Design and Optimization of FSO Mesh Networks over Atmospheric Turbulence and Misalignment Fading Channels

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***Abstract*—In this paper, we design and optimize free-space optics (FSO) mesh networks over atmospheric turbulence and misalignment fading channels. We propose an heuristic algorithm to choose the best sites to install the FSO transceivers and also the best topology for a given traffic load in a region. The algorithm aims to use the least of FSO links as possible since this allows to reduce the network costs. In addition, the algorithm takes into account the influence of turbulence, misalignment, and noise by choosing links with low bit-error rate (BER). The simulation results show that the calculation time of the proposed heuristic algorithm is much smaller than optimal integer linear programming (ILP) model. Since the proposed algorithm runs very fast, even with a large number of choices of FSO sites, it is practical to use it in building quickly a restoration communication networks for replacing a regular one after a disaster. The average BER of all links of the FSO network designed by using the proposed algorithm also meets the requirement of end-to-end BER threshold.**

Keywords*—Free Space Optics, Atmospheric Turbulence, Misalignment Fading Channels, Topology design

I. INTRODUCTION

Free Space Optics (FSO) refers to an optical communication technology that transmits data using a laser beam in free space between a pair of transceivers. FSO transceivers are now widely available in the market and a FSO link can be setup quickly in several minutes to hours. Contrast to the fiber-optic networks, FSO networks can be deployed without requiring to lay out physical cable. Therefore, FSO networks are promising candidate for densely populated urban areas, where the deployment of fiber optic infrastructure is impractical due to high costs or physical deployment difficulty [1]. For example in Fig. 1, FSO transceivers could be used to setup a backbone campus networks where there are a lot of tall buildings and it is not really convenient to run cable under the ground between buildings.

Nowadays, we observe often disasters such as earthquake, tsunami, flood which usually destroy the infrastructure of a whole region, including communication networks. FSO links could be used to setup a network in responding to the requirement of quick recovery of communication in the affected region after the disaster thanks to the good mobility of FSO transceivers and fast link deployment. FSO transceivers will be placed on some secured places, then they are aligned to see each other according to a topology for making a network. The

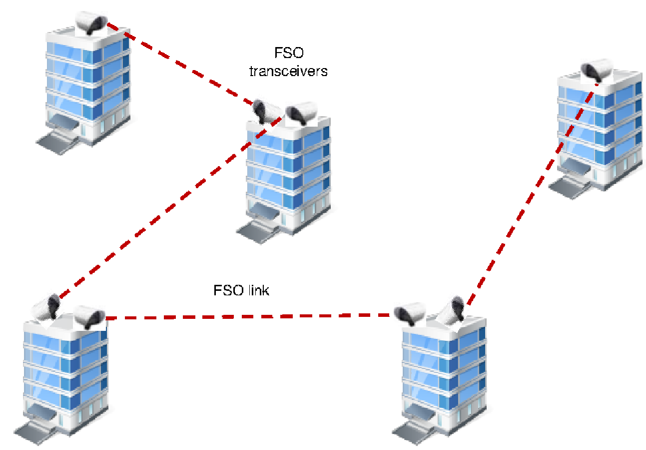

Fig. 1. Example of a FSO network

FSO transceivers of the same site can be wired together for exchanging the data between their links.

Mesh topology is a good choice for network architecture thanks to its advantages of high availability, enhanced capacity and network utilisation. The first FSO mesh network was proposed by A.S. Acampora et al. in [2]. In this study, the authors describe an approach that uses of FSO links to interconnect densely deployed packet-switching nodes in a multihop mesh topology. Next, a broadband access network based on FSO links in mesh architecture is proposed in [3]. Although the problem of designing wireline and RF wireless network topology has been largely studies but the problem of designing FSO mesh networks is still new. In [4], the authors consider the problem of designing a topology with strong connectivity and short diameter for FSO networks. Two centralised approaches including Delaunay triangulation and Closest Neighbor (CN) algorithms are presented in this study. As an extension of the work of [4], the authors in [5] proposed network topology design (NTD) algorithm, which is able to achieve not only high spatial diversity but also high reliability. This work is, however, limited to the case that at least three FSO transceivers are deployed at each site. The design and optimization problem for the tiered wireless access network with FSO links are studied in [6]. The authors consider two sub-problems including the optimized clustering problem in the underlying wireless mesh network and the topology optimization problem on designing the upper tier FSO network topology.

Although design and optimization of FSO mesh network have been studied profoundly, there are still unsolved problems. The impact of impairments caused by atmospheric channel on network design and optimization were often ignored in previous work [4],[5], except [6], where the impact of weak atmospheric turbulence is quantified into weight of an FSO link via received intensity. In order to build a more practical algorithm for designing and optimizing FSO networks, in this paper, the weight of an FSO link is measured via bit-error rate (BER), which is a practical and useful performance parameter. The link weight in our research, therefore, can reflect various factors including the received power, fading-induced atmospheric turbulence, misalignment fading, and noise. Instead of considering log-normal distributed fading channel as in [6], FSO channel is modelled as Gamma-Gamma fading channel so as to moderate-to-strong turbulence can be considered [7].

In building an FSO mesh network, especially restoration network for serving disaster recovery, there are two main concerns to be considered as follows: (1) network cost and (2) installation time. Since it is easy to install the FSO transceivers, the main network cost will be the cost of the transceivers. The network installation time includes the time placing FSO devices on site and time for aligning FSO links. In this paper, we therefore propose to take minimization of network cost and installation time as the optimization objectives. A heuristic algorithm will be used to choose the best sites to install the FSO transceivers and also the best topology for a given traffic load in a region. The algorithm aims to use the least of FSO links as possible since this allows to reduce both the network costs and the installation time.

The remainder of the paper is organised as follows. Section II states the problem of designing FSO mesh networks. Section III presents how atmospheric turbulence and misalignment fading channels affect BER of FSO links. Section IV proposes an heuristic solution to solve the FSO mesh network design problem. Section V shows the numerical results. Finally, Section VI concludes the paper.

II. Problem Formulation

In this paper, the problem of designing FSO mesh network is defined as following.
Given

- A set of possible sites for installing FSO transceivers.
- A traffic load that needs to be satisfied. The traffic load is a list of connection demands between sites.

Assume that

- All FSO links have identical capacity
- FSO devices of the same sites are connected to local wired networks and the quality of this network is not taken into account.

We need to seek for a network topology composing of FSO transceivers and links between them so that

- All the demanded traffic is satisfied.
- Every end-to-end FSO connection must have BER not exceed a threshold δ.
- The solution must minimize the cost of FSO transceiver and installation times.

Let the unit cost of a FSO transceiver be C_{FSO} and the number of FSO links in the topology be n, then the total network cost is $2n \times C_{FSO}$.

Let the average time to install a FSO link be T_{link}, then the total installation time of the network is $n \times T_{link}$.

It is clear that, in order to minimize the network cost and the installation time, we need only to minimize the number of FSO links in the network topology. Therefore, the optimization objective of the design turns to minimizing the total number of FSO links.

BER of a link between two FSO transceivers is calculated according to the channel model in Section III. Let BER_p be the BER of an end-to-end connection p through several FSO links and BER_ℓ be BER of a link $\ell \in p$, then the probability of non-error along p is the product of the probability of non-error of all links in p and hence can be given as

$$1 - \text{BER}_p = \prod_{\ell \in p} (1 - \text{BER}_\ell). \tag{1}$$

Therefore, the constraint limiting the end-to-end BER under threshold δ can be expressed as

$$\text{BER}_p = 1 - \prod_{\ell \in p} (1 - \text{BER}_\ell) \leq \delta. \tag{2}$$

III. Channel Model

FSO channel considered in this study is characterized by three parameters including channel loss, atmospheric turbulence-induced fading, and pointing errors. The mathematical model of channel state can be expressed as

$$h = h_l h_a h_p, \tag{3}$$

where h_l is the channel loss coefficient. h_a represents the intensity fluctuation due to atmospheric turbulence. h_p is the fraction of power collected by a photo-detector (PD), which depends on the relative distance between the PD and the center of the received optical beam. In one bit duration, it can be assumed that h_l is deterministic while h_a and h_p are random variables.

Optical signal is attenuated while traversing atmospheric channel due to absorption and scattering processes. Signal attenuation is caused by the variation of the concentrations of matter in the atmosphere, which depend on the weather conditions. According to Beers-Lambert law, the channel loss coefficient is described as [8]

$$h_l = \exp(-a_l z), \tag{4}$$

where a_l the attenuation coefficient and z is the transmission distance.

Intensity fluctuation (or fading) happens at the receiver due to atmospheric turbulence. In this study, we use Gamma-Gamma distribution in order to investigate the system performance in moderate-to-strong turbulence regime. The probability distribution function (PDF) of the intensity fluctuation is thus given by [7]

$$f_{h_a}(h_a) = \frac{2(\alpha\beta)^{\frac{(\alpha+\beta)}{2}}}{\Gamma(\alpha)\Gamma(\beta)} h_a^{\frac{(\alpha+\beta)}{2}-1} K_{\alpha-\beta}\left(2\sqrt{\alpha\beta h_a}\right), \quad (5)$$

where $K_v(.)$ is the modified Bessel function of the second kind and order v. $\Gamma(.)$ is the standard gamma function. The two parameters $\alpha > 0$ and $\beta > 0$ can be adjusted for wide range of turbulence conditions. In the case of spherical wave propagation, They are directly linked to physical parameters as [7]

$$\alpha = \left[exp\left(\frac{0.49\sigma_R^2}{\left(1+1.11\sigma_R^{12/5}\right)^{7/6}} \right) - 1 \right]^{-1} \quad (6)$$

$$\beta = \left[exp\left(\frac{0.51\sigma_R^2}{\left(1+0.69\sigma_R^{12/5}\right)^{5/6}} \right) - 1 \right]^{-1}, \quad (7)$$

where σ_R is the unitless Rytov variance, which represents the strength of the turbulence and is defined as

$$\sigma_R^2 = 1.23\left(\frac{2\pi}{\lambda}\right)^{7/6} C_n^2 z^{11/6}, \quad (8)$$

where λ is the wavelength. C_n^2 is the index of refraction structure parameter and z is the distance of a FSO link.

To compute the PDF of the fraction of power collected by a PD h_p, we use the assumptions and methodology described in [9], which assumes a circular detection aperture of radius r and a Gaussian beam. Consequently, the PDF of h_p can be derived as [9]

$$f_{h_p}(h_p) = \frac{{\gamma_p}^2}{A_0^{\gamma_p^2}} (h_p)^{\gamma_p^2 - 1}, \quad (9)$$

where $\gamma_p = \omega_{z_{eq}}/2\sigma_s$ is the ratio between the equipment beam radius and the jitter standard deviation σ_s of the mis-alignment. The parameter $\omega_{z_{eq}}$ can be calculated using the relations $v = \sqrt{\pi}r/\sqrt{2}\omega_z$, $A_0 = [erf(v)]^2$ and $\omega_{z_{eq}}^2 = \omega_z^2\sqrt{\pi}\mathrm{erf}(v)/2v\exp(-v^2)$, where erf(.) is the error function and ω_z is the beam waist (radius calculated at e^{-2}) at the distance z.

By expressing the $K_v(.)$ in terms of Meijers G-function and making simplification, the PDF of the channel state, $h = h_l h_a h_p$, is given as [10]

$$f_h(h) = \frac{\alpha\beta{\gamma_p}^2}{A_0 h_l \Gamma(\alpha)\Gamma(\beta)} G_{1,3}^{3,0}\left(\frac{\alpha\beta}{A_0 h_l}h \middle| \begin{matrix} {\gamma_p}^2 \\ {\gamma_p}^2-1, \alpha-1, \beta-1 \end{matrix}\right). \quad (10)$$

The signal-to-nosie ratio (SNR) of the links between two FSO transceivers is defined as follows

$$\gamma = \frac{{P_T}^2 \Re^2 h^2}{\sigma_n^2}, \quad (11)$$

where γ denotes the instantaneous SNR of the link between transmitter and receiver. P_T is the transmitted optical power, σ_n^2 is variance of additive white Gaussian noise (AWGN). $\Re$ is the responsivity of the photodetector.

Denoting $p(1)$ and $p(0)$ are the probabilities of sending bit "1" and bit "0", the BER of a FSO link using OOK is given by BER $= P_{FSO}(e) = p(1)\,p(e|1) + p(0)\,p(e|0)$, where $p(e|1)$ and $p(e|0)$ are the conditional bit error probabilities. Assuming that $p(1) = p(0) = 1/2$ and $p(e|1) = p(e|0)$, the conditional bit error probabilities can be computed as

$$\mathrm{BER} = P_{e,FSO}(e|h) = p(e|1,h) = p(e|0,h) = Q(\sqrt{\gamma}), \quad (12)$$

where Q(.) is the Gaussian Q function which is related to the complementary error function erfc(.) by erfc $(x) = 2Q(\sqrt{2}x)$. γ is the SNR of FSO link, which is derived from (11). The average BEP, $P_{FSO}(e)$, can be obtained by averaging (12) over the PDF of h as follows

$$\mathrm{BER} = \int_0^\infty f_h(h) P_{e,fso}(e|h)\,dh. \quad (13)$$

By substituting (3), (12) in (13) and expressing efrc(.) as Meijer's G–function, i.e., erfc $(x) = \frac{1}{\sqrt{\pi}} G_{1,2}^{2,0}\left(x \middle| \begin{matrix} 1 \\ 0, 1/2 \end{matrix}\right)$ [[11],Eq. (06.27.26.0006.01)], the extract-form BEP of FSO link is given by (14). Next, using [[12], Eq. (21)] and [[13], Eq. (9.31.1)] the closed-form of BEP can be expressed as (15).

IV. Heuristic Solution

Following notations are used in the description of the heuristic solution.

- $\mathbb{M}$ is the set of sites for installing FSO transceivers.
- $\mathbb{D} = \{(s, t, d_{st}) : s, t \in \mathbb{M}\}$ is the set of connection demands between sites. Each demand in $\mathbb{D}$ is represented by tuple (s, t, d_{st}), where $s \in \mathbb{M}$ and $t \in \mathbb{M}$ are the source and the destination of the demand and d_{st} is the demanded bandwidth.
- BER_{ij} is the BER of the link between two FSO transceivers if they are placed at site i and site j. It can also be denoted as BER_ℓ, where ℓ is the link. BER is calculated according to (15).
- $\mathbb{G}$ is the full graph made from all FSO sites in $\mathbb{M}$.
- $\mathbb{T}$ is the topology to be built.

The main idea of the heuristic algorithm is that the network is built gradually while seeking a connection path for each demand in the given traffic load. For that, we will browse the list of demands in $\mathbb{D}$ one after the other. For each demand, we will find a path through the FSO sites for it. In order to make sure that the end-to-end BER of each path does not exceed threshold δ, we try to route the demand over the path with the smallest end-to-end BER. The FSO links in the path will be included in the topology. The subsequent demands will be routed similarly but with the links already included in the topology will be prioritized to be used. In so doing, we can approach the objective of minimizing the total number of FSO links.

In order to find a path with the smallest end-to-end BER, we start from taking logarithm of the two sides of (1). Then we have

$$\log_{10}(1 - \mathrm{BER}_p) = \sum_{\ell \in p} \log_{10}(1 - \mathrm{BER}_\ell). \quad (16)$$

$$\mathrm{BER} = \frac{\alpha\beta\gamma_p^2}{A_0 h_l \Gamma(\alpha)\Gamma(\beta)} \int_0^\infty G_{1,3}^{3,0}\left(\frac{\alpha\beta}{A_0 h_l}h \middle| \begin{matrix} \gamma_p^2 \\ \gamma_p^2-1, \alpha-1, \beta-1 \end{matrix}\right) \frac{1}{2\sqrt{\pi}} G_{1,2}^{2,0}\left(\frac{P_t^2}{\sigma_1^2}h^2 \middle| \begin{matrix} 1 \\ 0, 1/2 \end{matrix}\right) dh. \tag{14}$$

$$\mathrm{BER} = \frac{2^{\alpha+\beta-3}\gamma_p^2\alpha\beta}{\sqrt{\pi^3}A_0 h_l \Gamma(\alpha)\Gamma(\beta)} G_{6,3}^{2,5}\left(\frac{16P_t^2A_0^2h_l^2}{\sigma_1^2\alpha^2\beta^2} \middle| \begin{matrix} -\frac{\gamma_p^2-2}{2}, \frac{-\alpha+1}{2}, \frac{-\alpha+2}{2}, \frac{-\beta+1}{2}, \frac{-\beta+2}{2}, 1 \\ 0, 1/2, -\gamma_p^2/2 \end{matrix}\right). \tag{15}$$

Since $1-\mathrm{BER}_p < 1$ then $\log_{10}(1-\mathrm{BER}_p) < 0$. In order make all the operands positive, we convert the equation to

$$(-\log_{10}(1-\mathrm{BER}_p)) = \sum_{\ell\in p}(-\log_{10}(1-\mathrm{BER}_\ell)). \tag{17}$$

We remark that the smaller BER_p is, the smaller $(-\log_{10}(1-\mathrm{BER}_p))$ is; and thus the smaller the right hand-side of (17) is. The right hand-side of (17) is in fact the sum of $(\log_{10}(1-\mathrm{BER}_\ell))$ of links of p. Consequently, if we assign the weight $(\log_{10}(1-\mathrm{BER}_\ell))$ to each edge of $\mathbb{G}$ then the path p with the smallest end-to-end BER between two sites is the shortest path in $\mathbb{G}$ between the two sites. From this idea, we propose the algorithm as following.

Initiation:

- $T = \emptyset$,
- Assign weight $-\log_{10}(1-\mathrm{BER}_{ij})$ to all edges of $\mathbb{G}$.

Then for each demand $(s, t, d_{st}) \in \mathbb{D}$, the algorithm goes through following steps:

1) Remove from $\mathbb{G}$ edges with insufficient bandwidth for the demand d_{st}.
2) Route the demand as the shortest path in $\mathbb{G}$. In so doing, the demand will take the route with the smallest end-to-end BER.
3) Check end-to-end BER constraint (2) for the path. If the path does not satisfy the constraint then search the path again in $\mathbb{G}$ where all edge weights are set back to the initial weight $-\log_{10}(1-\mathrm{BER}_{ij})$. If no path can be found, the problem is reported infeasible and the algorithm terminates.
4) Add edges of the path to $\mathbb{T}$ and subtract d_{st} from the available bandwidth of these links.
5) Assign new weights $-0.5\log_{10}(1-\mathrm{BER}_{ij})$ to these edges so that they will be prioritized in the next path finding.
6) Repeat the process until all demands are routed.

V. Numerical Results

The proposed algorithm have been implemented in Matlab. We also developed an exact solution for the design problem by using Integer Linear Programming (ILP). The ILP model takes the minimization of the number of FSO links of the topology as the primary objective and then the minimization of the total BER of links as the secondary objectives. Although the ILP model offers the optimal solution, this model needs to browse all feasible solutions to find the optimal one, so it not possible to achieve the result on a large set of FSO sites. Differently, the heuristic solution usually does not attain the optimal results, but it runs very fast in with a large set of FSO sites. In this paper in order to evaluate the quality of solutions given by the proposed heuristic algorithm, we compare its results again the optimal solutions given by ILP only in small network instances with few number of FSO sites. System parameters and constants used in our analysis are shown in Table I.

TABLE I. System Parameters and Constants.

Name	Symbol	Value
Bit rate	R_b	1 Gbps
Wavelength	λ	1550 nm
Refractive index structure coeff.	C_n^2	10^{-14} m$^{-2/3}$
Transmitted power	P_T	0 dBm
Noise variance	σ_n^2	10^{-19} A^2
Attenuation coefficient	a_l	0.1 km^{-1}
Receiver diameter	$2a$	20 cm
Beam radius at 1 km	ω_z	2.0 m
Beam divergence angle	θ	1 mrad
Jitter standard deviation	σ_s	10 cm
End-to-end BER threshold	δ	10^{-3}
Number of FSO sites	$\mathbb{M}$	$5 \sim 30$
Demanded bandwidth per connection	d_{st}	$100 \sim 300$ Mbps
Experimental space $S \times S \times 100m$	S	$2000 \sim 5000$ m
Density of FSO sites		$1.1 \sim 1.4$ sites /km^2

FSO sites are generated randomly in the experimental space of $S \times S \times 100$ m where S varies in range $2000 \sim 5000$ m. The number of sites $|\mathbb{M}|$ are chosen so that the density of FSO sites varies from 1.1 to 1.4 sites/km^2. With this density, the distances between FSO devices are around 1 km which is the practical distance of FSO links. The traffic demands in $\mathbb{D}$ are generated with sources and destinations taken randomly from $\mathbb{M}$ and the demanded bandwidth d_{st} varies between $100 \sim 300$ Mbps. The number of demands in $\mathbb{D}$ increases with the number of FSO sites.

The performance of the proposed algorithm will be evaluated regarding:

- The algorithm calculation time.
- The total number of links it uses for building the FSO networks because it reflects the network costs.
- The total BER of all links in the networks because it allows to see if the algorithm chooses good quality links.

First, Fig. 2 shows the calculation time of the proposed heuristic and ILP model versus the number of FSO sites. It is clear that the heuristic runs much faster. In large network instances, where the number of FSO sites $|\mathbb{M}| \geq 10$, we even cannot get the results from ILP model, while the heuristic takes around 100 seconds for resolving the topology for 30 sites within the space of 5000 m $\times$ 5000 m $\times$ 100 m.

Next, the numbers of links of the topologies designed by the heuristic algorithm and the ILP model are shown in Fig. 3. ILP model provides the optimal FSO network with the fewest links. Obviously, the heuristic cannot always give such

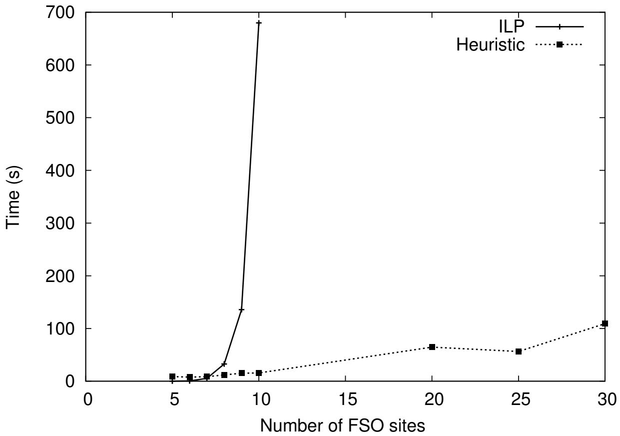

Fig. 2. Calculation time of the proposed heuristic and ILP model.

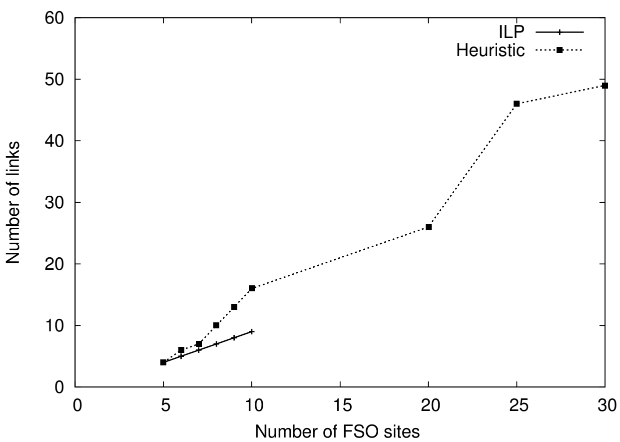

Fig. 3. Number of links of the topologies designed by the proposed heuristic and ILP model.

a good solution. The gap between the optimal topology and the heuristic designed topology increases with the number of FSO sites. It is reasonable since ILP has more chance to find good solution when the feasible solution space increases with the number of FSO sites.

Finally, Fig. 4 shows the average value BER of links in the topology designed by the proposed heuristic algorithm and by ILP model. Small BER is only secondary objectives of both heuristic algorithm and ILP model. In case of the heuristic, we can see that, in the worst cases, average BER of links is around 10^{-4}. This value is quite smaller than the BER tolerability of FSO transceiver, which is targeted at BER of 10^{-3} so that, the error-free communications can be guaranteed with the use of forward error correction (FEC).

VI. Conclusion

We have studied the problem of design and optimization of a FSO mesh network under the effects of atmospheric turbulence, misalignment fading, and noise. A heuristic algorithm was proposed to chooses the best sites to install the FSO devices and also the best topology for a given traffic load in a region. The objective of the algorithm is to minimize the

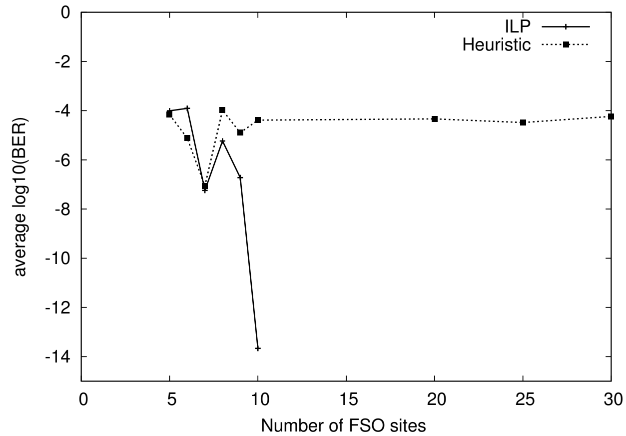

Fig. 4. Average value of $\log_{10}$ of BER of links in the topology designed by the proposed heuristic and ILP model.

network costs by using the least of FSO links as possible. In addition, the algorithm takes into account the influence of physical layer impairments by choosing links with low BER. The simulation results demonstrated the advantage of the proposed algorithm compared to the optimal solution by ILP model in terms of short calculation time. With the ability of running very fast, even with a large number of choices of FSO sites, it is practical to use the proposed heuristic algorithm in building quickly a restoration communication networks for replacing a regular one after a disaster. The performance of FSO links in the network designed by using the proposed algorithm also met the requirement of end-to-end BER, i.e., below the BER threshold of 10^{-3}.

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