



Design and Analysis  
of Algorithms I

# Master Method Examples

# The Master Method

If  $T(n) \leq aT\left(\frac{n}{b}\right) + O(n^d)$

then

$$T(n) = \begin{cases} O(n^d \log n) & \text{if } a = b^d \quad (\text{Case 1}) \\ O(n^d) & \text{if } a < b^d \quad (\text{Case 2}) \\ O(n^{\log_b a}) & \text{if } a > b^d \quad (\text{Case 3}) \end{cases}$$

# Example #1

## Merge Sort

$$\left. \begin{array}{l} a = 2 \\ b = 2 \\ d = 1 \end{array} \right\} b^d = a \Rightarrow \textit{Case 1}$$

$$T(n) = O(n^d \log n) = O(n \log n)$$

Where are the respective values of  $a, b, d$  for a binary search of a sorted array, and which case of the Master Method does this correspond to?

→ ☒ 1, 2, 0 [Case 1]

☐ 1, 2, 1 [Case 2]

☐ 2, 2, 0 [Case 3]

☐ 2, 2, 1 [Case 1]

$$a = b^d \Rightarrow T(n) = O(n^d \log n) = O(n \log n)$$

# Example #3

## Integer Multiplication Algorithm # 1

$$\left. \begin{array}{l} a = 4 \\ b = 2 \\ d = 1 \end{array} \right\} b^d = 2 < a \text{ (Case 3)}$$

$$\Rightarrow T(n) = O(n^{\log_b a}) = O(n^{\log_2 4})$$
$$= O(n^2)$$


Same as grade-school  
algorithm

Where are the respective values of  $a, b, d$  for Gauss's recursive integer multiplication algorithm, and which case of the Master Method does this correspond to?

☐ 2, 2, 1 [Case 1]

☐ 3, 2, 1 [Case 1]

☐ 3, 2, 1 [Case 2]

 ☒ 3, 2, 1 [Case 3]

Better than  
the grade-  
school  
algorithm!!!

$$a = 3, \quad b^d = 2 \quad a > b^d \quad (\text{Case 3})$$
$$\Rightarrow T(n) = O(n^{\log_2 3}) = O(n^{1.59})$$

# Example #5

## Strassen's Matrix Multiplication Algorithm

$$a = 7$$

$$b = 2$$

$$d = 2$$

$$\left. \begin{array}{l} b = 2 \\ d = 2 \end{array} \right\} b^d = 4 < a \quad (\text{Case 3})$$

$$\Rightarrow T(n) = O(n^{\log_2 7}) = O(n^{2.81})$$

$\Rightarrow$  beats the naïve iterative algorithm !

# Example #6

## Fictitious Recurrence

$$T(n) \leq 2T(n/2) + O(n^2)$$

$$\Rightarrow a = 2$$

$$\Rightarrow b = 2$$

$$\Rightarrow d = 2$$

$$\left. \begin{array}{l} \Rightarrow a = 2 \\ \Rightarrow b = 2 \\ \Rightarrow d = 2 \end{array} \right\} b^d = 4 > a \quad (Case\ 2)$$

$$\Rightarrow T(n) = O(n^2)$$