Chapter 3

Passive Components, Resonators and Impedance Matching

Trinh Xuan Dung, PhD
dung.trinh@hcmut.edu.vn

Department of Telecommunications
Faculty of Electrical and Electronics Engineering
Ho Chi Minh city University of Technology

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1. High frequencies Characteristics of Passive Components

❖ **Resistance of a wire above a ground plane:**

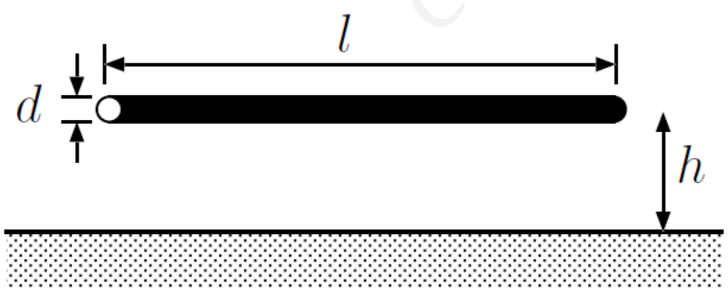
- Current tends to flow near the external surface of a conductor at high frequencies.
- The current density directed along the axis of a conducting wire is largest at the surface of the conductor, and falls to small values inside the conductor.
- Most of the current flows within the cylindrical shell within one skin depth from the surface. The **skin depth** is denoted by δ[m]:

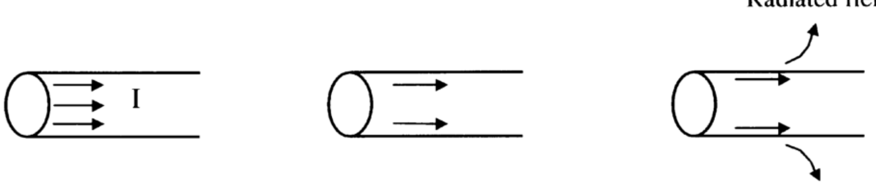
$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}}$$

where μ, σ are the permeability and conductivity of the conductor. $\mu_0 = 4\pi \times 10^{-7}\ (H/m)$

- The skin depth in copper at 100 kHz, 10 MHz, and 1 GHz is approximately 0.2mm, 0.02mm, and0.002 mm.

Material	σ S/m
Aluminum	3.5×10^{7}
Copper	5.8×10^{7}
Brass	1.5×10^{7}
Gold	4.1×10^{7}
Silver	6.1×10^{7}





1. High frequencies Characteristics of Passive Components

❖ Resistance of a wire above a ground plane:

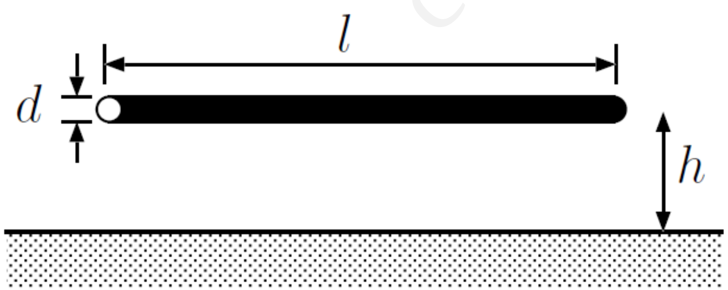
- At low frequency:

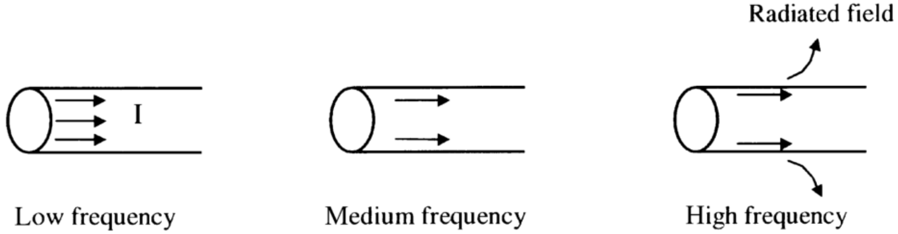
$$R_{DC} = \frac{l}{\sigma S} = \frac{l}{\pi r^2 \sigma}$$

- At higher frequencies, where the skin depth is small compared to the wire radius. The AC resistance can then be written as:

$$R_{AC} = R_{DC}\frac{\pi r^2}{\pi d\delta} = R_{DC}\frac{r}{2\delta}$$

- Obviously, the wire resistance at high frequency is much greater than its DC resistance, especially at high frequencies.





1. High frequencies Characteristics of Passive Components

- **Example 1:** Consider AWG 22 wire. Compute resistance per unit length of the wire R_{DC} and R_{AC} at 10MHz and 1GHz.
- **Solution:**

$$R_{DC} = \frac{l}{\sigma S} = \frac{l}{\sigma \pi r^2} = \frac{1}{5.8 \times 10^7 \times \pi \times (0.643/2)^2 \times 10^{-6}} = 0.053(\Omega/m)$$

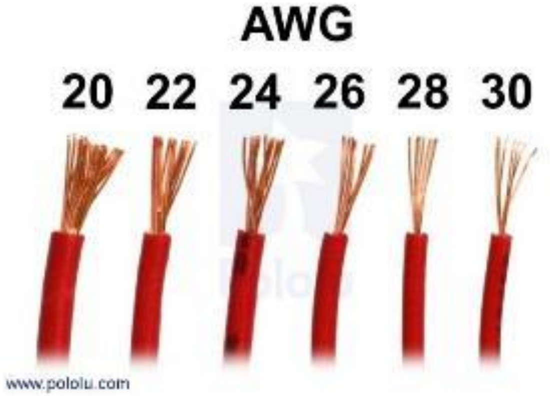
  - *At 10MHz:*

$$\delta = \frac{1}{\sqrt{\pi f \sigma \mu}} = \frac{1}{\sqrt{\pi \times 10^7 \times 5.8 \times 10^7 \times 4\pi \times 10^{-7}}} \simeq 21(\mu m)$$

$$R_{AC} = R_{DC}\frac{r}{2\delta} = 0.053 \times \frac{0.643/2 \times 10^{-3}}{2 \times 23 \times 10^{-6}} \simeq 0.4(\Omega/m)$$

  - *At 1GHz:* $R_{AC} \simeq 4(\Omega/m)$

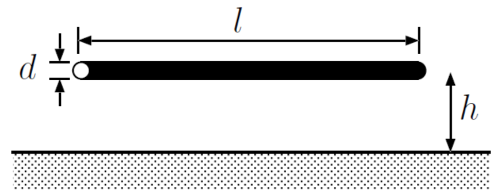
- AWG (American Wire Gauge) 22 gauge wire is a typical wire used for connections and component leads which has diameter $d = 25.3 mils$ or $0.643 mm$.



1. High frequencies Characteristics of Passive Components

❖ **Inductor of a wire above a ground plane:**

- The inductance, per unit length, of a wire with length l, diameter d, and distance h from a ground plane is (when $d \ll l$ and $h \ll l$):



$$L = \frac{\mu_0}{2\pi}\cosh^{-1}\frac{2h}{d}$$

- When $h/d > 1$ then the following approximation is useful: $L \simeq \frac{\mu_0}{2\pi}\ln\frac{4h}{d}$

  Note that $\cosh^{-1}(x) = \ln\left(x + \sqrt{x^2 - 1}\right)$

- For values of h/d in the range 1 to 100, above equation predicts that L ranges from 2.8nH/cm to 12 nH/cm.

❖ For short wires, the resistance is often small enough to be ignored, however the inductance of the wire is often significant. In a circuit where impedances are relatively low, the series impedance of even a short connecting lead may have a significant impact on circuit performance.

❖ This leads to a fundamental rule of RF circuit design:

- At high frequencies it is important to keep the length of interconnecting wires and circuit-board traces short in order to minimize lead inductance.
- When components are separated by significant distances, interconnections must be treated as distributed circuit elements, and transmission line models are used to model the conductors that interconnect components.

1. High frequencies Characteristics of Passive Components

❖ **Example 2:** A AWG 22 copper wire placed directly on top surface of a PCB with dielectric thickness of 1.57mm (standard thickness) and a ground plane on the bottom of the board. Compute the inductance of the wire per unit length, inductive reactance at 100MHz and 1GHz.

❖ **Solution:**

- We are given $h = 1.57mm$ and $d = 0.643mm$. Because $h/d > 1$, then:

$$L \simeq \frac{\mu_0}{2\pi}\ln\frac{4h}{d} \simeq \frac{4\pi \times 10^{-7}}{2\pi}\ln\frac{4 \times 1.57}{0.643} = 0.46(\mu H/m)$$

- At 100MHz:

$$Z_L = j\omega L = j2\pi \times 10^8 \times 0.46 \times 10^{-6} = j300(\Omega/m)$$

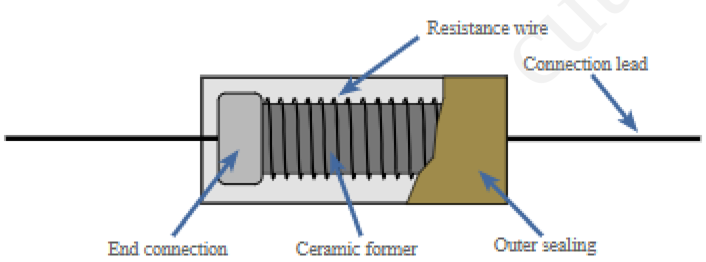
- At 1GHz:

$$Z_L = j\omega L = j2\pi \times 10^9 \times 0.46 \times 10^{-6} = j3\ (k\Omega/m)$$

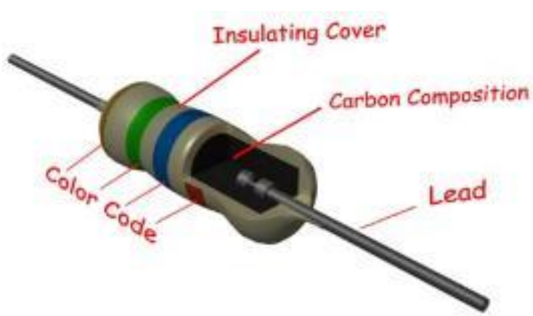
1. High frequencies Characteristics of Resistors

❖ Several types of resistors are used in RF circuits, including wire-wound, carbon composition, thick film, and thin film units:

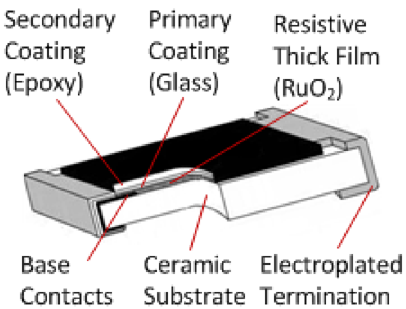
- Wire-wound resistors consist of a length of lossy wire that is coiled up to fit into a small package. This type of resistor is seldom used at RF because they have relatively large inductance.
- Carbon composition resistors consist of a lossy dielectric material sandwiched between two conducting electrodes.
- Thick or thin-film resistors consist of a film of conducting material deposited on an insulating substrate. Film resistors are available in cylindrical packages with attached connecting leads and also as surface mount devices (SMDs). Thick or thin film resistors in a surface mount package (aka, "chip" resistors) are the most common type for RF applications.



A wire wound resistor



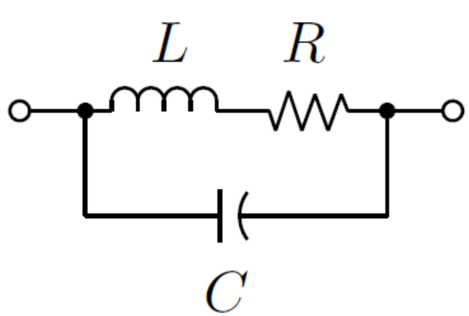
A carbon composition resistor



A Thick film resistor

1. High frequencies Characteristics of Resistors

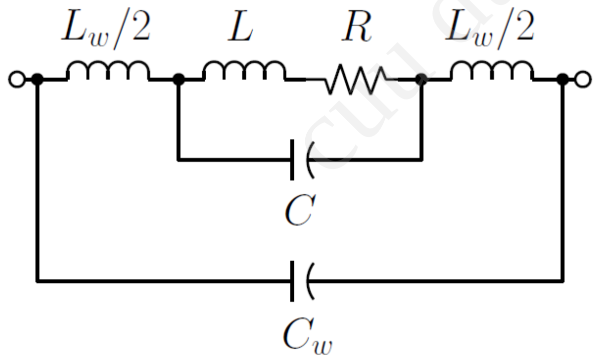
- Over a fairly wide frequency range, resistors can be modeled using the equivalent circuit shown in Fig.1.
  - The inductor represents the inductance associated with the current path through the resistor.
  - The capacitor represents the capacitance between the two electrodes used to connect the resistor to external circuitry.
- The inductance is primarily determined by the length of the current path within the element. The capacitance is determined by the size and separation of the contact electrodes, as well as the dielectric permittivity of the material between the electrodes.
- Generally, the inductance and capacitance associated with a miniature surface mount resistor package are on the order of 1 nH and 1 pF, respectively.

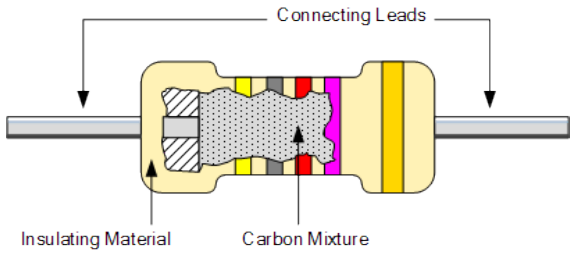


Standard equivalent circuit of a resistor

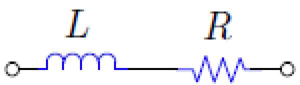
1. High frequencies Characteristics of Resistors

- Significantly higher inductance would be associated with a part in an axial package with wire leads.
- Even with very short leads, such a package would have a typical inductance value on the order of 10 nH.
- For many purposes, a simpler (standard) model can be used even for resistors with external connecting leads if the inductance is taken to be the sum of the resistor inductance and lead inductance and the shunt capacitance is taken to be the sum of package capacitance and capacitance between the leads.
- When the resistance is small (≪ 100Ω), and the frequency is not too high, the series LR brand of the model has a much lower impedance than the capacitance shunt to it. The capacitive reactance can be neglected.





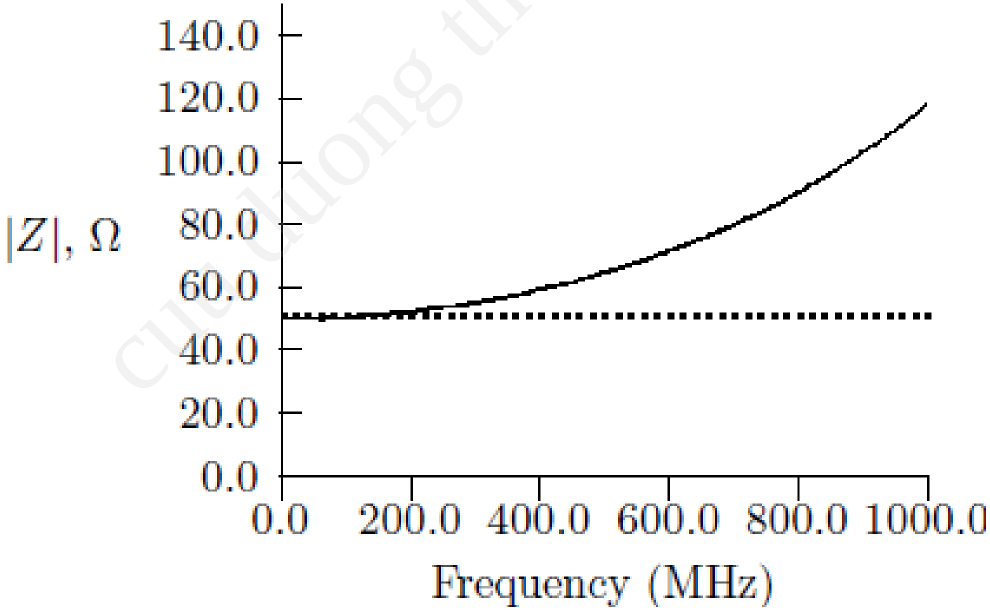
Equivalent circuit of a resistor with connecting leads



Equivalent circuit of a small resistor

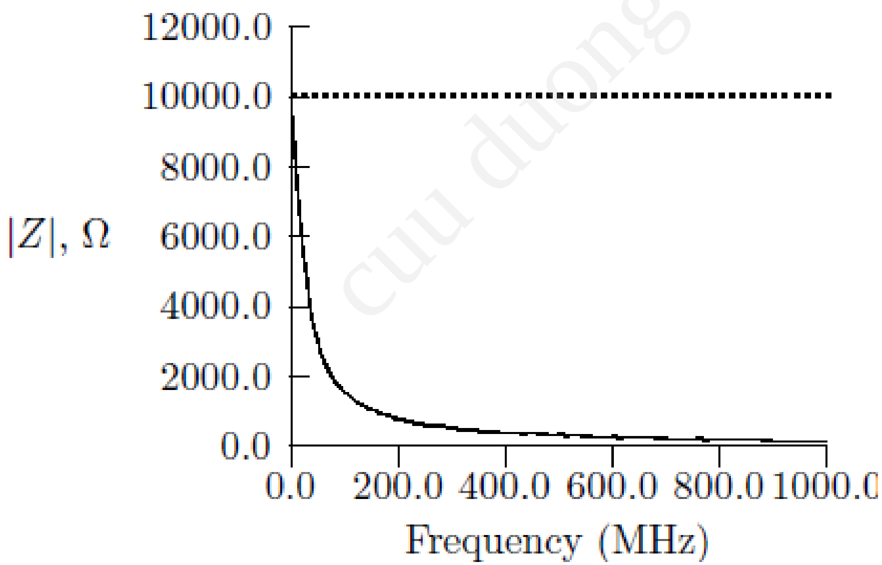
1. High frequencies Characteristics of Resistors

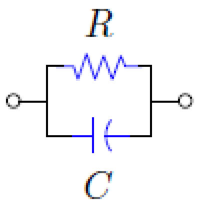
- **Example 3:** Consider a 50Ω resistor with a lead inductance of $L = 10nH$, shunt capacitor $C = 1pF$. Draw the magnitude of the impedance when the frequency varies from 1Hz to 1GHz.
- **Solution:**
  - The series inductance acts to increase the impedance at high frequencies.
  - The equivalent circuit of a small resistor can be applied to compute the impedance.



1. High frequencies Characteristics of Resistors

- **Example 4:** Consider a $10k\Omega$ resistor with a lead inductance of $L = 10nH$, shunt capacitor $C = 1pF$. Draw the magnitude of the impedance when the frequency varies from 1Hz to 1GHz.
- **Solution:**
  - When the resistance is large ($\gg 100\Omega$), and the frequency is not too high the inductive reactance will be small compared to R and can be neglected.
  - The shunt capacitance is dominant and tends to "short out" the resistance, resulting in a dramatic reduction in the impedance at high frequencies.





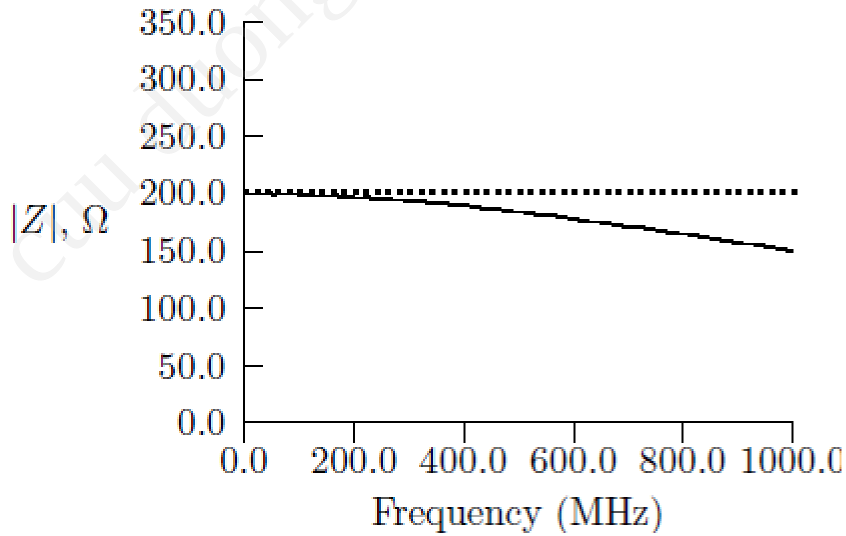
Equivalent circuit of a resistor with high resistance

1. High frequencies Characteristics of Resistors

❖ **Example 5:** Consider a 200Ω resistor with a lead inductance of $L = 10nH$, shunt capacitor $C = 1pF$. Draw the magnitude of the impedance when the frequency varies from 1Hz to 1GHz.

❖ **Solution:**

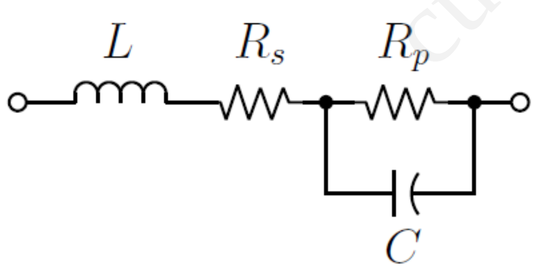
- In this case neither of the parasitic reactances is significant compared to 200Ω and the impedance variation is relatively small up through 1GHz.
- Notice that the impedance of a resistor can be much larger or smaller than the DC resistance, depending on the resistance value and frequency.



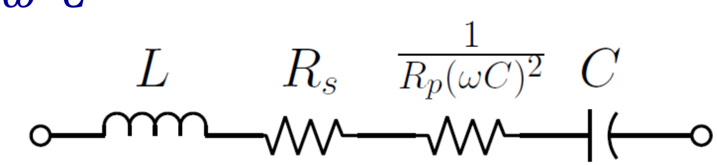
1. High frequencies Characteristics of Capacitors

- Capacitors are constructed by separating two conducting electrodes by an insulating medium such as air, or a low-loss dielectric material.
  - Loss in the dielectric is modeled as a resistance (R_p) in parallel with the intrinsic capacitance.
  - The inductance associated with the current path through the electrodes and any connected leads appears in series.
  - A resistance in series with the inductor (R_s) models losses in the electrodes and leads.
- If $R_p \gg {}^{1}/_{\omega C}$, the model can be transformed into a series RLC model.
- The sum of the ohmic resistance and the transformed dielectric loss resistance is termed the equivalent series resistance (ESR). The ESR of a capacitor is dominated by dielectric loss at sufficiently low frequencies:

$$R_{esr} = R_s + \frac{1}{R_p \omega^2 C^2}$$



Standard equivalent circuit of a capacitor



Equivalent RLC model of a capacitor, valid if $R_p \gg {}^{1}/_{\omega C}$

1. High frequencies Characteristics of Capacitors

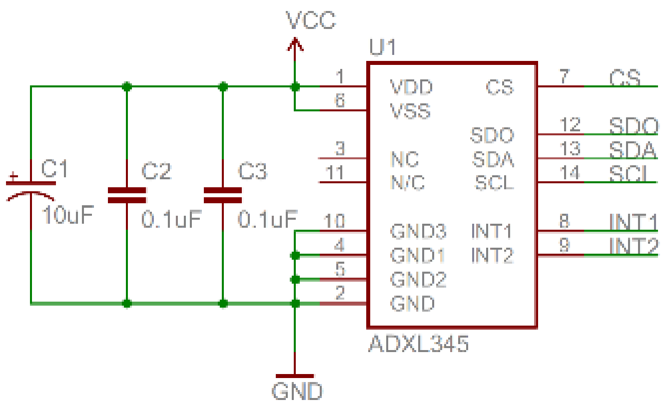
- The Q of a capacitor at any frequency is the ratio of the reactance and the ESR

$$Q = \frac{|X|}{R_{esr}}$$

where the reactance $X = \omega L - {}^{1}/_{\omega C}$. The dissipation factor, d, is the inverse of Q:

$$tan\delta = d = \frac{1}{Q} = \frac{R_{esr}}{|X|}$$

- The dissipation factor is also called the loss tangent.
- All capacitors will have a series resonant frequency, $f_s = {}^{1}/_{2\pi\sqrt{LC}}$. Above this frequency, the inductive dominates. Hence capacitors are inductive at frequency above f_s.
- Circuit designers make explicit use of the fact that capacitor impedance is smallest at and near the series resonant frequency. When a capacitor is used as a DC bias circuit decoupling element or as an inter-stage DC-blocking coupling element, it is sometimes possible to choose the capacitor so that the intended frequency of operation falls near the series resonant frequency of the capacitor, where the impedance is smallest.

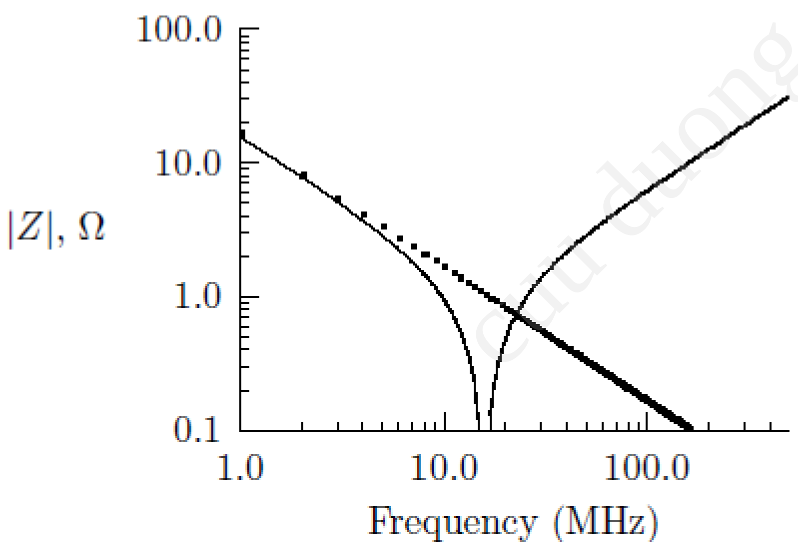


1. High frequencies Characteristics of Capacitors

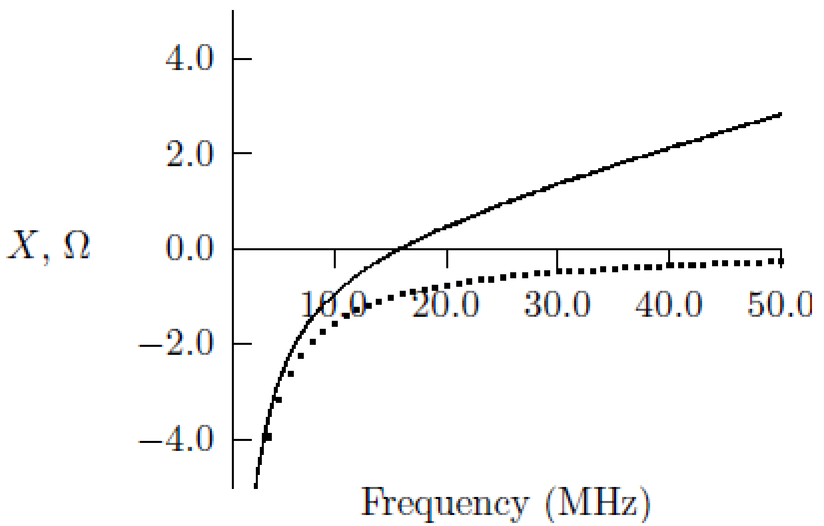
❖ **Example 6:** Consider a $0.01\mu F$ capacitor with total lead length of $1cm$. Given typical value of lead inductance is 10nH/cm. Compute resonant frequency of the capacitor. Draw the capacitor's impedance and reactance for the frequency range from 1Hz to 100MHz.

❖ **Solution:**

- Series resonant frequency of the capacitor is 15.9MHz.



Log amplitude of capacitor's impedance.



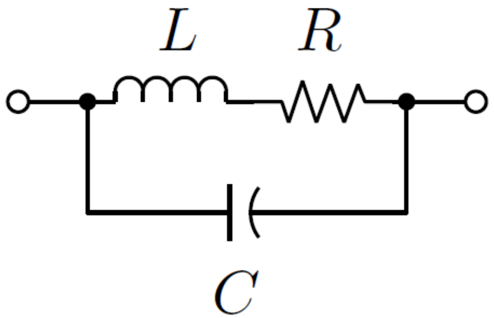
Reactance for a $0.01\mu F$ capacitor.

1. High frequencies Characteristics of Inductors

- A reasonably accurate equivalent circuit for an inductor consisting of a coil of wire wound as a solenoid, around a torus, or in a plane includes
  - A series resistance to model the ohmic loss in the wire.
  - A shunt capacitance to model distributed capacitance between the turns of the coil
- The inductance of the wire results in voltage differences between the different parts of the coil. This voltage difference sets up an electric field in the air and in any dielectric material near the coil.
- The effect of this stored electric energy can be modeled with a capacitance shunted across the terminals of the coil. This effective capacitance is called the distributed capacity of the coil.
- The coil resistance can be estimated from the AC resistance of an isolated wire having the length equal to the total length of wire in the coil.

$$R_{AC} = r_{DC}\frac{n\pi dD}{4\delta}$$

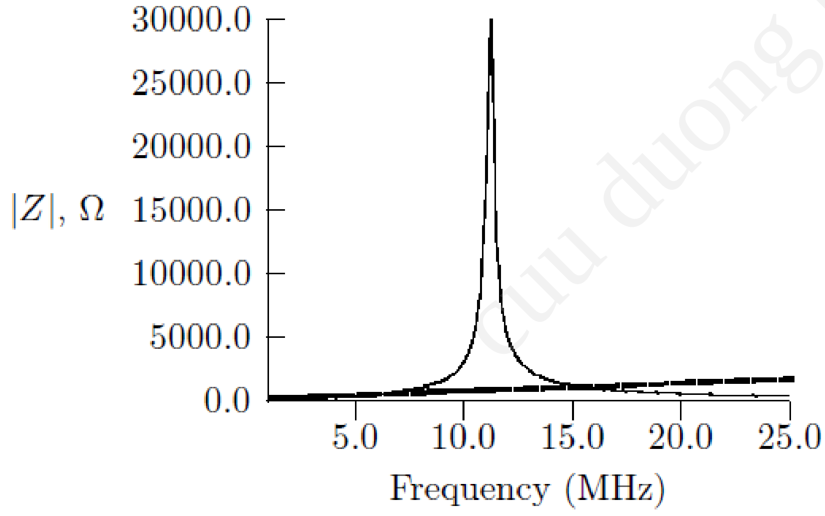
  - d is the wire diameter, D is the coil diameter, n is the number of turns in the coil and δ is the skin depth in the wire.



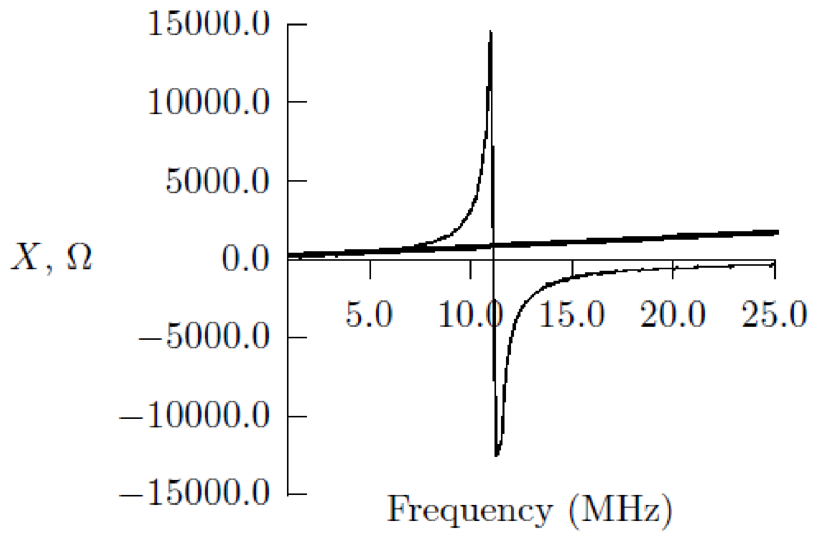
Equivalent circuit of an inductor

1. High frequencies Characteristics of Inductors

- **Example 7:** Consider a $10\mu H$ inductor with series resistance of 15Ω and distributed capacitor of 20pF. Compute the resonant frequency of the inductor. Draw the inductor's impedance and reactance for the frequency range from 1Hz to 100MHz.
- **Solution:**



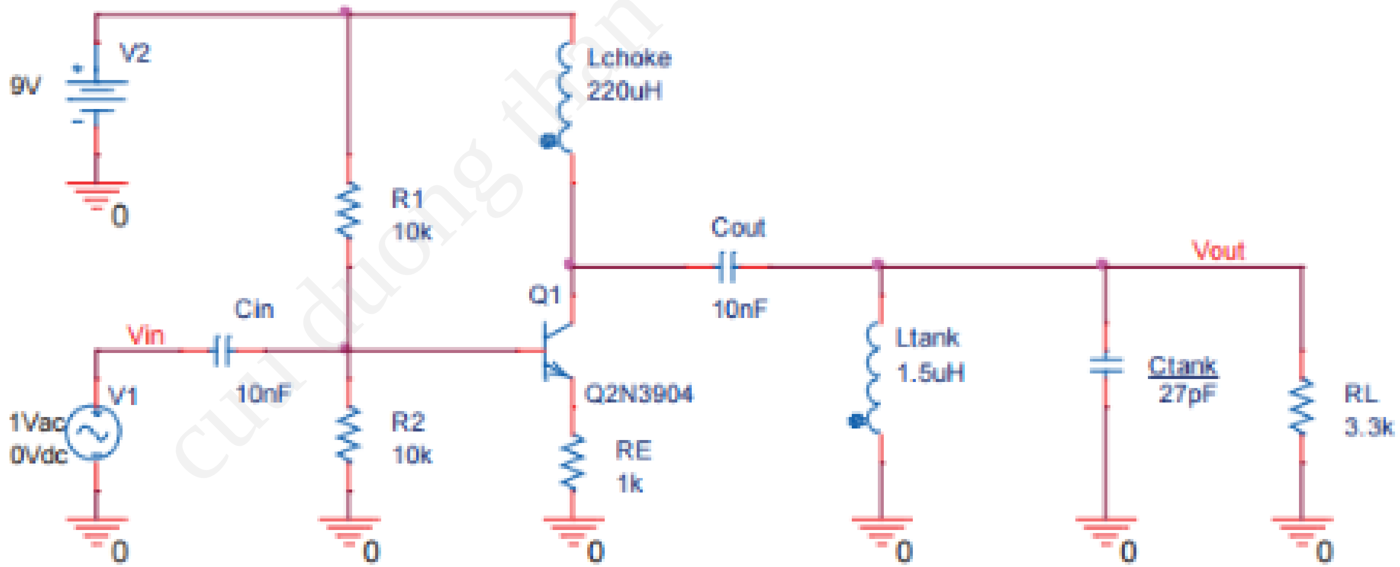
Impedance of the inductor



Reactance for the inductor.

1. High frequencies Characteristics of Inductors

- In many cases, an inductor is used to provide a DC bias signal to a circuit, but it is necessary to isolate the bias supply from the circuit at the operating frequency.
- In this case, an inductor with parallel resonant frequency near the operating frequency may be employed to provide a low impedance to DC and large impedance to RF signals. In such an application the inductor element is referred to as an RF choke.



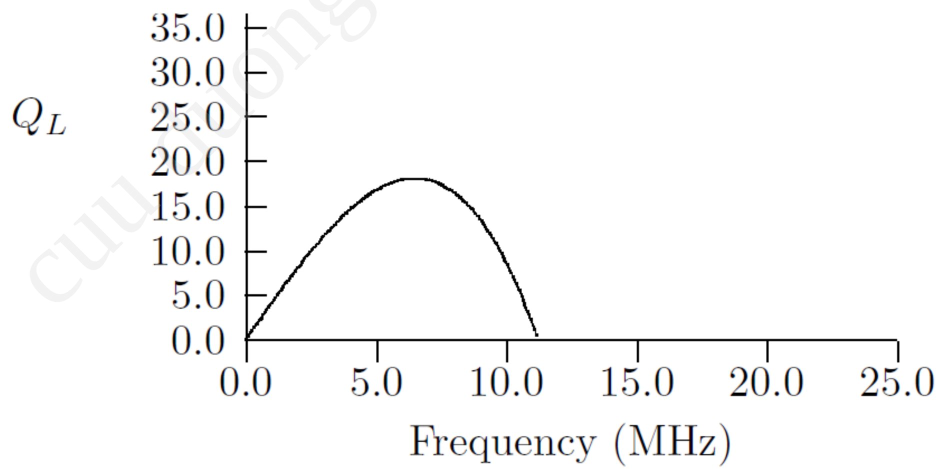
Example use of RFC

1. High frequencies Characteristics of Inductors

- When considering a particular inductor for use in a circuit, the designer needs to be aware of the parallel resonant frequency as well as the “Quality Factor,” or Q, of the inductor. The Q of an inductor is defined to be the ratio of inductive reactance and resistance associated with the component

$$Q = \frac{|X|}{R_s}$$

- The higher the Q, the better the inductor approximates an “ideal” component. The Q is an important parameter if the inductor is to be used in a resonant circuit, filter, or matching network.



The quality factor for the inductor in example 7

2. RLC Networks and Resonators

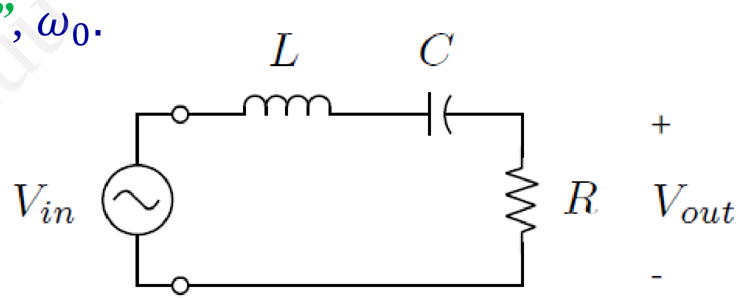
- In RF communication systems, resonant circuits are extensively used to select the wanted signal and reject the unwanted signal.
- Consider the series RLC circuit in a filter configuration where the output voltage is taken across the resistor. The voltage transfer function is

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{R}{R + sL + \frac{1}{sC}}$$

- Consider sinusoidal excitation under steady-state condition, the frequency response

$$H(j\omega) = \frac{R}{R + j\omega L + \frac{1}{i\omega C}} = \frac{R}{R + j\omega\left(L - \frac{1}{\omega^2 LC}\right)}$$

- When $\omega = \frac{1}{\sqrt{LC}}$, the phase shift of the transfer function is zero, this is called the ***"resonant frequency"***, ω_0.



Series RLC circuit as a filter

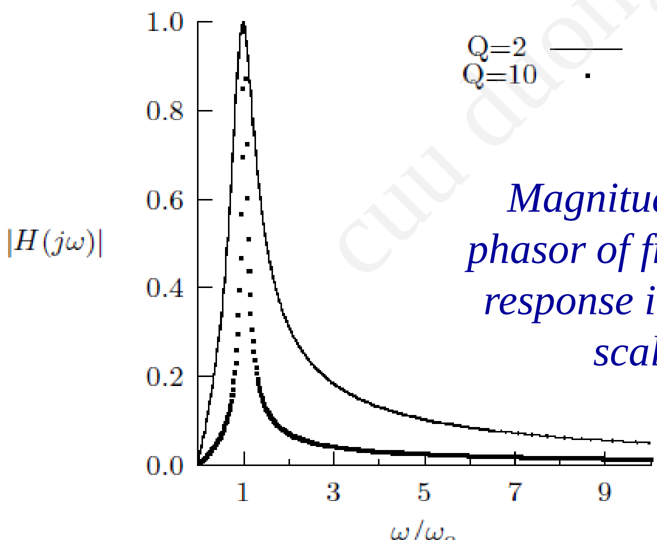
2. Series RLC Networks

- The transfer function depends on R,L and C but only two parameters. Define another quantity Q_s, where

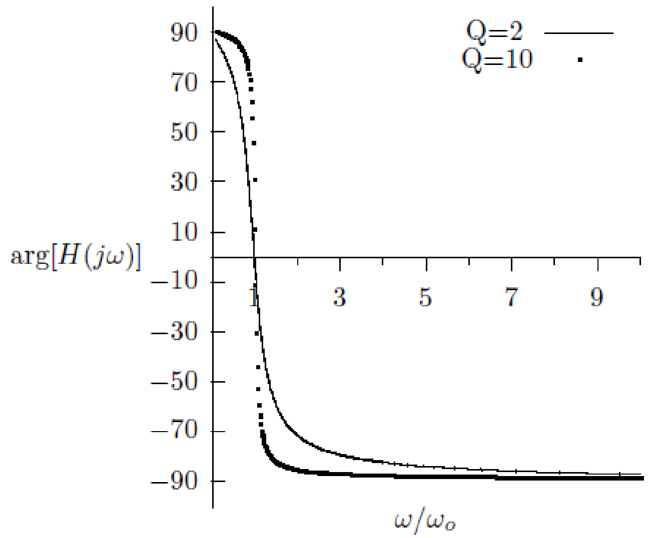
$$Q_s = \frac{\omega_0 L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

- The frequency transfer function can be written in terms of only ω_0 and Q_s

$$H(j\omega) == \frac{R}{R + j\omega\left(L - \frac{1}{\omega^2 LC}\right)} = \frac{1}{1 + jQ_s\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)}$$



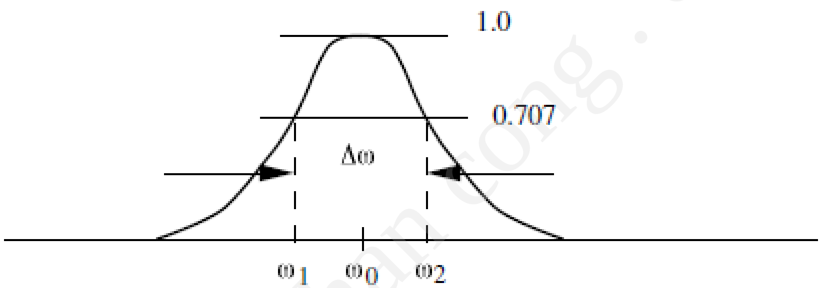
Magnitude and phasor of frequency response in linear scale



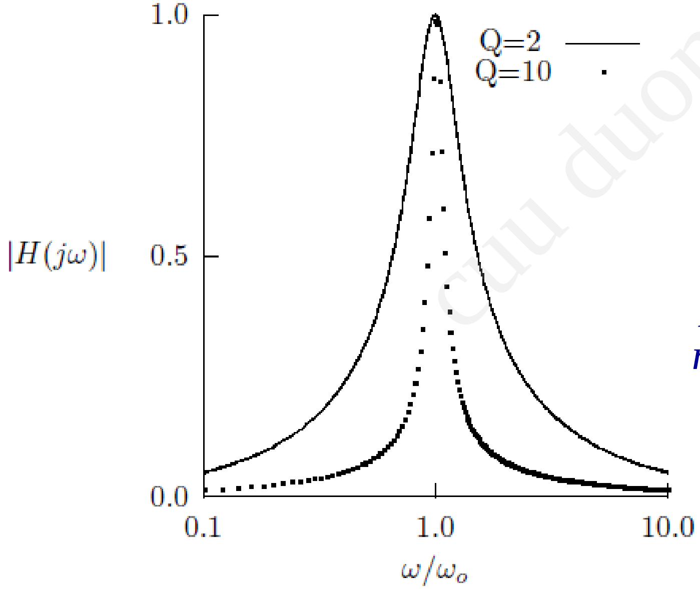
2. Series RLC Networks

❖ The RLC filter has a "bandpass" characteristic. The 3dB bandwidth of the filter is

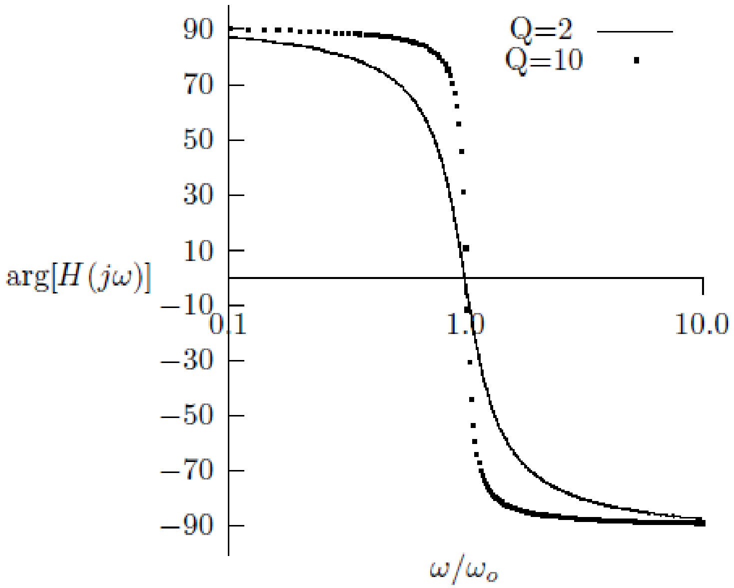
$$BW = \Delta\omega = \omega_2 - \omega_1 = \frac{\omega_0}{Q_s}$$



3dB bandwidth of a filter



Magnitude and phasor of frequency response in log scale



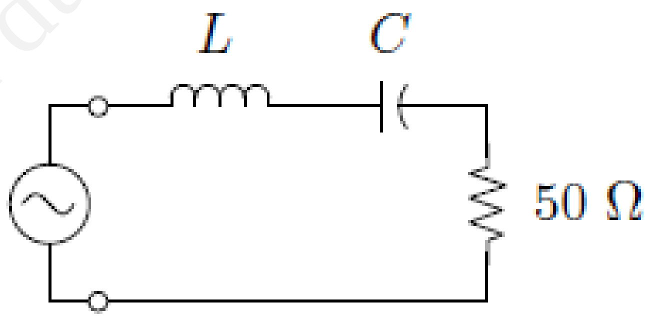
2. Series RLC Networks

- **Example 8:** Use a series RLC circuit to couple a voltage source with negligible source resistance to a 50Ω load as shown in following figure. The circuit should have a center frequency of 5 MHz and a 3dB bandwidth of 100 kHz. Determine value of L and C.
- **Solution:**

$$Q_s = \frac{f_0}{\Delta f} = \frac{5MHz}{100kHz} = 50$$

$$Q_s = \frac{\omega_o L}{R} \rightarrow L = \frac{Q_s R}{\omega_o} = \frac{50 \times 50}{2\pi \times 5 \times 10^6} = 79.6(\mu H)$$

$$C = \frac{1}{\omega_0^2 L} = \frac{1}{(2\pi \times 5 \times 10^6)^2 \times 79.6 \times 10^{-6}} = 12.7(pF)$$



2. Parallel RLC Networks

- A parallel RLC network being driven by an ideal source as shown in following figure. The input current and output voltage are related by an impedance function:

$$Z(s) = \frac{V_{out}}{I_{in}} = \left[\frac{1}{R_p} + \frac{1}{sL} + sC\right]^{-1}$$

- For sinusoidal steady-state excitation:

$$Z(j\omega) = \left[\frac{1}{R_p} + \frac{1}{j\omega L} + j\omega C\right]^{-1} = \frac{R_p}{1 + jQ_p\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)}$$

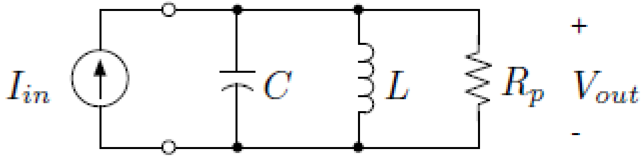
where:

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$Q_p = \frac{R_p}{\omega_0 L} = R_p\sqrt{\frac{C}{L}}$$

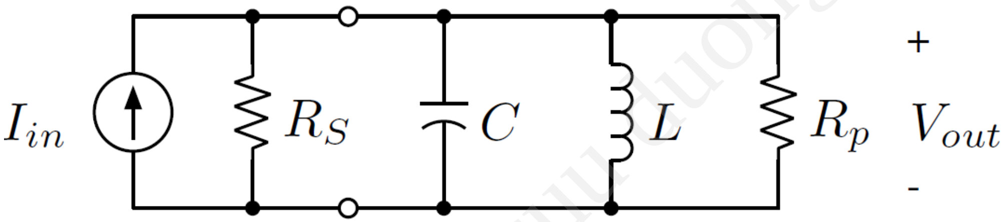
- The transfer function has exactly the same form as that of series RLC circuit except the scaling factor R_p. The 3dB bandwidth:

$$BW = \Delta\omega = \omega_2 - \omega_1 = \frac{\omega_0}{Q_p}$$



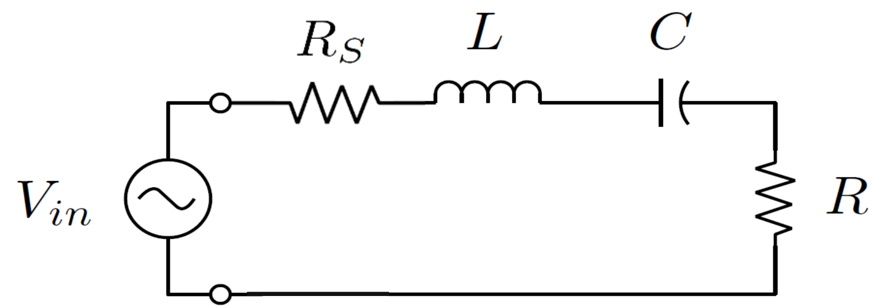
2. Unloaded and Loaded Q of RLC networks

- Compared to the case with infinite source impedance, the finite source impedance causes the Q to be reduced and, hence, the bandwidth to be increased.
- It is common practice to call the Q of the resonant circuit alone (either series or parallel RLC) the unloaded Q, and the Q of the composite circuit, which includes the source resistance and any other resistances that are external to the LC resonator, the loaded Q.
- The loaded Q is always smaller than the unloaded Q.



Loaded parallel RLC network

$$Q_p = \frac{R_p \parallel R_s}{\omega_0 L}$$



Loaded series RLC network

$$Q_s = \frac{\omega_0 L}{R + R_s}$$

2. Quality Factor Q

- Q describes the frequency selectivity of the Resonators and Circuit characteristics.
- The general definition of Q for a system is:

$$Q = 2\pi \frac{\text{Maximum instantaneous stored energy}}{\text{Energy dissipated per cycle}}$$

$$= 2\pi f \frac{\text{Maximum instantaneous stored energy}}{\text{Time} - \text{average power dissipated}}$$

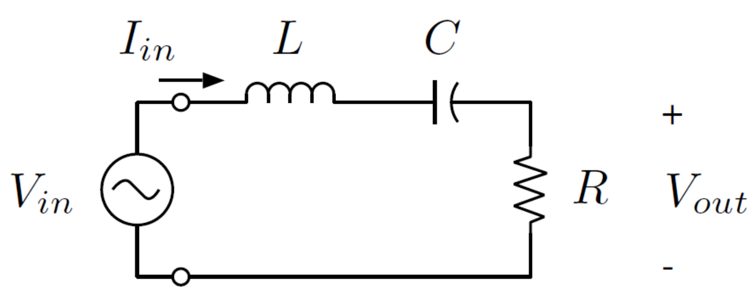
- This definition can be applied to resonant and non-resonant circuits. If this energy based definition is applied to resonant second-order RLC circuits the result is compatible with the (Q_s, Q_p) defined in the previous section.
- The energy definition is also commonly applied to characterize lossy inductors or capacitors which, by themselves, are non-resonant.
- At resonant frequency:

$$V_{out} = V_{in} = I_{in}R$$

- The maximum stored energy is:

$$E_{max} = \frac{1}{2}L|I_{in}|^2 = \frac{1}{2}L\frac{|V_{in}|^2}{R^2}$$

$$E_{max} = \frac{1}{2}C|V_C|^2 = \frac{1}{2}C\frac{|I_{in}|^2}{(\omega_0 C)^2}$$



2. Quality Factor Q

- The fact that the voltage across the capacitor is 90 degrees out of phase with the current through the inductor means that the current maximizes at the time when the capacitor voltage is zero.
- At that instant in time, all of the stored energy resides in the inductor and the magnitude of the current phasor (which is the peak current magnitude) can be used to calculate the total stored energy.
- Alternatively, at the time instant when the capacitor voltage is maximum then the current in the system is zero and all of the stored energy resides in the capacitor. The magnitude of the capacitor voltage phasor can then be used to calculate the stored energy at that time.
- The stored energy comes out the same either way. Actually, it can be shown that the total stored energy in this driven resonant RLC circuit is a constant, so that the maximum instantaneous stored energy is equal to the energy stored at any instant of time.

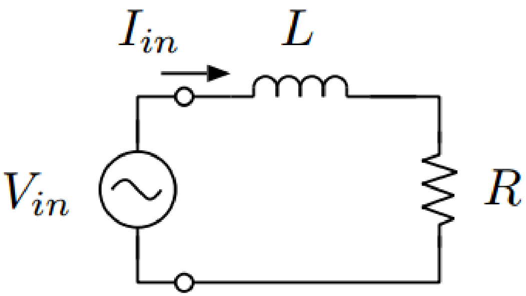
2. Quality Factor Q

- The time-averaged power delivered to (and dissipated in) the network is:

$$P_{avg} = \frac{1}{2}\mathbb{R}e\{V_{in}I_{in}^{*}\} = \frac{1}{2}\frac{|V_{in}|^2}{R}$$

- Using the energy definition of Q:

$$Q = 2\pi f_0 \frac{E_{max}}{P_{avg}} = \frac{\omega_0 L}{R}$$



- **For non-resonant circuit, consider RL circuit:**

$$I_{in} = \frac{V_{in}}{R + j\omega L}$$

$$E_{max} = \frac{1}{2}L|I_{in}|^2 = \frac{1}{2}L|V_{in}|^2\frac{1}{R^2 + \omega^2 L^2}$$

- The time-averaged power delivered is:

$$P_{avg} = \frac{1}{2}\mathbb{R}e\{V_{in}I_{in}^{*}\} = \frac{1}{2}|V_{in}|^2\mathbb{R}e\left(\frac{1}{R-j\omega L}\right) = \frac{1}{2}|V_{in}|^2\frac{R}{R^2+\omega^2 L^2}$$

- Then

$$Q = 2\pi f_0 \frac{E_{max}}{P_{avg}} = \frac{\omega_0 L}{R}$$

- The concept of component Q is often used to describe the properties of arbitrary circuit elements at a particular frequency.

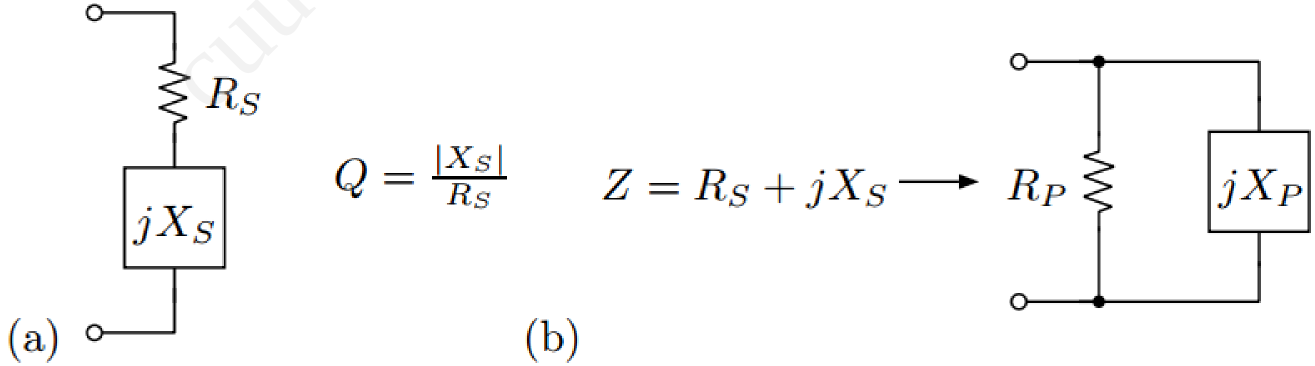
2. Series to Parallel Transformations

- Any circuit element has both a series and a parallel representation. Since the energy storage and dissipation properties of the element do not depend on how we represent it, the Q is the same for either representation.
- The circuit Q is given by: $Q = \frac{|X_s|}{R_s}$
- The equivalent parallel resistance and reactance are:

$$R_p = R_s(1 + Q^2)$$

$$X_p = X_s\left(1 + \frac{1}{Q^2}\right)$$

- If $Q \gg 1$, then: $R_p \simeq R_s Q^2$ and $X_p \simeq X_s$.
- A complex parallel impedance can be transformed to series by inversing the above equations.



2. Series RLC Networks

- **Example 9:** A $1\mu H$ inductor has a component Q of 100 at 10MHz. Find a parallel representation for the inductor.
- **Solution:**
  - At 10MHz:

$$Z_L = \omega L = 2\pi \times 10^7 \times 1 \times 10^{-6} = 62.8\ (\Omega)$$

$$Q_L = 100 = \frac{\omega L}{R} \to R = 0.628\ (\Omega)$$

  - $X_p = X_s = 62.8(\Omega)$
  - $R_p \simeq R_s Q^2 = 6.28(k\Omega)$

3. Impedance Matching Network

- A source with impedance $Z_S = R_S + jX_S$ is connected to a load $Z_L = R_L + jX_L$. The peak voltage of the source is V_S. The time-averaged real power delivered to the load:

$$P_L = \frac{1}{2}\mathbb{R}e\{V_L I_L^*\} = \frac{1}{2}|V_L|^2 \mathbb{R}e\left\{\frac{1}{Z_L^*}\right\} = \frac{1}{2}|V_S|^2 \frac{R_L}{|Z_L + Z_S|^2}$$

- The above equation can be used to determine what conditions are required to maximize the power delivered to the load. The problem is then to choose R_L and X_L to maximize P_L. P_L is maximized if

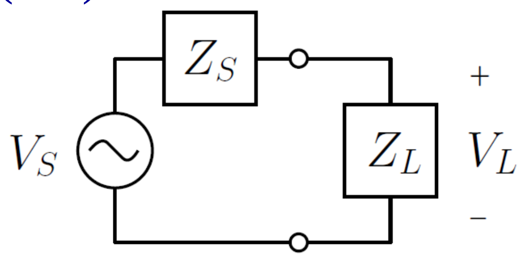
$$Z_L = Z_S^*$$

- The maximum power that the source can deliver to an external passive load is referred to as the power available from the source, P_{avs}

$$P_{avs} = P_{L|Z_L = Z_S^*} = \frac{|V_S|^2}{8R_S}$$

- The degree to which the actual power delivered to an arbitrary load is smaller than the available power can be quantified in terms of a mismatch factor (MF)

$$MF = \frac{P_L}{P_{avs}} = \frac{4R_S R_L}{|Z_L + Z_S|^2}$$



- Mismatch Loss: $ML = -20\log(MF)$

3. Impedance Matching Network

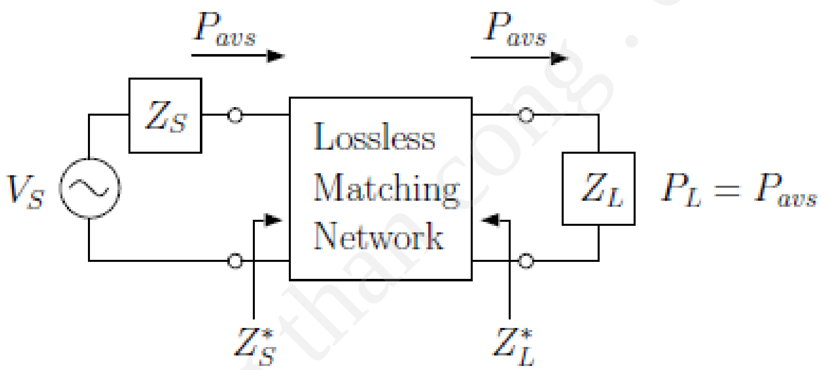
- **Example 10:** Suppose a 50Ω signal generator has available power of 1mW. The generator is to drive a load impedance of $250 + j100\Omega$. What is the power delivered to the load?
- **Solution:**

$$MF = \frac{P_L}{P_{avs}} = \frac{4R_S R_L}{|Z_L + Z_S|^2} = \frac{4 \times 50 \times 250}{|250 + j100 + 50|^2} = 0.5 = 3\ (dB)$$

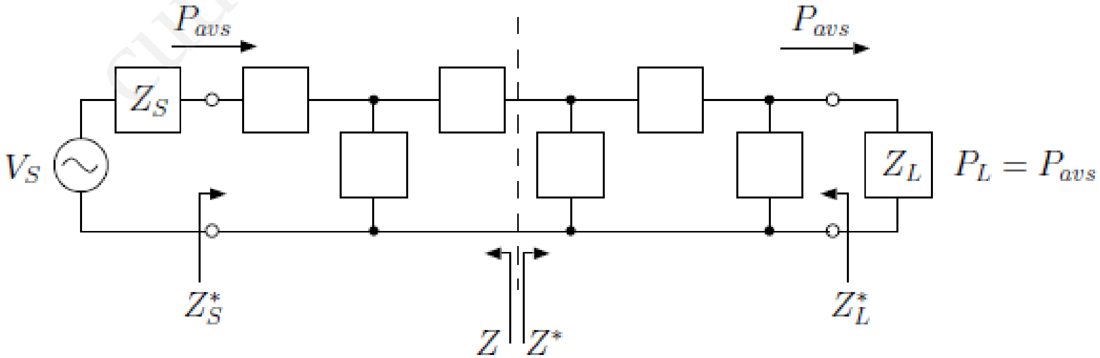
$$P_L = MF \times P_{avs} = 0.5 \times 1mW = 0.5\ (mW) = -3\ (dBm)$$

3. Impedance Matching Network

❖ In many applications it is desirable to maximize the transfer of power from the source to the load. This can be achieved by using a lossless 2-port network inserted between the source and the load.

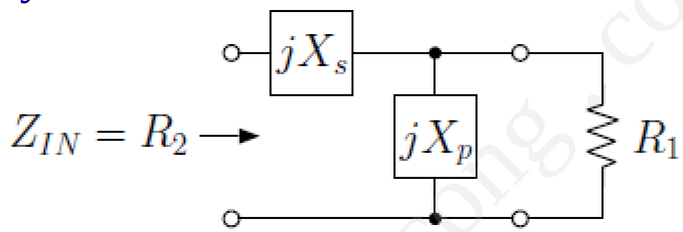


❖ Another way to look at this is to note that the original source plus the matching network can be viewed as a new source with available power P_{avs} and source impedance $Z_S = Z_L^*$. Most lumped-element matching networks are versions of ladder networks.

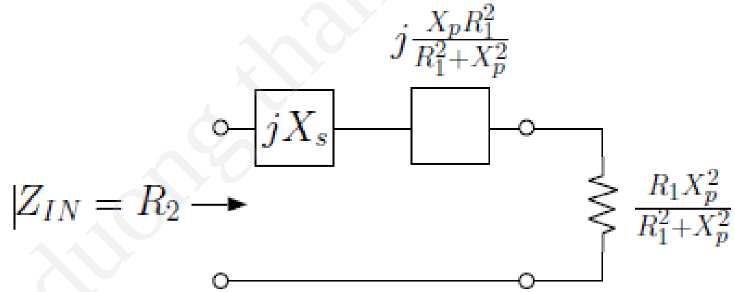


3. Impedance Matching with lossless L-networks

❖ Two resistances is to be matched with a lossless L-network. The goal is to transform R_1 to R_2 at one frequency.



❖ The unknown reactances X_S and X_P are easily found with the use of a parallel-to-series transformation.



❖ Solving for X_S and X_P gives

$$X_P = \pm R_1\sqrt{\frac{R_2}{R_1 - R_2}} \quad and \quad X_S = \mp\sqrt{R_1 R_2 - R_2^2}$$

❖ The solutions yield real values for X_S and X_P only if $R_1 > R_2$. This leads to a rule for using a lossless L-network to match two resistances.

The shunt arm of the L-network is connected across the larger of the two resistances.

3. Q of an L-network

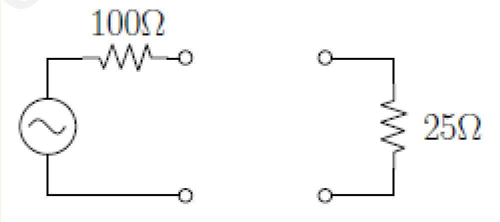
- The transformed L-network looks like a series resonant circuit. Treating this circuit in the same manner as a resonant RLC network, the Q of the network is:

$$Q = \left| \frac{X_P R_1^2 / R_1^2 + X_P^2}{R_1 X_P^2 / R_1^2 + X_P^2} \right| = \frac{R_1}{X_P} = \sqrt{\frac{R_1}{R_2} - 1}$$

- This suggests that the BW of the matching network depends only on the ratio $^{R_1}/_{R_2}$.
- We have defined the Q of the L-network by analogy with the series RLC network. This analogy works well only when $R_1 \gg X_P$ or equivalently, Q $\gg$ 1.
- For moderate or small values of Q the expression $Q = \frac{R_1}{X_P} = \sqrt{\frac{R_1}{R_2} - 1}$ is still valid. But a simple relationship between Q and bandwidth does not exist, since the series reactance that results from the parallel-to-series transformation has a different frequency dependence from that of a simple inductor or capacitor.
- Thus an L-network does not behave exactly like a series RLC circuit. Only when the Q is very large will the equivalent series reactance behave approximately like a capacitor or inductor.

3. Q of an L-network

- **Example 11:** Match a 100Ω source to a 25Ω load with a lossless L-network. Compare the power delivered to the load with and without the matching network in place. Suppose the peak voltage of the source is 1 Volt.

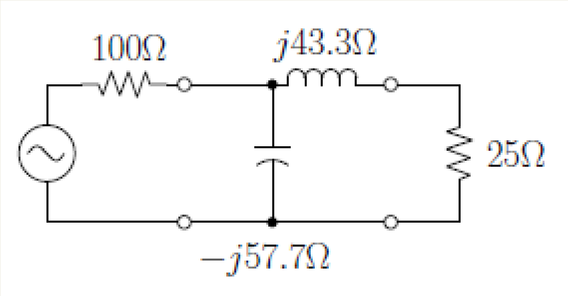


- **Solution:**

$$Q=\sqrt{\frac{R_{big}}{R_{small}}-1}=\sqrt{\frac{100}{25}-1}=\sqrt{3}$$

$$X_P=X_C=-\frac{R_{big}}{Q}=-\frac{100}{\sqrt{3}}=-57.7\ (\Omega)$$

$$X_S=X_L=QR_{small}=25\sqrt{3}=43.3\ (\Omega)$$



- The power delivered to the load without the matching network:

$$P_L=\frac{1}{2}\frac{|V_{out}|^2}{25}=\frac{1}{50}\times\left(\frac{1}{5}\right)^2=0.8\ (mW)$$

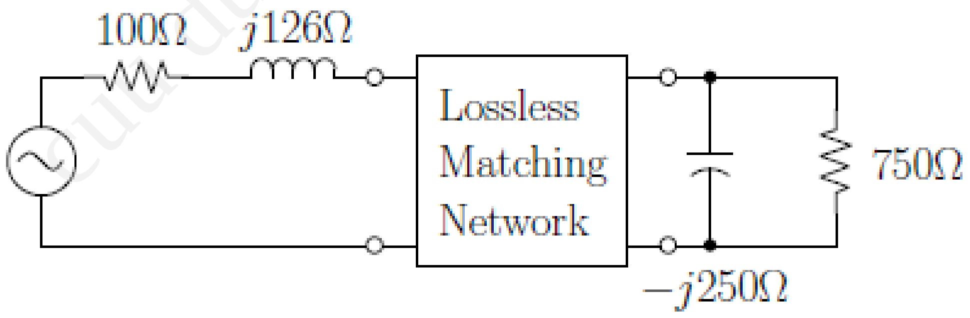
- With the matching network, the power delivered to the matching network is:

$$P_{in}=\frac{1}{2}\frac{|1/2|^2}{100}=1.25\ (mW)$$

3. Matching complex loads with a lossless L-network

- When complex source and loads are involved, there are two basic conceptual approaches that can be used:
  - Absorption - “absorb” the source or load reactance into the matching network.
  - Resonance - series or parallel resonate the source or load reactance at the frequency of interest.

- **Example 12:** Match the source and load of the following circuit at 100MHz with a lossless L-network.



3. Matching complex loads with a lossless L-network

❖ **Solution:**

- X_S' and X_P' can be found by using the design equations:

$$Q = \sqrt{\frac{750}{100} - 1} = 2.55$$

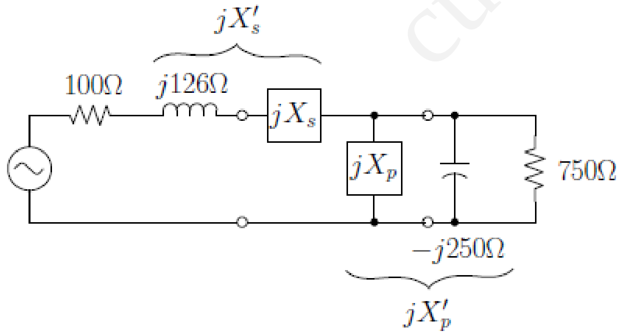
$$X_S' = \pm 100 \times 2.55 = \pm 255\ (\Omega)$$

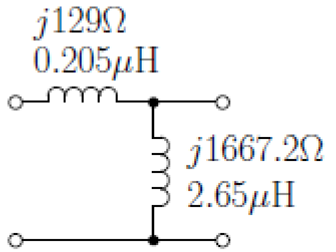
$$X_P' = \mp 750/2.55 = \mp 294.1(\Omega)$$

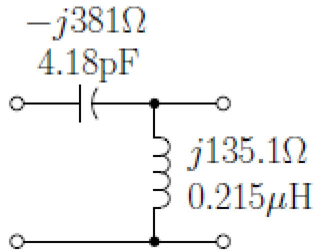
- The lump reactances can be obtained:

$$X_S = X_S' - 126 = \begin{cases} 129\ (\Omega) \\ -381\ (\Omega) \end{cases}$$

$$X_P = \frac{250X_P'}{250 + X_P'} = \begin{cases} 1667\ (\Omega) \\ 135\ (\Omega) \end{cases}$$

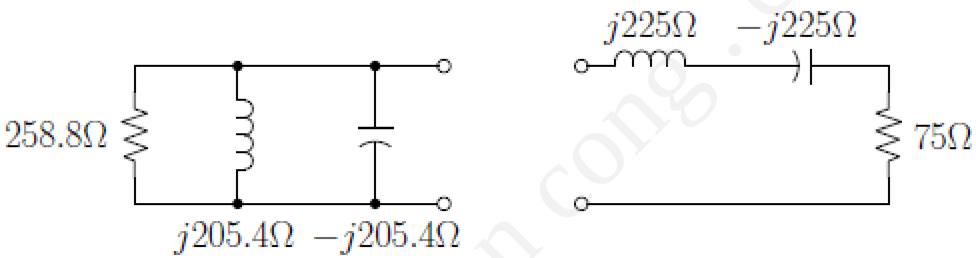




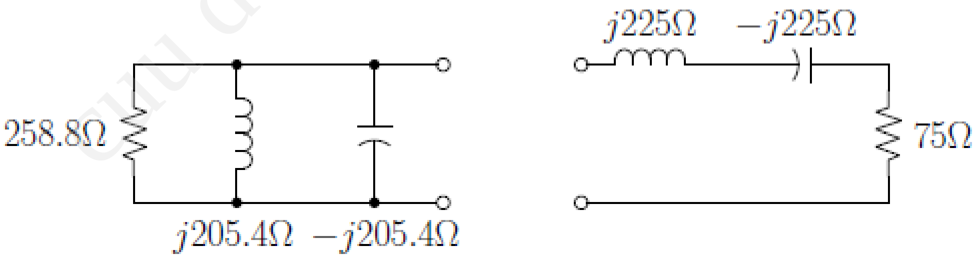


3. Matching complex loads with a lossless L-network

❖ To apply the resonance concept, the source and load are augmented with reactances that resonate with the source and load reactances.



❖ **Example 13:** Redesign the matching network in Example 12 with resonance method.



3. Matching complex loads with a lossless L-network

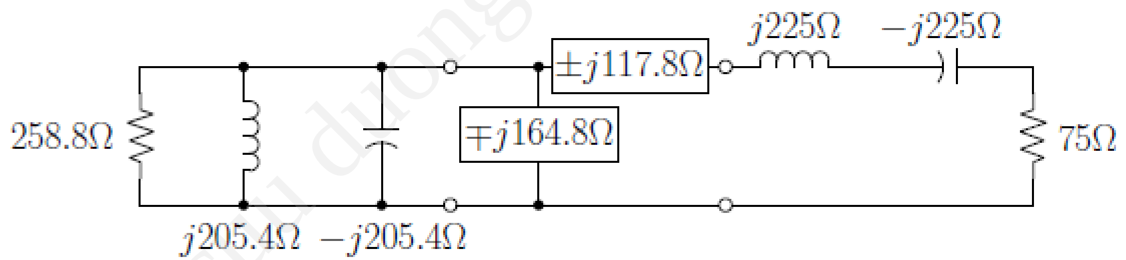
❖ **Solution:**

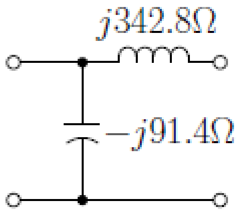
- An L-network is designed to match these two rea:

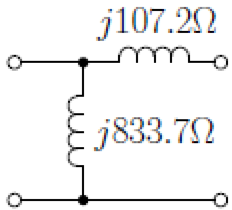
$$Q = \sqrt{\frac{258.76}{75} - 1} = 1.57$$

$$X_S' = \pm 100 \times 1.57 = \pm 117.8\ (\Omega)$$
$$X_P' = \mp 258.76/1.57 = \mp 164.8(\Omega)$$

- The solution of the design is as follows

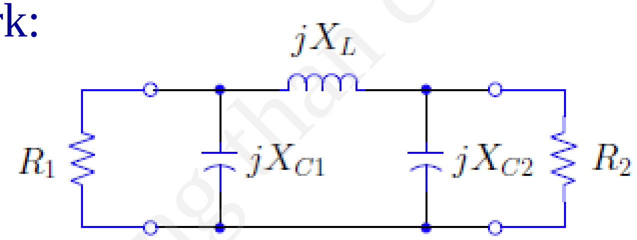




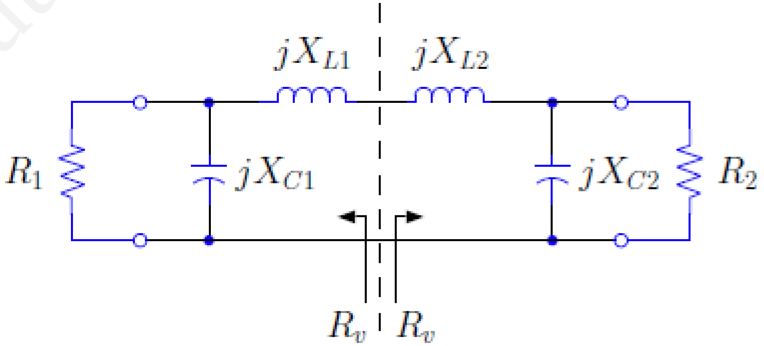


3. Three-element Matching Networks

- The L-network does not give the designer freedom to choose the Q (bandwidth) or phase shift of the matching network.
- The addition of a third matching element makes it possible to design for a match and a specified phase shift or Q. The Pi- and T-networks can be used to design matching circuits with specific bandwidth (Q).
- We consider a Pi-network:



- The Pi-network can be thought of as two back-to-back L-networks that act to match both R_1 and R_2 to a "virtual resistance" R_v.



3. Pi Matching Networks

- Because the series arms of both L-networks are connected to R_v, it is clear that R_v is smaller than R_1 and R_2. Define the Q's of the two L-networks to be Q_1 and Q_2:

$$Q_1 = \sqrt{\frac{R_1}{R_v} - 1} \quad \text{and} \quad Q_2 = \sqrt{\frac{R_2}{R_v} - 1}$$

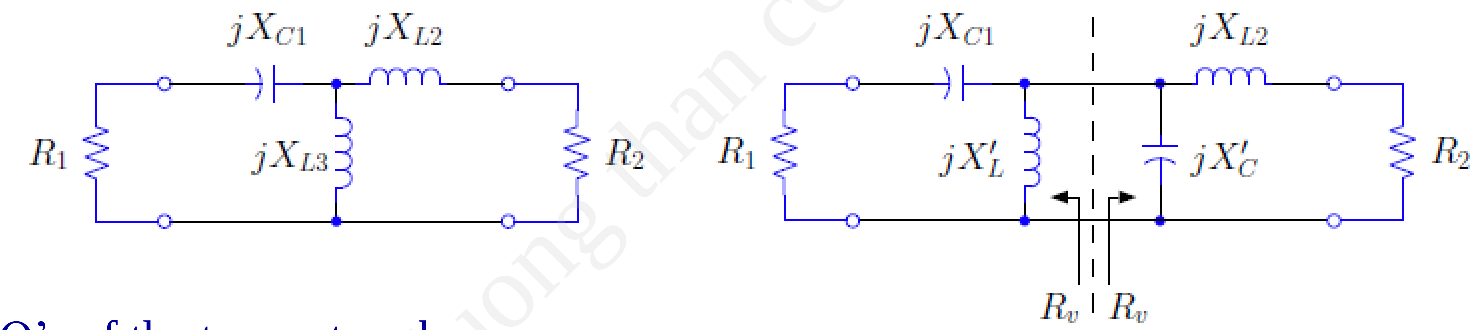
- Assume $R_2 > R_1$, therefore $Q_2 > Q_1$. For most practical purposes the Q of the Pi-network can be approximated by Q_2. This is especially true if $R_2 \gg R_1$.
- The design process is as follows:
  - Determine the required Q of the matching network. This Q is taken to be equal to Q_2, and thus the virtual resistance, R_v, is determined.
  - Once R_v is found, the values of X_{C1}, X_{L1}, X_{L2} and X_{C2} can be calculated:

$$X_{C2} = -\frac{R_2}{Q_2} \qquad X_{C1} = -\sqrt{\frac{R_1 R_2}{(Q_2^2+1) - \frac{R_2}{R_1}}}$$

$$X_L = \frac{R_2 Q_2 + R_2\sqrt{R_1/R_2\,(Q_2^2+1) - 1}}{Q_2^2 + 1}$$

3. T-Matching Networks

- The Pi-network is most useful for matching when the values of R_1 and R_2 are not too small. If R_1 and R_2 are small, the virtual resistance will be even smaller, and the capacitor values will turn out to be impractically large.
- If either terminating resistance is significantly less than 50Ω, the T-network will usually be a more practical choice.



- The Q's of the two networks are:

$$Q_1 = \sqrt{\frac{R_1}{R_v} - 1} \quad \text{and} \quad Q_2 = \sqrt{\frac{R_2}{R_v} - 1}$$

- The design formulas are: $X_{C1} = -R_1 Q_1$

$$X_{L2} = R_2 \sqrt{{}^{R_1}/_{R_2}\,(Q_1^2 + 1) - 1}$$

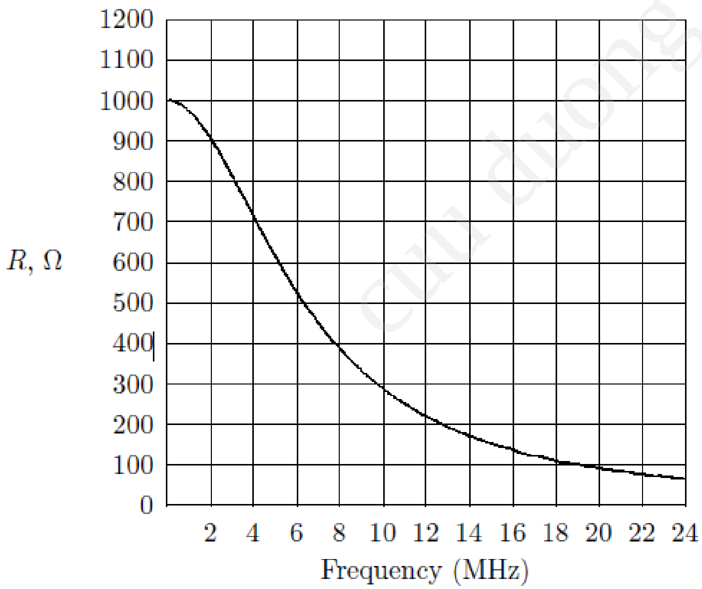
$$X_{L3} = \frac{R_1(Q_1^2 + 1)}{Q_1 - \sqrt{{}^{R_1}/_{R_2}\,(Q_1^2 + 1) - 1}}$$

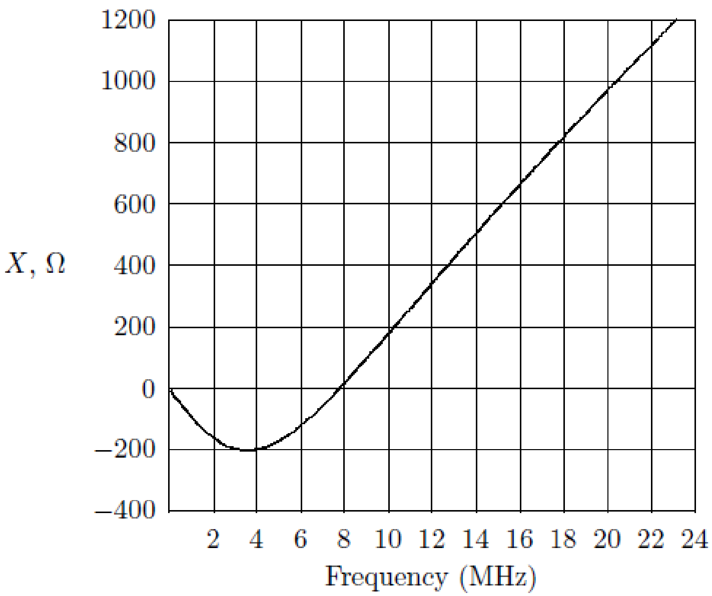
4. Exercises

Exercise 1: You are given a black box with two terminals. Suppose that you know that the box contains a passive circuit that is constructed from 3 elements: a resistor (R), lossless capacitor (C), and lossless inductor (L). Your task is to figure out how the elements are connected, and what their values are. You make some measurements of the impedance of the box $Z(f) = R(f) + jX(f)$. The results of the measurements are shown in following figures.

a. Sketch the circuit that is inside of the box. [R in parallel with C, all in series with L]

b. Estimate the element values. [$R = 1k\Omega$, $C = 24.36pF$, $L = 7.96\mu H$].



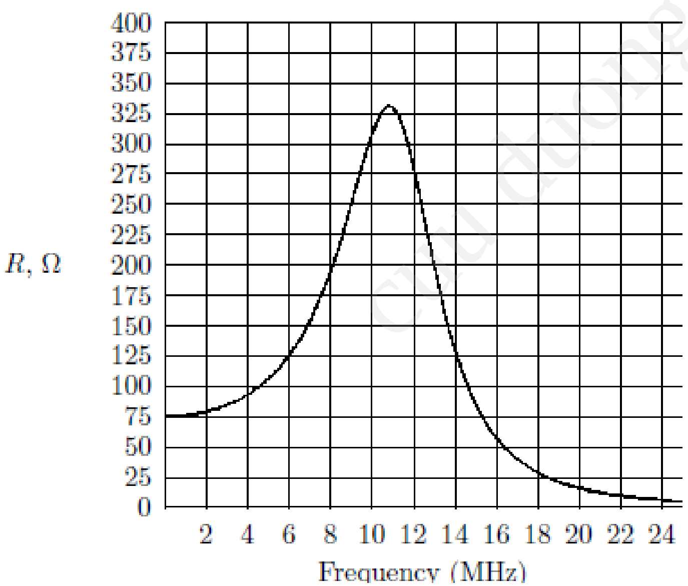


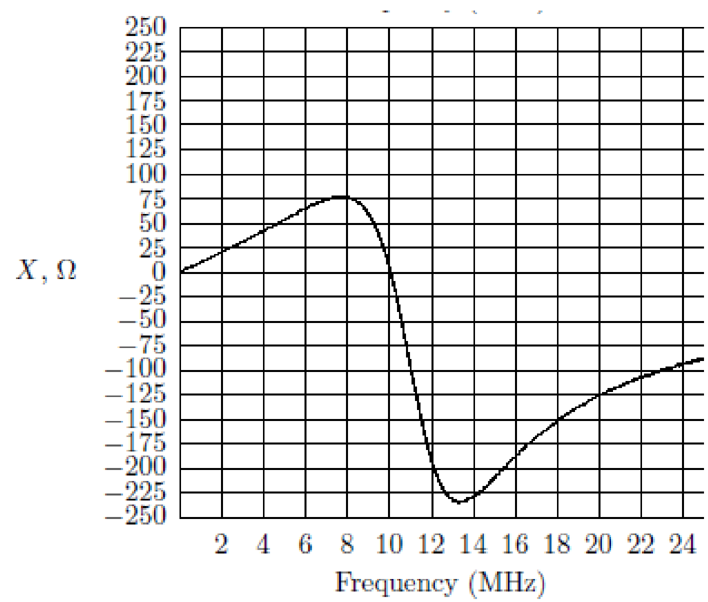
4. Exercises

Exercise 2: You are given a black box with two terminals. Suppose that you know that the box contains a passive circuit that is constructed from 3 elements: a resistor (R), lossless capacitor (C), and lossless inductor (L). Your task is to figure out how the elements are connected, and what their values are. You make some measurements of the impedance of the box $Z(f) = R(f) + jX(f)$. The results of the measurements are shown in following figures.

a. Sketch the circuit that is inside of the box..

b. Estimate the element values.





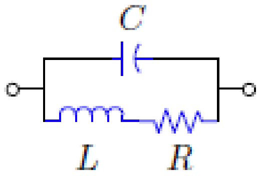
4. Exercises

Exercise 3: The circuit shown in following figure is used as a model for a realistic resistor or inductor. It can also be used to model a realistic parallel resonant circuit.

a. Find an expression for the impedance of this circuit.

b. The frequency at which the impedance is purely resistive is called the "resonant frequency." Under what conditions will there be a frequency (> 0) at which the impedance is purely resistive? Show that if this condition is satisfied, and if $CR^2/L \ll 1$, then the resonant frequency is given approximately by:

$$\omega_0 = \frac{1}{\sqrt{LC}}\left(1 - \frac{CR^2}{2L}\right)$$



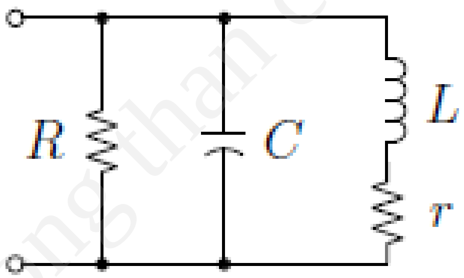
c. Assume that the term CR^2/L can be ignored and find the magnitude of the impedance at the parallel resonant frequency. How does it depend on R?

d. Find an approximate expression for the impedance valid when $\omega \ll \frac{1}{\sqrt{LC}}$ and $\frac{L}{R^2C} \gg 1$. Draw a simplified equivalent circuit that is valid under these conditions.

e. Find an approximate expression for the impedance valid when $\omega \ll \frac{1}{\sqrt{LC}}$ and $\frac{L}{R^2C} \ll 1$. Draw a simplified equivalent circuit that is valid under these conditions.

4. Exercises

Exercise 4: The circuit shown in following figure is usually a good model for a parallel LC circuit implemented with real, lossy components. The element values are $C = 800pF$, $L = 15\mu H$, $r = 1\Omega$, $R = 10k\Omega$. Use series to parallel transformations to transform this circuit into an equivalent parallel RLC circuit and find the approximate resonant frequency and Q_P of the equivalent circuit.



Exercise 5: The circuit in following figure is a model for the operation of a “ferrite loopstick” antenna that is commonly employed in AM broadcast band radios. The antenna is tuned to resonance by a capacitor C which is usually adjustable to allow the circuit to cover the entire broadcast band. This circuit also performs the function of the pre-selector. The voltage source V_S represents the emf induced in the coil as a result of an incident electromagnetic wave with frequency ω. The resistance r represents the losses in the coil and R represents the input impedance of the following stage.

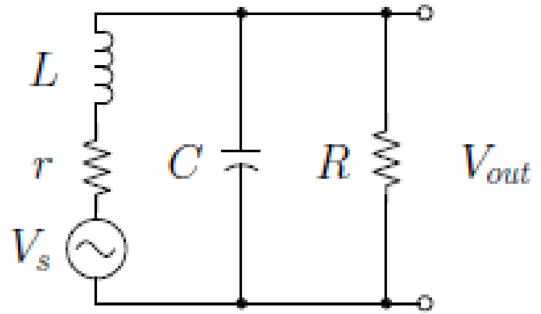
4. Exercises

Exercise 5 (cont):

a. Find an exact expression in terms of r, R, L, and C for the frequency where the output voltage is a maximum (the resonance frequency).

b. Suppose that $V_s = 1(mV)\cos(2\pi f_c t)$, $L = 100\mu H$, $R = 100k\Omega$, and $r = 6\Omega$. Find the range of values that C must cover in order for the circuit to tune the AM broadcast band (540-1700 kHz). Note: For this purpose you need an expression for the value of C that maximizes the voltage response at a given frequency. An approximate analysis is acceptable, but be sure to carefully state and justify your assumptions.

c. Now suppose that the source frequency, f_s, is swept from 540 to 1700 kHz. The zero-to-peak value of the source voltage is held constant at 1mV as the frequency is swept. Also suppose that the circuit is tuned to follow the frequency of the source so that the output voltage is always maximized. Plot the output voltage as a function of frequency.



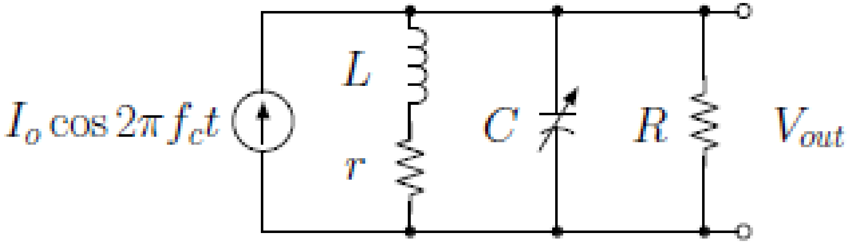
4. Exercises

Exercise 6: Consider the circuit shown in following figure. The current source has constant amplitude and frequency f_C, and it drives a bandpass filter consisting of a lossy inductor in parallel with a variable capacitor and a resistor. You may assume that any capacitance associated with the inductor has been incorporated into the variable capacitor indicated in the schematic. The variable capacitor C can be set to any value in the range 36 to 365pF, and $r = 10\Omega$, $R = 100k\Omega$. Suppose that the frequency of the current source can be adjusted to any frequency in the range 540-1700 kHz. For a given value of the source frequency, f_C, the variable capacitor will be tuned to maximize the output voltage. Approximate the filter as a parallel RLC circuit to answer the following questions:

a. Specify a single value of L that would allow the variable capacitor to tune the filter (i.e. maximize the output voltage) to any frequency in the range 540-1700kHz.

b. With the value of L that was determined in part (a), determine the approximate 3dB bandwidth of the filter when $f_C = 540kHz$ and C is adjusted to maximize the output voltage.

c. Same as part (b) but for $f_C = 1700kHz$.

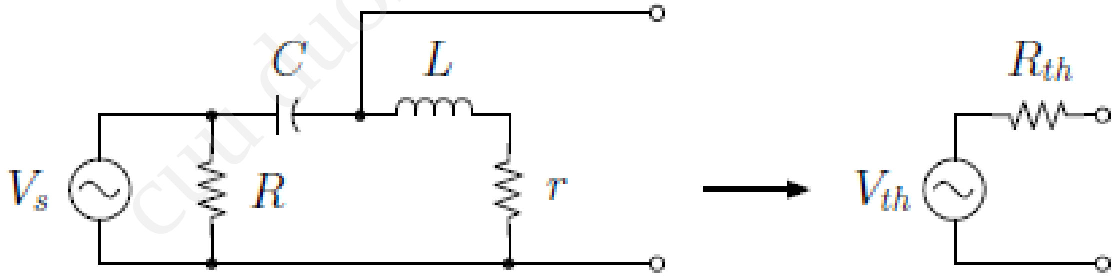


4. Exercises

Exercise 7: Consider the following circuit and its Thevenin equivalent circuit.

a. If $V_s = 2V$, $R = 1k\Omega$, $C = 1nF$, $L = 10\mu H$, $r = 1\Omega$, find the (non-zero) frequency, ω_0, where the indicated Thevenin equivalent circuit would be valid. In other words, find the frequency where the Thevenin impedance is purely resistive. Clearly state any assumptions or approximations that you make.

b. Find R_{th} and V_{th} at the frequency found in part (a). For V_{th}, specify magnitude and phase (in degrees).



4. Exercises

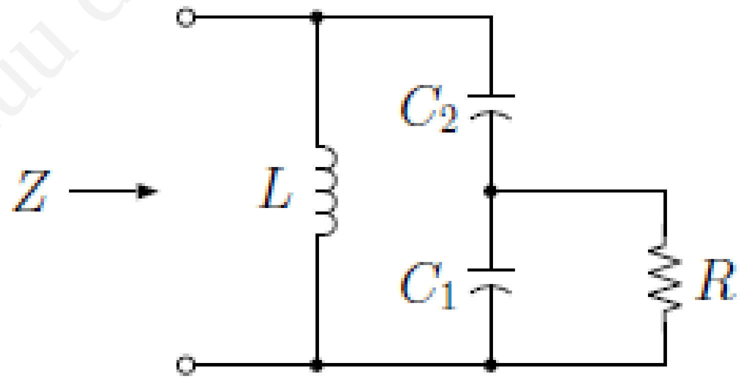
Exercise 8: The circuit shown in following figure is a capacitive transformer with resonating inductance L. Suppose that $R = 50\Omega, C_1 = 3183pF, C_2 = 3183p$. Use parallel-series and series-parallel transformations, with appropriate approximations, and find:

a. The inductance, L, required to resonate the circuit at 10 MHz. At resonance, the impedance Z will be purely real.

b. The input impedance, Z at 10 MHz.

c. The Q of the circuit can be approximated by the Q_P of the equivalent parallel RLC circuit (at 10 MHz). Find the approximate Q.

To save time, note that the reactances of C_1 and C_2 have magnitude 5Ω at 10MHz.



4. Exercises

Exercise 9: You are given a “black box” with two output terminals (a “1-port"). You play around with the box for awhile and make the following observations:

a. The output voltage from the box is sinusoidal.

b. The peak magnitude of the open circuit voltage at the output is found to be 5V.

c. You connect a 50Ω resistor across the terminals and find that the peak magnitude of the voltage across the resistor is 2.795V.

d. You short the output of the box and find the peak magnitude of the short circuit current is 100 mA.

Find the power available from the source. Express your result in dBm.

4. Exercises

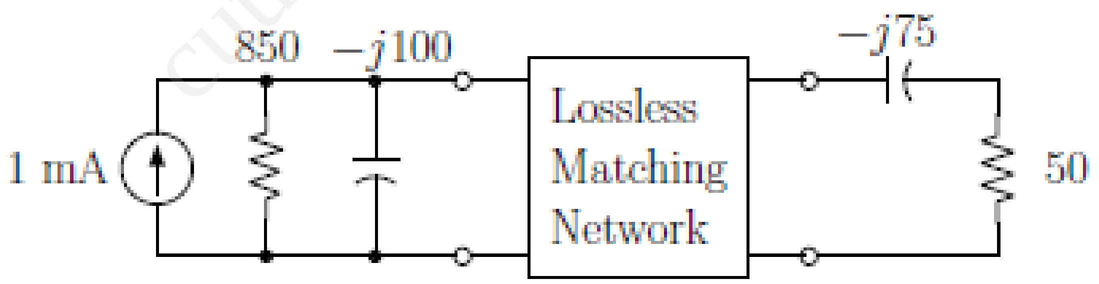
Exercise 10: Consider the design of a lossless L-network to match the source and load shown in following figure. All resistances and reactances are in ohms, and the current source magnitude is the peak value.

a. Find the power available from the source. Express your result in dBm.

b. How much power would be delivered to the load if a lossless matching network was not used, i.e., if the load is connected directly to the source? Express your result in dBm.

c. There are four possible solutions for the matching network if an L-network is used. Find all four. Sketch all solutions and indicate whether the elements are inductors or capacitors.

d. Verify two of your designs by plotting the path from the load to the source on a Smith Chart.



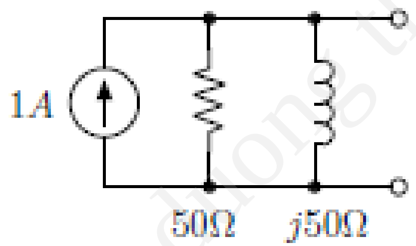
4. Exercises

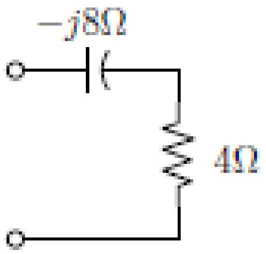
Exercise 11: Consider the source and load shown in the following figure.

a. Find the power available from the source. Express your answer in dBm.

b. Match this source and load using a lossless L-network.

c. Now suppose that you can only use a single lossless inductor or capacitor, either in series or in shunt, to couple the source to the load. If the goal is to maximize the power delivered to the load under this constraint, find the best possible solution.





Exercise 12: A 50Ω source has $P_{avs} = 4mW$. It is necessary to couple the source to a load with impedance $Z_L = 2 = j20\Omega$.

a. Find the power delivered to the load when the load is connected directly to the source. Express your answer in dBm.

b. There are 4 lossless LC L-networks that will match this source and load. Find the solution that has a capacitive series arm and the shunt arm connected across the load.

Q&A