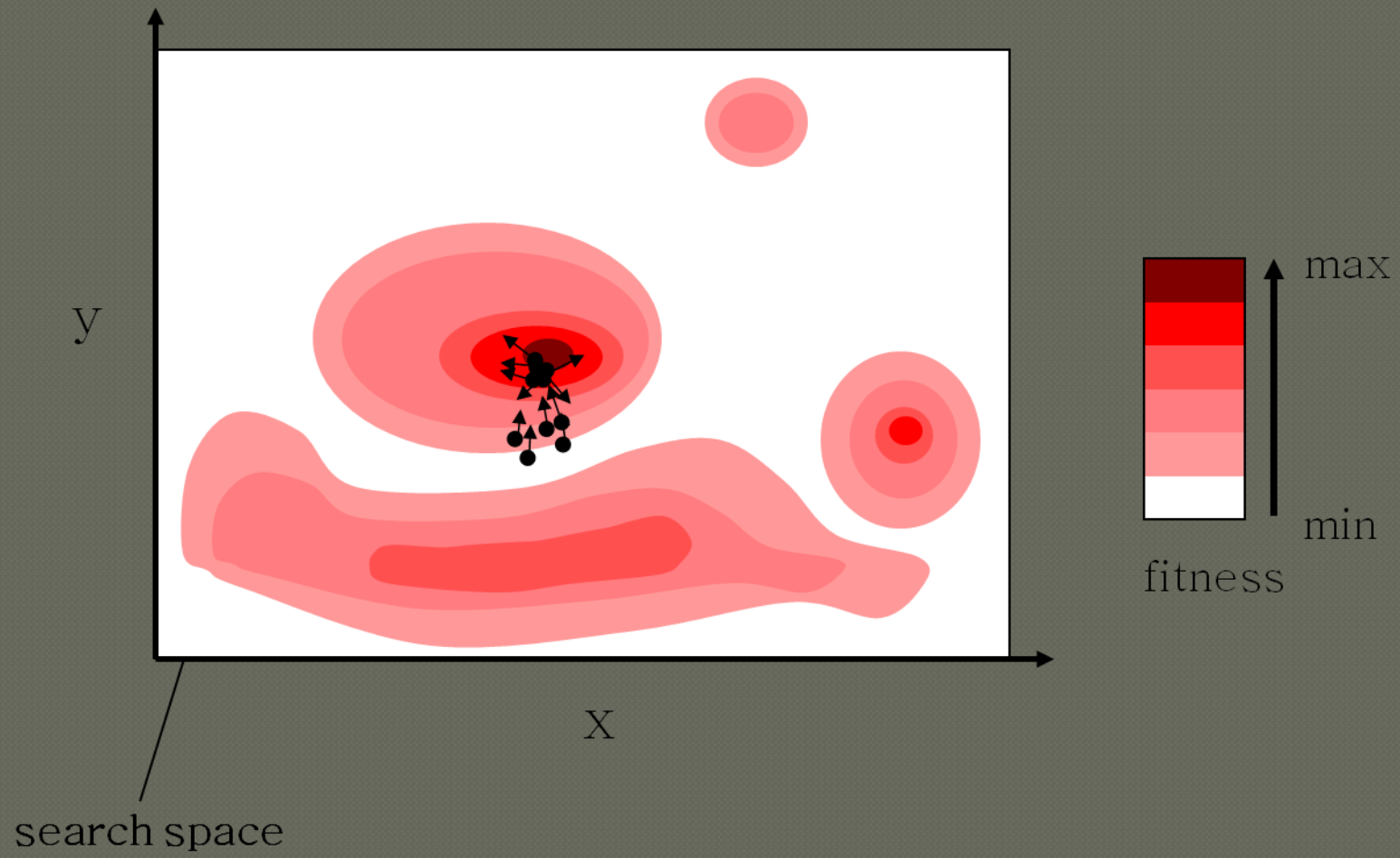
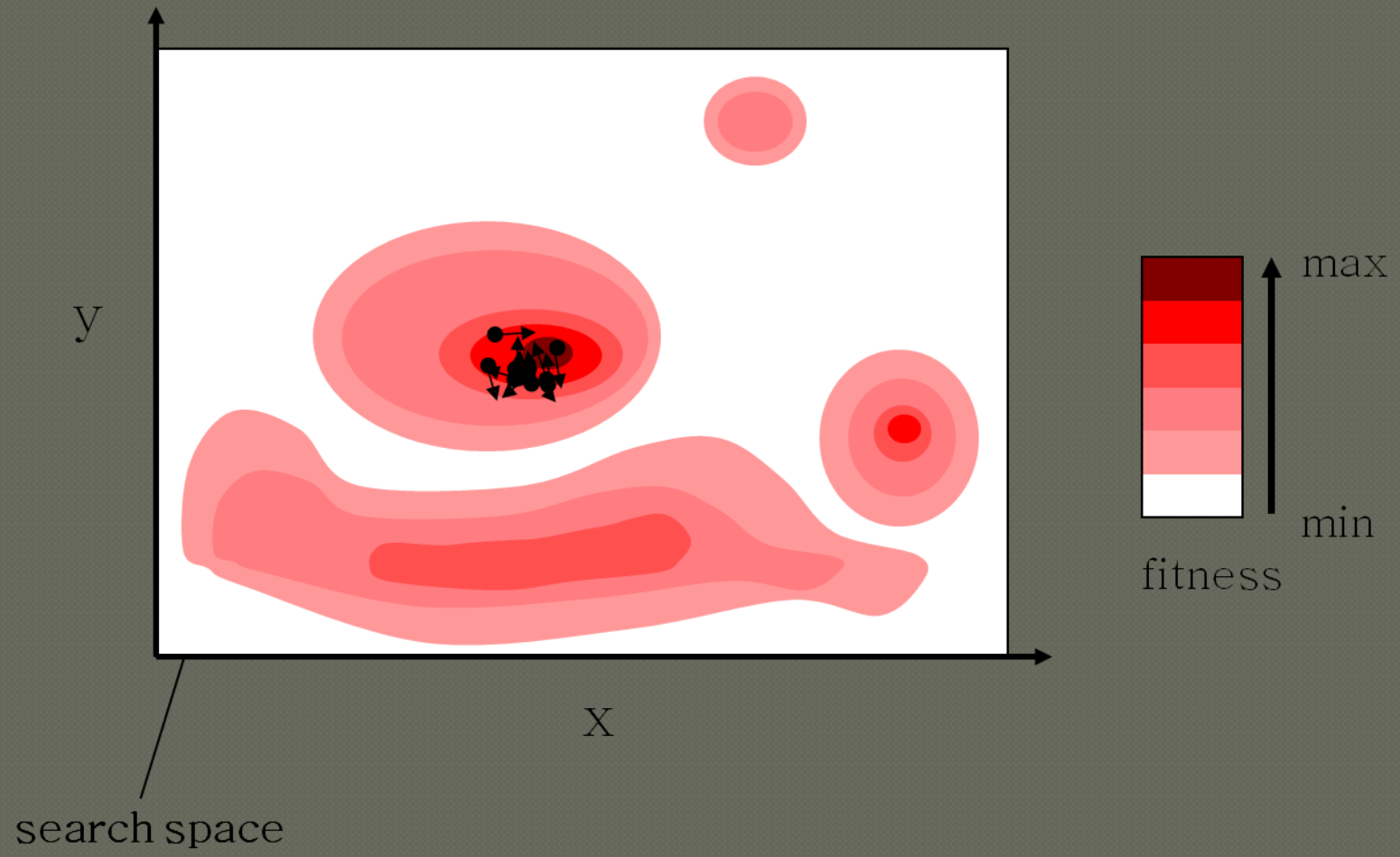


# Introduction to the PSO: **Algorithm - Example**



# Introduction to the PSO: Algorithm Characteristics

- **Advantages**
  - Insensitive to scaling of design variables
  - Simple implementation
  - Easily parallelized for concurrent processing
  - Derivative free
  - Very few algorithm parameters
  - Very efficient global search algorithm
- **Disadvantages**
  - Tendency to a fast and premature convergence in mid optimum points
  - Slow convergence in refined search stage (weak local search ability)

# Introduction to the PSO: **Different Approaches**

- **Several approaches**
  - *2-D Otsu PSO*
  - *Active Target PSO*
  - *Adaptive PSO*
  - *Adaptive Mutation PSO*
  - *Adaptive PSO Guided by Acceleration Information*
  - *Attractive Repulsive Particle Swarm Optimization*
  - *Binary PSO*
  - *Cooperative Multiple PSO*
  - *Dynamic and Adjustable PSO*
  - *Extended Particle Swarms*
  - *…*

Davoud Sedighizadeh and Ellips Masehian, "Particle Swarm Optimization Methods, Taxonomy and Applications". International Journal of Computer Theory and Engineering, Vol. 1, No. 5, December 2009

# PSO for the BPP: **Introduction**

*On solving Multiobjective Bin Packing Problem Using Particle Swarm Optimization*

D.S Liu, K.C. Tan, C.K. Goh and W.K. Ho
2006 - IEEE Congress on Evolutionary Computation

- First implementation of PSO for BPP

# PSO for the BPP: **Problem Formulation**

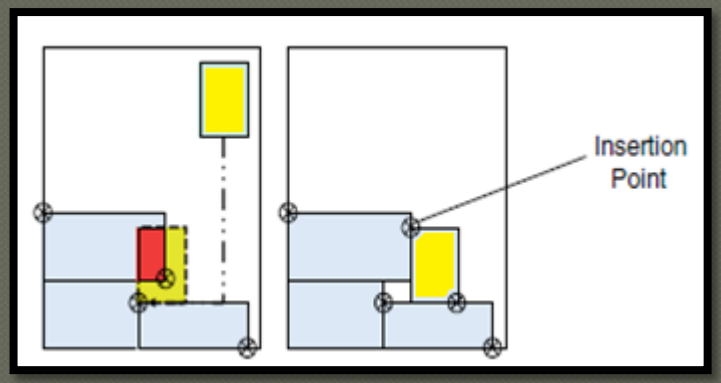
- Multi-Objective 2D BPP
- Maximum of $I$ bins with width $W$ and height $H$
- J items with $w_j \leq W, h_j \leq H$ and weight $\psi_j$

- Objectives
  - Minimize the number of bins used K
  - Minimize the average deviation between the overall centre of gravity and the desired one

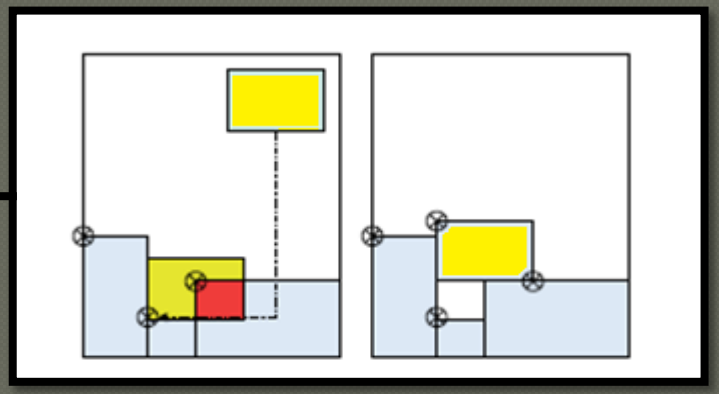
# PSO for the BPP: **Initialization**

- Usually generated randomly
- In this work:
  - Solution from Bottom Left Fill (BLF) heuristic
  - To sort the rectangles for BLF:
    - Random
    - According to a criteria (width, weight, area, perimeter..)

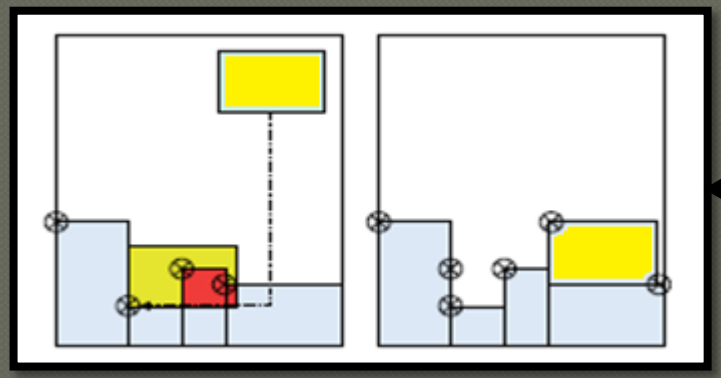
# PSO for the BPP: **Initialization BLF**




Item moved to the right if intersection detected at the top



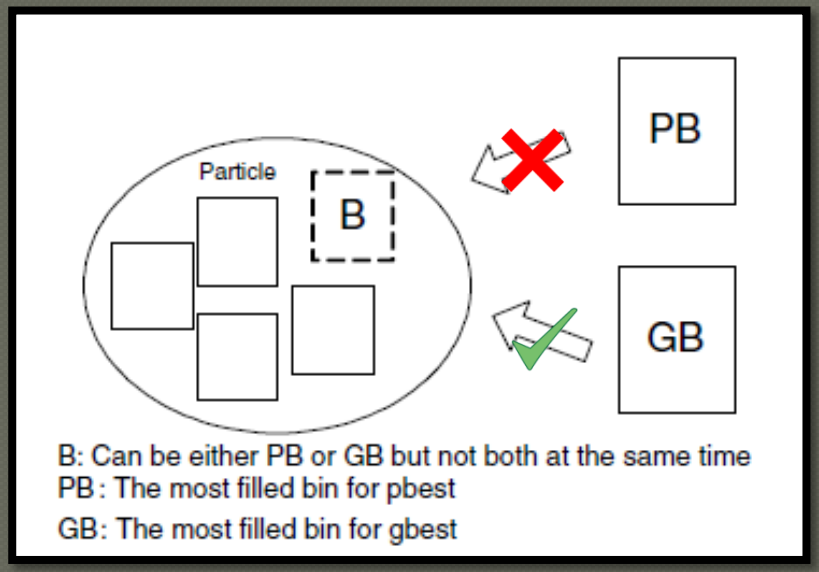
Item moved to the top if intersection detected at the right



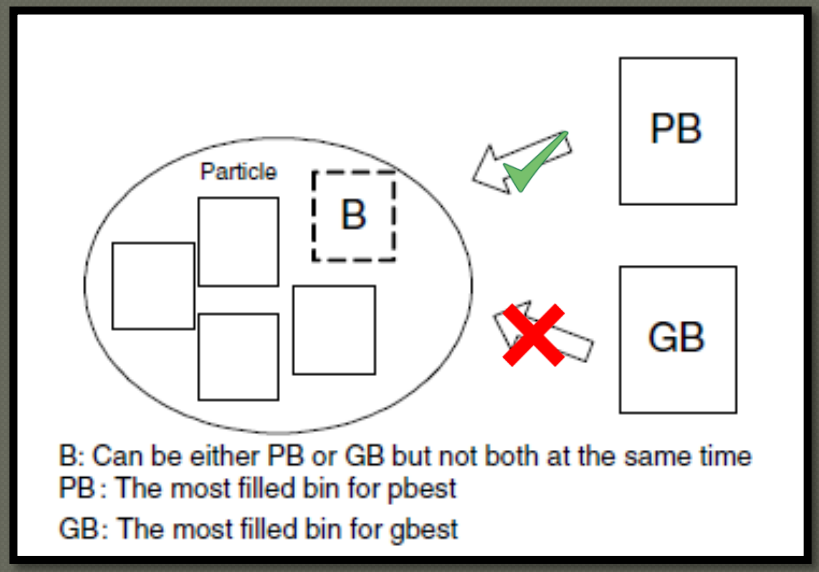
Item moved if there is a lower available space for insertion

# PSO for the BPP: **Algorithm**

- Velocity depends on either *pbest* **or** *gbest*: never both at the same time




OR



# PSO for the BPP: Algorithm

1st Stage:

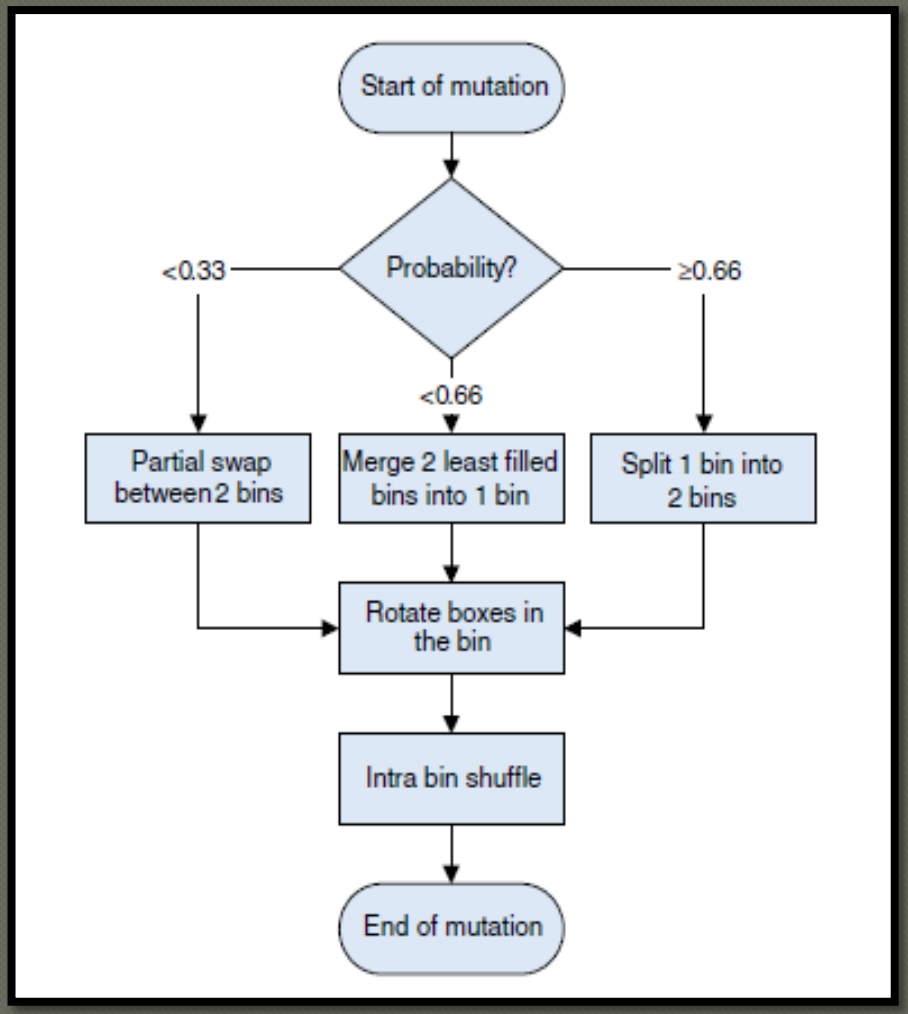
- Partial Swap between 2 bins
- Merge 2 bins
- Split 1 bin

2nd Stage:

- Random rotation

3rd Stage:

- Random shuffle




Mutation modes for a single particle

# PSO for the BPP: Algorithm

**H** hybrid

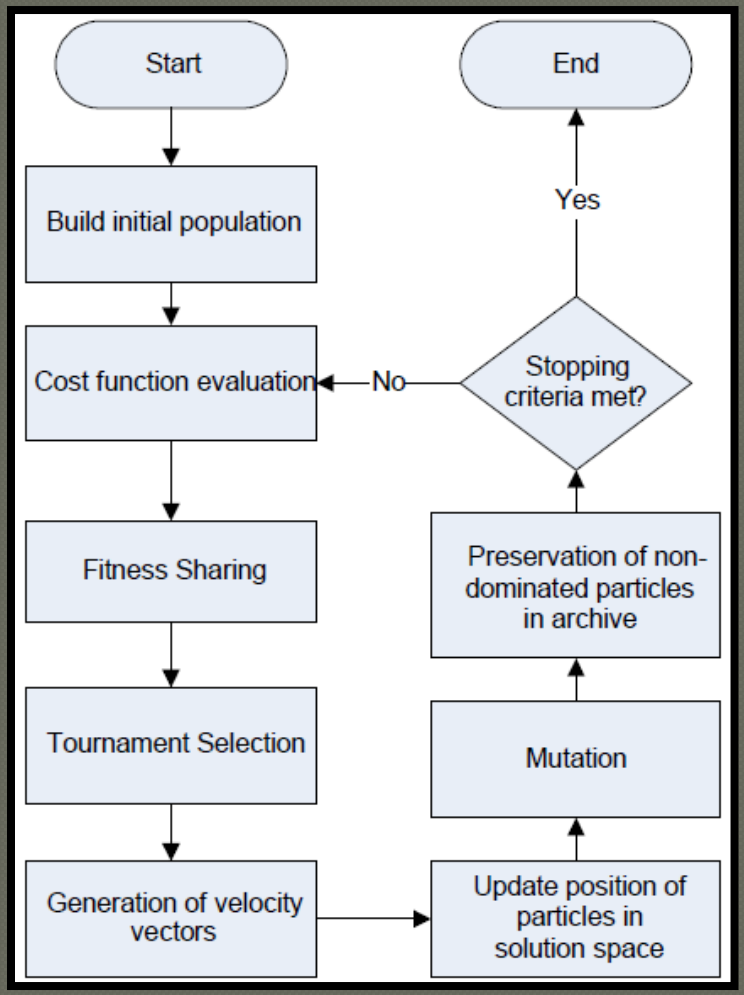
**M** multi

**O** objective

**P** particle

**S** swarm

**O** optimization




The flowchart of HMOPSO

# PSO for the BPP: **Problem Formulation**

- 6 classes with 20 instances randomly generated
- Size range:
  - Class 1: [0, 100]
  - Class 2: [0, 25]
  - Class 3: [0, 50]
  - Class 4: [0, 75]
  - Class 5: [25, 75]
  - Class 6: [25, 50]
- Class 2: small items → more difficult to pack

# PSO for the BPP: Simulation Results

- Comparison with 2 other methods
  - MOPSO (Multiobjective PSO) from [1]
  - MOEA (Multiobjective Evolutionary Algorithm) from [2]
- Definition of parameters:

| Parameter | MOPSO | MOEA | HMOPSO |
|---|---|---|---|
| crossover rate | - | 0.7 | - |
| PSO update rate | - | - | 0.7 |
| mutation rate | - | 0.4 | 0.4 |
| population size | 500 | 500 | 500 |
| generation size | 200 generations | | |
| niche radius | 0.1 | 0.1 | 0.1 |

[1] Wang, K. P., Huang, L., Zhou C. G. and Pang, W., "Particle Swarm Optimization for Traveling Salesman Problem," *International Conference on Machine Learning and Cybernetics, vol. 3, pp.* 1583-1585, 2003.

[2] Tan, K. C., Lee, T. H., Chew, Y. H., and Lee, L. H., "A hybrid multiobjective evolutionary algorithm for solving truck and trailer vehicle routing problems," *IEEE Congress on Evolutionary Computation, vol. 3, pp. 2134-2141, 2003.*

# PSO for the BPP: **Simulation Results**

- Comparison on the performance of metaheuristic algorithms against the branch and bound method (BB) on single objective BPP
- Results for each algorithm in 10 runs
- Proposed method (HMOPSO) capable of evolving more optimal solution as compared to BB in 5 out of 6 classes of test instances

# PSO for the BPP: Simulation Results

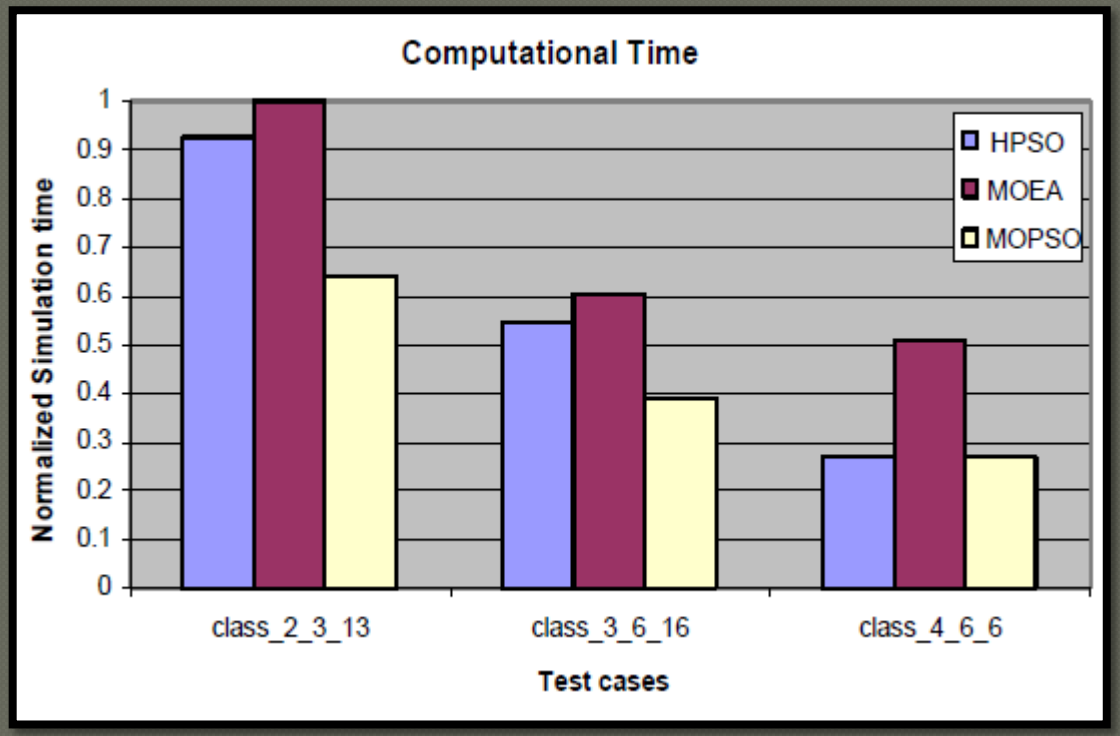
| Class 1 | BB | MOEA | MOPSO | HMOPSO |
|---|---|---|---|---|
| n=20 | 10 | 10 | 10 | 10 |
| n=40 | 10 | 10 | 9 | 10 |
| n=60 | 7 | 6 | 5 | 6 |
| n=80 | 3 | 10 | 5 | 10 |
| n=100 | 1 | 6 | 3 | 7 |
| Total | 31 | **42** | **32** | **43** |
| **Class 2** | | | | |
| n=20 | 10 | 10 | 10 | 10 |
| n=40 | 10 | 9 | 9 | 9 |
| n=60 | 4 | 10 | 9 | 10 |
| n=80 | 10 | 9 | 8 | 9 |
| n=100 | 10 | 10 | 9 | 10 |
| Total | 44 | **48** | **45** | **48** |
| **Class 3** | | | | |
| n=20 | 9 | 10 | 10 | 10 |
| n=40 | 9 | 6 | 5 | 6 |
| n=60 | 5 | 5 | 1 | 6 |
| n=80 | 0 | 4 | 1 | 4 |
| n=100 | 0 | 5 | 1 | 5 |
| Total | 23 | **30** | **18** | **31** |

| Class 4 | BB | MOEA | MOPSO | HMOPSO |
|---|---|---|---|---|
| n=20 | 10 | 10 | 10 | 10 |
| n=40 | 10 | 10 | 10 | 10 |
| n=60 | 7 | 8 | 8 | 8 |
| n=80 | 10 | 7 | 7 | 7 |
| n=100 | 10 | 8 | 7 | 8 |
| Total | 47 | 43 | 42 | 43 |
| **Class 5** | | | | |
| n=20 | 10 | 9 | 10 | 9 |
| n=40 | 10 | 8 | 9 | 10 |
| n=60 | 8 | 5 | 5 | 6 |
| n=80 | 0 | 3 | 3 | 3 |
| n=100 | 1 | 3 | 0 | 2 |
| Total | 29 | 28 | 27 | **30** |
| **Class 6** | | | | |
| n=20 | 10 | 10 | 10 | 10 |
| n=40 | 5 | 6 | 6 | 9 |
| n=60 | 10 | 9 | 9 | 6 |
| n=80 | 10 | 10 | 10 | 10 |
| n=100 | 2 | 8 | 7 | 8 |
| Total | 37 | **43** | **42** | **43** |

Number of optimal solution obtained

# PSO for the BPP: Simulation Results

- Computational Efficiency
  - stop after 1000 iterations or no improvement in last 5 generations
  - MOPSO obtained inferior results compared to the other two



# PSO for the BPP: Conclusions

- Presentation of a mathematical model for MOBPP-2D
- MOBPP-2D solved by the proposed HMOPSO
- BLF chosen as the decoding heuristic
- HMOPSO is a robust search optimization algorithm
  - Creation of variable length data structure
  - Specialized mutation operator
- HMOPSO performs consistently well with the best average performance on the performance metric
- Outperforms MOPSO and MOEA in most of the test cases used in this paper

# The Particle Swarm Optimization Algorithm