Chapter 4

The Analytical Model

This chapter proposes an analytic model to investigate the Multiple-Event-based Charging Reservation (MECR) procedure at the OCS. Figure 4.1 illustrates the timing diagram for the PCEF credit maintenance cycle, which consisting of the MECR procedure period and the idle period. The cycle starts at t_0, the moment when the previous MECR period ends, and completes at $t_1 + T_{\texttt{MECR}}$, the moment the current MECR period ends. In this figure, the MTC records arrive at t_1, t_2, $\ldots$, t_i and t_{i+1} respectively. Let τ_i denote the inter-arrival time between the i`th` and the $i+1$`st` MTC records. The first arrival of the MTC record occurs at t_1. We denote the interval (t_0, t_1) as the idle period, where the OCS reserves no credit unit for MTC services at this period. Let $\tau_{\texttt{idle}} = t_1 - t_0$. The MECR procedure starts the Reserve Units operation at t_1 (i.e., PCEF sends the CCR(INITIAL-REQUEST) message to OCS) and executes the Debit Units operation at $t_1 + T_{\texttt{MECR}}$ (i.e., PCEF sends the CCR(TERMINATE-REQUEST) message to OCS). We have $T_P = \tau_{\texttt{idle}} + T_{\texttt{MECR}}$. $T_{\texttt{MECR}}$ denotes the length of the MECR procedure session time. T_P denotes the PCEF credit maintenance cycle time.

Upon receipt of an CCR(INITIAL-REQUEST) message (Step 2 in Figure 3.3), the OCS needs to determine how many credit should be reserved. Then, the OCS grants θ reserved credit units to PCEF (Steps 3 and 4 in Figure 3.3). It is essential to select appropriate θ value to increase the efficiency of the MECR procedure. We consider three output measures:

- $E[N]$: The expected number of MTC records in a CDR which is generated by

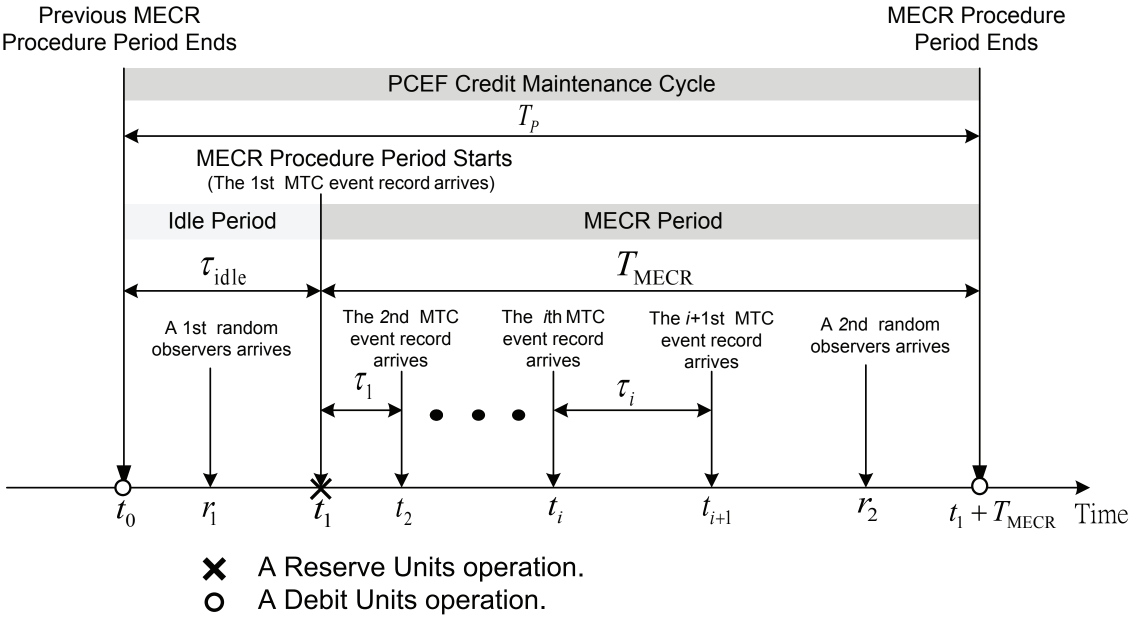


Figure 4.1: Timing diagram for a PCEF Credit Maintenance Cycle.

the EBCF.

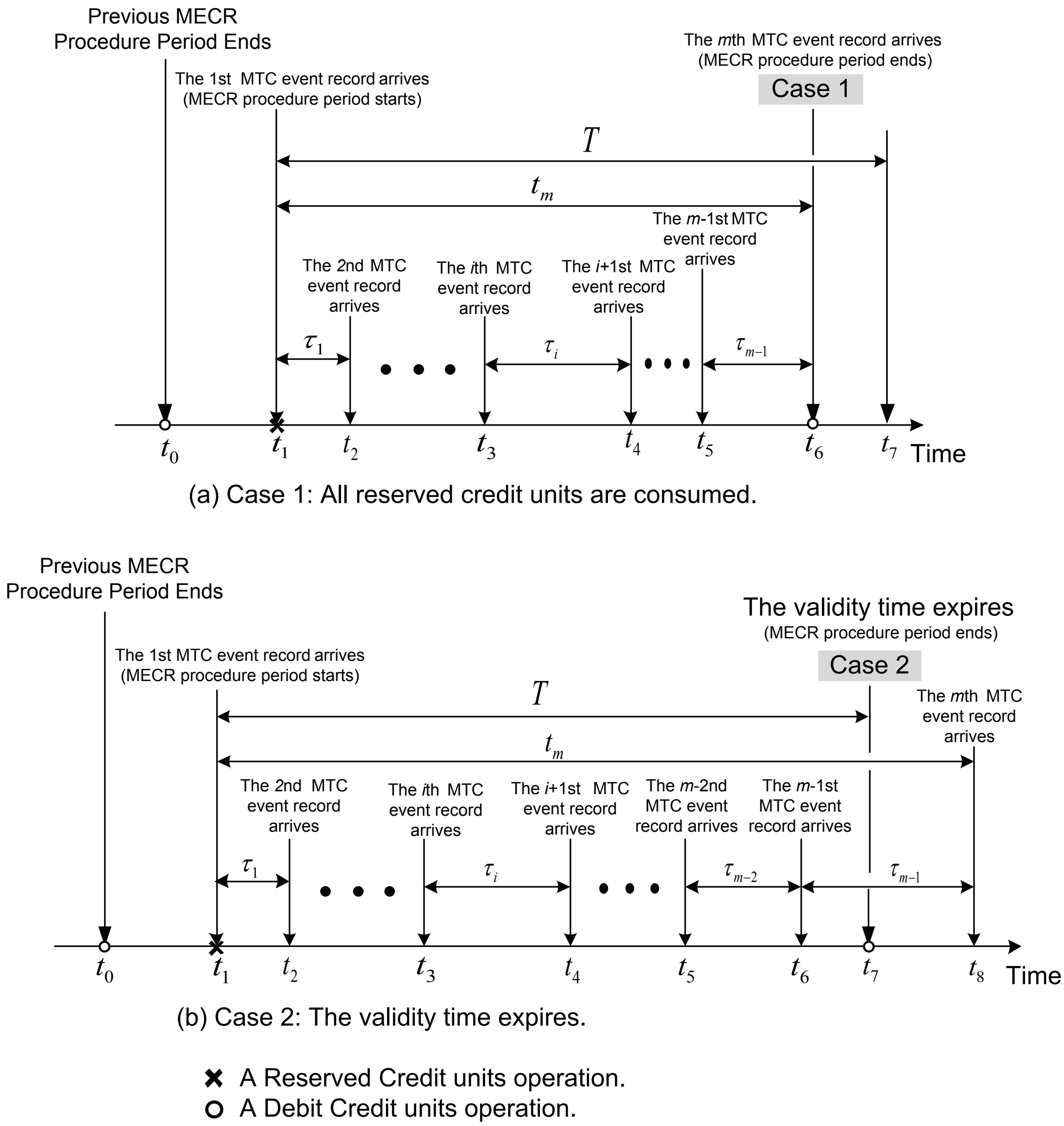
- $E[C_u]$: The expected amount of unused reserved credit units return to the OCS at each MECR procedure.

- $E[C_{ru}]$: The expected amount of unused reserved credit units found at any random checkpoint.

It is clear that the larger the $E[N]$, the lower the signaling overhead in OCS. The smaller the $E[C_u]$ or the $E[C_{ru}]$ values are, the more flexible an OCS handles the credits. The following sections formulate an analytical model for deriving $E[N]$, $E[C_u]$ and $E[C_{ru}]$ for our proposed MECR procedure.

4.1 Derivation for $E[N]$ and $E[C_u]$

This section derives the average number $E[N]$ of MTC records served, and the expected number of unused reserved credit units $E[C_{ru}]$ returned by each MECR procedure.



(a) Case 1: All reserved credit units are consumed.

(b) Case 2: The validity time expires.

Figure 4.2: Timing diagram for MECR Procedure.

In Figure 4.2, we consider the MTC records arrivals follow a Poisson distribution with rate λ. From the property of Poisson process, the inter-arrival time τ_i follows an exponential distribution with mean $\frac{1}{\lambda}$, which and density function

$$f(\tau_i) = \lambda e^{-\lambda \tau_i} \tag{4.1}$$

Let $t_m = \sum_{i=1}^{m-1} \tau_i$ denote the elapsed time between the `1st` and the `mth` records in the PCEF credit maintenance cycle. According to the Poisson process property, the t_m follows an Erlang distribution with mean $\frac{1}{\lambda}$. The density function $f_m(t_m)$ of t_m is expressed by

$$f_m(t_m) = \lambda e^{-\lambda t_m} \frac{(\lambda t_m)^{m-2}}{(m-2)!} \tag{4.2}$$

When the first MTC record arrives at t_1, the PCEF issues the CCR message with CC-Request-Type "INITIAL-REQUEST" to carry out the MECR procedure. MECR procedure sets validity time T and reserve θ credit units which can support m MTC records transmission, i.e., $\theta = m * c_{\texttt{mtc}}$. For the discussion purpose, we set the $c_{\texttt{mtc}} = 1$ (i.e., $\theta = m$). There are two cases that a MECR procedure will be terminated.

Case 1. In Figure 4.2(a), when the reserved credit units θ are depleted before the timer expiry, the PCEF sends the CCR message with CC-Request-Type "TERMINATION-REQUEST" to the OCS. In this case, the MECR procedure starts at t_1 and completes at t_6. Hence, the length of the MECR period is $t_6 - t_1$ is denoted by t_m.

Case 2. In Figure 4.2(b), when the validity time expires before the credits are consumed, the PCEF sends the CCR message with CC-Request-Type "TERMINATION-REQUEST" to the OCS. In this case, the MECR procedure starts at t_1 and completes at t_7. Hence, the length of the MECR period is T.

Therefore, the MECR procedure session time $T_{\texttt{MECR}} = \min(t_m, T)$ is

$$T_{\texttt{MECR}} = \begin{cases} t_m, & t_m \le T, \text{ (Case 1)} \\ T, & t_m > T. \text{ (Case 2)} \end{cases} \tag{4.3}$$

From Eq. (4.2) and Eq. (4.3), the expected value of $T_{\texttt{MECR}}$ is computed by

$$\begin{aligned}
E[T_{\texttt{MECR}}] =& E[\min(t_m, T)] \\
=& \int_0^T t_m f_m(t_m) dt_m + \int_T^\infty T f_m(t_m) dt_m \\
=& \frac{(m-1)}{\lambda} \int_0^T \lambda e^{-\lambda t_m} \left[\frac{(\lambda t_m)^{m-1}}{(m-1)!} \right] dt_m + T \int_T^\infty \lambda e^{-\lambda t_m} \left[\frac{(\lambda t_m)^{m-2}}{(m-2)!} \right] dt_m \\
=& \frac{(m-1)}{\lambda} \left\{ 1 - \sum_{k=0}^{m-1} e^{-\lambda T} \left[\frac{(\lambda T)^k}{k!} \right] \right\} + T \left\{ \sum_{k=0}^{m-2} e^{-\lambda T} \left[\frac{(\lambda T)^k}{k!} \right] \right\}
\end{aligned} \tag{4.4}$$

From Eq. (4.4), the average number of MTC records served by a MECR procedure can be expressed as

$$\begin{aligned}
E[N] &= 1 + \lambda E[T_{\texttt{MECR}}] \\
&= 1 + \lambda \left\{ \frac{(m-1)}{\lambda} \left[1 - \sum_{k=0}^{m-1} e^{-\lambda T} \frac{(\lambda T)^k}{k!} \right] + T \left[\sum_{k=0}^{m-2} e^{-\lambda T} \frac{(\lambda T)^k}{k!} \right] \right\}
\end{aligned} \tag{4.5}$$

From Eq. (4.5), we obtain the expected amount of unused reserved credit units returned by the MECR procedure as follows

$$\begin{aligned}
E[C_u] =& \theta - (1 + \lambda E[T_{\texttt{MECR}}]) \\
=& \theta - \left\{ 1 + \lambda \left\{ \frac{(m-1)}{\lambda} \left[1 - \sum_{k=0}^{m-1} e^{-\lambda T} \frac{(\lambda T)^k}{k!} \right] + T \left[\sum_{k=0}^{m-2} e^{-\lambda T} \frac{(\lambda T)^k}{k!} \right] \right\} \right\}
\end{aligned} \tag{4.6}$$

4.2 Derivation for $E[C_{ru}]$

This section derives the expected amount of unused reserved credit units $E[C_{ru}]$ observed by a random observer. By considering whether the CCR(TERMINATION-REQUEST) message is occurred at $t_6 = t_1 + t_m$ (see Figure 4.2(a)) or $t_7 = t_1 + T$ (see Figure 4.2(b)), we express $E[C_{ru}]$ as

$$E[C_{ru}] = E[C_{ru}|t_m \leq T] \Pr[t_m \leq T] + E[C_{ru}|t_m > T] \Pr[t_m > T] \tag{4.7}$$

Suppose that OCS grants $\theta = m * c_{\texttt{mtc}}$ credit units to PCEF at each MECR procedure. Without loss of generality, let $c_{\texttt{mtc}} = 1$ (i.e., one credit units cost per MTC record). Hence, Eq. (4.7) is re-written as

$$\begin{aligned} E[C_{ru}] = &\sum_{j=1}^{m-1} j \Pr[C_{ru} = j | t_m \le T] \Pr[t_m \le T] \\ &+ \sum_{j=1}^{m-1} j \Pr[C_{ru} = j | t_m > T] \Pr[t_m > T] \end{aligned} \tag{4.8}$$

$\Pr[C_{ru} = j|t_m \le T]$ and $\Pr[C_{ru} = j|t_m > T]$ are denote the conditional probability mass function that exactly j unused credit units remained in the PCEF at arbitrary time in the credit maintenance cycle.

In Eq. (4.8), the $\Pr[t_m \le T]$ and $\Pr[t_m > T]$ derived as follows. Let $N(t)$ denote the number of MTC records occur during time period with length t. Since the MTC record arrivals follow a Poisson distribution with rate λ, we have

$$\Pr[N(t) = n] = e^{-\lambda T} \frac{(\lambda t)^n}{n!} \tag{4.9}$$

We first derive $P[t_m \le T]$. In Figure 4.2(a), the first MTC record arrives at t_1. The PCEF performs a debit unit operation at t_6 (i.e., $t_1 + t_m$). Notice that the arrival of the $m\texttt{th}$ record will be smaller than t_7 if and only if there are at least $m - 1$ record arrivals occur in (t_1, t_7). Therefore,

$$\Pr[t_m \le T] = \Pr[N(T) \ge m - 1] = 1 - \Pr[N(T) \le m - 2] \tag{4.10}$$

From Eq. (4.9) and Eq. (4.10), we have

$$\Pr[t_m \le T] = \Pr[N(T) \ge m - 1] = 1 - \sum_{k=0}^{m-2} e^{-\lambda T} \left[\frac{(\lambda T)^k}{k!}\right] \tag{4.11}$$

Then, in Figure 4.2(b), the probability that a validity time expiry occurs before the credit depletion is

$$\Pr[t_m > T] = 1 - \Pr[t_m \le T] = \sum_{k=0}^{m-2} e^{-\lambda T} \left[\frac{(\lambda T)^k}{k!}\right]. \tag{4.12}$$

From Eq. (4.11) and Eq. (4.12), Eq. (4.8) is re-written as

$$\begin{aligned} E[C_{ru}] = &\sum_{j=1}^{m-1} j \Pr[C_{ru}=j|t_m \le T] \left\{ 1 - \sum_{k=0}^{m-2} e^{-\lambda T} \frac{(\lambda T)^k}{k!} \right\} \\ &+ \sum_{j=1}^{m-1} j \Pr[C_{ru}=j|t_m > T] \left\{ \sum_{k=0}^{m-2} e^{-\lambda T} \frac{(\lambda T)^k}{k!} \right\} \end{aligned} \tag{4.13}$$

In Eq. (4.13), $\Pr[C_{ru}=j|t_m \le T]$ and $\Pr[C_{ru}=j|t_m > T]$ are derived as follows. For $j=0$, random check occurs in interval (t_0,t_1) shown in Figure 4.1. Therefore,

$$\Pr[C_{ru}=0] = \frac{E[\tau_{\mathtt{idle}}]}{E[T_P]}. \tag{4.14}$$

For $j>0$ $(1 \le j \le m-1)$, we discuss it in two cases given Eq. (4.3). In Figure 4.2(a), $N(T) \ge m-1$ and $T_{\mathtt{MECR}} = t_m$. There are m MTC records occur in interval $[t_1,t_6]$. The first and the last MTC records occur at t_1 and t_6, respectively. There are $m-2$ records arrive in (t_1,t_6). Since the MTC records arrive at PDN-GW according to a Poisson process, the occurrence times for these MTC records are independent uniformly distributed over (t_1,t_6). Let $\tau_{\mathtt{mtc}}$ denote the average inter-arrival time for MTC records occurring in period (t_1,t_6) given that $t_m \le T$. We have

$$\tau_{\mathtt{mtc}} = \frac{E[t_m|t_m \le T]}{m-1} \tag{4.15}$$

$E[t_m|t_m \le T]$ denotes the average MECR period in case 1. The average length of the PCEF credit maintenance cycle $E[T_P|\mathtt{Case1}] = \mathtt{E}[\tau_{\mathtt{idle}}] + \mathtt{E}[\mathtt{t_m}|\mathtt{t_m} \le \mathtt{T}]$. Then, the conditional probability of $\Pr[C_{ru}=j|t_m \le T]$ that exactly j unused credit units remained in the PCEF credit maintenance cycle can be computed by

$$\Pr[C_{ru}=j|t_m \le T] = \frac{\tau_{\mathtt{mtc}}}{E[T_P|\mathtt{Case1}]}. \tag{4.16}$$

In Figure 4.2(b), $N(T) < m-1$ and $T_{\mathtt{MECR}} = T$. There are at most $m-1$ MTC records occur in interval $[t_1,t_7)$ and $E[T_{\mathtt{P}}|\mathtt{Case2}] = \mathtt{E}[\tau_{\mathtt{idle}}] + \mathtt{T}$. The first MTC record arrives at t_1 and there are k record arrivals $(0 \le k \le m-2)$ occur in interval (t_1,t_7). For

$(0 \leq k \leq m-2)$, the conditional probability mass function represented by

$$\Pr[N(T)=k|t_m > T] = \frac{\Pr[N(T)=k]}{\sum_{i=0}^{m-2}\Pr[N(T)=i]} = \frac{1}{B}\frac{(\lambda T)^k}{k!} \tag{4.17}$$

where $B = \sum_{i=0}^{m-2}\frac{(\lambda T)^i}{i!}$. These occurrence times are independent uniformly distributed on (t_1, t_7). In this case, the average inter-arrival time of the MTC records arrivals during (t_1, t_7) becomes $\frac{T}{k+1}$. Then, the conditional probability of $\Pr[C_{ru} = j|t_m > T]$ that exactly j unused credit units remained in the PCEF credit maintenance cycle can be computed by

$$\Pr[C_{ru}=j|t_m > T] = \sum_{k=m-j-1}^{m-2}\frac{T/(k+1)}{E[T_P|\texttt{Case2}]}\frac{e^{\lambda T}}{B}\Pr[N(T)=k]. \tag{4.18}$$

Substituting Eq. (4.16) and Eq. (4.18) into (4.13), we have

$$\begin{aligned} E[C_{ru}] = &\left\{\sum_{j=1}^{m-1} j\frac{\tau_{\texttt{mtc}}}{E[T_P|\texttt{Case1}]}\right\}\left\{1-\sum_{k=0}^{m-2}e^{-\lambda T}\frac{(\lambda T)^k}{k!}\right\} \\ &+\left\{\sum_{j=1}^{m-1} j \sum_{k=m-j-1}^{m-2}\frac{T/(k+1)}{E[T_P|\texttt{Case2}]}\frac{e^{\lambda T}}{B}\Pr[N(T)=k]\right\}\left\{\sum_{k=0}^{m-2}e^{-\lambda T}\frac{(\lambda T)^k}{k!}\right\} \end{aligned} \tag{4.19}$$

The analytic model developed in this chapter is validated against the simulation model. The simulation model is developed by a C++ discrete-event simulation in the next chapter.