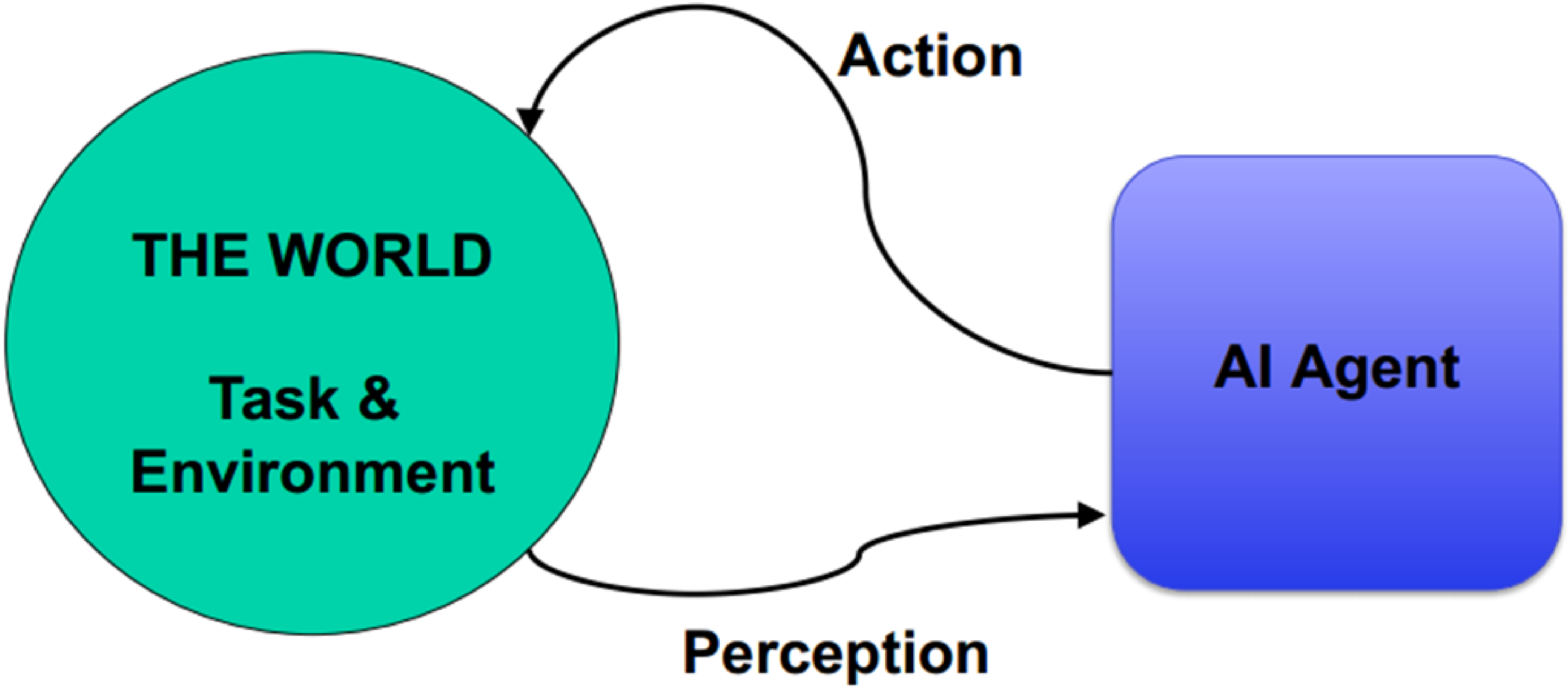
REPRESENT & PROCESS KNOWLEDGE

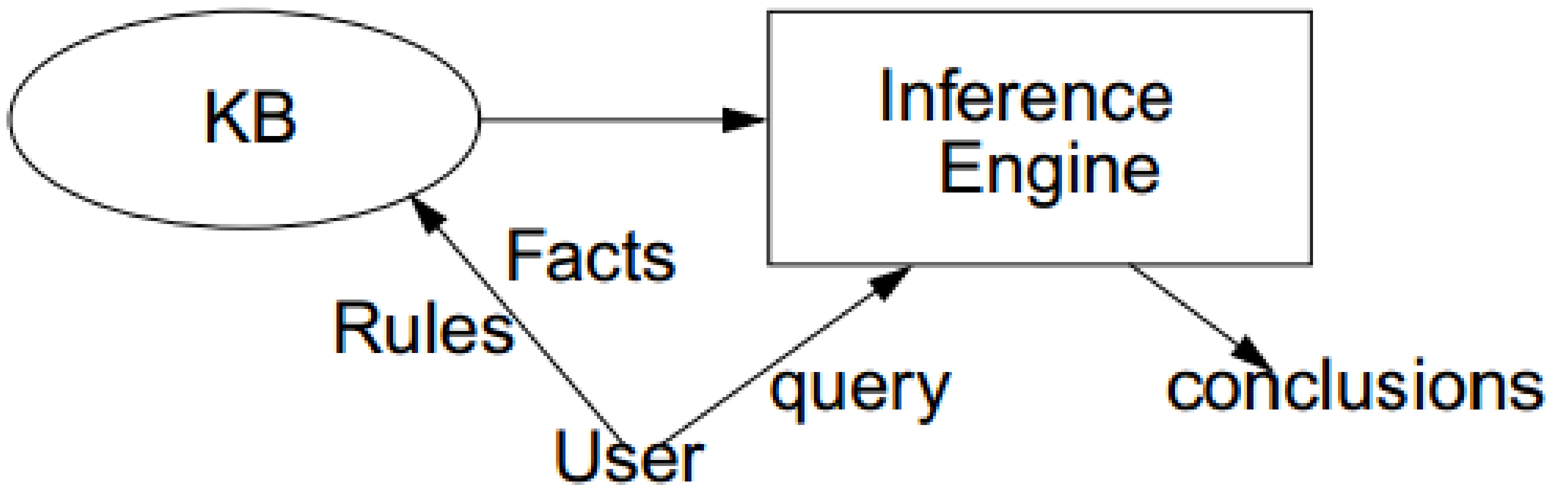
Nguyễn Thị Hải Bình, PhD

AI agent



Knowledge Representation

- A knowledge-based agent has a knowledge base (KB).
- The KB contains a set of sentences.



How to represent knowledge?

Propositional Logic

Predicate Logic

PROPOSITIONAL LOGIC

Definition of Proposition

- A proposition:
  - is a sentence, written in a language,
  - that has a truth value (i.e., it is true or false) in a world.

- Examples:
  - P = “Hanoi is the capital of Vietnam”
  - Q = “6 is a prime number”
  - R = $(x+2 \geq y)$

Syntax

- An atomic proposition, or just an atom, is a symbol.
- Propositions can be built from simpler propositions using logical connectives.
- Order of logical connectives:
  - $\neg, \wedge, \vee, \Rightarrow, \Leftrightarrow$

$\neg p$	"not p"	**negation** of p
$p \wedge q$	"p and q"	**conjunction** of p and q
$p \vee q$	"p or q"	**disjunction** of p and q
$p \rightarrow q$	"p implies q"	**implication** of q from p
$p \leftarrow q$	"p if q"	**implication** of p from q
$p \leftrightarrow q$	"p if and only if q"	**equivalence** of p and q

Syntax

- Examples:

$$\neg p$$

$$(\neg p) \wedge \text{true}$$

$$\neg((\neg p) \vee \text{false})$$

$$(\neg p) \Rightarrow (\neg((\neg p) \vee \text{false}))$$

$$(p \wedge (q \vee r)) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$$

$$p \wedge \neg q \Rightarrow r$$

Semantics of the Propositional Logic

- Semantics defines the meaning of the sentences of a language.

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
False	*False*	*True*	*False*	*False*	*True*	*True*
False	*True*	*True*	*False*	*True*	*True*	*False*
True	*False*	*False*	*False*	*True*	*False*	*False*
True	*True*	*False*	*True*	*True*	*True*	*True*

Semantics of the Propositional Logic

- A knowledge base is a set of propositions that are stated to be true.

- An element of the knowledge base is an axiom.

- A model of a knowledge base KB is an interpretation in which all the propositions in KB are true.

Example 1:

- Complete the truth table of $(P \Rightarrow Q) \wedge S$
- Find a model of the given proposition.

P	Q	S	$P \Rightarrow Q$	$(P \Rightarrow Q) \wedge S$

Example 2:

- Complete the truth table of $(p \wedge \neg p)$
- Find a model of the given proposition.

Example 3:

- Complete the truth table of $((P \vee H) \wedge \neg H) \Rightarrow P$
- Find a model of the given proposition.

Example 4:

- Complete the truth table of $\neg p \lor (q \land r)$
- Find a model of the given proposition.

Laws of Propositional Logic

1. Double-negation law:

$$\neg(\neg P) \equiv \mathrm{P}$$

2. Complement law:
   a. $P \vee \neg P \equiv True$
   b. $P \wedge \neg P \equiv False$
3. Identity law:
   a. $P \vee False \equiv P$
   b. $P \wedge True \equiv P$
4. Conditional identities:
   a. $P \Rightarrow Q \equiv \neg P \vee Q$
   b. $P \Rightarrow Q \equiv \neg Q \Rightarrow \neg P$
5. Conditional identities:

$$P \Leftrightarrow Q \equiv (P \Rightarrow Q) \wedge (Q \Rightarrow P)$$

6. De Morgan's law:
   a. $\neg(P \vee Q) \equiv \neg P \wedge \neg Q$
   b. $\neg(P \wedge Q) \equiv \neg P \vee \neg Q$
7. Distributive law:
   a. $R \vee (P \wedge Q) \equiv (R \vee P) \wedge (R \vee Q)$
   b. $R \wedge (P \vee Q) \equiv (R \wedge P) \vee (R \wedge Q)$
8. Commutative law:
   a. $P \wedge Q \equiv Q \wedge P$
   b. $P \vee Q \equiv Q \vee P$

Laws of Propositional Logic

9. Associative law:
   a. $(P \wedge Q) \wedge R \equiv P \wedge (Q \wedge R)$
   b. $(P \vee Q) \vee R \equiv P \vee (Q \vee R)$
10. Idempotent law:
    a. $P \wedge P \equiv P$
    b. $P \vee P \equiv P$
11. Domination law:
    a. $P \wedge False \equiv False$
    b. $P \vee True \equiv True$
12. Absorption law:
    a. $P \vee (P \wedge Q) \equiv P$
    b. $P \wedge (P \vee Q) \equiv P$

Laws of propositional logic can be proof using truth table.

Exercise

Using laws of propositional logic to proof:

- $P \Rightarrow (\neg P \Rightarrow P) \equiv \text{True}$
- $(P \land Q) \Rightarrow P \equiv \text{True}$
- $P \Rightarrow (Q \Rightarrow (P \land Q)) \equiv \text{True}$
- $\neg(Q \Rightarrow P) \lor (P \land Q) \equiv Q$
- $P \lor \big((P \land Q) \lor (P \land \neg R)\big) \equiv P \land ((\neg Q \Rightarrow R) \lor \neg(Q \lor (R \land S) \lor (R \land \neg S)))$

Let’s consider a propositional language where:

- p means “Paola is happy”,
- q means “Paola paints a picture”,
- r means “Renzo is happy”.

Formalize the following sentences:

1. “if Paola is happy and paints a picture then Renzo isn’t happy”
2. “if Paola is happy, then she paints a picture”
3. “Paola is happy only if she paints a picture”

Aladdin finds two trunks A and B in a cave. He knows that each of them either contains a treasure or a fatal trap.

On trunk A is written: “At least one of these two trunks contains a treasure.”

On trunk B is written: “In A there’s a fatal trap.”

Aladdin knows that either both the inscriptions are true, or they are both false.

Can Aladdin choose a trunk being sure that he will find a treasure?

If this is the case, which trunk should he open?

Conjunctive Normal Form (CNF)

- CNF = conjunction of disjunction of literals.
- Example of CNF:
  - $(A \vee \neg B \vee \neg C) \wedge (\neg D \vee E \vee F)$
  - $(A \vee B) \wedge (C)$
  - $(A \vee B)$
  - (A)
- Convert a formula into a CNF:
  - Conditional identities
  - De Morgan's law
  - Distributive law

Conjunctive Normal Form (CNF) - Example

- Convert the following formular into CNF: $(P \Rightarrow Q) \vee \neg(R \vee \neg S)$

Logical Inference Problem

- Given:
  - A knowledge base KB (a set of sentences)
  - A sentence H (called a theorem)
- Does the KB semantically entail H? $(KB \vDash H)$

  In other words: In all interpretations in which sentences in the KB are true, is also H true?

Inference Rules for Propositional Logic

- Modus Ponens

$$\frac{\alpha \Rightarrow \beta, \alpha}{\beta}$$

- Modus Tollens:

$$\frac{\alpha \Rightarrow \beta, \neg\beta}{\neg\alpha}$$

- Hypothetical syllogism

$$\frac{\alpha \Rightarrow \beta, \beta \Rightarrow \gamma}{\alpha \Rightarrow \gamma}$$

- Simplification

$$\frac{\alpha_1 \wedge \cdots \wedge \alpha_i \wedge \cdots \wedge \alpha_m}{\alpha_i}$$

- Conjunction

$$\frac{\alpha_1, \dots, \alpha_i, \dots, \alpha_m}{\alpha_1 \wedge \cdots \wedge \alpha_i \wedge \cdots \wedge \alpha_m}$$

- Addition

$$\frac{\alpha_i}{\alpha_1 \vee \cdots \vee \alpha_i \vee \cdots \vee \alpha_m}$$

- Resolution

$$\frac{\alpha \vee \beta, \neg\beta \vee \gamma}{\alpha \vee \gamma}$$

- Unit resolution

$$\frac{\alpha \vee \beta, \neg\beta}{\alpha}$$

Proof by Resolution

- **Purpose**: Proof $KB \vDash H$
- **Các bước của thuật toán:**
  1. Add ¬H to KB;
  2. Convert all sentences in KB into CNF;
  3. Create DNF from CNF created in step 2;
  4. Loop until no new sentence generated:
     a) Apply rule of resolution for all DNF created in step 3;
     b) If a contradiction (empty clause) is reached, return TRUE .

        *** Contradiction is obtained when apply resolution to a clause and its negation.*
  5. Return FALSE

Example 1

- KB:
    1. $\neg A \vee \neg B \vee P$
    2. $\neg C \vee \neg D \vee P$
    3. $\neg E \vee C$
    4. A
    5. E
    6. D
- Theorem: P.

Example 2

- KB:
  1. $\neg A \vee \neg B \vee P$
  2. $\neg C \vee \neg D \vee P$
  3. $\neg E \vee C$
  4. A
  5. E
  6. D
- Theorem: $\neg P$

Example 3

- KB:
  1. $P \Rightarrow Q$
  2. $Q \Rightarrow R$
  3. $R \Rightarrow T$
  4. P
- Theorem: T.

Example 4

- KB:
  1. $P \wedge Q \Rightarrow R$
  2. $Q \wedge R \Rightarrow S$
- Theorem: $P \wedge Q \Rightarrow S$