A DENOISING METHOD BASED ON TOTAL VARIATION

Thanh Dang N.H.
Tula State University
92 Lenin Ave., Tula,Russian Federation
+7.9520170459
dnhthanh@hueic.edu.vn

Dvoenko Sergey D.
Tula State University
92 Lenin Ave., Tula, Russian Federation
+7.9105523290
dsd@tsu.tula.ru

Dinh Viet Sang
Hanoi University of Science and Technology
1 Dai Co Viet, Hanoi, Vietnam
+84.964131714
sangdv@soict.hust.edu.vn

ABSTRACT

Today large amounts of digital images are created by various modern devices such as digital cameras, X-Ray scanners, and so on. Noise reduces image quality and result of the processing. For example, biomedical images are a type of digital images. In these images, there is a combination of two types of noises: Gaussian noise and Poisson noise. In this paper, we propose a method to remove these noises. This method is based on the total variation of an image intensity (brightness) function. We combine two famous denoising models to remove this combination of noises.

CCS Concepts

•**Mathematics of computing** → *Continuous optimization;* •**Computing methodologies** → **Image processing;**

Keywords

Total variation; ROF model; Gaussian noise; Poisson noise; image processing; biomedical image; Euler-Lagrange equation

1. INTRODUCTION

In the modern fields of science, digital images are an important type of information. Digital images are created by many different devices such as digital cameras, X-Ray scanners, and so on. The implementing of digital devices in different conditions causes many different defects including noises.

Image denoising has attracted a lot of attention in recent years. In order to suppress noise efficiently, we need to know its type. There are many types of noises, for example, Gaussian noise (almost digital images), Poisson noise (X-Ray images), Speckle noise (ultra sonograms) and so on.

Until now, there are many methods to remove noise in different cases. One of the most famous and efficient methods is the total variation model [2–7, 13, 15, 17, 19, 22, 26, 28].

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The first person that suggested using total variation to solve denoising problem is Rudin [19]. He used the total variation as a universal tool to solve many problems of image processing. His denoising model was named as the ROF model [6, 19]. The ROF model is targeted to efficiently remove Gaussian noise.

The ROF model is used to remove not only Gaussian noise, but also other types of noise. However, in this case the result is not very good. For example, another type of noise that usually appears in X-Ray images is Poisson noise. The ROF model suppresses this noise not so efficiently. Therefore, Le T. [12] proposed another model, known as the modified ROF model to remove Poisson noise.

Both noises (Gaussian and Poisson) are popular, but their combination is important too [14]. This type of noise always appears in biomedical images, for example electronics microscopy images [10, 11].

In order to suppress the combination of noises, we can combine two different models: ROF and modified ROF. It is supposed the obtained model can remove the mixed noise better than ROF or modified ROF models separately, because we consider the proportion between Gaussian and Poisson noises in the mixed noise.

In experiments, we use real images and add noise into them. The quality of denoising is compared with some other denoising methods, such as: ROF and modified ROF models, Beltrami regularization method [29] as PDE-based (Partial Differential Equations) ones; Wiener filter [1] and median filter [23] as non-PDE-based ones.

In order to compare image quality after restoration, we use criteria like *PSNR* (Peak Signal-to-Noise Ratio), *MSE* (Mean Square Error), *SSIM* (Structure SIMilarity) [24, 25]. The *PSNR* is the most important criterion in this case, because it is always used to evaluate the quality of images and signals. The *MSE* criterion directly related to *PSNR*. These criteria are classical and quantitative methods to evaluate signal and image quality. The *SSIM* criterion is more exact with the actual feeling of human vision and it is a qualitative method to evaluate an image quality.

2. DENOISING MODEL FOR MIXED POISSON-GAUSSIAN NOISE

Let in $\mathbb{R}^2$ space a bounded domain $\Omega \in \mathbb{R}^2$ be given. Let us call functions $u(x, y) \in \mathbb{R}^2$ and $v(x, y) \in \mathbb{R}^2$, respectively, ideal (without noise) and observed images (noisy), where $(x, y) \in \Omega$. If the function u is smooth, then its total variation is defined by

$$V_T[u] = \int\limits_{\Omega} |\nabla u| dxdy,$$

where $\nabla u = (u_x, u_y)$ is a gradient (nabla operator), and $u_x = \partial u/\partial x$, $u_y = \partial u/\partial y$, $|\nabla u| = \sqrt{u_x^2 + u_y^2}$. In this paper, we only consider that function u always has limited its total variation $V_T[u] < \infty$.

According to [2, 5, 6, 19, 20], image smoothness is characterized by total variation of image intensity function. The total variation of noisy image is always greater than the total variation of the corresponding smooth image.

In order to solve the problem $V_T[u] \to min$, we need to use the following condition

$$\int\limits_{\Omega} (v-u)^2 dxdy = const.$$

Hence, we obtain the ROF model to remove Gaussian noise in image [19]:

$$u^* = \arg\min_u \left(\int\limits_{\Omega} |\nabla u| dxdy + \frac{\lambda}{2} \int\limits_{\Omega} (v-u)^2 dxdy \right),$$

where $\lambda > 0$ is Lagrange multiplier. This is a solution of the unconstrained optimization problem.

In order to remove Poisson noise, Le T. built another model based on ROF model [12]. That model is a result of the optimization problem $V_T[u] \to min$ with the following constraint

$$\int\limits_{\Omega} \ln(p(v|u)) dxdy = \int\limits_{\Omega} (u - v\ln(u)) dxdy = const.$$

This model resulted in the following unconstrained optimization problem

$$u^* = \arg\min_u \left(\int\limits_{\Omega} |\nabla u| dxdy + \beta \int\limits_{\Omega} (u - v\ln(u)) dxdy \right),$$

where $\beta > 0$ is a coefficient of regularization. This model is known as the modified ROF model to remove Poisson noise.

In order to build a model that can effectively remove the mixed Poisson-Gaussian noise, we also solve the same optimization problem $V_T[u] \to min$, but with a different constraint which is described as follows.

This constraint is very similar to constraints above. We assume that the noise variance is unchangeable with respect to the given image (Poisson noise is not changed and Gaussian noise is only depends on noise variance):

$$\int\limits_{\Omega} \ln(p(v|u)) dxdy = const, \tag{1}$$

where $p(v|u)$ is a conditional probability of observation of real image v with given ideal image u.

The probability density function of Gaussian noise is

$$p_1(v|u) = \exp\left(-\frac{(v-u)^2}{2\sigma^2} \right) / (\sigma\sqrt{2\pi}),$$

and the probability density function of Poisson noise is

$$p_2(v|u) = \frac{\exp(-u)u^v}{v!}.$$

We have to notice that values of intensity function of images u and v are integer (for example, for 8-bits grayscale image the range of intensity is from 0 to 255).

In order to combine Gaussian and Poisson noises, we consider the following linear combination

$$\ln(p(v|u)) = \lambda_1 \ln(p_1(v|u)) + \lambda_2 \ln(p_2(v|u)),$$
$$\lambda_1 > 0,\ \lambda_2 > 0,\ \lambda_1 + \lambda_2 = 1.$$

According to (1), we obtain the denoising problem as a constrained optimization problem

$$\begin{cases} u^* = \arg\min\limits_u \int\limits_{\Omega} |\nabla u| dxdy, \\ \int\limits_{\Omega} \left(\frac{\lambda_1}{2\sigma^2}(v-u)^2 + \lambda_2(u - v\ln(u)) \right) dxdy = \kappa, \end{cases}$$

where κ is a constant value.

We transform this problem into unconstrained optimization problem by using Lagrange functional

$$L(u,\tau) = \int\limits_{\Omega} |\nabla u| dxdy + \tau \left(\frac{\lambda_1}{2\sigma^2} \int\limits_{\Omega} (v-u)^2 dxdy + \right.$$
$$\left. \lambda_2 \int\limits_{\Omega} (u - v\ln(u)) dxdy - \kappa \right)$$

to find the solution as

$$(u^*, \tau^*) = \arg\min_{u,\,\tau} L(u,\tau), \tag{2}$$

where τ is Lagrange multiplier.

In the proposed model if $\lambda_1 = 0$ and $\beta = \lambda_2\tau$, we obtain the modified ROF model to remove Poisson noise. And if $\lambda_2 = 0$ and $\lambda = \lambda_1/(2\sigma^2)$, we obtain the ROF model to remove Gaussian noise. If $\lambda_1 > 0$, $\lambda_2 > 0$, we obtain the model to remove mixed Poisson-Gaussian noise.

3. DISCRETE DENOISING MODEL

The problem (2) can be solved by using Lagrange multipliers method [8, 18, 27]. In this paper, we use Euler-Lagrange equation in [27].

Let a function $f(x,y)$ be defined in a limited domain $\Omega \in \mathbb{R}^2$ and be second-order continuously differentiated by x and y, where $(x,y) \in \Omega$. Let $F(x,y,f,f_x,f_y)$ be a convex functional, where $f_x = \partial f/\partial x$, $f_y = \partial f/\partial y$. Then the solution of the following optimization problem

$$\int\limits_{\Omega} F(x,y,f,f_x,f_y) dxdy \to \min$$

satisfies the following Euler-Lagrange equation

$$F_f(x,y,f,f_x,f_y) - \frac{\partial}{\partial x} F_{f_x}(x,y,f,f_x,f_y) -$$
$$\frac{\partial}{\partial y} F_{f_y}(x,y,f,f_x,f_y) = 0,$$

where $F_f = \partial F/\partial f$, $F_{f_x} = \partial F/\partial f_x$, $F_{f_y} = \partial F/\partial f_y$.

We use the above result to solve the obtained model. Then the solution of problem (2) satisfies the following Euler-

Lagrange equation

$$-\frac{\lambda_1}{\sigma^2}(v-u)+\lambda_2(1-\frac{v}{u})-$$
$$\mu\frac{\partial}{\partial x}\left(\frac{u_x}{\sqrt{u_x^2+u_y^2}}\right)-\mu\frac{\partial}{\partial y}\left(\frac{u_y}{\sqrt{u_x^2+u_y^2}}\right)=0, \qquad (3)$$

where $\mu = 1/\tau$.

We rewrite equation (3) in the form

$$\frac{\lambda_1}{\sigma^2}(v-u)-\lambda_2(1-\frac{v}{u})+$$
$$\mu\frac{u_{xx}u_y^2-2u_xu_yu_{xy}+u_x^2u_{yy}}{(u_x^2+u_y^2)^{3/2}}=0, \qquad (4)$$

where

$$u_{xx}=\frac{\partial^2 u}{\partial x^2}, u_{yy}=\frac{\partial^2 u}{\partial y^2}, u_{xy}=\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y}\right)=\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}\right)=u_{yx}.$$

In order to obtain discrete form of the model (4), we add an artificial time parameter and consider the function $u = u(x, y, t)$. The equation (4) corresponds to the following diffusion equation

$$u_t=\frac{\partial u}{\partial t}=\frac{\lambda_1}{\sigma^2}(v-u)-\lambda_2(1-\frac{v}{u})+$$
$$\mu\frac{u_{xx}u_y^2-2u_xu_yu_{xy}+u_x^2u_{yy}}{(u_x^2+u_y^2)^{3/2}}. \qquad (5)$$

Let the size of the image be $N_1 \times N_2$. Then the discrete form of the equation (5) is

$$u_{ij}^{k+1}=u_{ij}^k+\xi\left(\frac{\lambda_1}{\sigma^2}\,(v_{ij}-u_{ij}^k)-\right.$$
$$\left.\lambda_2(1-\frac{v_{ij}}{u_{ij}^k})+\mu\varphi_{ij}^k\right), \qquad (6)$$

$$\varphi_{ij}^k=\frac{\nabla_{xx}(u_{ij}^k)(\nabla_y(u_{ij}^k))^2}{((\nabla_x(u_{ij}^k))^2+(\nabla_y(u_{ij}^k))^2)^{3/2}}+$$
$$\frac{-2\nabla_x(u_{ij}^k)\nabla_y(u_{ij}^k)\nabla_{xy}(u_{ij}^k)+(\nabla_x(u_{ij}^k))^2\nabla_{yy}(u_{ij}^k)}{((\nabla_x(u_{ij}^k))^2+(\nabla_y(u_{ij}^k))^2)^{3/2}},$$
$$\nabla_x(u_{ij}^k)=\frac{u_{i+1,j}^k-u_{i-1,j}^k}{2\Delta x},$$
$$\nabla_y(u_{ij}^k)=\frac{u_{i,j+1}^k-u_{i,j-1}^k}{2\Delta y}, \nabla_{xx}(u_{ij}^k)=\frac{u_{i+1,j}^k-2u_{ij}^k+u_{i-1,j}^k}{(\Delta x)^2},$$
$$\nabla_{yy}(u_{ij}^k)=\frac{u_{i,j+1}^k-2u_{ij}^k+u_{i,j-1}^k}{(\Delta y)^2},$$
$$\nabla_{xy}(u_{ij}^k)=\frac{u_{i+1,j+1}^k-u_{i+1,j-1}^k-u_{i-1,j+1}^k+u_{i-1,j-1}^k}{4\Delta x\Delta y},$$
$$u_{0j}^k=u_{1j}^k;\;\; u_{N_1+1,j}^k=u_{N_1,j}^k;\;\; u_{i0}^k=u_{i1}^k;\;\; u_{i,N_2+1}^k=u_{i,N_2}^k;$$
$$i=1,...,N_1;\; j=1,...,N_2;$$
$$k=0,1,...,K;\;\; \Delta x=\Delta y=1;\;\; 0<\xi<1.$$

K is enough great number, $K = 500$.

4. OPTIMAL PARAMETERS

The procedure (6) can remove noise in image, if values of parameters λ_1, λ_2, μ, σ are given. In practice, these parameters are usually unknown, and we need to define them. Then we have to change λ_1, λ_2, μ in procedure (6) into λ_1^k, λ_2^k, μ^k on every k-step. In the new procedure, these parameters are calculated on every iteration step.

4.1 Optimal Parameters λ_1 and λ_2

Let (u, τ) be a solution of problem (2). Then we obtain the following condition

$$\frac{\partial L(u,\tau)}{\partial u}=0.$$

This condition gives us optimal parameters λ_1, λ_2:

$$\lambda_1=\frac{\int\limits_\Omega(1-\frac{v}{u})dxdy}{\frac{1}{\sigma^2}\int\limits_\Omega(v-u)dxdy+\int\limits_\Omega(1-\frac{v}{u})dxdy}, \lambda_2=1-\lambda_1.$$

Its discrete form is

$$\lambda_1^k=\frac{\sum\limits_{i=1}^{N_1}\sum\limits_{j=1}^{N_2}(1-\frac{v_{ij}}{u_{ij}^k})}{\sum\limits_{i=1}^{N_1}\sum\limits_{j=1}^{N_2}(\frac{v_{ij}-u_{ij}^k}{\sigma^2}+1-\frac{v_{ij}}{u_{ij}^k})}, \lambda_2^k=1-\lambda_1^k,$$

where $k = 0, 1, ..., K$.

4.2 Optimal Parameter μ

In order to find optimal parameter μ, we multiply (3) by $(v - u)$ and integrate by parts over domain Ω. Finally, we obtain the formula to find the optimal parameter μ:

$$\mu=\frac{\int\limits_\Omega(-\frac{\lambda_1}{\sigma^2}(v-u)^2-\lambda_2\frac{(v-u)^2}{u})dxdy}{\int\limits_\Omega(\sqrt{u_x^2+u_y^2}-\frac{u_xv_x+u_yv_y}{\sqrt{u_x^2+u_y^2}})dxdy}.$$

Its discrete form is

$$\mu^k=\frac{\sum\limits_{i=1}^{N_1}\sum\limits_{j=1}^{N_2}(-\frac{\lambda_1^k}{\sigma^2}(v_{ij}-u_{ij}^k)^2-\lambda_2^k\frac{(v_{ij}-u_{ij}^k)^2}{u_{ij}^k})}{\sum\limits_{i=1}^{N_1}\sum\limits_{j=1}^{N_2}\eta_{ij}^k},$$

where

$$\eta_{ij}^k=\sqrt{(\nabla_x(u_{ij}^k))^2+(\nabla_y(u_{ij}^k))^2}-$$
$$\frac{\nabla_x(u_{ij}^k)\nabla_x(v_{ij})+\nabla_y(u_{ij}^k)\nabla_y(v_{ij})}{\sqrt{(\nabla_x(u_{ij}^k))^2+(\nabla_y(u_{ij}^k))^2}},$$
$$\nabla_x(u_{ij}^k)=\frac{u_{i+1,j}^k-u_{i-1,j}^k}{2\Delta x}, \nabla_y(u_{ij}^k)=\frac{u_{i,j+1}^k-u_{i,j-1}^k}{2\Delta y},$$
$$\nabla_x(v_{ij}^k)=\frac{v_{i+1,j}^k-v_{i-1,j}^k}{2\Delta x}v, \nabla_y(v_{ij}^k)=\frac{v_{i,j+1}^k-v_{i,j-1}^k}{2\Delta y},$$
$$u_{0j}^k=u_{1j}^k;\;\; u_{N_1+1,j}^k=u_{N_1,j}^k;\;\; u_{i0}^k=u_{i1}^k;\;\; u_{i,N_2+1}^k=u_{i,N_2}^k;$$
$$v_{0j}=v_{1j};\;\; v_{N_1+1,j}=v_{N_1j};\;\; v_{i0}=v_{i1};\;\; v_{i,N_2+1}=v_{i,N_2};$$
$$i=1,...,N_1;\;\; j=1,...,N_2;$$
$$k=0,1,...,K;\;\; \Delta x=\Delta y=1.$$

4.3 Optimal Parameter σ

In order to calculate parameter σ, we use the method of Immerker [9]:

$$\sigma = \frac{\sqrt{\pi/2}}{6(N_1-2)(N_2-2)} \sum_{i=1}^{N_1}\sum_{j=1}^{N_2} |u_{ij} * \Lambda|, \text{where}$$

$$\Lambda = \begin{pmatrix} 1 & -2 & 1 \\ -2 & 4 & -2 \\ 1 & -2 & 1 \end{pmatrix} \text{is the mask of an image.}$$

Operator * is a convolution, where

$$u_{ij} * \Lambda = u_{i-1,j-1}\Lambda_{33} + u_{i,j-1}\Lambda_{32} + u_{i+1,j-1}\Lambda_{31} + u_{i-1,j}\Lambda_{23} + u_{ij}\Lambda_{22} + u_{i+1,j}\Lambda_{21} + u_{i-1,j+1}\Lambda_{13} + u_{i,j+1}\Lambda_{12} + u_{i+1,j+1}\Lambda_{11}$$

$$i = 1, ..., N_1;\ j = 1, ..., N_2;$$

$$u_{ij} = 0, \text{if } i = 0, \text{or } j = 0, \text{or } i = N_1 + 1, \text{or } N_2 + 1.$$

Parameter σ is calculated at the first step of iteration.

4.4 Initial Solution

Of course, in the iteration procedure (6), the result of image restoration depends on the values of initial parameters $\lambda_1^0, \lambda_2^0, \mu^0$.

If parameters $\lambda_1^0, \lambda_2^0, \mu^0$ are given first, then its unsuitable values define not so good solution u_{ij} and later, not so good evaluation of a probability distribution parameters.

If parameters $\lambda_1^0, \lambda_2^0, \mu^0$ are randomized, the result will be unacceptable too, because of the additional noise added in the image.

Of course, initial values of $\lambda_1^0, \lambda_2^0, \mu^0$ need to be closed to required values. Hence, we evaluate parameters $\lambda_1^0, \lambda_2^0, \mu^0$ as the average values of neighbor pixels of image, for example, by method of Immerker.

5. IMAGE QUALITY EVALUATION

In order to evaluate image quality after denoising, we use criteria *PSNR* (Peak Signal-to-Noise Ratio), *MSE* (mean squared error) and *SSIM* (Structure SIMilarity) [24, 25]:

$$Q_{PSNR} = 10 \lg \left(N_1 N_2 L^2 / \sum_{i=1}^{N_1}\sum_{j=1}^{N_2} (v_{ij} - u_{ij})^2 \right),$$

$$Q_{MSE} = \frac{1}{N_1 N_2} \sum_{i=1}^{N_1}\sum_{j=1}^{N_2} (v_{ij} - u_{ij})^2,$$

$$Q_{SSIM} = \frac{(2\bar{u}\,\bar{v} + C_1)(2\sigma_{uv} + C_2)}{(\bar{u}^2 + \bar{v}^2 + C_1)(\sigma_u^2 + \sigma_v^2 + C_2)},$$

where

$$\bar{u} = \frac{1}{N_1 N_2} \sum_{i=1}^{N_1}\sum_{j=1}^{N_2} u_{ij}, \bar{v} = \frac{1}{N_1 N_2} \sum_{i=1}^{N_1}\sum_{j=1}^{N_2} v_{ij},$$

$$\sigma_u^2 = \frac{1}{N_1 N_2 - 1} \sum_{i=1}^{N_1}\sum_{j=1}^{N_2} (u_{ij} - \bar{u})^2,$$

$$\sigma_v^2 = \frac{1}{N_1 N_2 - 1} \sum_{i=1}^{N_1}\sum_{j=1}^{N_2} (v_{ij} - \bar{v})^2,$$

$$\sigma_{uv} = \frac{1}{N_1 N_2 - 1} \sum_{i=1}^{N_1}\sum_{j=1}^{N_2} (u_{ij} - \bar{u})(v_{ij} - \bar{v}),$$

$$C_1 = (K_1 L)^2,\ C_2 = (K_2 L)^2;\ K_1 \ll 1;\ K_2 \ll 1.$$

For example, $K_1 = K_2 = 10^{-6}$, L is an image intensity with $L = 2^8 - 1 = 255$ for 8-bits grayscale image.

The greater the value of Q_{PSNR}, the better image quality. If the value of Q_{PSNR} belongs to interval from 20 to 25, then the image quality is acceptable, for example, for wireless transmission [21].

The value of Q_{MSE} is used to evaluate the difference between two images. Q_{MSE} is a mean squared error. The lower the value of Q_{MSE}, the better the result of restoration. The value of Q_{MSE} directly related to the value of Q_{PSNR}.

The value of Q_{SSIM} is used to evaluate an image quality by comparing the similarity of two images. This value is between -1 and 1. The greater the value of Q_{SSIM}, the better the image quality.

6. EXPERIMENTS

For first example, we use a human skull image [16] with the size of 300×300 pixels (Fig. 1a). Other images (Fig. 1b - 1h) show the zoomed in part of it. Here we test the proposed model for a real image with artificial noise. It is supposed, such a situation is really closed to be similar to the case of noised real images.

In this case, we generate Gaussian noise, Poisson noise and mixed noise to add into real image. The case of coefficients $\lambda_1 = 1, \lambda_2 = 0$ is used for Gaussian noise. The case of coefficients $\lambda_1 = 0, \lambda_2 = 1$ is used for Poisson noise only. We specify non-zeros coefficients of linear combination for the case of mixed noise.

Table 1: Image quality comparison of denoising methods for mixed noise case

Method	Q_{PSNR}	Q_{SSIM}	Q_{MSE}
Noisy image	21.4168	0.4246	427.9526
ROF	26.5106	0.8465	145.2183
Modified ROF	26.3153	0.6885	151.8976
Median filter	25.6477	0.7871	177.1364
Wiener filter	24.2657	0.6596	243.5077
Beltrami method	26.8549	0.6678	134.1484
Proposed method with $\lambda_1 = 0.8$, $\lambda_2 = 0.2$, $\mu = 0.0857$, $\sigma = 40.2412$	**27.4315**	0.8198	**117.4713**
Proposed method with automatically defined parameters $\lambda_1 = 0.8095$, $\lambda_2 = 0.1905$, $\mu = 0.0970$, $\sigma = 38.2310$	27.2567	**0.8383**	122.2941

In order to obtain the noisy image, we add Gaussian noise (Fig. 1c), and Poisson noise (Fig. 1e). Fig. 1g shows the image with mixed noise by using parameters $\lambda_1 = 0.8, \lambda_2 = 0.2$.

Parameters of linear combination λ_1 and λ_2 are defined as follows. We consider Poisson noise with probability density $p_2(v|u)$ and variation $\sigma_2 = \sqrt{u_{ij}}$, corresponding with u_{ij} at every pixels (i,j), $i = 1, ..., N_1$; $j = 1, ..., N_2$. Values of the intensity function $v^{(2)}$ need to be between 0 and 255.

If this value is out of the interval from 0 to 255, then it needs to be reset to $v_{ij}^{(2)} = u_{ij}$. For this real image, the number of pixels, which are out of this interval, is 5 (0.0056%). Variance of Poisson noise is defined as average value $\overline{\sigma}_2 = 10.0603$.

Next, we specify variance of Gaussian noise to be four times greater, than variance of Poisson noise. It means $\sigma_1 = 40.2412$. We denote the intensity function of this image as $v^{(1)}$. As we explain above, values of the intensity function $v^{(1)}$ also need to be between 0 to 255. In this case, there are 5780 pixels out of this interval (6.42%).

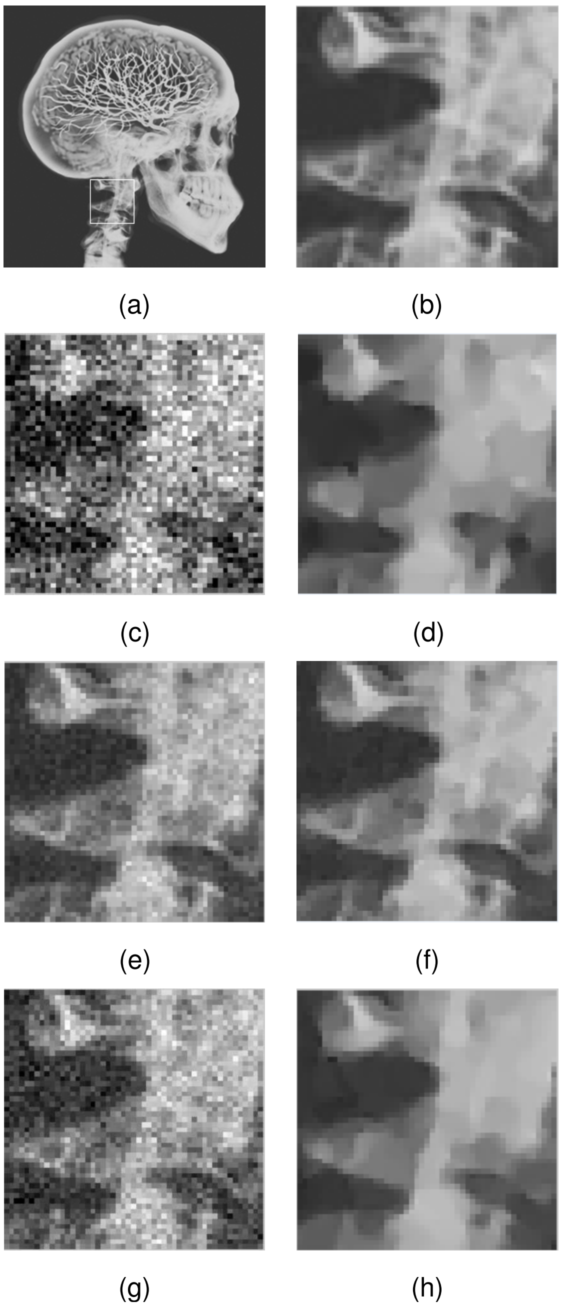

Figure 1: Noise removal in real image: a) original image, b) zoomed in part of image, c) with Gaussian noise, d) after Gaussian noise removal, e) with Poisson noise, f) after Poisson noise removal, g) with mixed noise, h) after mixed noise removal

Table 2: Image quality comparison of denoising methods for Gaussian noise case

Method	Q_{PSNR}	Q_{SSIM}	Q_{MSE}
Noisy image	16.5386	0.2516	1442.900
ROF	25.0181	0.7194	204.7700
Modified ROF	21.2356	0.4536	489.2402
Median filter	23.1412	0.6314	315.4741
Wiener filter	22.5138	0.5059	364.5051
Beltrami method	20.4575	0.3745	585.2284
Proposed method with $\lambda_1 = 1$, $\lambda_2 = 0$, $\mu = 0.0978$, $\sigma = 40.2412$	**25.0200**	**0.7735**	**204.6811**
Proposed method with automatically defined parameters $\lambda_1 = 0.9738$, $\lambda_2 = 0.0262$, $\mu = 0.0954$, $\sigma = 38.9036$	24.9681	0.7389	207.1441

Table 3: Image quality comparison of denoising methods for Poisson noise case

Method	Q_{PSNR}	Q_{SSIM}	Q_{MSE}
Noisy image	27.7349	0.6902	109.5442
ROF	32.0548	0.9355	40.5131
Modified ROF	**33.6101**	**0.9501**	**35.5310**
Median filter	27.7349	0.6902	109.5442
Wiener filter	25.0410	0.8113	203.6962
Beltrami method	31.6356	0.9425	44.6195
Proposed method with $\lambda_1 = 0$, $\lambda_2 = 1$, $\mu = 0.0853$, $\sigma = 0.0001$	33.5213	0.9452	36.2235
Proposed method with automatically defined parameters $\lambda_1 = 0.0045$, $\lambda_2 = 0.9955$, $\mu = 0.0797$, $\sigma = 2.7797$	32.6244	0.9362	45.3455

The final noisy image is created by two noisy images: Gaussian noisy image and Poisson noisy image with proportion 0.5 for $v^{(1)}$ and 0.5 for $v^{(2)}$. This means $v = 0.5v^{(1)} + 0.5v^{(2)}$. Finally, we have the proportion

$$\lambda_1/\lambda_2 = \frac{40.2412 \times 0.5}{10.0603 \times 0.5} = 4/1.$$

Table 4: Dependency of mixed noise removal on initial solution

Parameter	(a)	(b)	(c)	(d)
λ_1	0.8095	0.8114	0.9256	0.8069
λ_2	0.1905	0.1886	0.0744	0.1931
μ	0.0970	0.0985	0.1026	0.0965
σ	38.2310			
Q_{PSNR}	27.2567	27.1327	26.4279	**27.2571**
Q_{MSE}	122.2941	125.8371	148.0081	**121.632**
Q_{SSIM}	0.8383	0.8381	**0.8497**	0.8384

Hence, we obtain values of coefficients of linear combination $\lambda_1 = 4/5 = 0.8$, and $\lambda_2 = 1/5 = 0.2$.

For noisy image, values of Q_{PSNR}, Q_{MSE}, and Q_{SSIM} are, respectively, 21.4168, 427.9526, and 0.4246.

Tables 1-3 show results of denoising for the following two cases: given parameters and automatically calculated parameters.

It should be noticed, that for this image, the value of Q_{PSNR} in the case of given parameters is better than the value of Q_{PSNR} in the case of automatically calculated parameters, but for the value of Q_{SSIM} such a situation can be inverted.

In order to find the initial solution, we use the convolution operator. Table 4 shows dependency of processing results on different initial solutions, where:

(a) initial parameters are given as $\lambda_1^0 = 0, \lambda_2^0 = 1, \mu = 1$;

(b) initial parameters are given as $\lambda_1^0 = \lambda_2^0 = 0.5, \mu = 1$;

(c) initial solution u^0 is given as randomized matrix;

(d) initial solution u^0 given as average value of neighbor pixels by using convolution operator with

$$A = \frac{1}{9}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}.$$

Table 4 shows that the best result of mixed noise removal is the case (d) based on quality criteria *PSNR* and *MSE*.

Next, we would like to show that the proposed method could achieve best results based on all specified criteria for the case of minimal variability of an image itself.

For second example, we use the artificial image with the size of 300×300 pixels (Fig. 2). It consists of six bars with the size of 50×300 pixels for every bar with grey levels as 110, 130, and 160 respectively twice (Fig. 2a). Hence, the number 2×1500 of pixels for every grey level is the same. Fig. 2b shows zoomed in fragment.

We create again one noisy image by adding Gaussian noise (Fig. 2c) and another noisy image by adding Poisson noise (Fig. 2d). Poisson noise intensity is unchangeable. Therefore, we can control variance of Gaussian noise only.

Variance of Poisson noise is calculated as average value $\bar{\sigma}_2 = (\sqrt{110} + \sqrt{130} + \sqrt{160}) = 11.5130$, because this image has three intensity grey levels with the same number of pixels. As before, the intensity values of $v^{(2)}$ need to be between 0 and 255. If it is not so, they need to be reset to $v_{ij}^{(2)} = u_{ij}$.

In this case, there are not pixels out of this interval.

Next, we again specify variance of Gaussian noise to be four times greater $\sigma_1 = 46.052$, than variance of Poisson noise. As in the case above, intensity value of $v^{(1)}$ also needs to be between 0 and 255. In this case, there are 1063 pixels out of this interval, respectively 1.2% of all image pixels.

The final noisy image is created (Fig. 2e) by combining first and second noisy images with proportion 0.6 for $v^{(1)}$ and 0.4 for $v^{(2)}$, with $v = 0.6v^{(1)} + 0.4v^{(2)}$. Hence:

$$\lambda_1/\lambda_2 = \frac{46.052 \times 0.6}{11.513 \times 0.4} = \frac{6}{1}.$$

As a result: $\lambda_1 = 6/7 = 0.8571$,$\lambda_2 = 1/7 = 0.1429$.

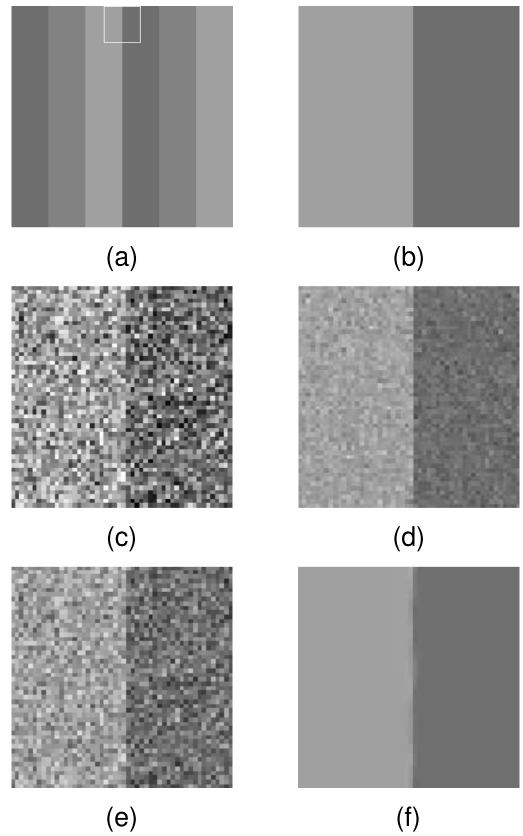

Figure 2: The noise initialization and denoising: a) original image, b) zoomed in fragment of image, c) with Gaussian noise, d) with Poisson noise, e) with mixed noise, f) after denoising.

Values of Q_{MSE}, Q_{PSNR} and Q_{SSIM} of the final noisy image are respectively 718.8782, 19.5643, and 0.1036. Fig. 2f shows the result of denoising. Table 5 shows results of denoising given by criteria of image quality. For automatically calculated parameters, we obtained values of $\lambda_1 = 0.8102$, $\lambda_2 = 0.1898$, $\mu = 0.3846, \sigma = 45.4523$. These values are closed enough to exact ones. Moreover, all criteria are completely coordinated for the best result and for next one.

7. CONCLUSIONS

In this paper, we proposed a novel method that can effectively remove the mixed Poisson-Gaussian noise. Furthermore, our proposed method can be also used to remove Gaussian or Poisson noises separately. This method is based on variational approach.

Although we combine ROF and modified ROF models to remove mixed noise, the denoising result of our model some-

Table 5: Image quality comparison of denoising methods for the artificial image with artificial noise

Method	Q_{PSNR}	Q_{SSIM}	Q_{MSE}
Noisy image	19.5643	0.1036	718.8782
ROF	35.1284	0.9130	19.9635
Modified ROF	33.6751	0.8834	33.4512
Beltrami method	39.5761	0.9753	11.6734
Median filter	31.4844	0.7797	46.1996
Wiener filter	30.1502	0.6018	62.8146
Proposed method with $\lambda_1 = 0.8571$, $\lambda_2 = 0.1429$, $\mu = 0.4738$, $\sigma = 46.0520$	**42.8237**	**0.9902**	**3.3940**
Proposed method with automatically defined parameters $\lambda_1 = 0.8102$, $\lambda_2 = 0.1898$, $\mu == 0.3846$, $\sigma = 45.4523$	42.7795	0.9900	3.4287

times is better than denoising result of ROF model itself for Gaussian noise, because our method to estimate the noise variance gives higher evaluation than the method that Rudin have used before. However, this difference is not too much and depends on some characteristics of images.

The denoising result of our method depends mainly on values of coefficients of linear combination λ_1 and λ_2. These values can be set manually or can be defined automatically. When processing real images, we can use the proposed method with automatically defined parameters.

The time in all tests is always less than 5 seconds. This value depends on the number of iterations (or the given accuracy). For our tests, we use the number of iterations $K = 500$. We can increase the speed of calculations by using the split method of Bregman [7]. This is very useful method for our denoising model.

As it is shown in the paper, the proposed method is promising to process real images. In future investigations it is supposed to test it on image data bases in public access and on images with natural unknown noise.

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