Modulation techniques with emphasis on analog systems

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Outlines

- Linear modulation
- Angle modulation
- Interference
- Feedback demodulators: PLL
- Analog pulse modulation
- Delta modulation and PCM
- Multiplexing

Introduction

- The modulation process commonly translates an information-bearing signal to a new spectral location depending upon the intended frequency for transmission.
- Two basic types of analog modulation:
  - *Continuous-wave modulation*: a parameter of a high-frequency carrier is varied proportionally to the message signal such that a one-to-one correspondence exists between the parameter and the message signal. (AM, FM, PM)
  - *Analog pulse modulation*: the amplitude, width, or position of a pulse is varied in one-to-one correspondence with the values of the samples of the message signal. (PAM, PWM, PPM)

Linear modulation

- A linearly modulated carrier is expressed as

$$x_c(t) = A(t)\cos(2\pi f_c t)$$

where $A(t)$ varies in one-to-one correspondence with the message signal.

- Double-sideband (DSB) modulation
  – $A(t)$ is proportional to the message signal $m(t)$

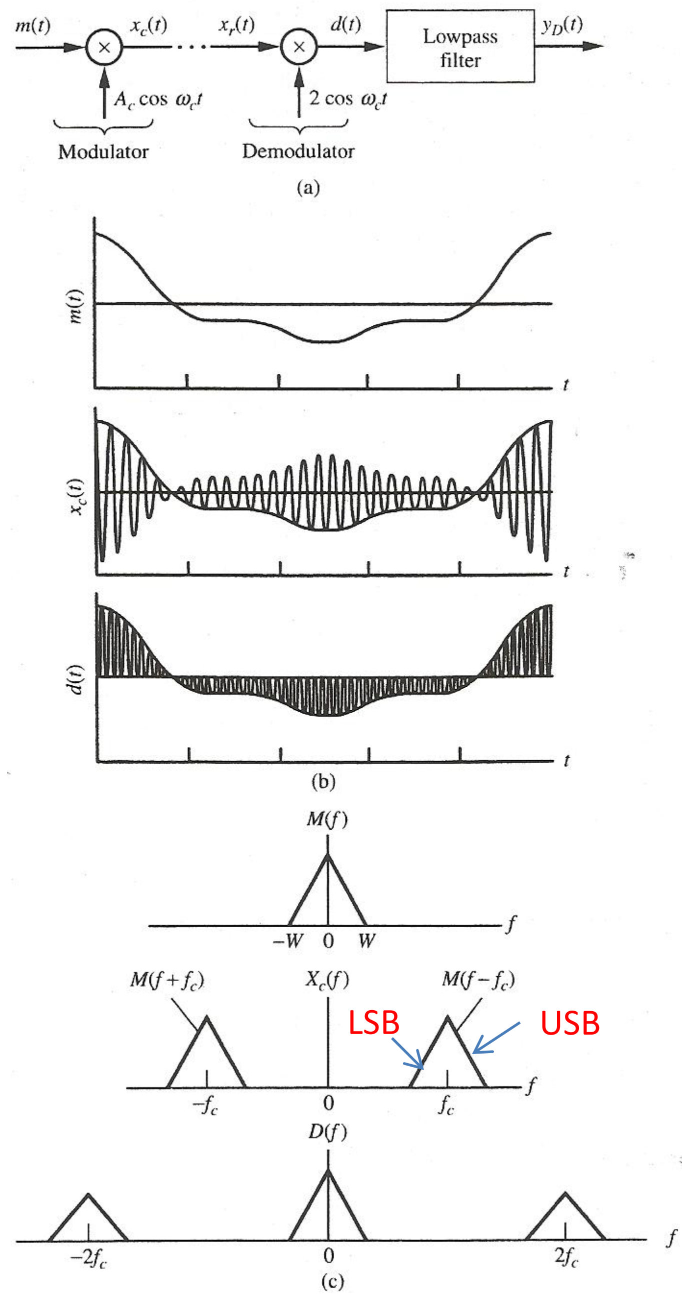
$$x_c(t) = A_c m(t)\cos(2\pi f_c t)$$

  – Spectrum of a DSB signal

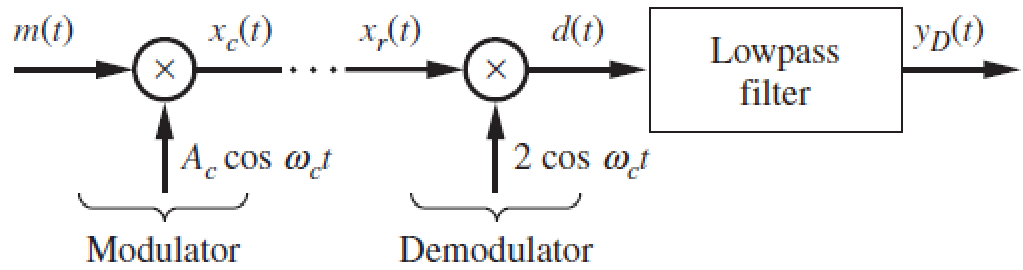
$$X_c(f) = \frac{1}{2}A_c M(f + f_c) + \frac{1}{2}A_c M(f - f_c)$$

– DSB is 100% power efficient since all of the transmitted power lies in the sidebands and it is the sidebands that carry *m*(*t*).

– Demodulation of DSB is difficult since phase coherent with the carrier used for modulation at the transmitter is required at the receiver. (*coherent* or *synchronous demodulation*)



Linear modulation



- DSB
  - A demodulation carrier: $2\cos[2\pi f_c t+\theta(t)]$ phase error
    - Then $d(t) = A_c m(t)\cos\theta(t)+A_c m(t)\cos[4\pi f_c t+\theta(t)]$ and $y_D(t) = m(t)\cos\theta(t)$
    - $\theta(t)$ is a constant $\rightarrow$ the effect of the phase error is an attenuation of the demodulated message signal $\rightarrow$ no distortion.
    - $\theta(t)$ is time varying $\rightarrow$ distortion
  - The spectrum of DSB signal does not contain a discrete spectral component at the carrier frequency unless $m(t)$ has a DC component $\rightarrow$ *suppressed carrier systems*

Linear modulation

$$m_n(t) = \frac{m(t)}{\max|m(t)|}$$

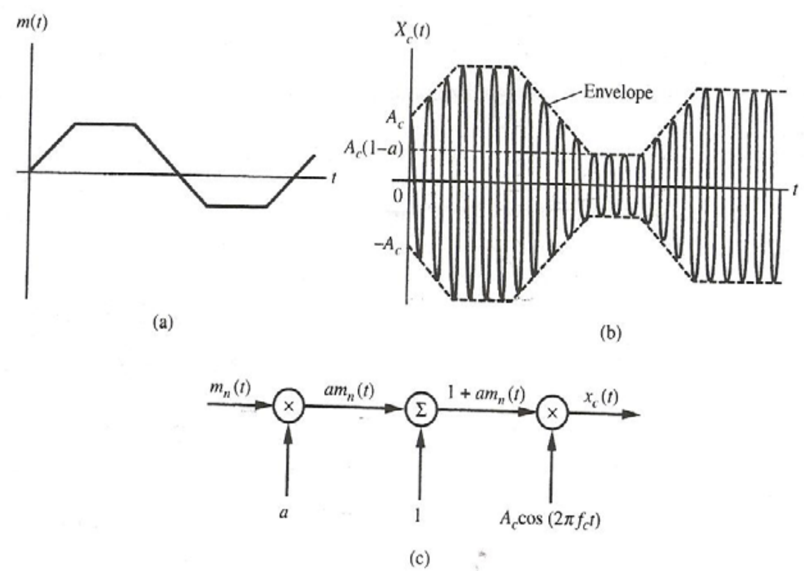
- Amplitude modulation (AM)
  - An AM signal:

$$x_c(t) = [1 + a m_n(t)] A_c \cos(2\pi f_c t)$$

Modulation index

Normalized message signal

Unmodulated carrier



Linear modulation

- AM
  - All information in the modulator output is contained in the sidebands $\rightarrow A_c \cos\omega_c t$ is wasted power.
  - Total power of the AM signal

$$\left\langle x_c^2(t) \right\rangle = \left\langle A_c^2 \left[1 + a m_n(t)\right]^2 \cos^2(2\pi f_c t) \right\rangle$$

$$= \left\langle A_c^2 \left[1 + a m_n(t)\right]^2 \left[\frac{1}{2} + \frac{1}{2}\cos(4\pi f_c t)\right] \right\rangle$$

$$= \left\langle \frac{1}{2} A_c^2 \left[1 + 2a m_n(t) + a^2 m_n^2(t)\right] \right\rangle; m_n(t) \text{ slowly varies w.r.t. the carrier}$$

$$= \frac{1}{2} A_c^2 + \frac{1}{2} A_c^2 a^2 \left\langle m_n^2(t) \right\rangle; m_n(t) \text{ having zero average value}$$

Linear modulation

- AM
  - Efficiency of the modulation process

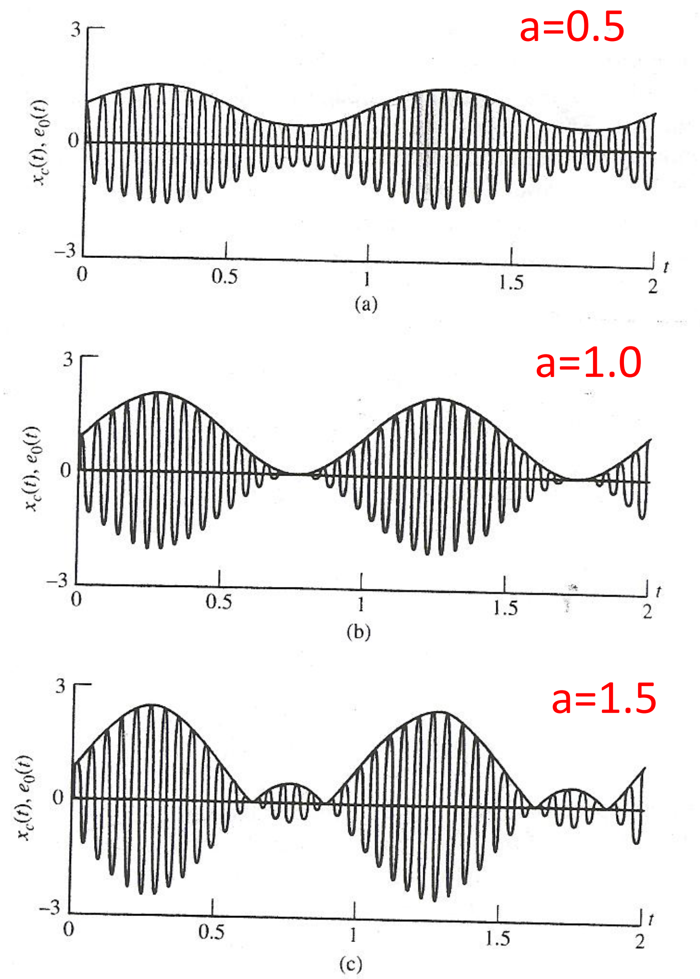
$$E_{ff} = \frac{a^2 \left\langle m_n^2(t) \right\rangle}{1 + a^2 \left\langle m_n^2(t) \right\rangle}$$

  - The main advantage of AM is that since a coherent demodulation is not needed for $a \leq 1$, the demodulator is simple and inexpensive.

Linear modulation

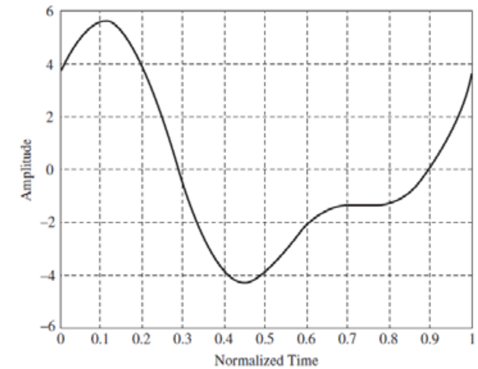
- AM
  - Ex: determine the efficiency and the output spectrum for an AM modulator operating with a modulation index of 0.5. The carrier power is 50W, and the message signal is

$$m(t) = 4\cos\left(2\pi f_m t - \frac{\pi}{9}\right) + 2\sin(4\pi f_m t)$$



The first step is to determine the minimum value of $m(t)$. There are a number of ways to accomplish this. Perhaps the easiest way is to simply plot $m(t)$ and pick off the minimum value. MATLAB is very useful for this purpose as shown in the following program.

```
%File:  c3ex1.m
fmt = 0:0.0001:1;
m = 4*cos(2*pi*fmt-pi/9) + 2*sin(4*pi*fmt);
[minmessage,index] = min(m);
plot(fmt,m,'k'),
grid, xlabel('Normalized Time'), ylabel('Amplitude')
minmessage, mintime = 0.0001*(index-1)
%End of script file.
```



Executing the program yields the plot of the message signal, the minimum value of $m(t)$, and the occurrence time for the minimum value as follows:

```
  c3ex1
minmessage = -4.3642
mintime = 0.4352
```

The message signal as generated by the MATLAB program is shown in Figure 3.5(a). Note that the time axis is normalized by dividing by f_m. As shown, the minimum value of $m(t)$ is -4.364 and occurs at $f_m t = 0.435$, as shown. The normalized message signal is therefore given by

$$m_n(t) = \frac{1}{4.364}\left[4\cos\left(2\pi f_m t - \frac{\pi}{9}\right) + 2\sin(4\pi f_m t)\right] \tag{3.16}$$

or

$$m_n(t) = 0.9166\cos\left(2\pi f_m t - \frac{\pi}{9}\right) + 0.4583\sin(4\pi f_m t) \tag{3.17}$$

The mean-square value of $m_n(t)$ is

$$\langle m_n^2(t)\rangle = \frac{1}{2}(0.9166)^2 + \frac{1}{2}(0.4583)^2 = 0.5251 \tag{3.18}$$

Thus, the efficiency is

$$E_{ff} = \frac{(0.25)(0.5251)}{1+(0.25)(0.5251)} = 0.116 \tag{3.19}$$

or 11.6%.

Since the carrier power is 50 W, we have

$$\frac{1}{2}(A_c)^2 = 50 \tag{3.20}$$

from which

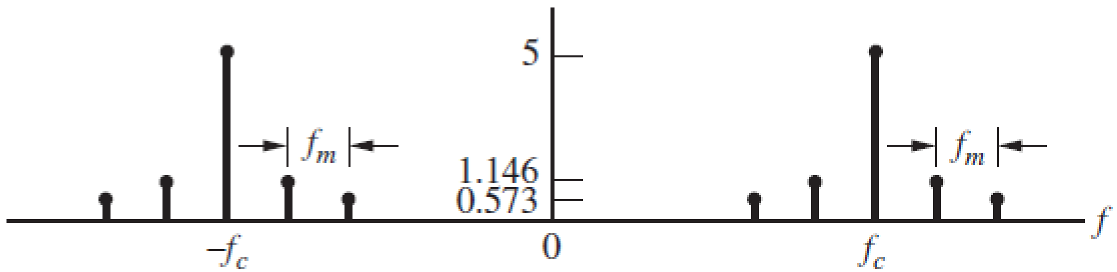
$$A_c = 10 \tag{3.21}$$

Also, since $\sin x = \cos(x - \pi/2)$, we can write $x_c(t)$ as

$$x_c(t) = 10\left\{1 + 0.5\left[0.9166\cos\left(2\pi f_m t - \frac{\pi}{9}\right) + 0.4583\cos\left(4\pi f_m t - \frac{\pi}{2}\right)\right]\right\}\cos(2\pi f_c t) \tag{3.22}$$

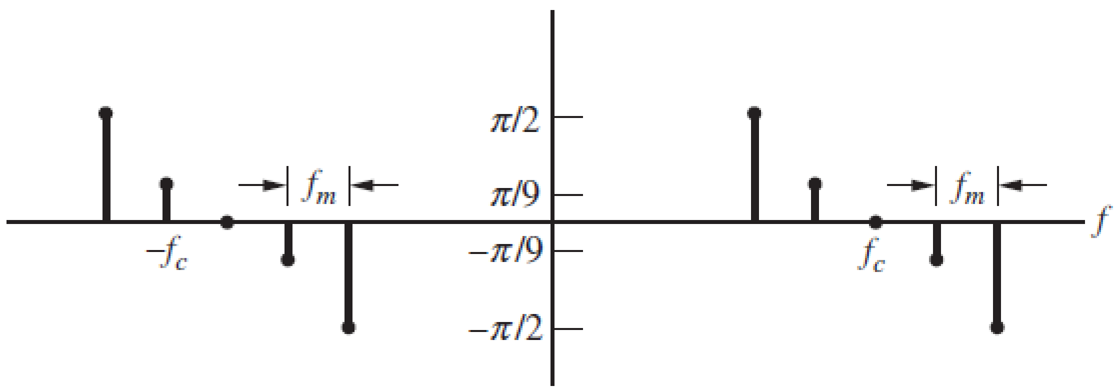
In order to plot the spectrum of $x_c(t)$, we write the preceding equation as

$$\begin{aligned} x_c(t) &= 10\cos(2\pi f_c t) \\ &\quad + 2.292\left\{\cos\left[2\pi(f_c + f_m)t - \frac{\pi}{9}\right] + \cos\left[2\pi(f_c + f_m)t + \frac{\pi}{9}\right]\right\} \\ &\quad + 1.146\left\{\cos\left[2\pi(f_c + 2f_m)t - \frac{\pi}{2}\right] + \cos\left[2\pi(f_c + 2f_m)t + \frac{\pi}{2}\right]\right\} \end{aligned} \tag{3.23}$$



(b)

Amplitude

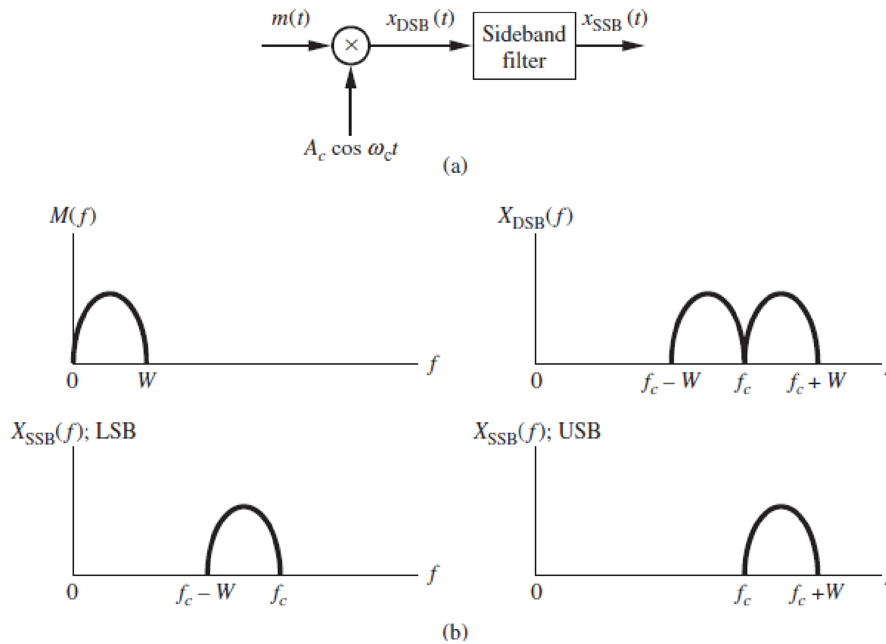


(c)

Phase

Linear modulation

- Single-sideband modulation (SSB)
  - SSB signal corresponds LSB or USB signal of the DSB signal → contains sufficient information to reconstruct *m*(*t*) and reduces the bandwidth to W.



Sideband filtering

Linear modulation

$$X_{\mathrm{DSB}}(f) = \frac{1}{2}A_c M(f+f_c) + \frac{1}{2}A_C M(f-f_c)$$

- SSB
  - Lower-sideband SSB

$$H_L(f) = \frac{1}{2}\left[\mathrm{sgn}(f+f_c) - \mathrm{sgn}(f-f_c)\right]$$

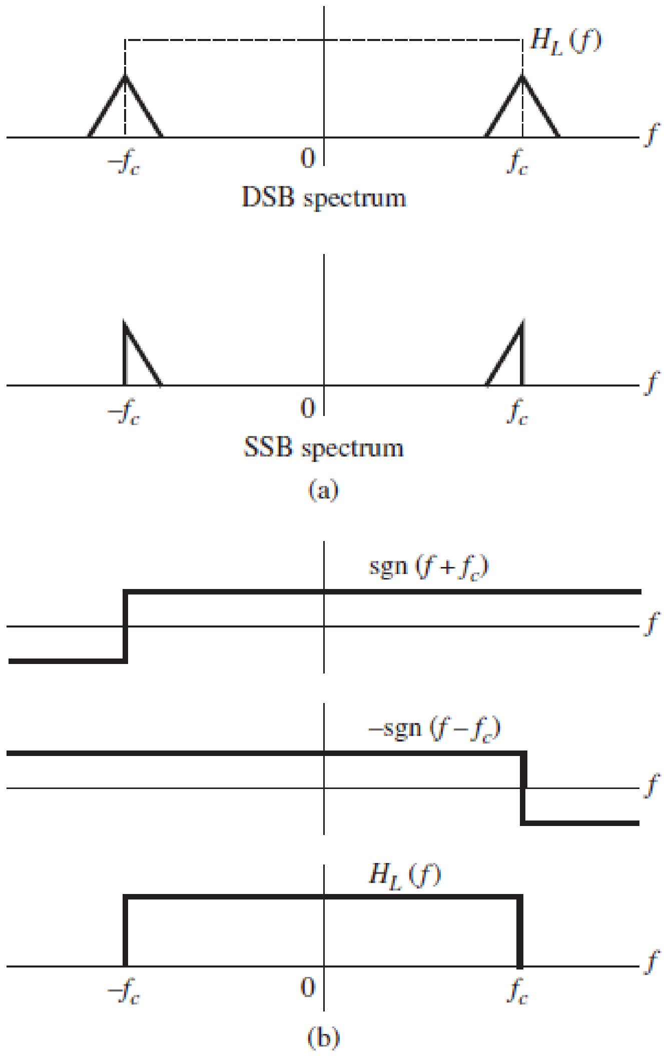
$$X_c(f) = \frac{1}{4}A_c\left[M(f+f_c) + M(f-f_c)\right]$$

$$+\frac{1}{4}A_c\begin{bmatrix} M(f+f_c)\,\mathrm{sgn}(f+f_c) - \\ M(f-f_c)\,\mathrm{sgn}(f-f_c) \end{bmatrix}$$

$$x_c(t) = \frac{1}{2}A_c m(t)\cos(2\pi f_c t)$$

$$+\frac{1}{2}A_c \hat{m}(t)\sin(2\pi f_c t)$$

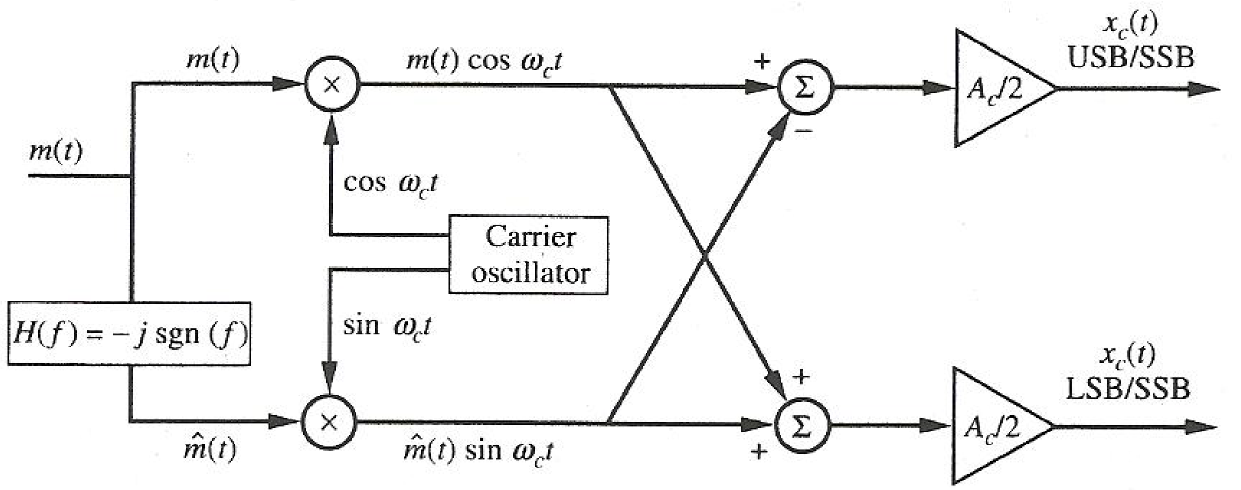
Hilbert transform of *mt*)



Linear modulation

- SSB
  - Upper-sideband SSB

$$x_c(t) = \frac{1}{2} A_c m(t) \cos(2\pi f_c t) - \frac{1}{2} A_c \hat{m}(t) \sin(2\pi f_c t)$$



Linear modulation

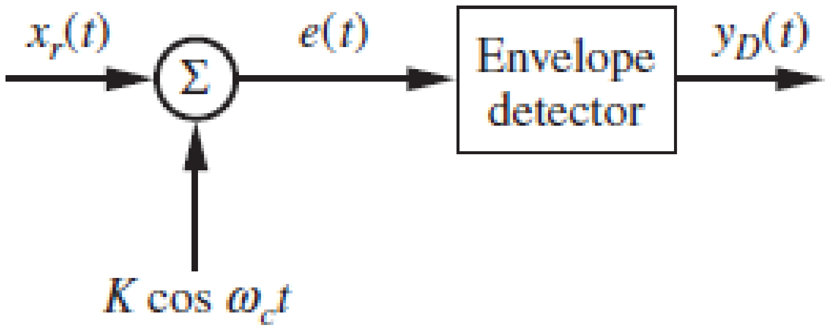
- SSB
  - SSB demodulation
    - Lowpass filtering

$$
\begin{aligned}
d(t) &= x_c(t)\,4\cos\left[2\pi f_c t+\theta(t)\right] \\
&= \left\{\frac{1}{2}A_c m(t)\cos\left(2\pi f_c t\right)\pm\frac{1}{2}A_c\hat{m}(t)\sin\left(2\pi f_c t\right)\right\}4\cos\left[2\pi f_c t+\theta(t)\right] \\
&= A_c m(t)\cos\theta(t)+A_c m(t)\cos\left[4\pi f_c t+\theta(t)\right] \\
&\quad \mp A_c\hat{m}(t)\sin\theta(t)\pm A_c\hat{m}(t)\sin\left[4\pi f_c t+\theta(t)\right]
\end{aligned}
$$

$$
y_D(t)=m(t)\cos\theta(t)\mp\hat{m}(t)\sin\theta(t)
$$

Linear modulation

- SSB
  - SSB demodulation
    - Carrier reinsertion



$$e(t) = \left[\frac{1}{2}A_c m(t) + K\right]\cos(2\pi f_c t) \pm \frac{1}{2}A_c \hat{m}(t)\sin(2\pi f_c t)$$

$$= R(t)\cos\left[2\pi f_c t + \theta(t)\right]$$

$$R(t) = \sqrt{\left[\frac{1}{2}A_c m(t) + K\right]^2 + \left[\frac{1}{2}A_c \hat{m}(t)\right]^2}; \theta(t) = \tan^{-1}\left(\frac{A_c \hat{m}(t)}{2\left(0.5A_c m(t) + K\right)}\right)$$

$$y_D(t) = \sqrt{\left[\frac{1}{2}A_c m(t) + K\right]^2 + \left[\frac{1}{2}A_c \hat{m}(t)\right]^2}$$

$$\left[\frac{1}{2}A_c m(t) + K\right]^2 >> \left[\frac{1}{2}A_c \hat{m}(t)\right]^2 \text{ for large } K$$

$$y_D(t) \sim \frac{1}{2}A_c m(t) + K$$

Linear modulation

- Vestigial-sideband (VSB) modulation
  - VSB overcomes difficulties present in SSB:
    - Eliminate the need for sharp cutoff at the carrier frequency
    - Improve low-frequency response
  - In VSB, small amount or vestige of the unwanted sideband to appear at the output of an SSB modulator.
  - The slight increase in bandwidth required for VSB over that required for SSB but implementation is simpler (envelope detection)

Linear modulation

- VSB
  - Ex: the message signal

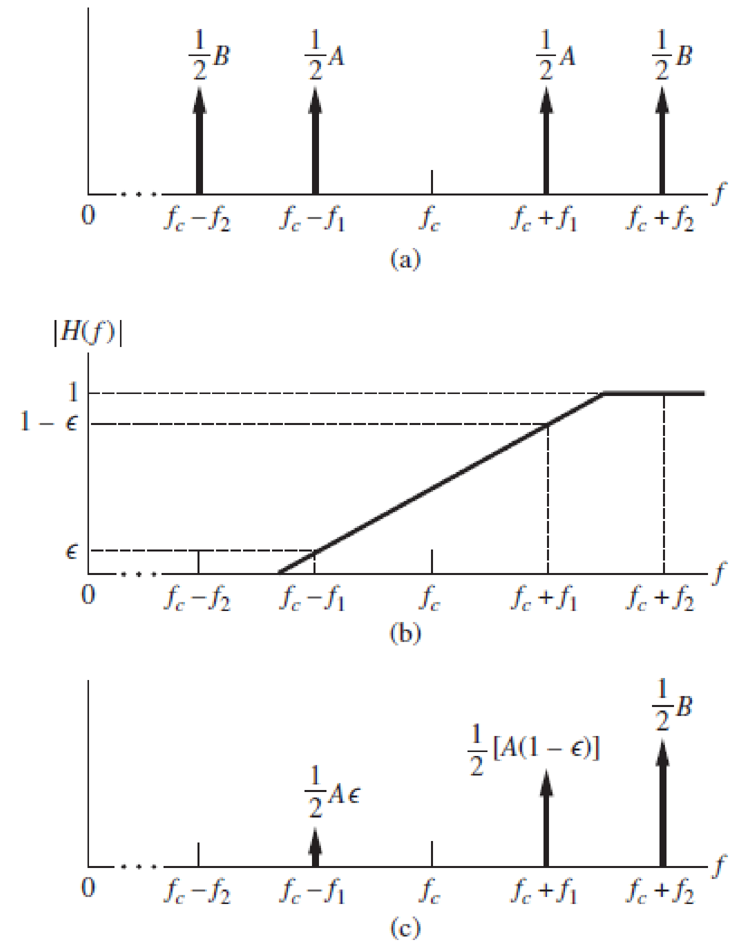
$$m(t) = A\cos(2\pi f_1 t) + B\cos(2\pi f_2 t)$$

- DSB signal

$$e_{DSB}(t) = \frac{1}{2}A\cos[2\pi(f_c - f_1)t] + \frac{1}{2}A\cos[2\pi(f_c + f_1)t]$$
$$+\frac{1}{2}B\cos[2\pi(f_c - f_2)t] + \frac{1}{2}B\cos[2\pi(f_c + f_2)t]$$

- VSB filter output

$$x_c(t) = \frac{1}{2}A\varepsilon\cos[2\pi(f_c - f_1)t]$$
$$+\frac{1}{2}A(1-\varepsilon)\cos[2\pi(f_c + f_1)t] + \frac{1}{2}B\cos[2\pi(f_c + f_2)t]$$



Linear modulation

- VSB
  - Demodulated by multiplying by $4\cos(2\pi f_c t)$ and lowpass filtering

$$e(t) = A\varepsilon\cos(2\pi f_1 t) + A(1-\varepsilon)\cos(2\pi f_1 t) + B\cos(2\pi f_2 t)$$
$$= A\cos(2\pi f_1 t) + B\cos(2\pi f_2 t)$$

– Ex: VSB filter

$$H(f_c - f_1) = \varepsilon e^{-j\theta_a} \quad H(f_c + f_1) = (1-\varepsilon)e^{-j\theta_b} \quad H(f_c + f_2) = e^{-j\theta_c},\ f > 0$$

  - VSB filter input is the DSB signal

$$x_{DSB}(t) = \mathrm{Re}\left\{\left(\frac{A}{2}e^{-j2\pi f_1 t} + \frac{A}{2}e^{j2\pi f_1 t} + \frac{B}{2}e^{-j2\pi f_2 t} + \frac{B}{2}e^{j2\pi f_2 t}\right)e^{-j2\pi f_c t}\right\}$$

Linear modulation

- VSB
  - VSB signal

$$x_c(t) = \mathrm{Re}\left\{\left(\frac{A}{2}\varepsilon e^{-j(2\pi f_1 t+\theta_a)} + \frac{A}{2}(1-\varepsilon)e^{j(2\pi f_1 t-\theta_b)} + \frac{B}{2}e^{j(2\pi f_2 t-\theta_c)}\right)e^{j2\pi f_c t}\right\}$$

  - Demodulation

$$\begin{aligned} e(t) &= \mathrm{Re}\left\{x_c(t)2e^{-j2\pi f_c t}\right\} \\ &= A\varepsilon\cos(2\pi f_1 t+\theta_a) + A(1-\varepsilon)\cos(2\pi f_1 t-\theta_b) + B\cos(2\pi f_2 t-\theta_c) \end{aligned}$$

  - To combine the first two terms, we must satisfy

$$\theta_a = -\theta_b \longrightarrow$$

Phase response must have odd symmetry about f_c

  - Then

$$e(t) = A\cos(2\pi f_1 t-\theta_b) + B\cos(2\pi f_2 t-\theta_c)$$

Linear modulation

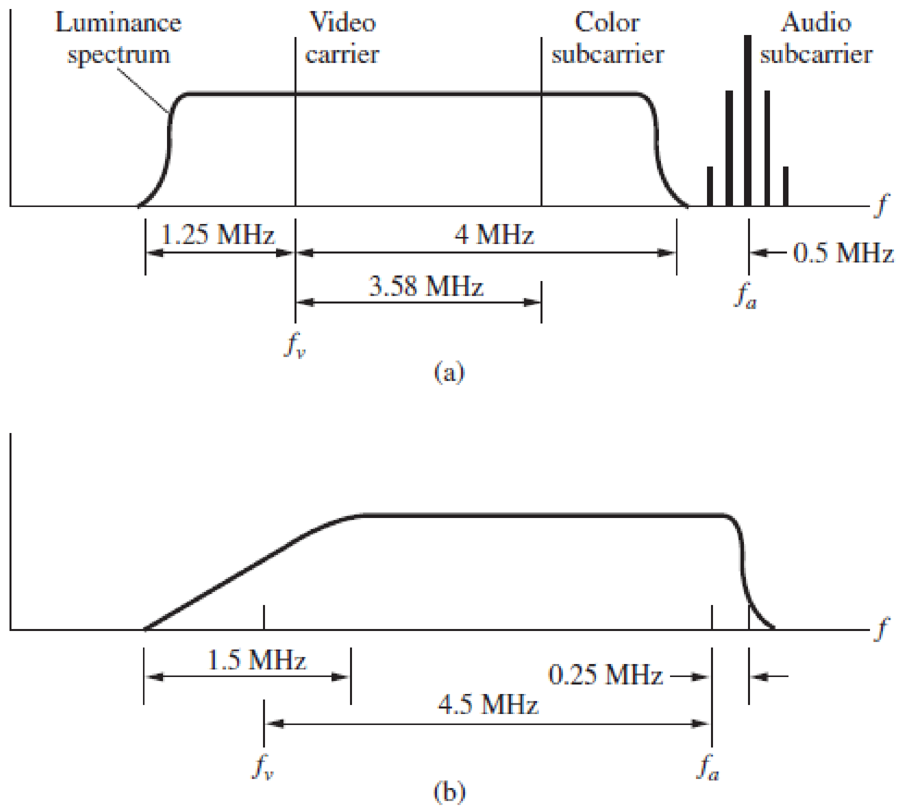
- VSB
  - For the demodulated signal *e*(*t*) to be an undistorted version of *m*(*t*): $e(t) = Km(t-\tau)$
  - Clearly *K*=1.
  - Time delay τ: $e(t) = A\cos\left[2\pi f_1(t-\tau)\right] + B\cos\left[2\pi f_2(t-\tau)\right]$
  - Matching: $\theta_b = 2\pi f_1\tau \qquad \theta_c = 2\pi f_2\tau$
  - For no phase distortion, the time delay must be the same for both components of *e*(*t*):

$$\theta_c = \frac{f_2}{f_1}\theta_b$$

  - The phase response of the VSB filter must be linear over the bandwidth of the input signal.

Linear modulation

- VSB
  - Analog commercial TV broadcasting



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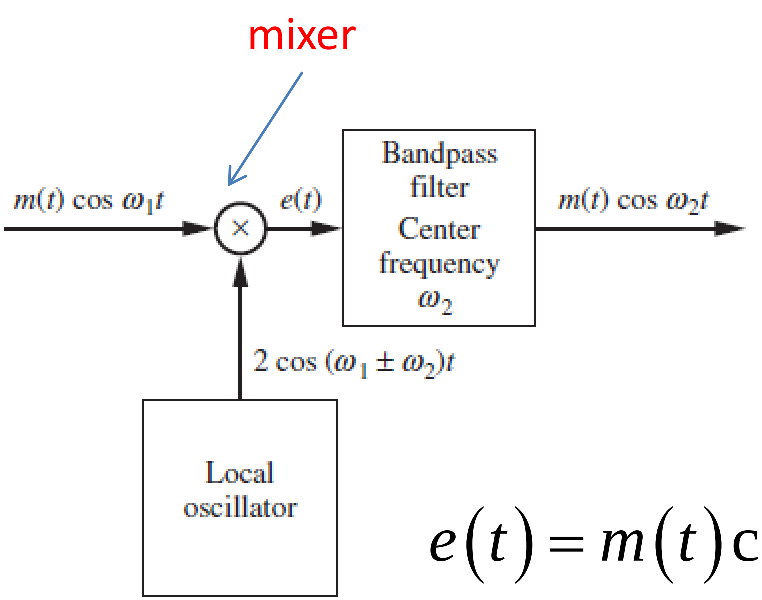
Linear modulation

- Frequency translation and mixing
  - Frequency translation: translate a bandpass signal to a new center frequency.



$$e(t) = m(t)\cos(2\pi f_2 t) + m(t)\cos(4\pi f_1 \pm 2\pi f_2)t$$

$f_1 \to f_2$

Linear modulation

- Frequency translation and mixing
  - A common problem with mixers is that two different input signals can be translated to the same frequency, f_2.
  - Ex: $$k(t)\cos\left[2\pi\left(f_1 \pm 2f_2\right)t\right]$$
    $$e(t) = 2k(t)\cos\left[2\pi\left(f_1 \pm 2f_2\right)t\right]\cos\left[2\pi\left(f_1 \pm f_2\right)t\right]$$
    $$= k(t)\cos\left(2\pi f_2 t\right) + k(t)\cos\left[2\pi\left(2f_1 \pm 3f_2\right)t\right]$$
    - The input frequency $f_1 \pm 2f_2$, which results in an output at f_2, is referred to as the image frequency of the desired frequency f_1.

Linear modulation

- Frequency translation and mixing
  - Superheterodyne receiver has good sensitivity (the ability to detect weak signals) and selectivity (the ability to separate closely spaced signals)

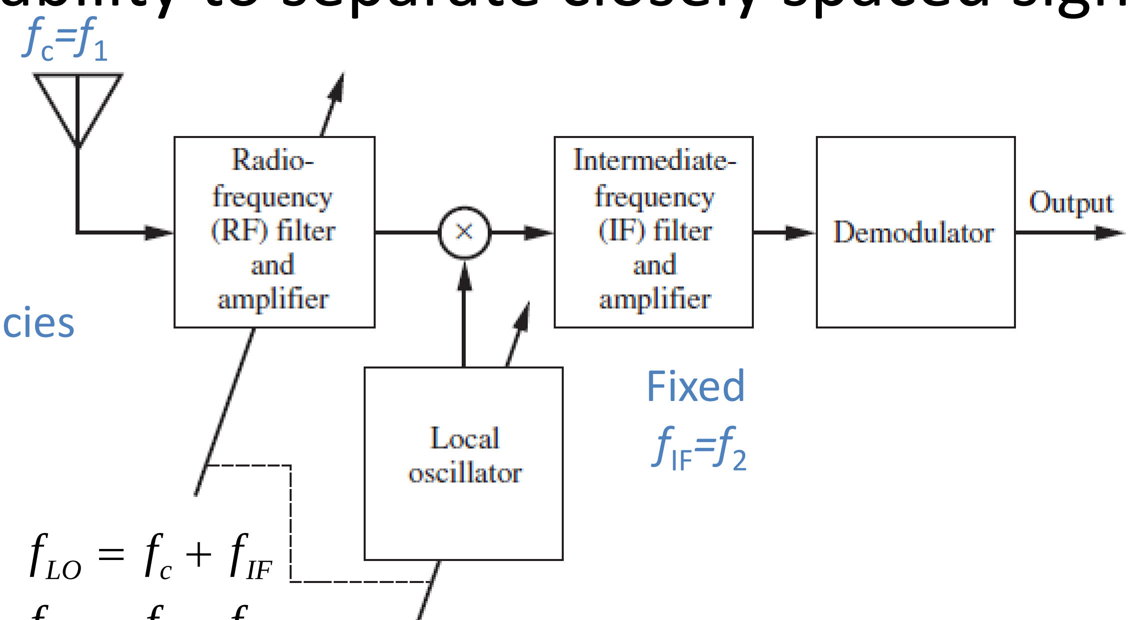
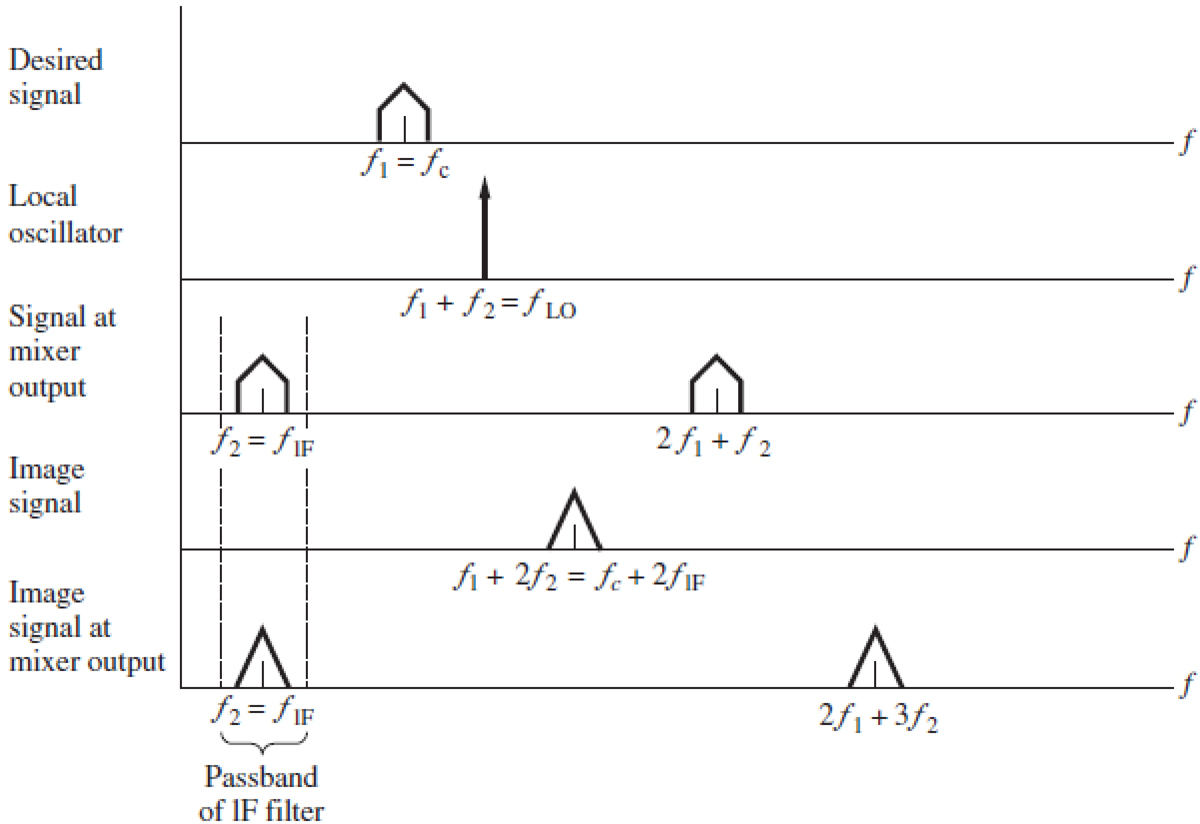


Image frequencies
$f_2 \pm 2 f_{IF}$

$$f_c \text{ and } \begin{cases} f_c + 2f_{IF}, & f_{LO} = f_c + f_{IF} \\ f_c - 2f_{IF}, & f_{LO} = f_c - f_{IF} \end{cases}$$

Linear modulation

- Frequency translation and mixing
  - Superheterodyne receiver



Linear modulation

- Frequency translation and mixing
  - Superheterodyne receiver
    - The radio-frequency (RF) filter eliminates the image frequency.
    - Ex:
      - A standard IF frequency for AM radio is 455kHz $\rightarrow$ the image frequency is separated from the desired signal by almost 1MHz $\rightarrow$ the RF filter need not to be narrowband.
      - The AM broadcast occupies the frequency range 540kHz to 1.6MHz, it is apparent that a tunable RF filter is not required, provided that stations at the high end of the band are not located geographically near stations at the low end of the band $\rightarrow$ inexpensive receiver.

Linear modulation

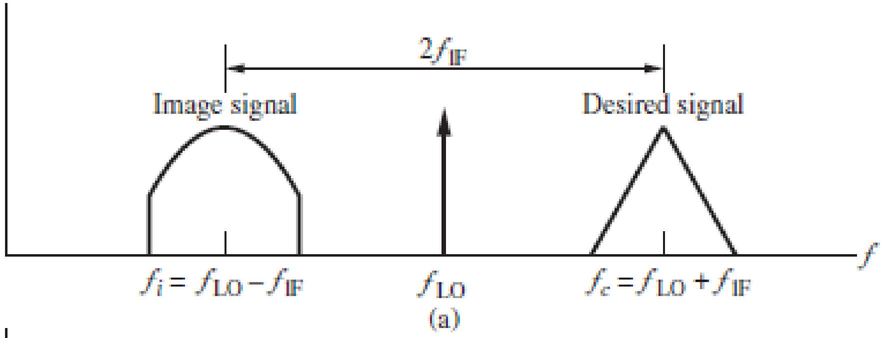
- Frequency translation and mixing
  - Superheterodyne receiver
    - The frequency of the local oscillator is to below/ above the frequency of the input carrier (*low-side tuning/high-side tuning*)?
    - Oscillators whose frequency must vary over a large ratio are much more difficult to implement than are those whose frequency varies over a small ratio.

Table 3.1 Low-Side and High-Side Tuning for AM Broadcast Band with $f_{IF} = 455$ kHz

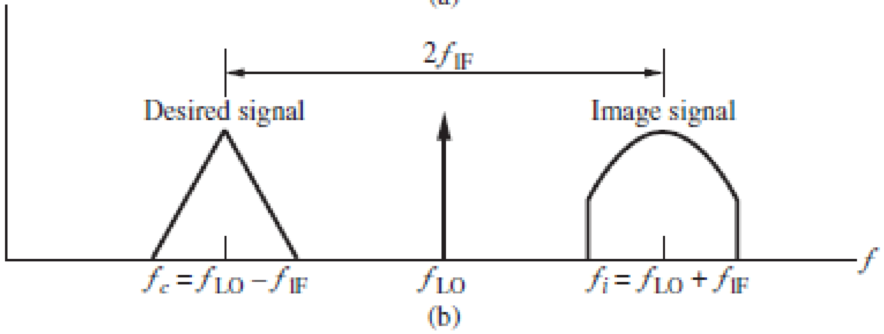
	Lower frequency	Upper frequency	Tuning range of local oscillator
Standard AM broadcast band	540 kHz	1600 kHz	
Frequencies of local oscillator for low-side tuning	540 kHz − 455 kHz = 85 kHz	1600 kHz − 455 kHz = 1145 kHz	13.47 to 1
Frequencies of local oscillator for high-side tuning	540 kHz + 455 kHz = 995 kHz	1600 kHz + 455 kHz = 2055 kHz	2.07 to 1

Linear modulation

- Frequency translation and mixing
  - Superheterodyne receiver



Low-side turning



High-side turning

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Angle modulation

- To generate angle modulation, the amplitude of the modulated carrier is held constant and either the phase or the time derivative of the phase of the carrier is varied linearly with the message signal *m*(*t*).
- General angle-modulated signal

frequency deviation

$$x_c(t) = A_c \cos\left[2\pi f_c t + \phi(t)\right]$$

Phase deviation

$$\theta_i(t) = 2\pi f_c t + \phi(t)$$

Instantaneous phase

$$f_i(t) = \frac{1}{2\pi}\frac{d\theta_i(t)}{dt} = f_c + \frac{1}{2\pi}\frac{d\phi(t)}{dt}$$

Instantaneous frequency

Angle modulation

- Phase modulation (PM): the phase deviation of the carrier is proportional to the message signal

Deviation constant in radian per unit of $m(t)$

$$\phi(t) = k_p m(t)$$

- Frequency modulation (FM): the frequency deviation of the carrier is proportional to the message signal

Frequency deviation constant in radian per second per unit of $m(t)$

$$\frac{d\phi(t)}{dt} = k_f m(t)$$

Angle modulation

- The phase deviation of a frequency-modulated carrier

Phase deviation at $t = t_0$

$$\phi(t) = k_f \int_{t_0}^{t} m(\alpha)\, d\alpha + \phi_0$$

- Frequency deviation constant f_d in hertz per unit of $m(t)$

$$k_f = 2\pi f_d$$

Angle modulation

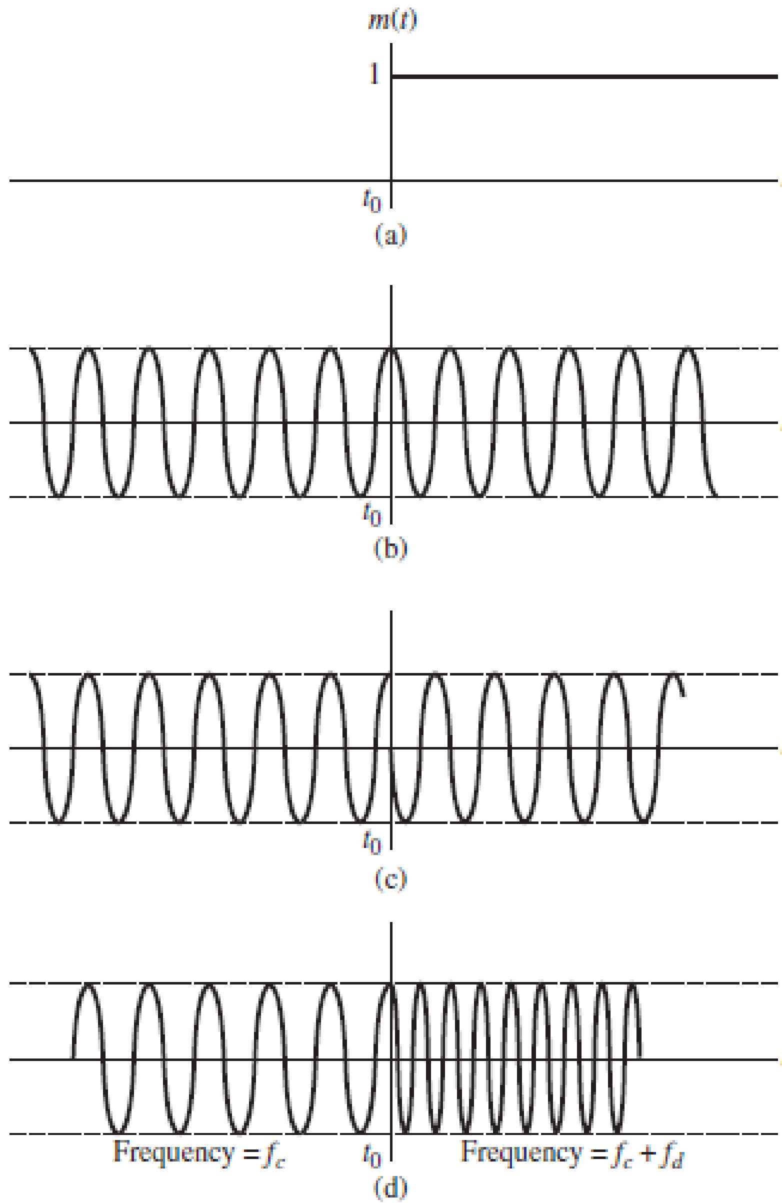
- The phase modulator output

$$x_c(t) = A_c \cos\left[2\pi f_c t + k_p m(t)\right]$$

- Frequency modulator output

$$x_c(t) = A_c \cos\left[2\pi f_c t + 2\pi f_d \int^{t} m(\alpha)\, d\alpha\right]$$

Not specified



Message signal

Unmodulated carrier

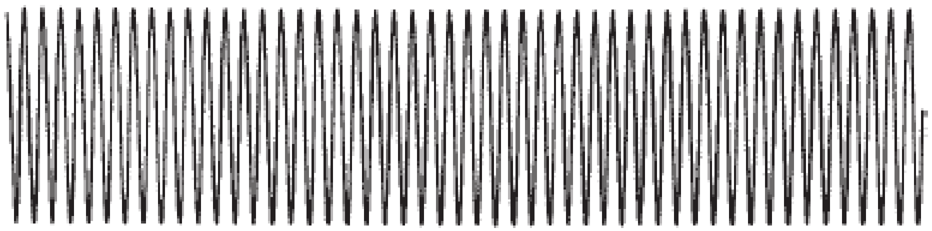
$$k_p = \frac{\pi}{2}$$

Phase modulator output

Frequency modulator output

(a)

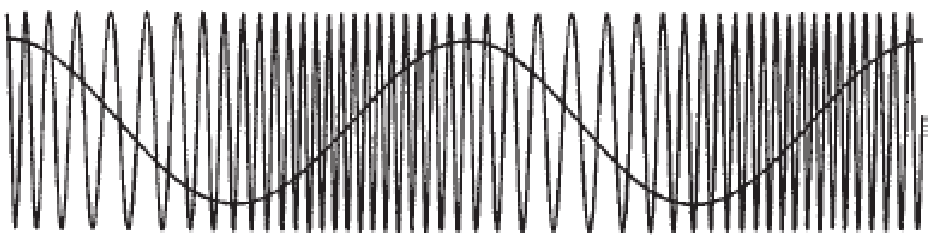
Message signal



(b)

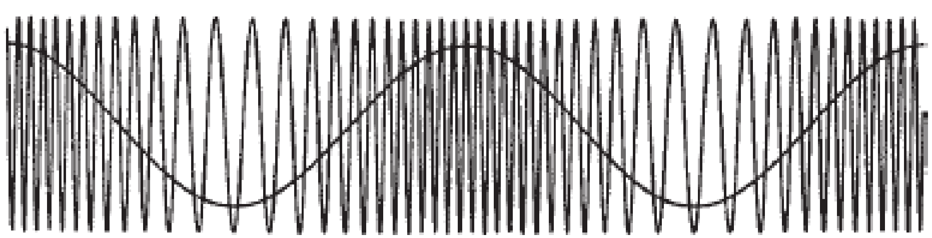
Unmodulated carrier

$$x_c(t) = A_c \cos(2\pi f_c t)$$



(c)

Phase modulator output



(d)

Frequency modulator output

Angle modulation

- Narrowband angle modulation
  - Exponential form: $x_c(t) = \mathrm{Re}\left(A_c e^{j\phi(t)} e^{j2\pi f_c t}\right)$
  - Expanding $e^{j\phi(t)}$ in a power series

$$x_c(t) = \mathrm{Re}\left(A_c\left[1 + j\phi(t) - \frac{\phi^2(t)}{2!} - \dots\right] e^{j2\pi f_c t}\right)$$

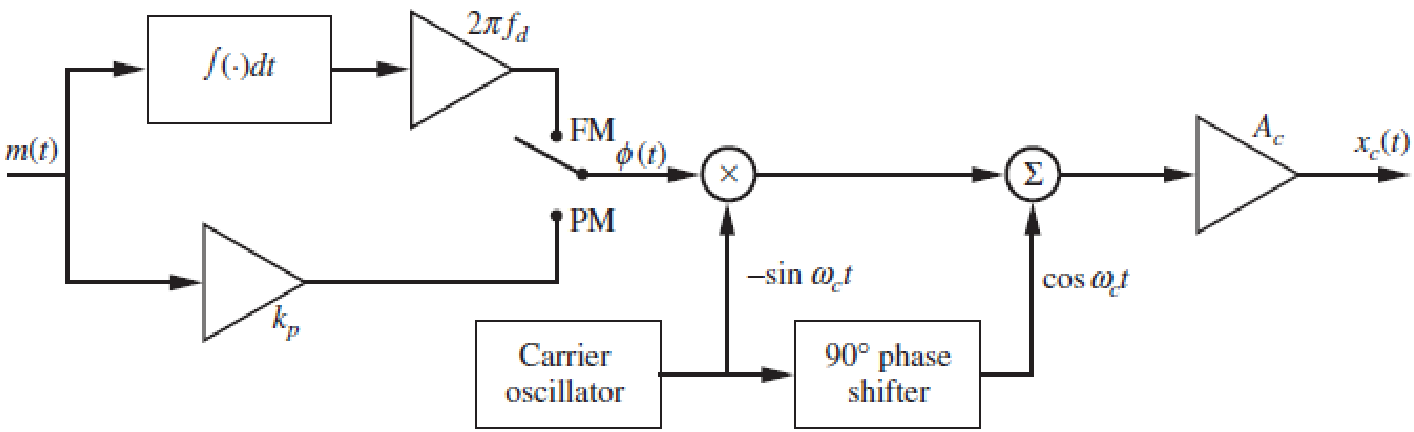
  - If max$|\phi(t)|$<<1, then

$$x_c(t) \approx \mathrm{Re}\left(A_c e^{j2\pi f_c t} + jA_c\phi(t) e^{j2\pi f_c t}\right)$$

$$= A_c \cos(2\pi f_c t) - A_c\phi(t)\sin(2\pi f_c t)$$

Angle modulation

- Narrowband angle modulation



  – Ex: FM signal? Its approximation? Similarity between FM and AM signals?

$$m(t) = A\cos(2\pi f_m t)$$

Angle modulation

- Spectrum of an angle-modulated signal
  - Consider a sinusoidal message signal.
  - Assume

$$\phi(t) = \beta \sin(2\pi f_m t)$$

where β is the *modulation index* and is the maximum value of phase deviation for both FM and PM.

  - Modulated carrier signal

$$x_c(t) = A_c \cos\left[2\pi f_c t + \beta \sin(2\pi f_m t)\right] = \operatorname{Re}\left(\underbrace{A_c e^{j\beta \sin(2\pi f_m t)}}_{\tilde{x}_c(t)} e^{j2\pi f_c t}\right)$$

Complex envelope

Angle modulation

- Spectrum of an angle-modulated signal
  - The complex envelope is periodic with frequency f_m and can be expanded in a Fourier series with the Fourier coefficients

$$f_m \int_{-1/2 f_m}^{1/2 f_m} e^{j\beta \sin(2\pi f_m t)} e^{-j2\pi n f_m t} dt = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{-(jnx - j\beta \sin x)} dx = J_n(\beta)$$

Bessel function of the first kind of order n and argument β

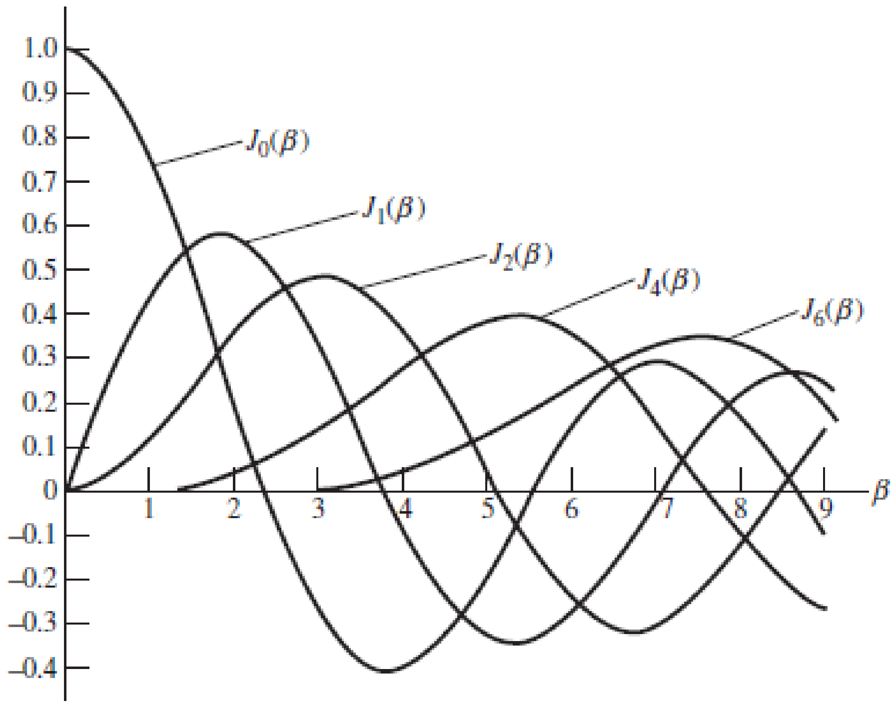
  - The Fourier series

$$e^{j\beta \sin(2\pi f_m t)} = \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j2\pi n f_m t}$$

Table 3.2 Bessel Functions

n	$\beta=0.05$	$\beta=0.1$	$\beta=0.2$	$\beta=0.3$	$\beta=0.5$	$\beta=0.7$	$\beta=1.0$	$\beta=2.0$	$\beta=3.0$	$\beta=5.0$	$\beta=7.0$	$\beta=8.0$	$\beta=10.0$
0	0.999	0.998	0.990	0.978	0.938	0.881	0.765	0.224	−0.260	−0.178	0.300	0.172	−0.246
1	0.025	0.050	0.100	0.148	0.242	0.329	0.440	0.577	0.339	−0.328	−0.005	0.235	0.043
2		0.001	0.005	0.011	0.031	0.059	0.115	0.353	0.486	0.047	−0.301	−0.113	0.255
3				0.001	0.003	0.007	0.020	0.129	0.309	0.365	−0.168	−0.291	0.058
4						0.001	0.002	0.034	0.132	0.391	0.158	−0.105	−0.220
5								0.007	0.043	0.261	0.348	0.186	−0.234
6								0.001	0.011	0.131	0.339	0.338	−0.014
7									0.003	0.053	0.234	0.321	0.217
8										0.018	0.128	0.223	0.318
9										0.006	0.059	0.126	0.292
10										0.001	0.024	0.061	0.207
11											0.008	0.026	0.123
12											0.003	0.010	0.063
13											0.001	0.003	0.029
14												0.001	0.012
15													0.005
16													0.002
17													0.001

$$\begin{cases} J_{-n}(\beta) = J_n(\beta), & n \text{ even} \\ J_{-n}(\beta) = -J_n(\beta) & n \text{ odd} \end{cases}$$



$$J_{n+1}(\beta) = \frac{2n}{\beta} J_n(\beta) + J_{n-1}(\beta)$$

- For β<<1, $J_0(\beta)$ predominates, giving rise to narrowband angle modulation.
- $J_n(\beta)$ oscillates for increasing β but the amplitude of oscillation decreases with increasing β.
- Maximum value of $J_n(\beta)$ decreases with increasing n.

Angle modulation

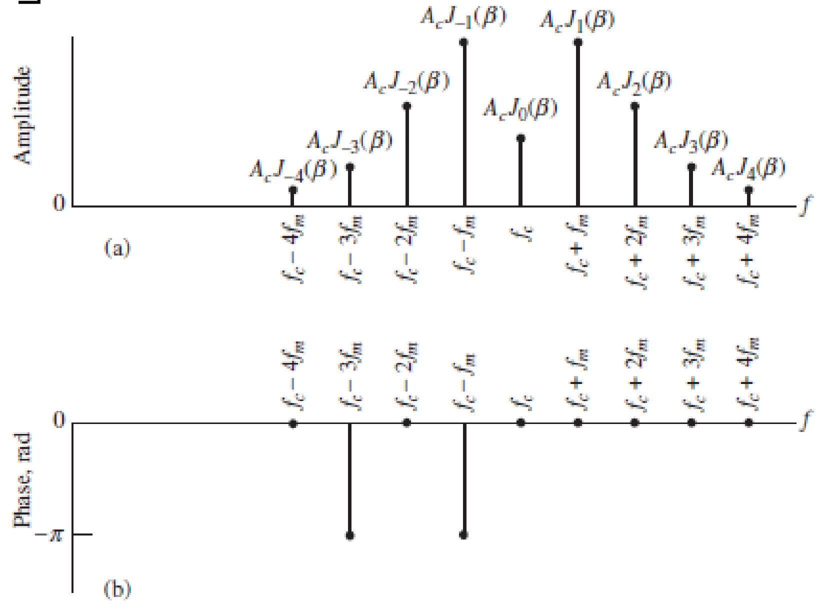
- Spectrum of an angle-modulated signal

$$x_c(t) = \mathrm{Re}\left(\left[A_c \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j2\pi n f_m t}\right] e^{j2\pi f_c t}\right)$$

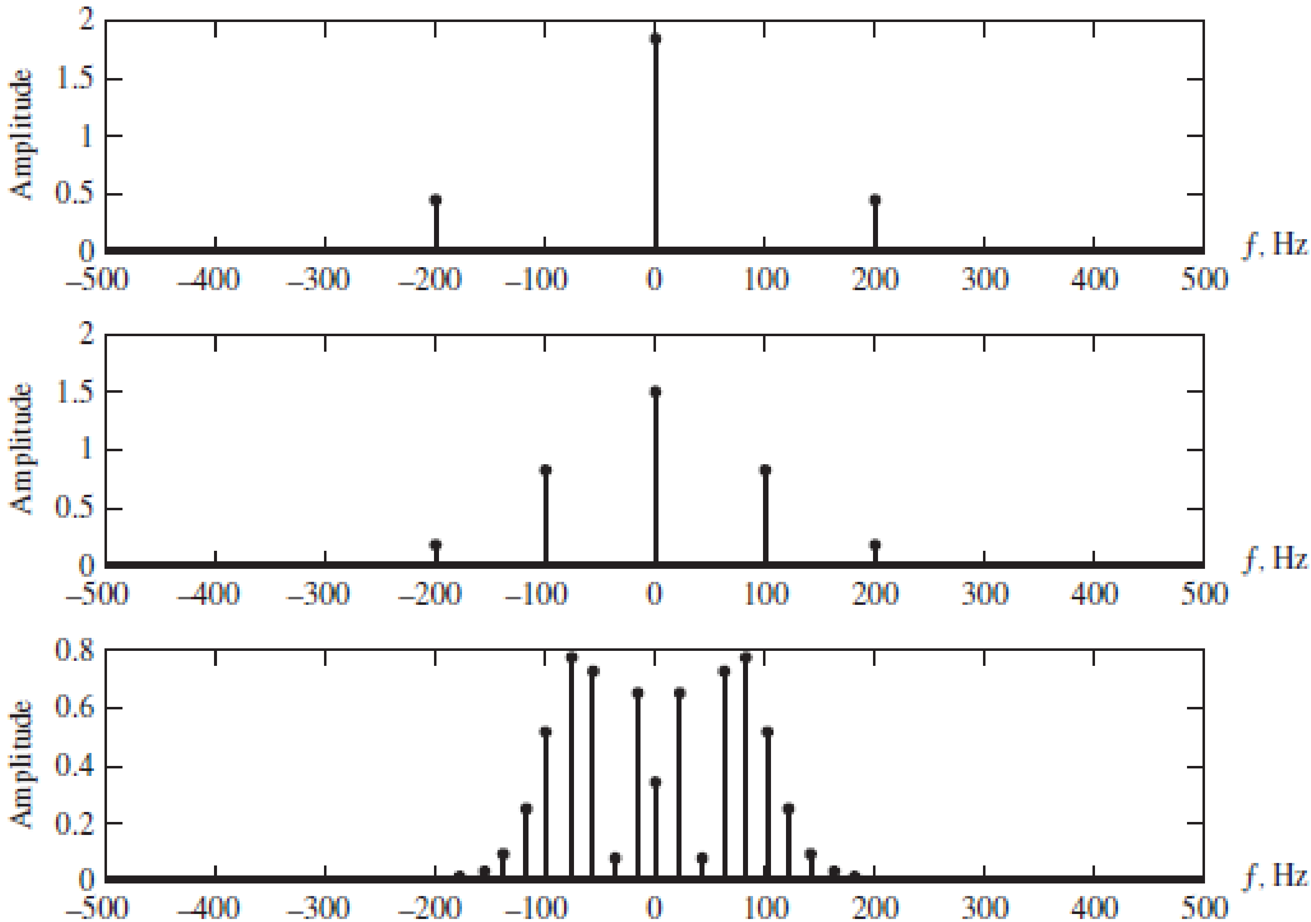
$$= A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos\left[2\pi\left(f_c + n f_m\right)t\right]$$

PM modulator $\begin{cases} m(t) = A\sin(\omega_m t) \\ \beta = k_p A \end{cases}$

FM modulator $\begin{cases} m(t) = A\cos(\omega_m t) \\ \beta = \dfrac{2\pi f_d A}{\omega_m} = \dfrac{f_d A}{f_m} \end{cases}$



FM



- ❑ $f_d A$ = constant
- ❑ Increasing β (decreasing f_m)

- Large values of f_m, the signal is narrowband FM.
- Small values of f_m, many sidebands have significant value.

Angle modulation

- Power in an angle-modulated signal

$$\left\langle x_c^2(t) \right\rangle = A_c^2 \left\langle \cos^2\left[\omega_c t + \phi(t)\right] \right\rangle$$
$$= \frac{1}{2} A_c^2 + \frac{1}{2} A_c^2 \left\langle \cos\left[2\left\{\omega_c t + \phi(t)\right\}\right] \right\rangle$$
$$= \frac{1}{2} A_c^2$$

  - Constant transmitted power, independent of the message signal, is one important difference between angle modulation and linear modulation

Angle modulation

- Bandwidth of angle-modulated signals
  - Strictly speaking, the bandwidth of angle-modulated signals is infinite.
  - For large *n*:

$$J_n(\beta) \approx \frac{\beta^n}{2^n n!} \xrightarrow{\text{fixed } \beta} \lim_{n\to\infty} J_n(\beta) = 0$$

  - The bandwidth of angle-modulated signals can be defined by considering only those terms that contain significant power.

Angle modulation

- Bandwidth of angle-modulated signals
  - The power ratio P_r is defined as the ratio of the power contained in the carrier (n=0) component and the k components on each side of the carrier to the total power in $x_c(t)$.

$$P_r = \frac{\frac{1}{2} A_c^2 \sum_{n=-k}^{k} J_n^2(\beta)}{\frac{1}{2} A_c^2} = \sum_{n=-k}^{k} J_n^2(\beta) = J_0^2(\beta) + 2\sum_{n=1}^{k} J_n^2(\beta)$$

Angle modulation

- Bandwidth of angle-modulated signals
  - Bandwidth for a particular application is often determined by defining an acceptable power ratio, solving for the required value *k*

$$B = 2kf_m$$

  - For $P_r \geq 0.98$, *n* equals the integer part of 1+β

$$B \approx 2\left(1+\beta\right) f_m$$

Angle modulation

- Bandwidth of angle-modulated signals
  - Deviation ratio

$$D = \frac{\text{peak frequency deviation}}{\text{bandwidth of } m(t)} = \frac{f_d}{W} \max\left|m(t)\right|$$

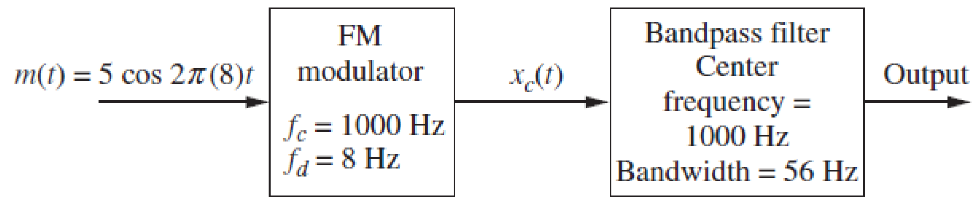
  - *Carson's rule (for arbitrary m*(*t*))

$$B = 2(1 + D)W$$

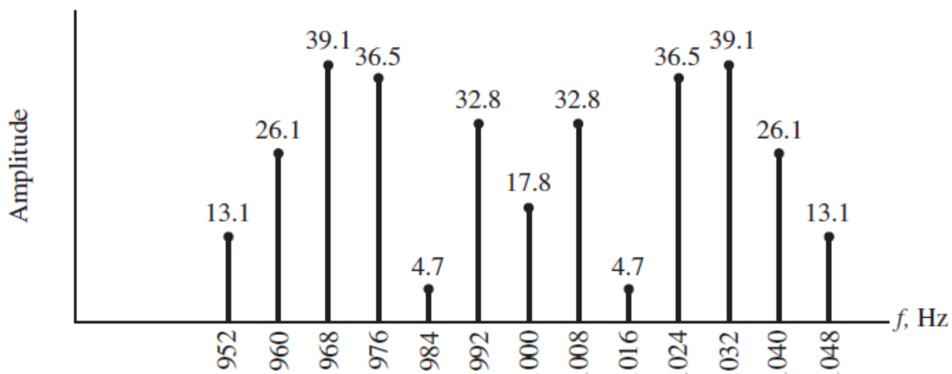
$$B \approx \begin{cases} 2W \text{ if } D << 1\text{: narrowband angle-modulated signal} \\ 2DW = 2 f_d \max\left|m(t)\right| \text{ if } D >> 1\text{: wideband angle-modulated signal} \end{cases}$$

Angle modulation

- Ex: Find the power of the filter output



$A_c = 100$



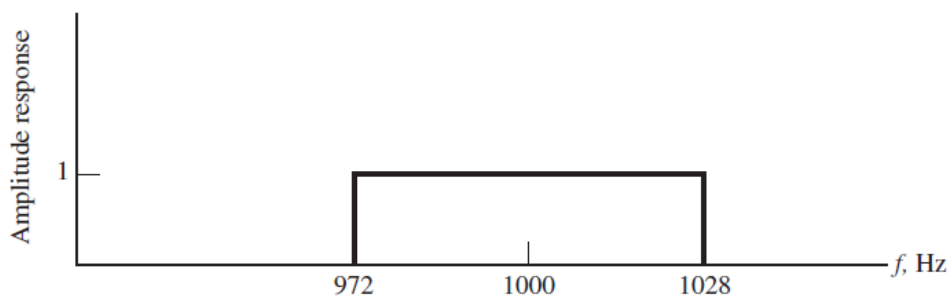


Table 3.2 Bessel Functions

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14												0.001	0.012
15													0.005
16													0.002
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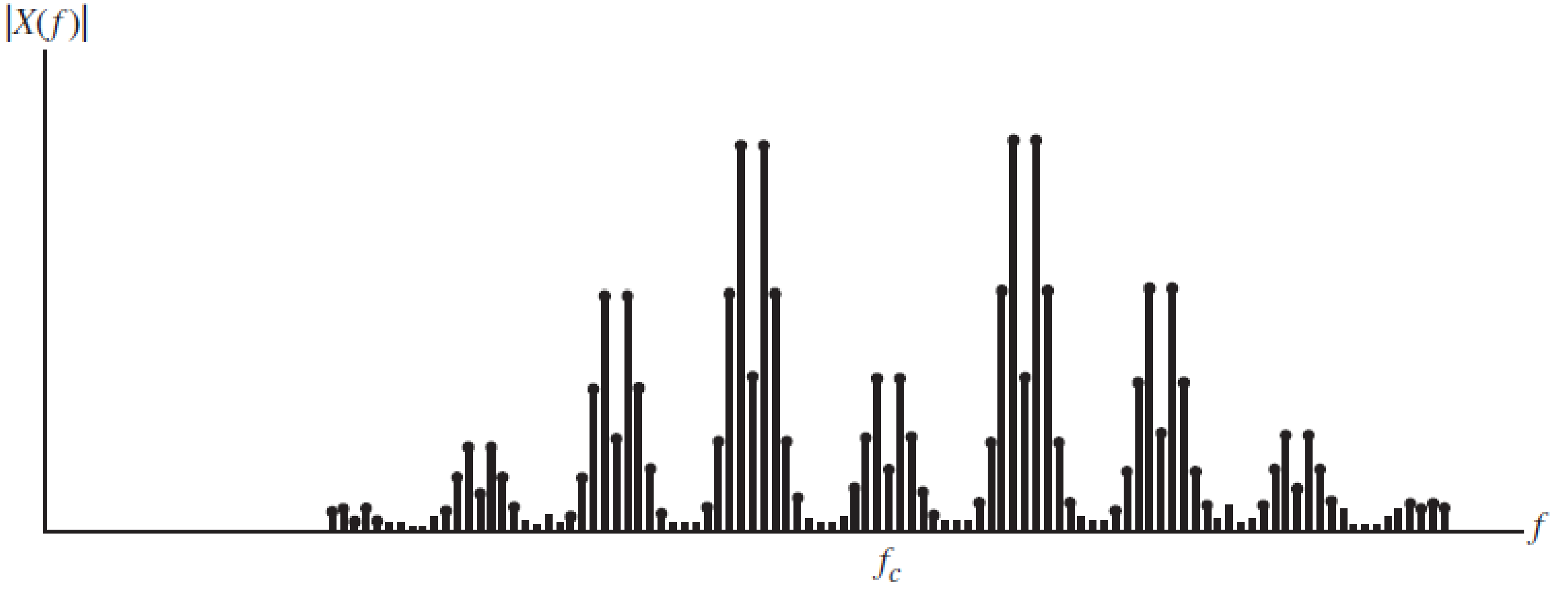
Angle modulation

- Ex: Find the spectrum of the FM signal, given

$$m(t) = A\cos(2\pi f_1 t) + B\cos(2\pi f_2 t)$$



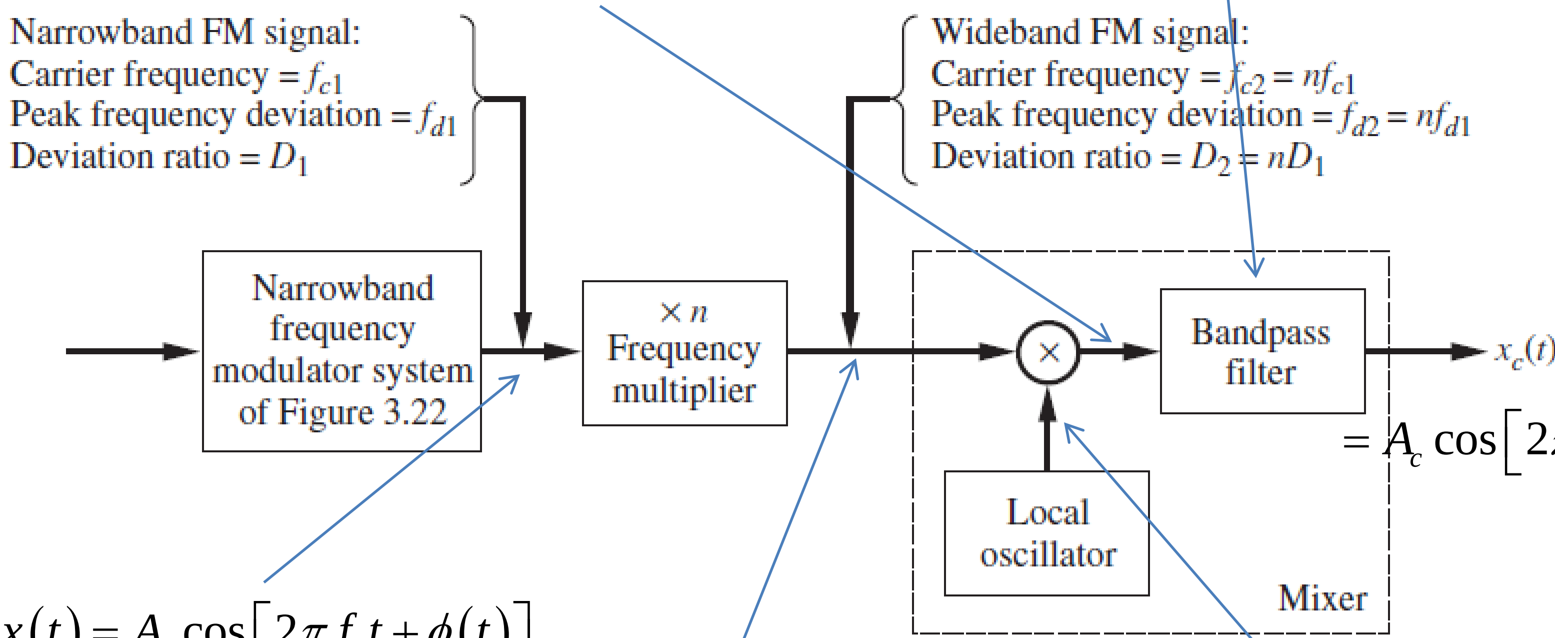
$$x_c(t) = A_c \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} J_n(\beta_1) J_m(\beta_2) \cos[2\pi(f_c + n f_1 + m f_2)t]$$

Angle modulation

- Narrowband-to-wideband conversion

$$e(t) = A_c \cos\left[2\pi\left(nf_0 + f_{LO}\right)t + n\phi(t)\right]$$
$$+A_c \cos\left[2\pi\left(nf_0 - f_{LO}\right)t + n\phi(t)\right]$$

center frequency $f_c = \begin{cases} nf_0 + f_{LO} \\ nf_0 - f_{LO} \end{cases}$



$$= A_c \cos\left[2\pi n f_c t + n\phi(t)\right]$$

$$x(t) = A_c \cos\left[2\pi f_0 t + \phi(t)\right]$$

$$y(t) = A_c \cos\left[2\pi n f_0 t + n\phi(t)\right]$$

$$e_{LO}(t) = 2\cos\left(2\pi f_{LO} t\right)$$

Angle modulation

- Narrowband-to-wideband conversion
  - The central idea in narrowband-to-wideband conversion is that the frequency multiplier changes both the carrier frequency and the deviation ratio by a factor of *n*.
  - Ex: f_0 = 100kHz, the peak frequency deviation of $\phi(t)$ is 50Hz, the bandwidth of $\phi(t)$ is 500Hz. The wideband output $x_c(t)$ is to have a carrier frequency of 85MHz and a deviation ratio of 5. Determine the frequency multiplier factor *n*, two possible oscillator frequencies, the center frequency and the bandwidth of the bandpass filter.

Angle modulation

- Demodulation of angle-modulated signals
  - *Frequency discriminator* for the demodulation of an FM signal yields an output proportional to the frequency deviation of the input.
  - The input and output of the ideal discriminator:

$$x_r(t) = A_c \cos\left[2\pi f_c t + \phi(t)\right]$$

Discriminator constant

$$y_D(t) = \frac{1}{2\pi} K_D \frac{d\phi}{dt}$$

  - FM

$$\phi(t) = 2\pi f_d \int^t m(\alpha)\,d\alpha \rightarrow y_D(t) = K_D f_d m(t)$$

Angle modulation

- Demodulation of angle-modulated signals
  - *Frequency discriminator* for the demodulation of an FM signal yields an output proportional to the frequency deviation of the input.
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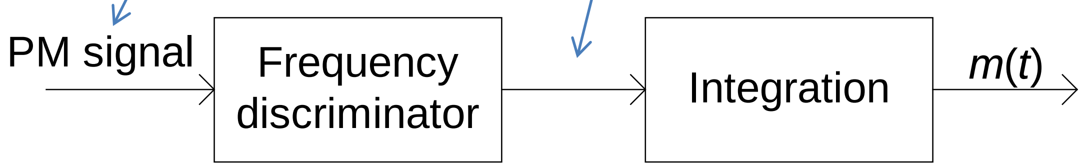
  - FM

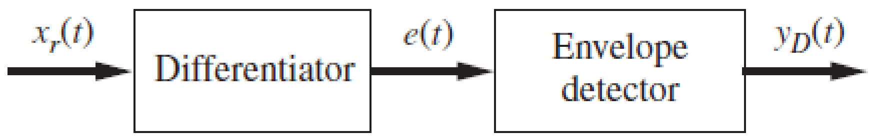
$$\phi(t) = 2\pi f_d \int^{t} m(\alpha)\, d\alpha \rightarrow y_D(t) = K_D f_d m(t)$$

Angle modulation

- Demodulation of angle-modulated signals
  - PM: $\phi(t) = k_p m(t)$ $\quad \approx \frac{d\phi}{dt} = \frac{dm}{dt}$



  - Frequency modulation discriminator

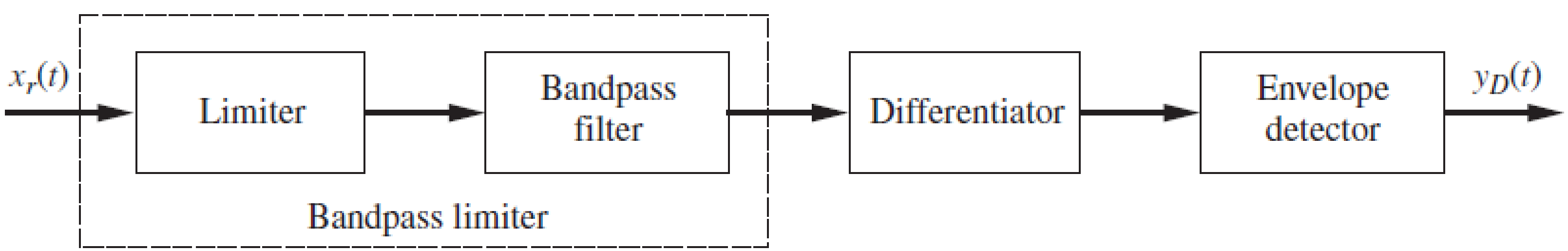


$$x_r(t) = A_c \cos\left[2\pi f_c t + \phi(t)\right] \qquad e(t) = -A_c\left(2\pi f_c + \frac{d\phi(t)}{dt}\right)\sin\left[2\pi f_c t + \phi(t)\right]$$

$$y(t) = A_c\left(2\pi f_c + \frac{d\phi(t)}{dt}\right) \xrightarrow[\text{remove DC term}]{f_c > \frac{1}{2\pi}\frac{d\phi(t)}{dt}} y_D(t) = A_c \frac{d\phi(t)}{dt} = 2\pi A_c f_d m(t)$$

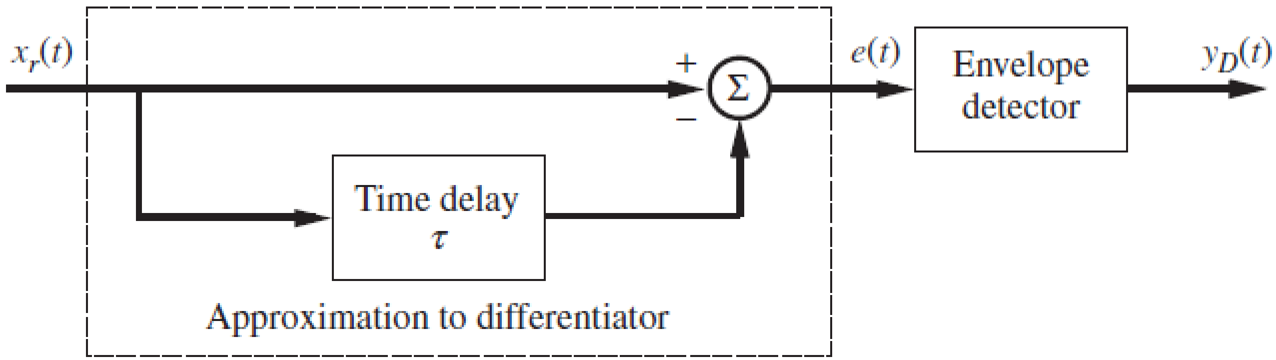
Angle modulation

- Demodulation of angle-modulated signals
  - Discriminator constant: $K_D = 2\pi A_c$
  - Interference and channel noise perturb the amplitude A_c of $x_r(t)$. To ensure the amplitude at the input to the differentiator is constant, a limiter is placed before the differentiator. The output of the limiter is a signal of square-wave type, $K\text{sgn}[x_r(t)]$. A bandpass filter having center frequency f_c is then placed after the limiter to convert the signal back to the sinusoidal.



Angle modulation

- Discriminator

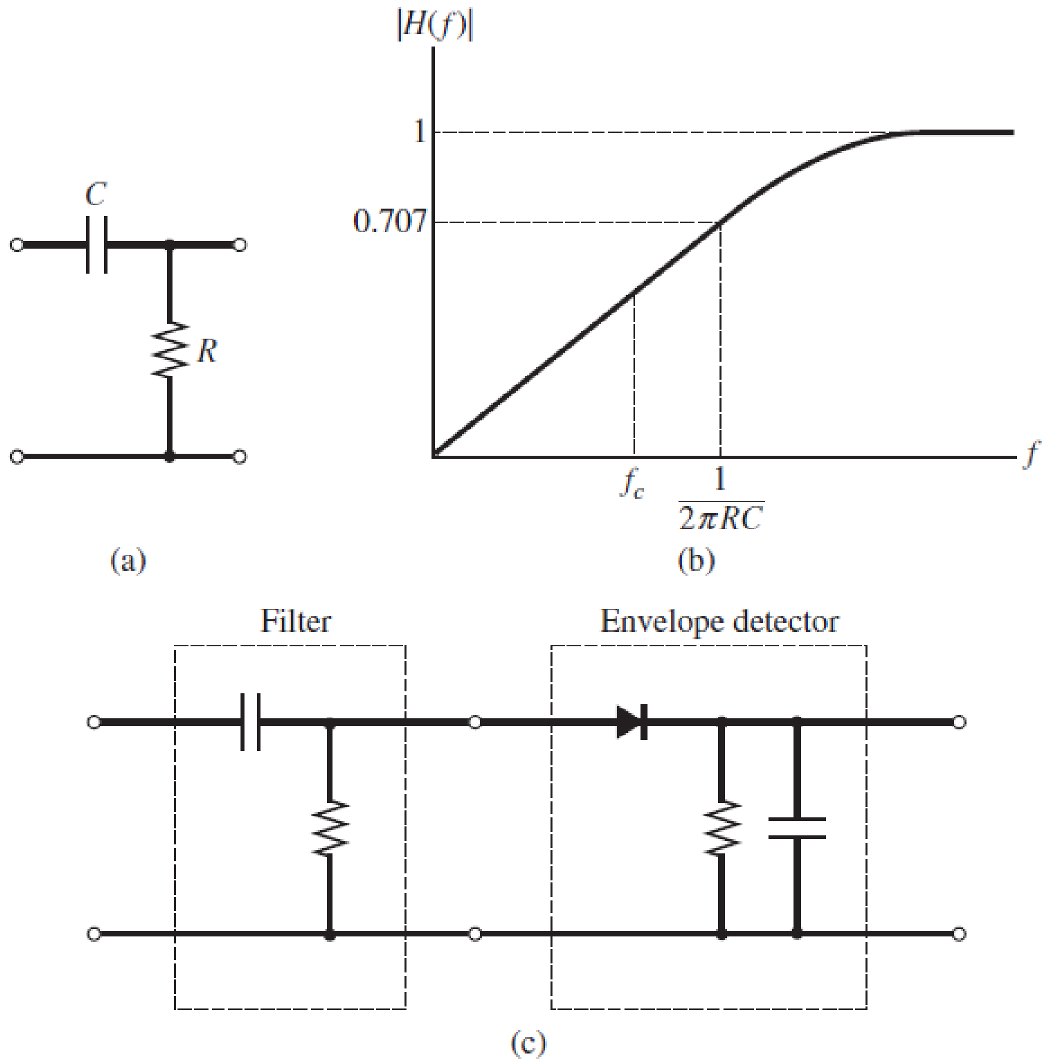


$$e(t) = x_r(t) - x_r(t-\tau) \rightarrow \frac{e(t)}{\tau} = \frac{x_r(t) - x_r(t-\tau)}{\tau}$$

$$\lim_{\tau \to 0} \frac{e(t)}{\tau} = \lim_{\tau \to 0} \frac{x_r(t) - x_r(t-\tau)}{\tau} = \frac{dx_r(t)}{dt} \rightarrow e(t) \approx \tau \frac{dx_r(t)}{dt}$$

Angle modulation

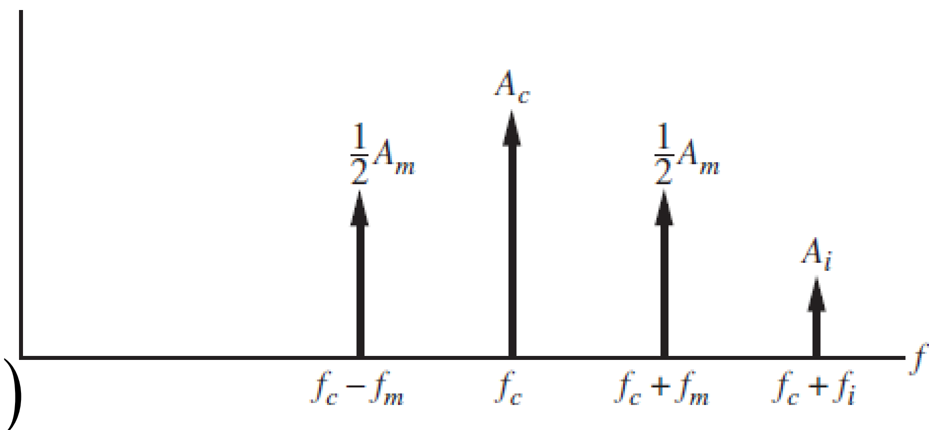
- Ex



Interference

- Interference occurs from various sources, such as RF emissions from transmitters having carrier frequency close to that of the carrier being modulated.
- Interference in linear modulation
  - Received signal

$$x_c(t) = A_c \cos(2\pi f_c t) + A_i \cos(2\pi [f_c + f_i] t) + A_m \cos(2\pi f_m t) \cos(2\pi f_c t)$$

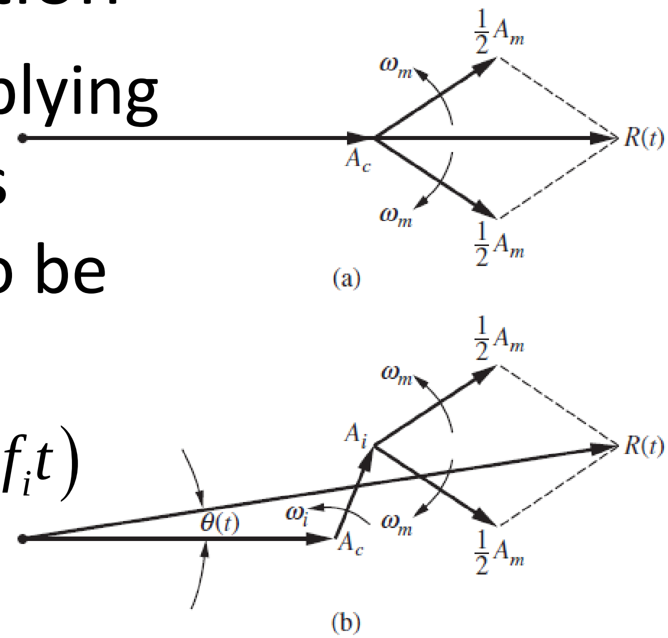


Interference

- Interference in linear modulation
  - Coherent demodulation: multiplying $x_c(t)$ by $2\cos(2\pi f_c t)$ and lowpass filtering (DC term is assumed to be blocked)

$$y_D(t) = A_m \cos(2\pi f_m t) + A_i \cos(2\pi f_i t)$$

  - Envelope detection:



$$x_r(t) = \mathrm{Re}\left[\left(A_c + A_i e^{j2\pi f_i t} + \frac{1}{2}A_m e^{j2\pi f_m t} + \frac{1}{2}A_m e^{-j2\pi f_m t}\right)e^{j2\pi f_c t}\right]$$

Interference

- Interference in linear modulation
  - Envelope detection:

$$x_r(t) = A_c \cos(2\pi f_c t) + A_m \cos(2\pi f_m t)\cos(2\pi f_c t) +$$
$$A_i\left[\cos(2\pi f_i t)\cos(2\pi f_c t) - \sin(2\pi f_i t)\sin(2\pi f_c t)\right]$$
$$= \left[A_c + A_m \cos(2\pi f_m t) + A_i \cos(2\pi f_i t)\right]\cos(2\pi f_c t) -$$
$$A_i \sin(2\pi f_i t)\sin(2\pi f_c t)$$

    - If $A_c >> A_i$, which is the usual case of interest (DC is assumed to be blocked)

$$y_D(t) \approx A_m \cos(2\pi f_m t) + A_i \cos(2\pi f_i t)$$

Interference

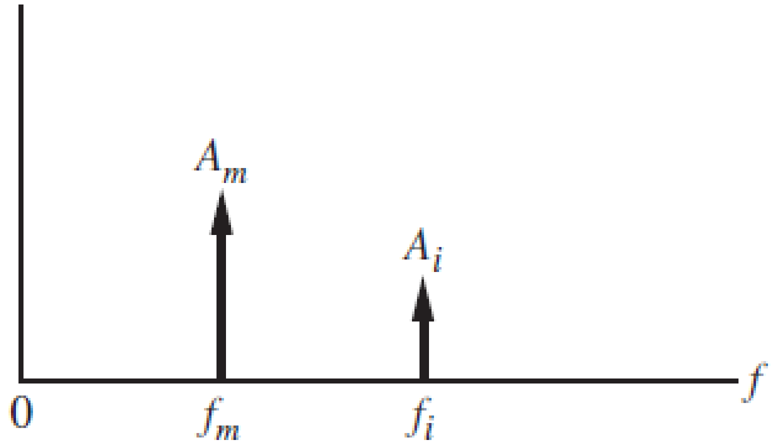
- Interference in linear modulation
  - Envelope detection:
    - If $A_c << A_i$

$$
\begin{aligned}
x_r(t) &= A_c\cos\left(2\pi\left[f_c + f_i - f_i\right]t\right) + A_i\cos\left(2\pi\left[f_c + f_i\right]t\right) + \\
&\quad A_m\cos\left(2\pi f_m t\right)\cos\left(2\pi\left[f_c + f_i - f_i\right]t\right) \\
&= A_c\left\{\cos\left(2\pi\left[f_c + f_i\right]t\right)\cos\left(2\pi f_i t\right) + \sin\left(2\pi\left[f_c + f_i\right]t\right)\sin\left(2\pi f_i t\right)\right\} \\
&\quad + A_i\cos\left(2\pi\left[f_c + f_i\right]t\right) + \\
&\quad A_m\cos\left(2\pi f_m t\right)\left\{\cos\left(2\pi\left[f_c + f_i\right]t\right)\cos\left(2\pi f_i t\right) + \sin\left(2\pi\left[f_c + f_i\right]t\right)\sin\left(2\pi f_i t\right)\right\} \\
&= \left[A_i + A_c\cos\left(2\pi f_i t\right) + A_m\cos\left(2\pi f_m t\right)\cos\left(2\pi f_i t\right)\right]\cos\left(2\pi\left[f_c + f_i\right]t\right) \\
&\quad + \left[A_c\sin\left(2\pi f_i t\right) + A_m\cos\left(2\pi f_m t\right)\sin\left(2\pi f_i t\right)\right]\sin\left(2\pi\left[f_c + f_i\right]t\right)
\end{aligned}
$$

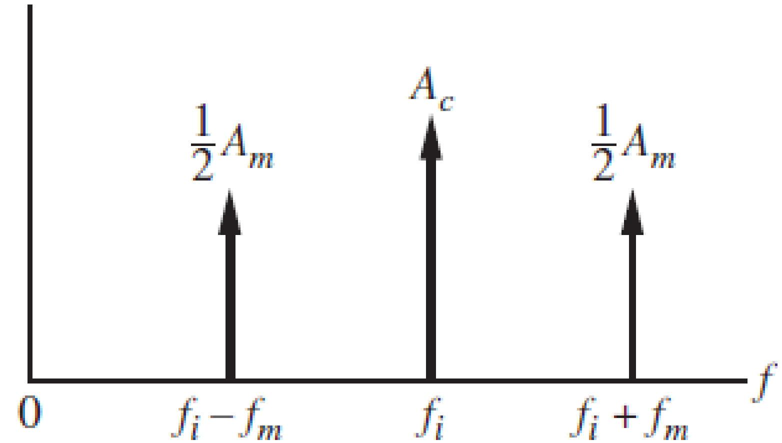
$$
y_D(t) \approx A_c\cos\left(2\pi f_i t\right) + A_m\cos\left(2\pi f_m t\right)\cos\left(2\pi f_i t\right)
$$

Interference

- Interference in linear modulation
  - Envelope detection



$A_c >> A_i$



$A_c << A_i$

Interference

- Interference in angle modulation
  - Assume that the input to a PM or FM ideal discriminator

$$x_r(t) = A_c \cos\left(2\pi f_c t\right) + A_i \cos\left[2\pi\left(f_c + f_i\right)t\right]$$

$$= A_c \cos\left(2\pi f_c t\right) + A_i \cos\left(2\pi f_c t\right)\cos\left(2\pi f_i t\right)$$

$$-A_i \sin\left(2\pi f_c t\right)\sin\left(2\pi f_i t\right)$$

$$= R(t)\cos\left[2\pi f_c t + \psi(t)\right]$$

$$R(t) = \sqrt{\left[A_c + A_i \cos\left(2\pi f_i t\right)\right]^2 + \left[A_i \sin\left(2\pi f_i t\right)\right]^2}$$

$$\psi(t) = \tan^{-1}\left(\frac{A_i \sin\left(2\pi f_i t\right)}{A_c + A_i \cos\left(2\pi f_i t\right)}\right)$$

Interference

- Interference in angle modulation
  - If $A_c >> A_i$

$$R(t) = A_c + A_i \cos(2\pi f_i t)$$

$$\psi(t) = \frac{A_i}{A_c} \sin(2\pi f_i t)$$

$$x_r(t) = A_c \left[1 + \frac{A_i}{A_c} \cos(2\pi f_i t)\right] \cos\left[2\pi f_c t + \frac{A_i}{A_c} \sin(2\pi f_i t)\right]$$

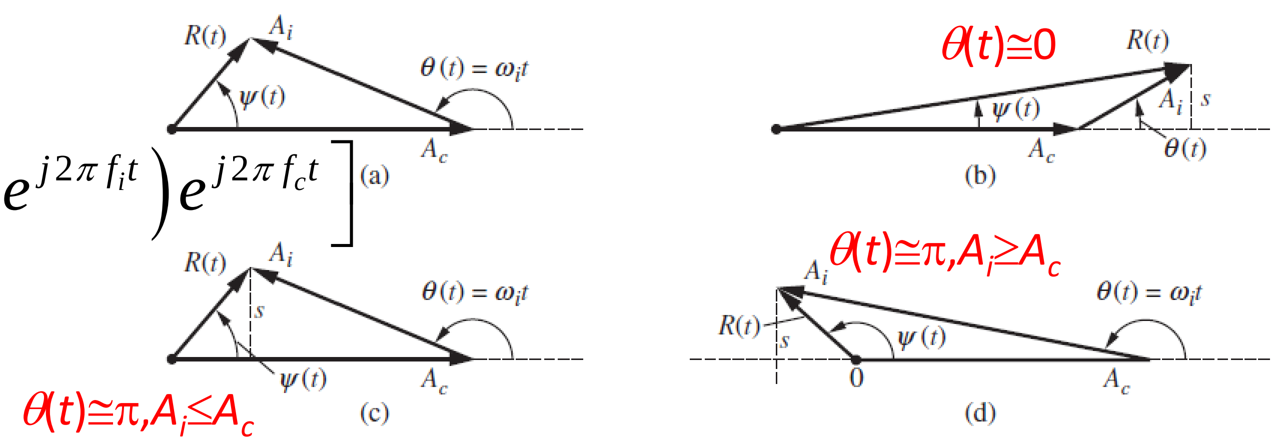
- Ideal discriminator output

$$y_D(t) = \begin{cases} K_D \dfrac{A_i}{A_c} \sin(2\pi f_i t), & \text{PM} \\ \dfrac{1}{2\pi} K_D \dfrac{d}{dt}\left(\dfrac{A_i}{A_c} \sin(2\pi f_i t)\right) = K_D \dfrac{A_i}{A_c} f_i \cos(2\pi f_i t), & \text{FM} \end{cases}$$

Interference

- Interference in angle modulation
  - If $A_c >> A_i$
    - FM: the amplitude of the discriminator output is proportional to the frequency f_i.
      - For small f_i, the interfering tone has less effect on the FM system than on the PM system.
      - For large f_i, the opposite is true.
  - Larger A_i

$$x_r(t) = \mathrm{Re}\left[\left(A_c + A_i e^{j2\pi f_i t}\right) e^{j2\pi f_c t}\right]$$



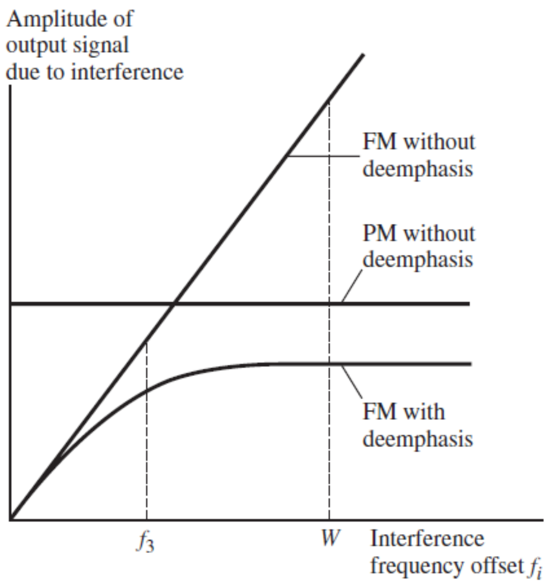
Interference

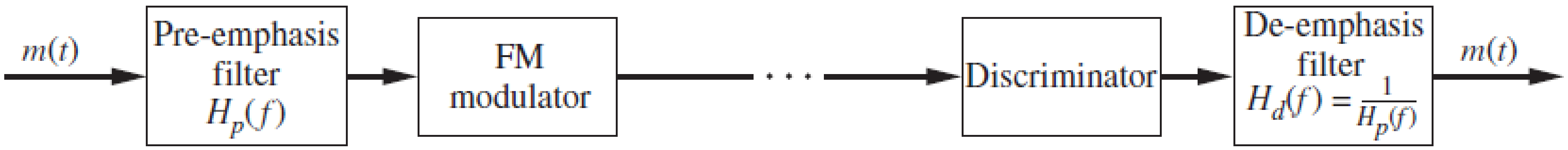
- Interference in angle modulation
  - Larger A_i

$$
\begin{aligned}
y_D(t) &= K_D \frac{A_i}{A_c + A_i} f_i, \quad \theta(t) \approx 0 \\
&= -K_D \frac{A_i}{A_c - A_i} f_i, \quad \theta(t) \approx \pi \ \text{ and } \ A_i \le A_c \\
&= -K_D \frac{A_i}{A_c - A_i} f_i, \quad \theta(t) \approx \pi \ \text{ and } \ A_i \ge A_c
\end{aligned}
$$

Interference

- Interference in angle modulation
  - For $A_i << A_c$, the severe effect of interference on FM for large f_i can be reduced by placing a *de-emphasis* filter (simply, RC lowpass filter with a 3-dB frequency considerably less than the modulation bandwidth W) at the FM discriminator output.

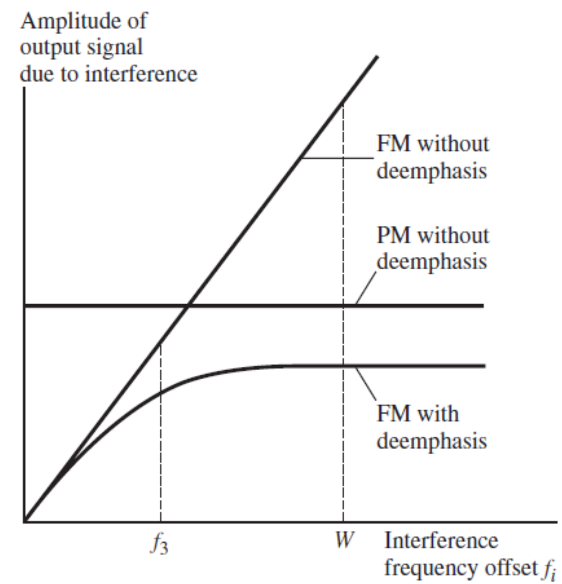


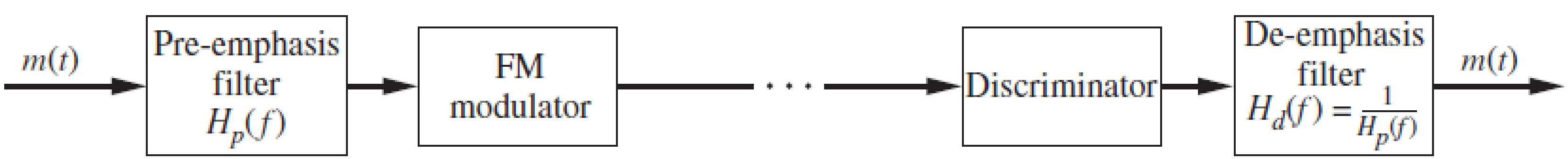


Interference

$$y_D(t) = \begin{cases} K_D \dfrac{A_i}{A_c} \sin(2\pi f_i t), & \text{PM} \\ \dfrac{1}{2\pi} K_D \dfrac{d}{dt}\left(\dfrac{A_i}{A_c} \sin(2\pi f_i t)\right) = K_D \dfrac{A_i}{A_c} f_i \cos(2\pi f_i t), & \text{FM} \end{cases}$$

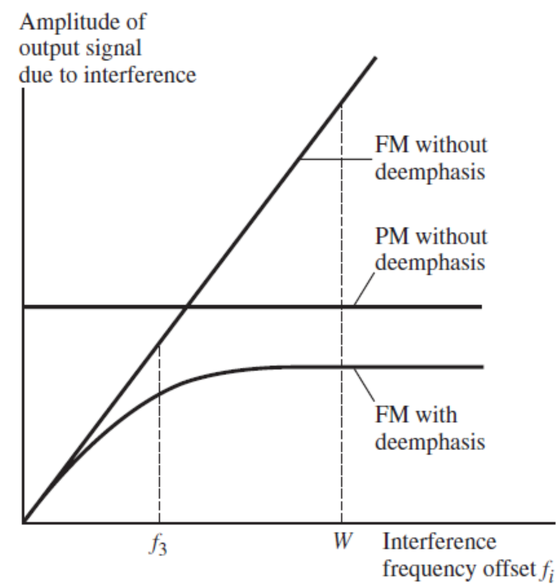
- Interference in angle modulation
  - The *de-emphasis* filter reduces the interference for large f_i.
  - For large frequencies, the magnitude of the transfer function of a first-order filter is approximately $1/f$.
  - Since the amplitude of the interference increases linearly with f_i for FM, the output is constant for large f_i.

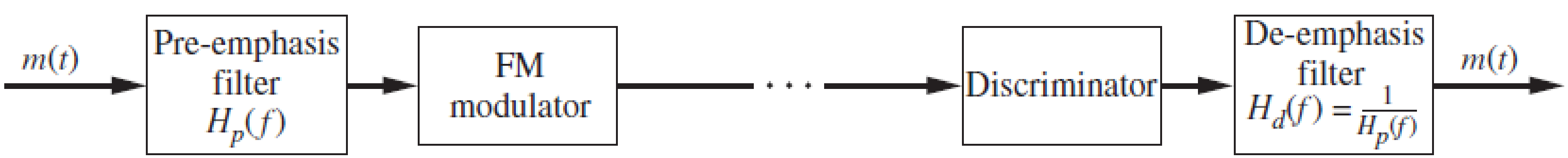




Interference

- Interference in angle modulation
  - Since f_i<W, the lowpass de-emphasis filter distorts the message signal.
  - The distortion can be avoided by passing the message through a highpass *pre-emphasis* filter that has a transfer function equal to the reciprocal of the transfer function of the lowpass de-emphasis filter.
  - Since the transfer function of the cascade combination of the pre-emphasis and de-emphasis filters is unity, there is no detrimental effect on the modulation.





Quiz

Feedback demodulators: PLL

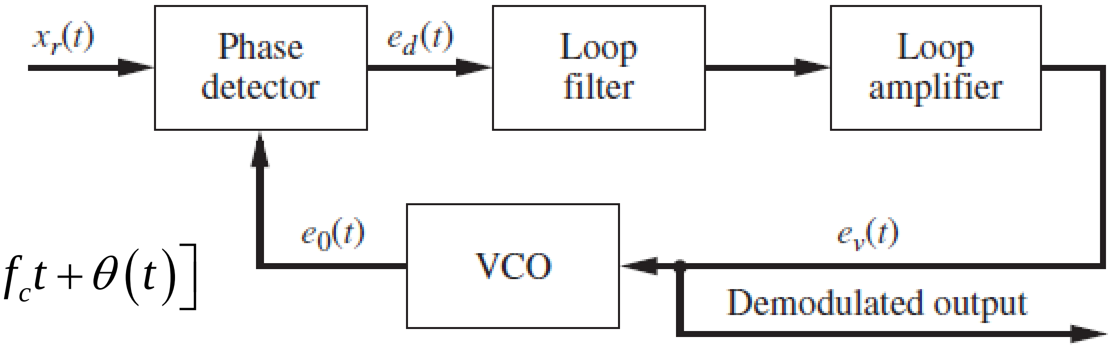
- Phase-locked loops (PLL) are widely used in today's communication systems, not only for demodulation of angle modulated signals but also for carrier and symbol synchronization, for frequency synthesis, and as the basic building block for a variety of digital demodulators.
- PLL are flexible in that they can be used in a wide variety of applications, are easily implemented, and give superior performance to many other techniques.

Feedback demodulators: PLL

- PLL for FM and PM demodulation

$$x_r(t) = A_c \cos\left[2\pi f_c t + \phi(t)\right]$$

$$e_0(t) = A_v \sin\left[2\pi f_c t + \theta(t)\right]$$



  - Phase detector is a multiplier followed by a lowpass filter to remove the second harmonic of the carrier, and includes an inverter to remove the minus sign resulting from the multiplication.
    - Output of the phase detector

Phase detector constant

$$e_d(t) = \frac{1}{2} A_c A_v K_d \sin\left[\phi(t) - \theta(t)\right] = \frac{1}{2} A_c A_v K_d \sin\psi(t)$$

Phase error

Feedback demodulators: PLL

- PLL for FM and PM demodulation
  - Phase detector:
    - For small phase error the two inputs to the multiplier are approximately orthogonal so that the result of the multiplication is an odd function of the phase error $\rightarrow$ a necessary requirement so that the phase detector can distinguish between positive and negative phase errors.
    - The output of the phase detector is filtered, amplified, and applied to the VCO (Voltage-controlled oscillator).
  - VCO is essentially a frequency modulator in which the frequency deviation of the output, $d\theta/dt$, is proportional to the VCO input signal.

VCO constant

$$\frac{d\theta}{dt} = K_v e_v(t) \quad \text{rad/s} \;\rightarrow\; \theta(t) = K_v \int^t e_v(\alpha)\, d\alpha$$

Feedback demodulators: PLL

- PLL for FM and PM demodulation
  - $F(s)$ is the transfer function of the loop filter

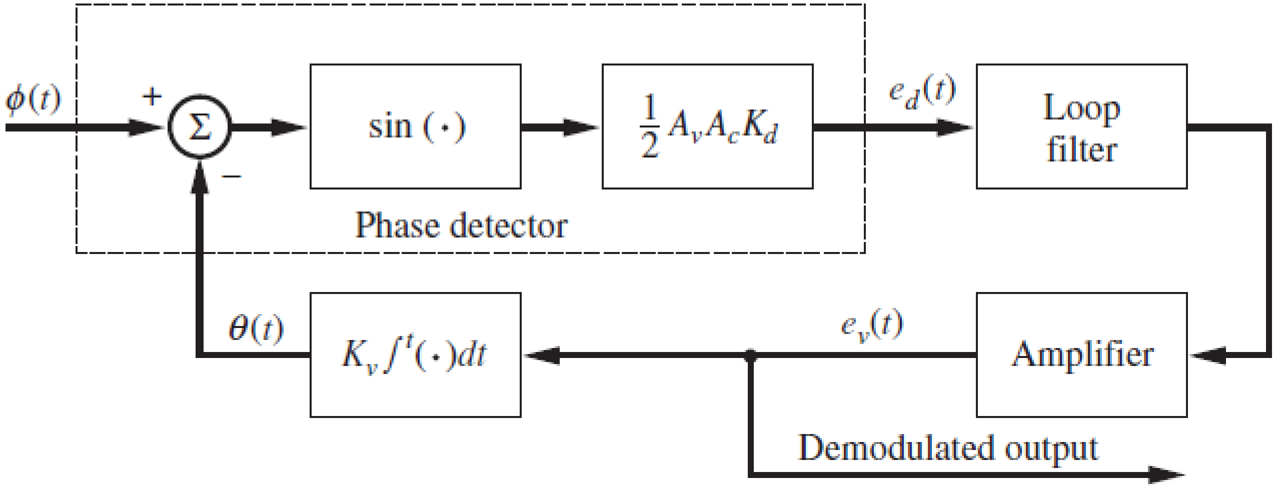
$$E_v(s) = F(s)E_d(s) \leftrightarrow e_v(t) = \int^t e_d(\lambda) f(\alpha - \lambda) d\lambda$$

  - Total loop gain: $K_t = \frac{1}{2} A_v A_c K_d K_v$

$$\theta(t) = K_t \int^t \int^\alpha \sin\left[\phi(\lambda) - \theta(\lambda)\right] f(\alpha - \lambda) d\lambda d\alpha$$

Feedback demodulators: PLL

- PLL for FM and PM demodulation



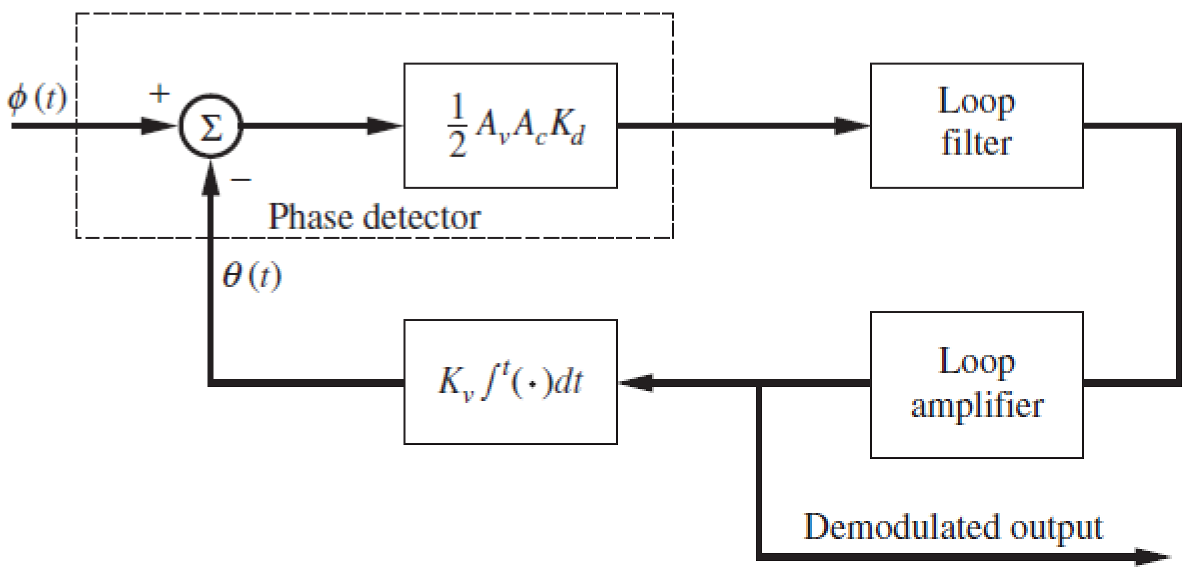
Nonlinear PLL model

  - PLL operation:
    - Acquisition mode: acquire a signal by synchronizing the frequency and phase of the VCO with the input signal (the phase errors are typically large)

Feedback demodulators: PLL

- PLL for FM and PM demodulation
  - PLL operation:
    - Tracking mode: the phase error $\phi(t)$-$\theta(t)$ is small

$$\theta(t) = K_t \int^{t} \int^{\alpha} \left[\phi(\lambda) - \theta(\lambda) \right] f\,(\alpha - \lambda)\, d\lambda\, d\alpha \longrightarrow \text{Linear PLL}$$



Feedback demodulators: PLL

- PLL for FM and PM demodulation
  - If $\phi(t) \approx \theta(t)$,

$$\frac{d\theta(t)}{dt} \approx \frac{d\phi(t)}{dt}$$

    - The VCO frequency deviation is a good estimate of the input frequency deviation.
    - For FM, the frequency deviation of the PLL input signal is proportional to the message signal $m(t)$
    - Since the VCO frequency deviation is proportional to the VCO input $e_v(t)$, $e_v(t)$ is proportional to $m(t)$
    - $e_v(t)$ is demodulated output for FM systems.

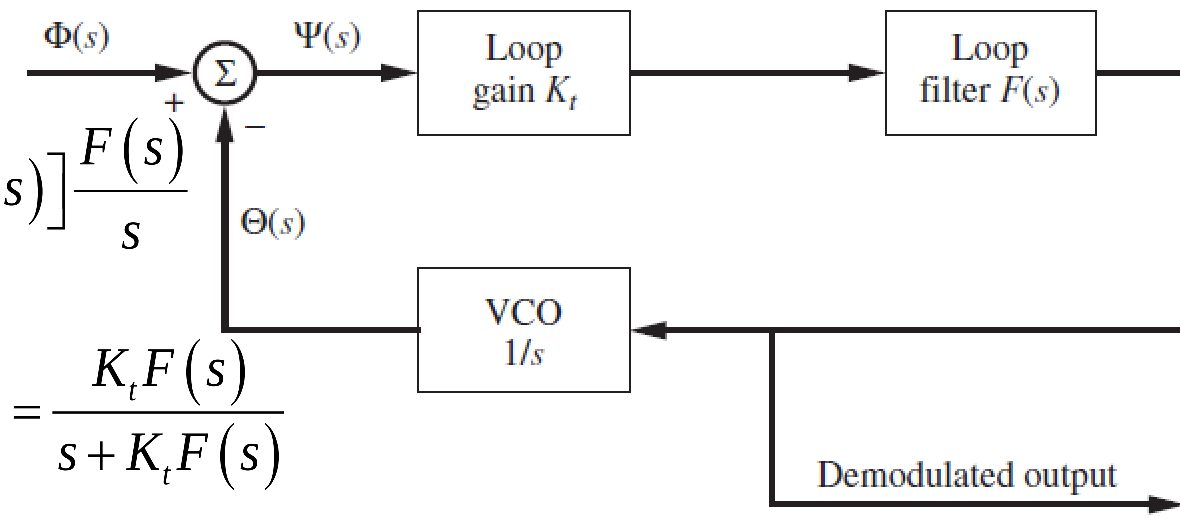
Feedback demodulators: PLL

- PLL for FM and PM demodulation
  - Loop filter transfer function

PLL order	Loop filter transfer function, $F(s)$
1	1
2	$1 + a/s = (s + a)/s$
3	$1 + a/s + b/s^2 = (s^2 + as + b)/s^2$

Feedback demodulators: PLL

- PLL operation in tracking mode: linear model
  - Transfer function relating the VCO phase to the input phase



$$\Theta(s)=K_t\left[\Phi(s)-\Theta(s)\right]\frac{F(s)}{s}$$

$$H(s)=\frac{\Theta(s)}{\Phi(s)}=\frac{K_tF(s)}{s+K_tF(s)}$$

  - Transfer function relating the phase error to the input phase

$$G(s)=\frac{\Psi(s)}{\Phi(s)}=\frac{\Phi(s)-\Theta(s)}{\Phi(s)}=1-H(s)=\frac{s}{s+K_tF(s)}$$

Feedback demodulators: PLL

- PLL operation in tracking mode: linear model
  - Assume the phase deviation as

$$\phi\left(t\right)=\pi Rt^{2}+2\pi f_{\Delta}t+\theta_{0},t>0$$

  - Frequency deviation

$$\frac{1}{2\pi}\frac{d\phi\left(t\right)}{dt}=Rt+f_{\Delta},t>0$$

  - Laplace transform of $\phi(t)$

$$\Phi\left(s\right)=\frac{2\pi R}{s^{3}}+\frac{2\pi f_{\Delta}}{s^{2}}+\frac{\theta_{0}}{s}$$

  - Steady-state phase error

$$\psi_{ss}=\lim_{s\to 0}s\left[\frac{2\pi R}{s^{3}}+\frac{2\pi f_{\Delta}}{s^{2}}+\frac{\theta_{0}}{s}\right]G\left(s\right)$$

Feedback demodulators: PLL

- PLL operation in tracking mode: linear model
  - Consider the third-order filter transfer function

$$F(s)=\frac{1}{s^2}\left(s^2+as+b\right)$$

PLL order	$\theta_0 \neq 0$ $f_\Delta = 0$ $R = 0$	$\theta_0 \neq 0$ $f_\Delta \neq 0$ $R = 0$	$\theta_0 \neq 0$ $f_\Delta \neq 0$ $R \neq 0$
1 ($a=0,\ b=0$)	0	$2\pi f_\Delta / K_t$	∞
2 ($a\neq 0,\ b=0$)	0	0	$2\pi R/K_t$
3($a\neq 0,\ b\neq 0$)	0	0	0

$$G(s)=\frac{s^3}{s^3+K_t s^2+K_t as+K_t b}$$

  - Steady-state phase error

$$\psi_{ss}=\lim_{s\to 0}\frac{s\left(\theta_0 s^2+2\pi f_\Delta s+2\pi R\right)}{s^3+K_t s^2+K_t as+K_t b}$$

Feedback demodulators: PLL

- PLL operation in tracking mode: linear model
  - Ex: Find the transfer function for a first-order PLL
  - Ex: Determine the demodulated output using a first-order PLL, assuming the input to an FM modulator $m(t) = Au(t)$

Feedback demodulators: PLL

- Second-order PLL: loop natural frequency and damping factor

$$F(s) = 1 + \frac{a}{s} \qquad H(s) = \frac{\Theta(s)}{\Phi(s)} = \frac{K_t(s+a)}{s^2 + K_t s + K_t a} \qquad G(s) = \frac{\Psi(s)}{\Phi(s)} = \frac{s^2}{s^2 + K_t s + K_t a}$$

  - Standard form for a second-order system

$$\frac{\Psi(s)}{\Phi(s)} = \frac{s^2}{s^2 + 2\varsigma\omega_n s + \omega_n^2}$$

  - Natural frequency:

$$\omega_n = \sqrt{K_t a}$$

  - Damping factor:

$$\varsigma = \frac{1}{2}\sqrt{\frac{K_t}{a}}$$

Feedback demodulators: PLL

- PLL operation in acquisition mode
  - In the acquisition mode, we must determine that the PLL actually achieves phase lock and the time required for the PLL to achieve phase lock.
  - Consider a first-order PLL: $F(s) = 1$ or $f(t) = \delta(t)$

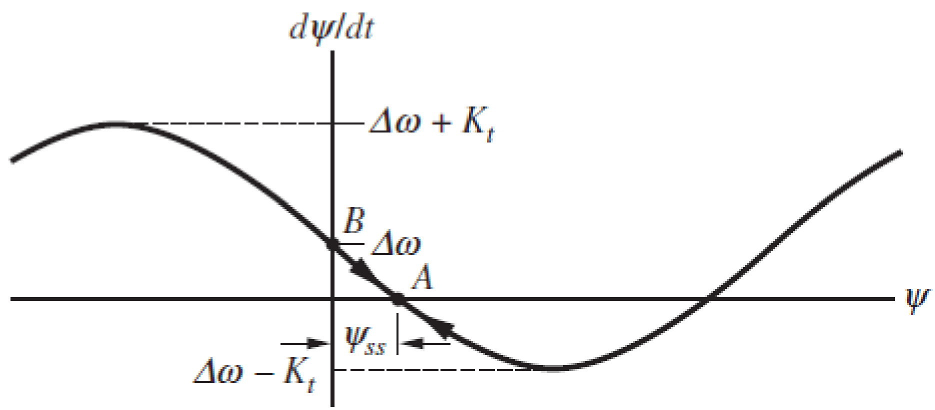
$$\theta(t) = K_t \int^{t} \sin\left[\phi(\alpha) - \theta(\alpha)\right] d\alpha \rightarrow \frac{d\theta(t)}{dt} = K_t \sin\left[\phi(t) - \theta(t)\right]$$

    - Assume that the input to the FM modulator is a unit step so that the frequency deviation $d\phi/dt$ is a unit step of magnitude $2\pi\Delta f = \Delta\omega$
    - Phase error: $\psi(t) = \phi(t) - \theta(t)$

Feedback demodulators: PLL

- PLL operation in acquisition mode

$$\frac{d\theta}{dt}=\frac{d\phi}{dt}-\frac{d\psi}{dt}=\Delta\omega-\frac{d\psi}{dt}=K_t\sin\psi\left(t\right),t>0\rightarrow\frac{d\psi}{dt}+K_t\sin\psi\left(t\right)=\Delta\omega$$



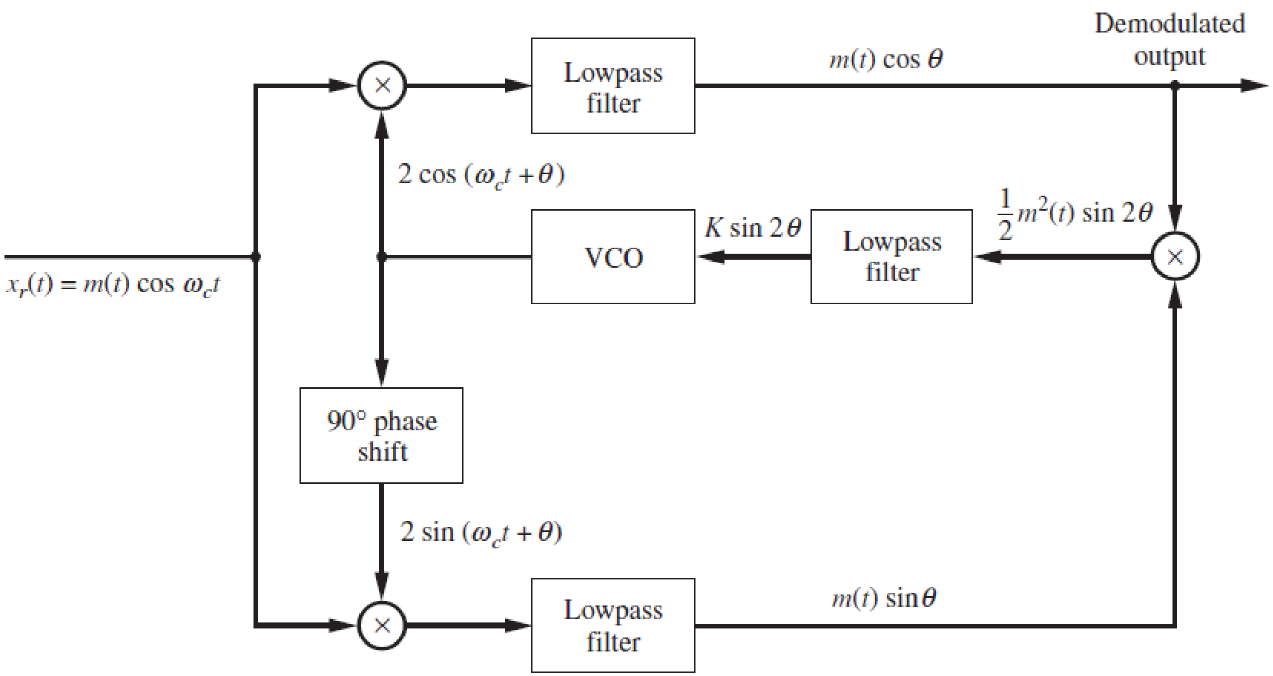
  - To demonstrate that the PLL achieves lock, assume that the PLL is operating with zero phase and frequency error prior to the application of the frequency step.

Feedback demodulators: PLL

- PLL operation in acquisition mode
  - When the step in frequency is applied, the frequency error becomes $\Delta\omega$ (point B).
  - Operating point must move from B to A.
  - When the operating point attempts to move from A by a small amount, it is forced back to A.
  - A is a stable operating point and is the steady-state operating point of the system.
  - The steady-state phase error is ψ_{ss} and the steady-state frequency error is 0.

Feedback demodulators: PLL

- Costas PLL
  - A feedback system can be used to generate the coherent demodulation carrier necessary for the demodulation of DSB signals.



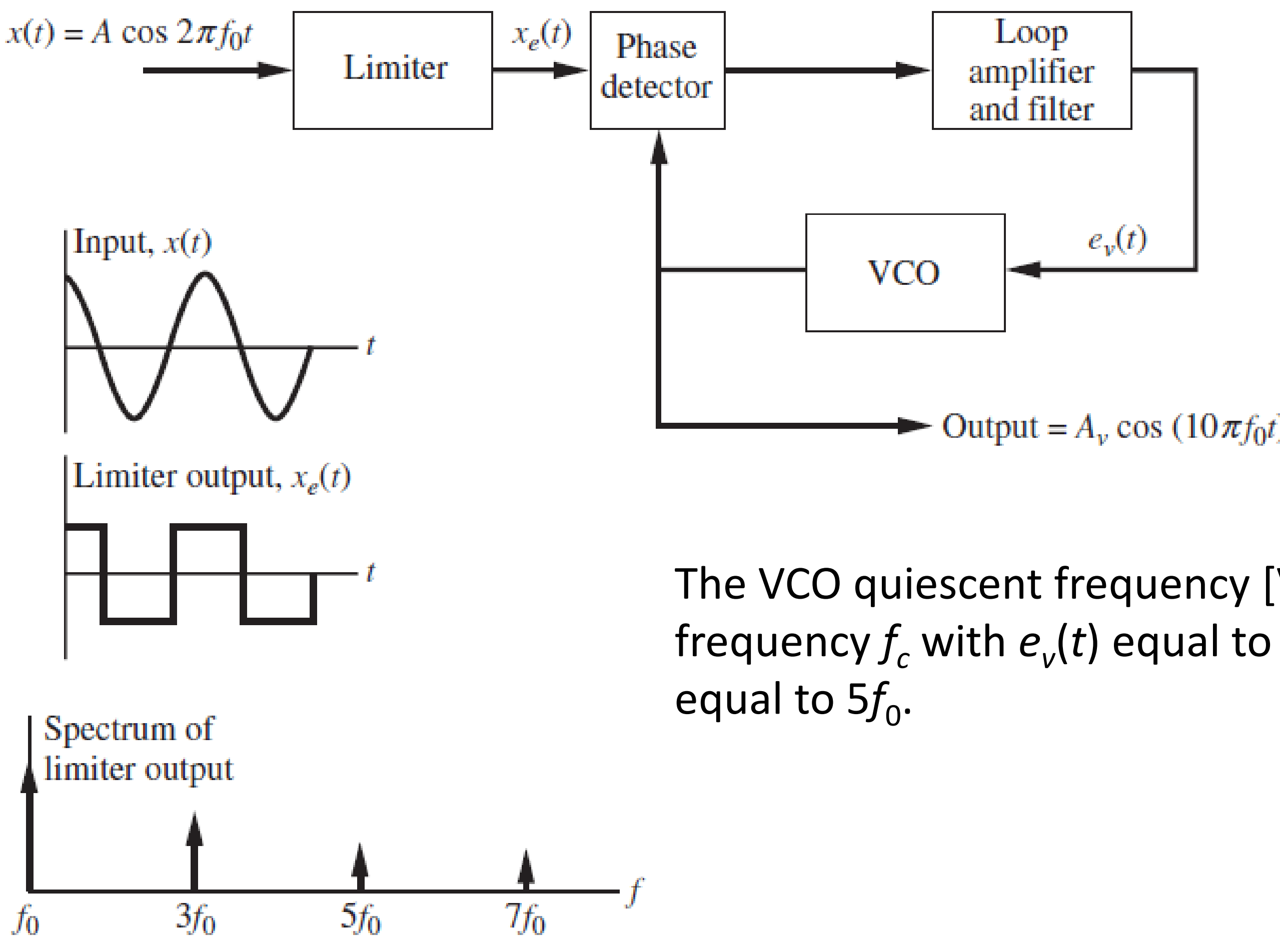
θ=0 when PLL achieves lock

Feedback demodulators: PLL

- PLL allows for simple implementation of frequency multipliers and dividers.
- Two basic schemes:
  - Harmonics of the input are generated and the VCO tracks one of these harmonics: frequency multipliers.
  - Generate harmonics of the VCO output and phase lock one of these frequency components to the input: frequency multipliers and frequency dividers.

Feedback demodulators: PLL

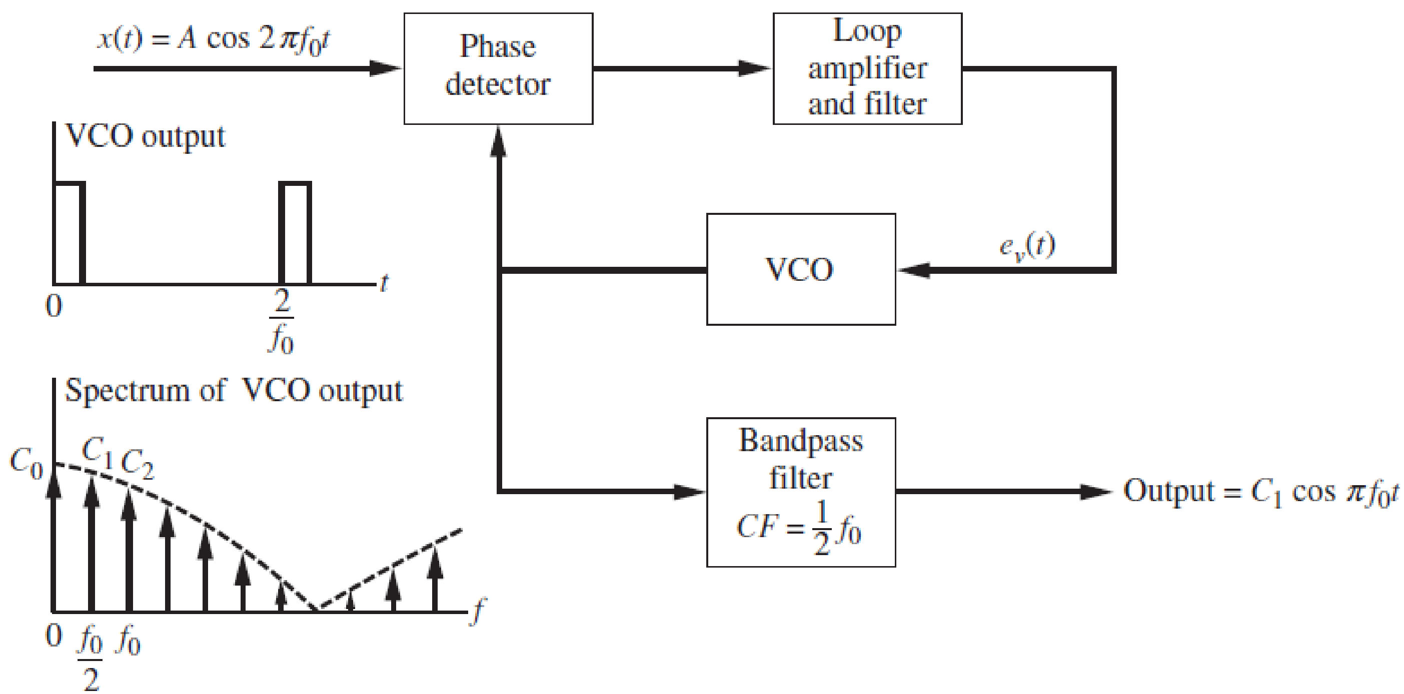
- Frequency multiplier



The VCO quiescent frequency [VCO output frequency f_c with $e_v(t)$ equal to 0] is set equal to $5f_0$.

Feedback demodulators: PLL

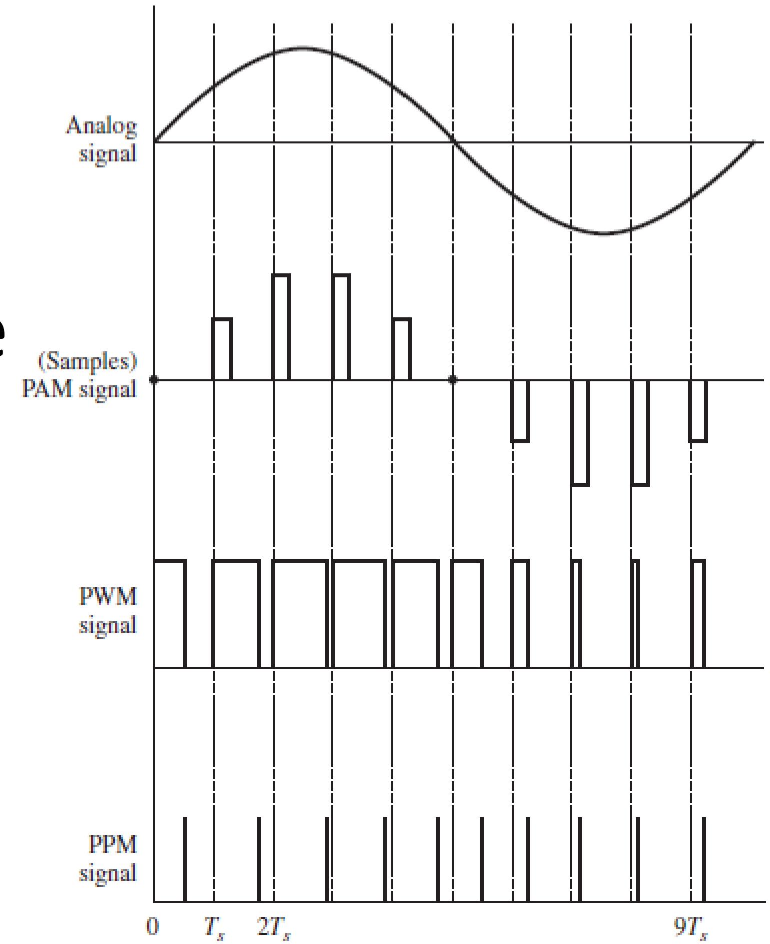
- Frequency divider



The VCO quiescent frequency is $f_0/2$.

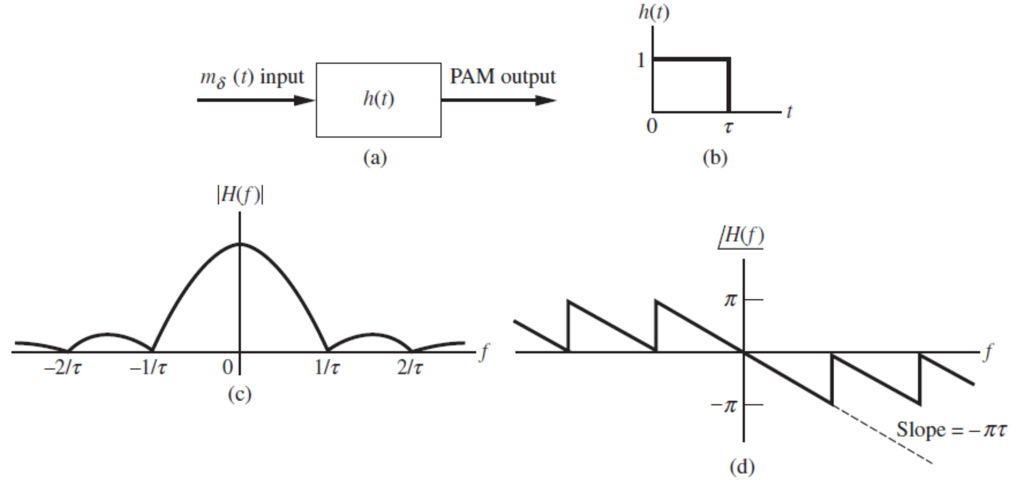
Analog pulse modulation

- Analog pulse modulation results when some attribute of a pulse varies continuously in one-to-one correspondence with a sample value.
- Three attributes: amplitude (PAM), width (PWM), position (PPM)



Analog pulse modulation

- PAM
  - Ideal holding circuit



$$h(t) = \Pi\left(\frac{t-0.5\tau}{\tau}\right)$$

$$H(f) = \tau \operatorname{sinc}(f\tau)e^{-j\pi f\tau}$$

  - Impulse function samples

$$m_\delta(t) = \sum_{n=-\infty}^{\infty} m(nT_S)\delta(t-nT_S)$$

  - PAM waveform

$$m_c(t) = \sum_{n=-\infty}^{\infty} m(nT_S)\Pi\left(\frac{t-(nT_S+0.5\tau)}{\tau}\right)$$

Analog pulse modulation

- PWM
  - A PWM waveform consists of a sequence of pulses with each pulse having a width proportional to the values of a message signal at the sampling instants
  - If sample value is 0, the PWM pulse width is typically $T_s/2$.
  - If sample value is negative, the PWM pulse width is less than $T_s/2$.
  - If sample value is positive, the PWM pulse width is greater than $T_s/2$.

Analog pulse modulation

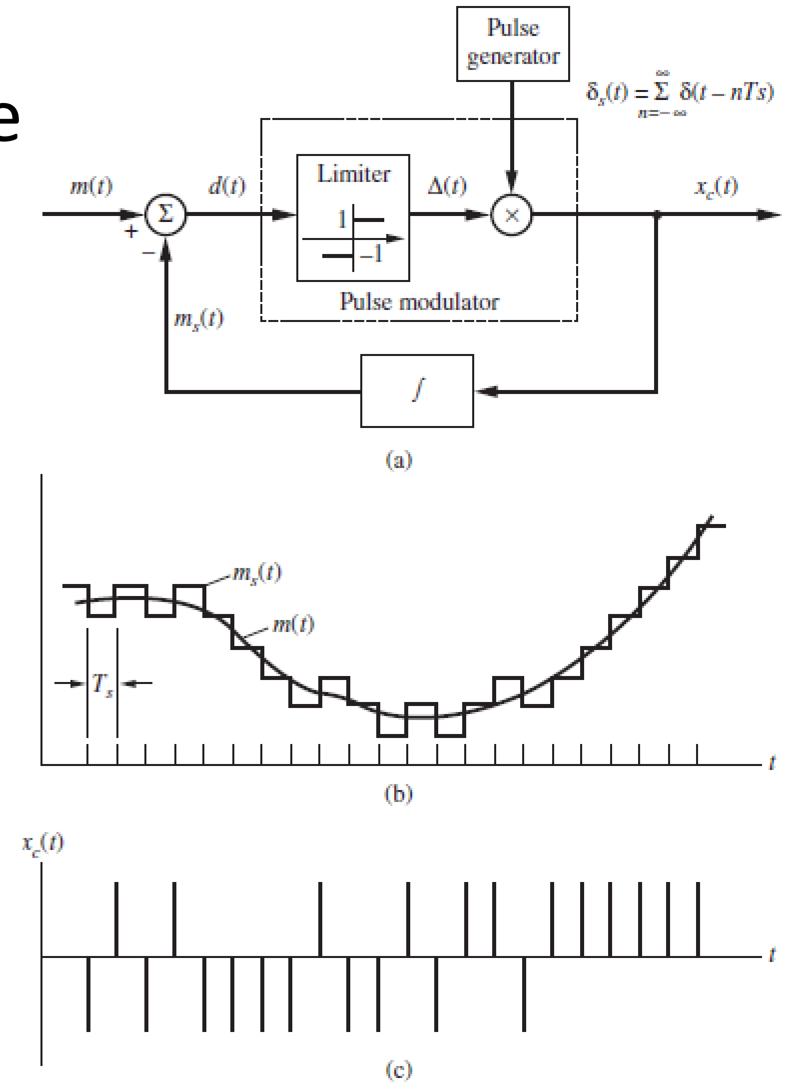
- PPM
  - A PPM signal consists of a sequence of pulses in which the pulse position from a specified time reference is proportional to the sample values of the message signal.

$$x(t) = \sum_{n=-\infty}^{\infty} g(t - t_n)$$

  - $g(t)$ represents the shape of the individual pulses
  - t_n is the occurrence time related to $m(nT_s)$

Delta modulation and PCM

- Digital pulse modulation: the transmitted samples take on only discrete values.
- Delta modulation: the message signal is encoded into a sequence of binary symbols which are represented by the polarity of impulse functions at the modulator output.



Delta modulation and PCM

- Delta modulation

$$d(t) = \underbrace{m(t)}_{\text{message signal}} - \underbrace{m_s(t)}_{\text{reference waveform}}$$

$$x_c(t) = \Delta(t) \sum_{n=-\infty}^{\infty} \delta(t - nT_s) = \sum_{n=-\infty}^{\infty} \Delta(nT_s)\,\delta(t - nT_s)$$

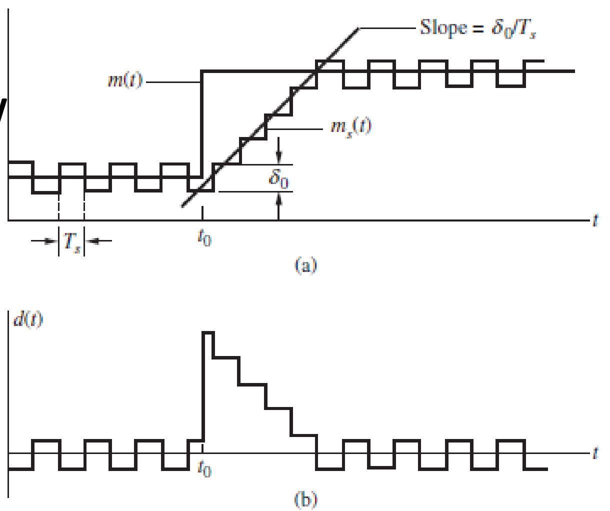
  – The output of the delta modulator is a series of impulses, each having positive or negative polarity depending on the sign of *d*(*t*) at the sampling instant.
  – Reference signal $m_s(t) = \sum_{n=-\infty}^{\infty} \Delta(nT_s) \int^{t} \delta(t - nT_s)\,dt$

Delta modulation and PCM

- Delta demodulation
  - Integrate $x_c(t)$ to form the stairstep approximation $m_s(t)$; and lowpass filter to suppress the discrete jumps in $m_s(t)$.
  - Since a lowpass filter approximates an integrator, it is often possible to eliminate the integrator portion of the demodulator.
- *Slope overload* occurs when $m(t)$ has a slope greater than can be followed by $m_s(t)$

Delta modulation and PCM

- Slope overload
  - Assume each pulse in $x_c(t)$ has weight δ_0, the maximum slope can be followed by $m_s(t)$ is δ_0/T_s.
  - Assume $m(t) = A\sin(2\pi f_1 t)$
    - Maximum slope that $m_s(t)$ can follow



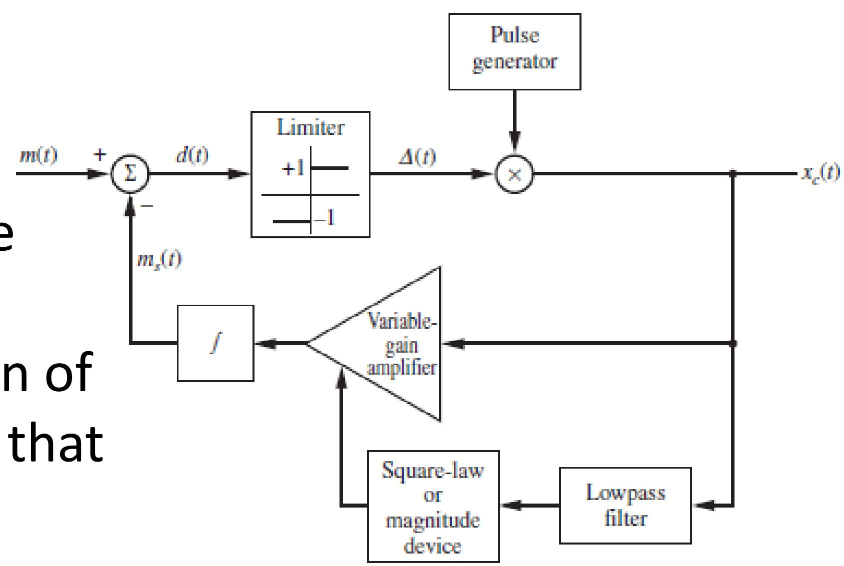
$$S_m = \frac{\delta_0}{T_S}$$

$$\frac{d}{dt}m(t) = 2\pi f_1 A\cos(2\pi f_1 t)$$

    - $m_s(t)$ can follow $m(t)$ without slope overload if $\dfrac{\delta_0}{T_S} \geq 2\pi f_1 A$

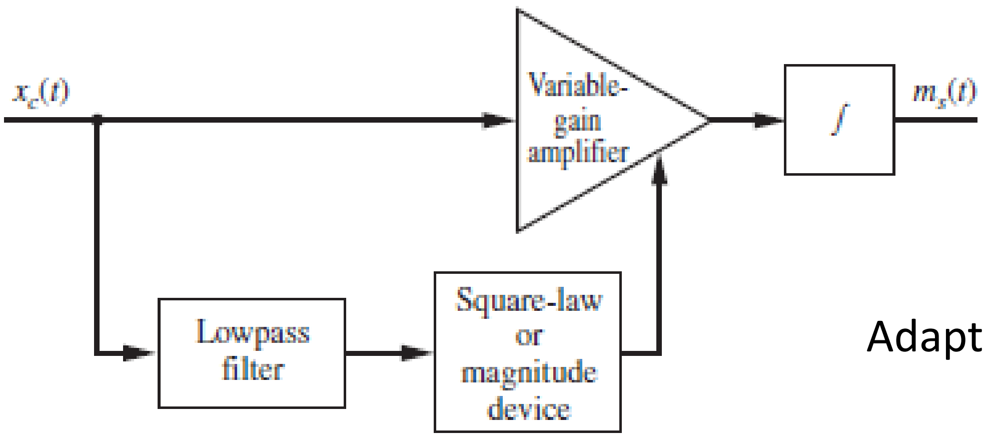
Delta modulation and PCM

- Adaptive delta modulation
  - Weight δ_0 can be very small if $m(t)$ is nearly constant.
  - A rapidly changing $m(t)$ requires a larger value of δ_0 if slope overload is to be avoided.
  - Use a lowpass filter:
    - If m(t) is constant or nearly constant, the pulse constituting $x_c(t)$ will alternate in sign $\rightarrow$ DC value, determined over the time constant of LPF, is nearly zero. This small value controls the gain of the variable-gain amplifier such that it is very small. Thus, δ_0 is made small at the integrator input.



Delta modulation and PCM

- Adaptive delta modulation
  - If m(t) is increasing or decreasing, the pulse $x_c(t)$ will have the same polarity over this period. Thus, the magnitude of the output of LPF will be relatively large $\rightarrow$ the increase in the gain of the variable-gain amplifier, and consequently an increase in δ_0.



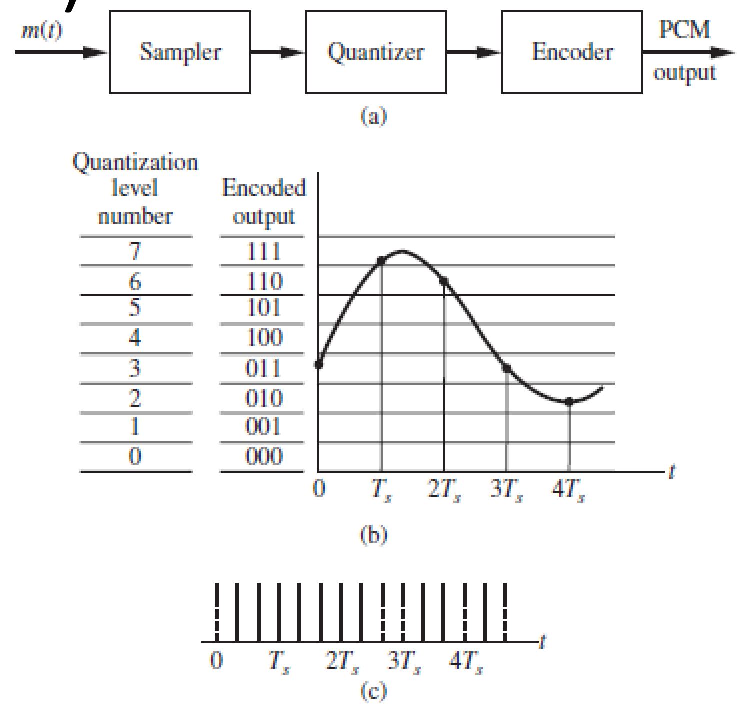
Adaptive DM receiver

DM and PCM

- PCM (Pulse-code modulation)
  - The output is a binary sequence of "1" and "0"
  - The bandwidth of PCM system

$$B = 2knW$$

    - $n = \log_2 q$: the word length
    - q: quantization level
    - W: the bandwidth of message signal
    - $2W$: sampling rate
    - k: constant of proportionality



Multiplexing

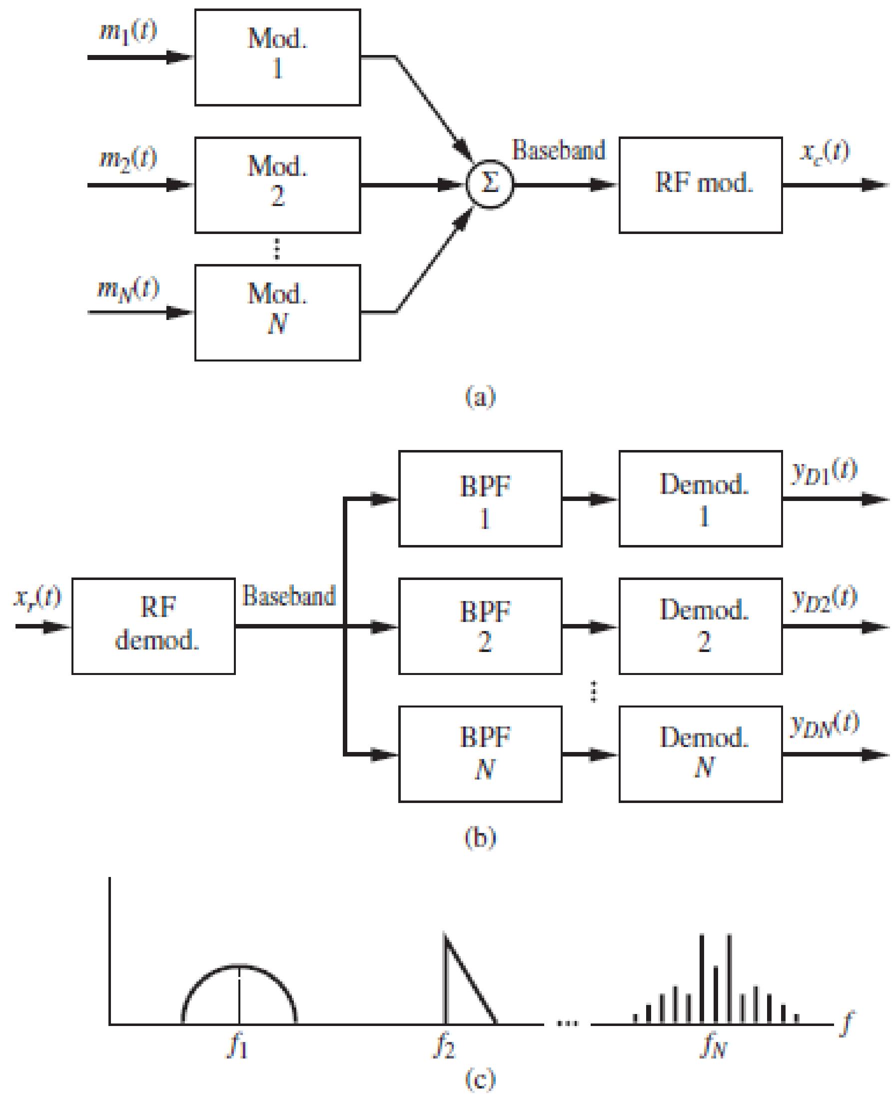
- In many applications, a large number of data sources are located at a common point, and it is desirable to transmit these signals simultaneously using a single communication channel $\rightarrow$ multiplexing
- Frequency division multiplexing (FDM)
  - Several signals are translated, using modulation, to different spectral locations and added to form a baseband signal.

Multiplexing

- FDM
  - The bandwidth

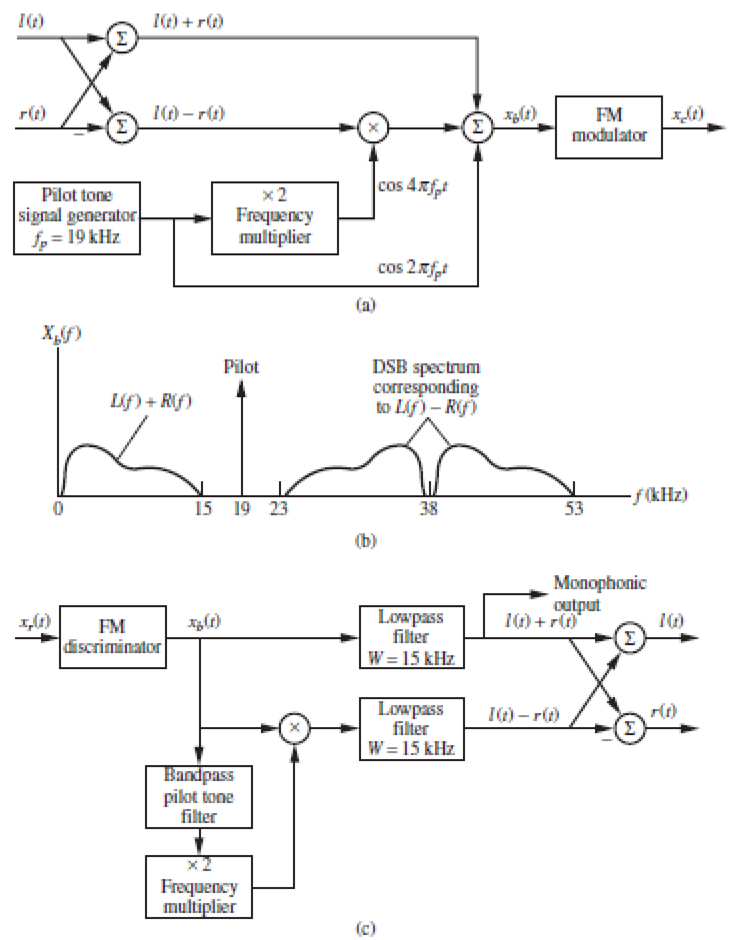
$$B = \sum_{i=1}^{N} W_i$$

W_i is the bandwidth of $m_i(t)$.



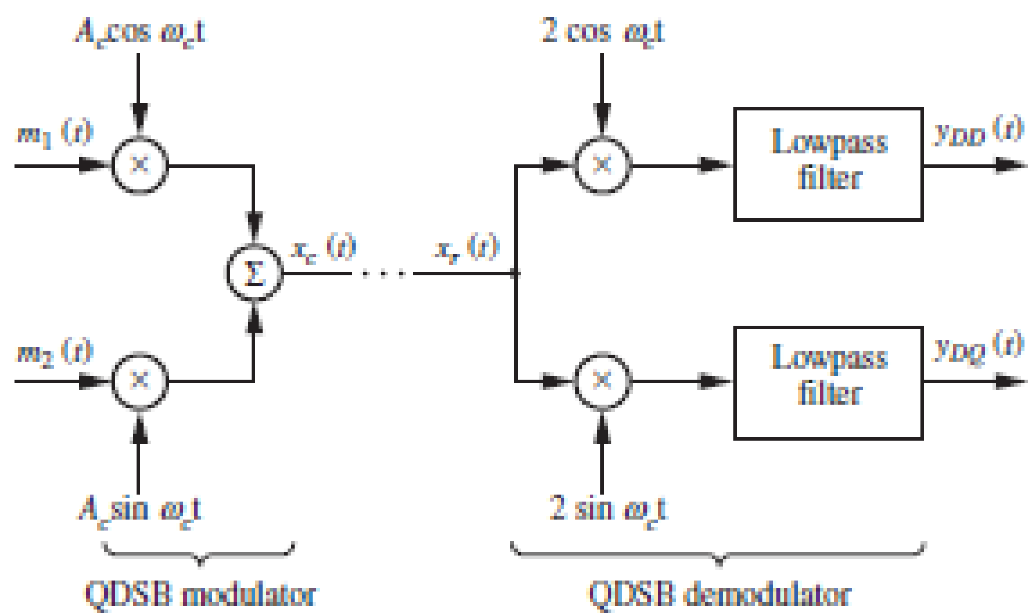
Multiplexing

- Example of FDM: stereophonic FM



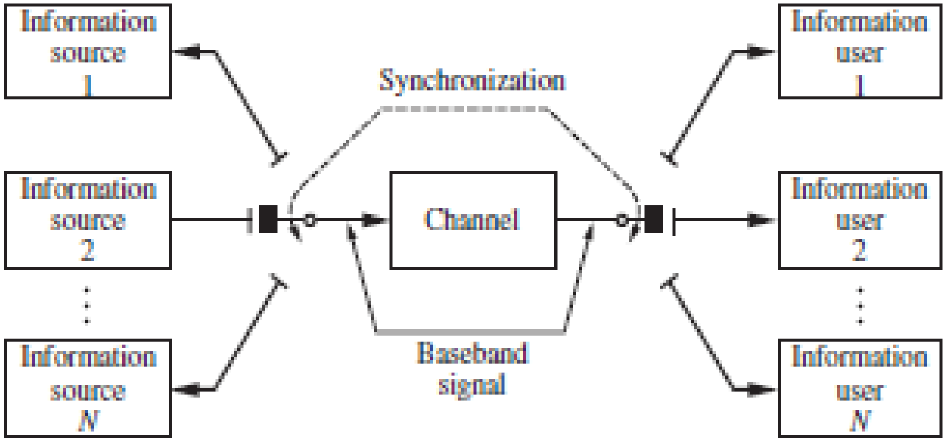
Multiplexing

- Quadrature multiplexing: quadrature carriers are used for frequency translation.
  - Spectra of two components in $x_c(t)$ overlap in frequency if the spectra of $m_1(t)$ and $m_2(t)$ overlap.
  - QM is not FDM since 2 channels do not occupy disjoint spectral location

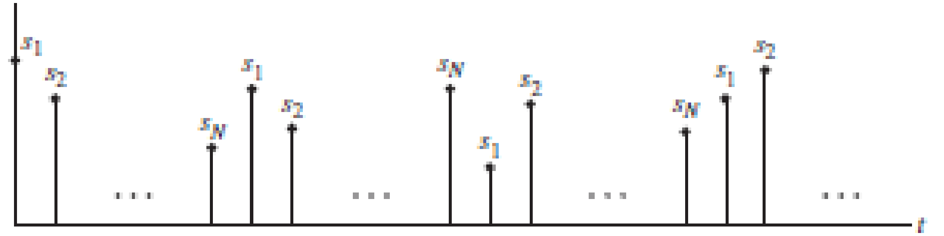


Multiplexing

- Time division multiplexing (TDM)
  – Data sources are sampled at the Nyquist rate or higher.
  – Commutator interlaces the samples to form the baseband signal.



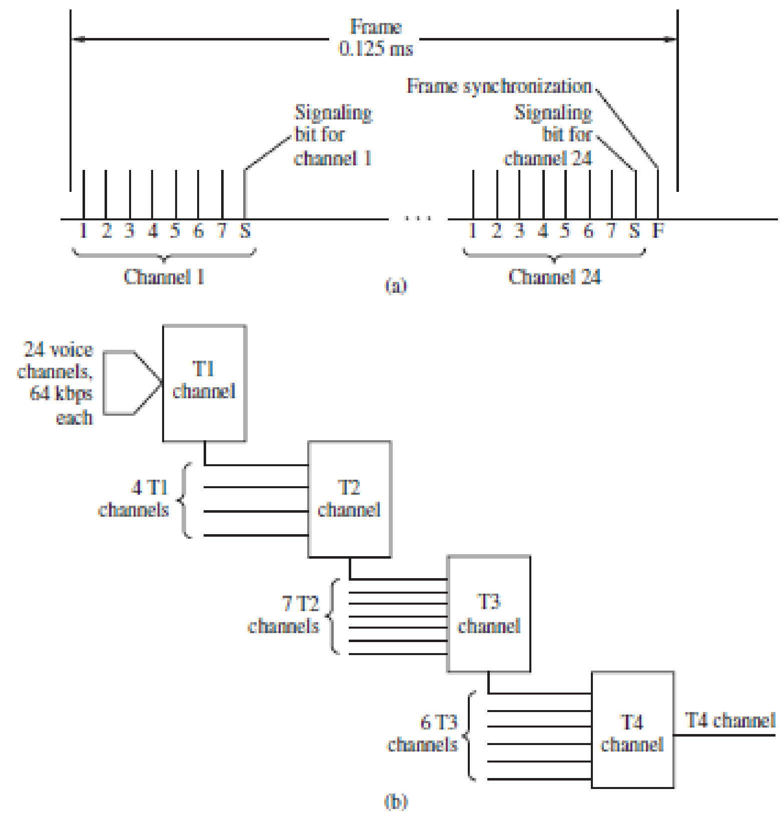
(a)



(b)

Multiplexing

- Example of TDM: digital telephone system
  - f_s = 8000 samples/s
  - Each sample is quantized into 7 bits.
  - 1 *signaling bit* is added to the basic 7 bits (call establishment and synchronization)
  - 8 bits for each sample $\rightarrow$ 64kbps
  - T1 frame = 24 of 64kbps voice channels + 1 *extra bit for frame synchronization* = 24x8+1=193bits $\rightarrow$ T1 data rate = 8000 (frames/s) x 193 (bits/frame) = 1.544Mbps.
  - Frame duration = 1/8000 = 0.125ms



C3_Quiz4.pptx

Assignments

- 3.7, 3.9, 3.10, 3.11, 3.15, 3.17
- 3.23, 3.25, 3.28, 3.31, 3.36
- 3.51, 3.52, 3.53

3.18. A mixer is used in a short-wave superheterodyne receiver. The receiver is designed to receive transmitted signals between 5 and 25 MHz. High-side tuning is to be used. Determine the tuning range of the local oscillator for IF frequencies varying between 400 kHz and 2 MHz. Plot the ratio defined by the tuning range over this range of IF frequencies as in Table 3.1.

3.19. A superheterodyne receiver uses an IF frequency of 455 kHz. The receiver is tuned to a transmitter having a carrier frequency of 1120 kHz. Give two permissible frequencies of the local oscillator and the image frequency for each. Repeat assuming that the IF frequency is 2500 kHz.

3.23. The power of an unmodulated carrier signal is 50 W, and the carrier frequency is $f_c = 50$ Hz. A sinusoidal message signal is used to FM modulate it with index $\beta = 10$. The sinusoidal message signal has a frequency of 5 Hz. Determine the average value of $x_c(t)$. By drawing appropriate spectra, explain this apparent contradiction.

3.36. A sinusoidal message signal has a frequency of 150 Hz. This signal is the input to an FM modulator with an index of 10. Determine the bandwidth of the modulator output if a power ratio, P_r, of 0.8 is needed. Repeat for a power ratio of 0.9.

3.31. An FM modulator has $f_c = 2000$ Hz and $f_d = 14$ Hz/V. The modulator has input $m(t) = 5\cos 2\pi(10)t$.

a. What is the modulation index?

b. Sketch, approximately to scale, the magnitude spectrum of the modulator output. Show all frequencies of interest.

c. Is this narrowband FM? Why?

d. If the same $m(t)$ is used for a phase modulator, what must k_p be to yield the index given in (a)?