Tutorial 10

- Paper
- Pen
- Calculator

Tutorial No. 10 for Digital Signal Processing

Flow Graphs; Filter Realizations and Frequency Response

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Question 1.

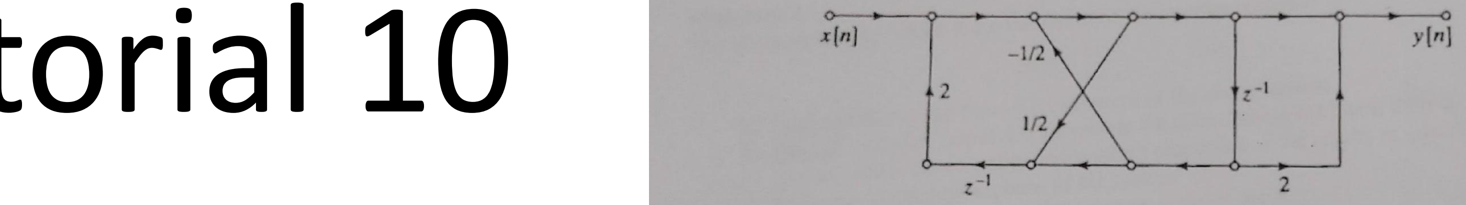
Suppose a digital filter has the transfer function as follows,

$$H(z) = \frac{1+\frac{7}{8}z^{-1}}{1-\frac{1}{2}z^{-1}}. \qquad |z| > 1/2$$

a. Find the poles and zeros then sketch the pole/zero pattern
b. Find the h(n) and conclude about the causality and stability of the function?
c. Sketch the frequency response of the filter
d. Realize the filter in the direct form and canonical form.

Question 2:

The flow graph shown in the figure is an implementation of a causal, LTI system



a. Draw the transpose of the signal flow graph
b. Determine the difference equation relating to the input signal x(n) to the y(n)
c. Find and sketch the pole/zero pattern of the system. Is the system stable?
d. Realize the system in the direct and canonical form.
e. Determine y(2) if x(n)=(1/2)n u(n).

Question 3:

Consider a causal LTI system whose system function is

$$H(z) = \frac{1-\frac{1}{5}z^{-1}}{\left(1-\frac{1}{2}z^{-1}+\frac{1}{3}z^{-2}\right)\left(1+\frac{1}{4}z^{-1}\right)}$$

a. Draw the signal flow graphs for implementations of the system in each of following forms: Direct form I; Direct form II; Cascade form; Parallel form; Transpose direct form II.
b. Write the difference equations
c. Sketch Pole and zero pattern
d. Conclusion about the stability of the system?

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Q1 (answer)

pole = ½
zero = -⅞

$H(z) = A + B/(1 - 0.5z^{-1}) \longrightarrow h(n) = A\delta(n) + B \times 0.5^n u(n)$

$H(w) = H(z=e^{jw})$

- $H(z) = \ldots = (b_0 + b_1.z^{-1} + b_2.z^{-2} + \ldots)/(1 + a_1.z^{-1} + a_2.z^{-2} + \ldots)$

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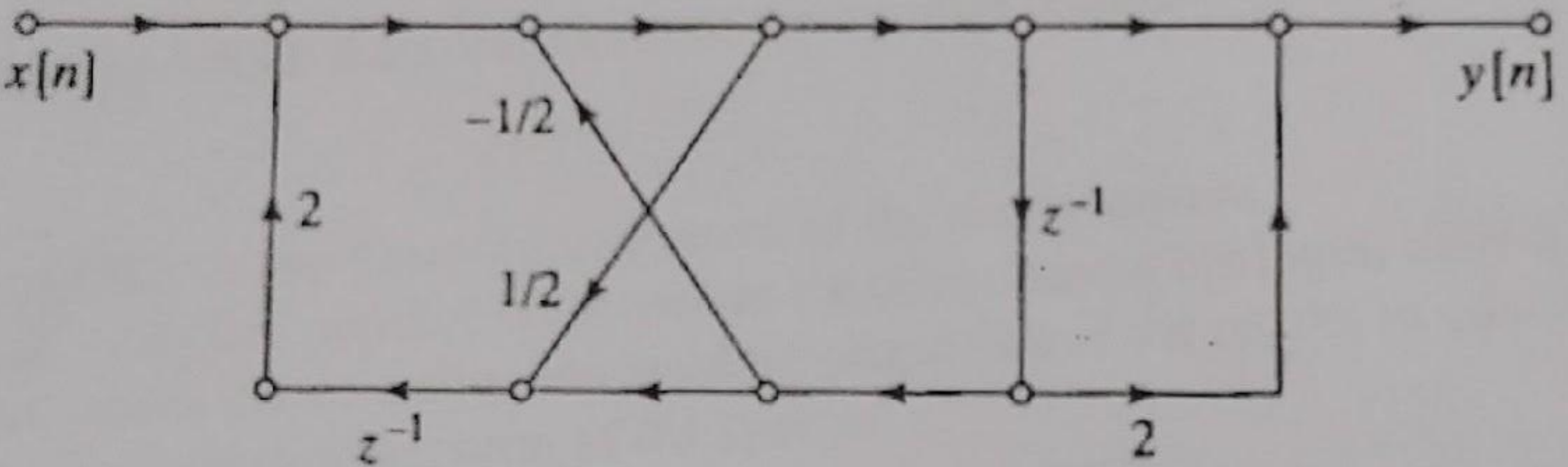

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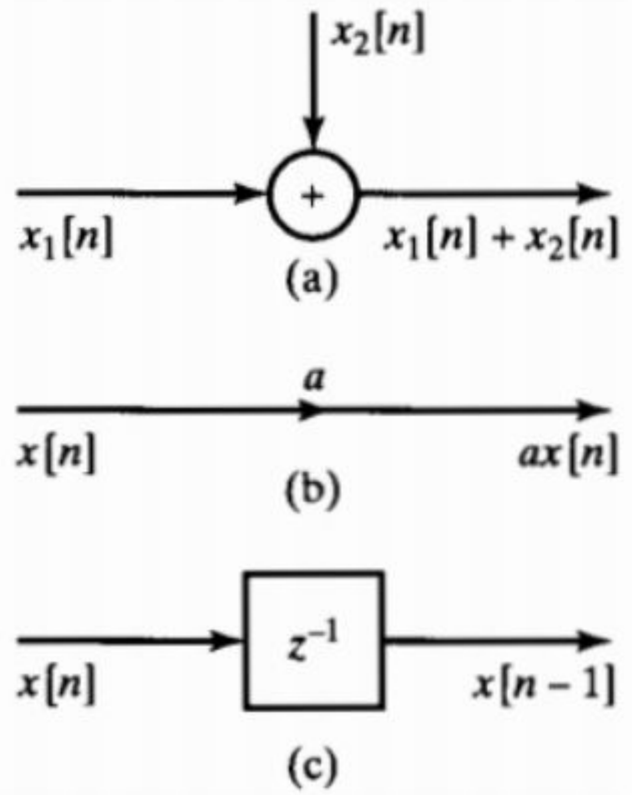
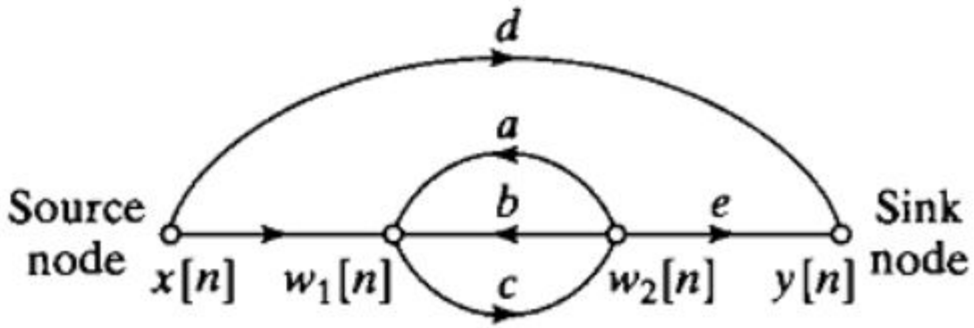


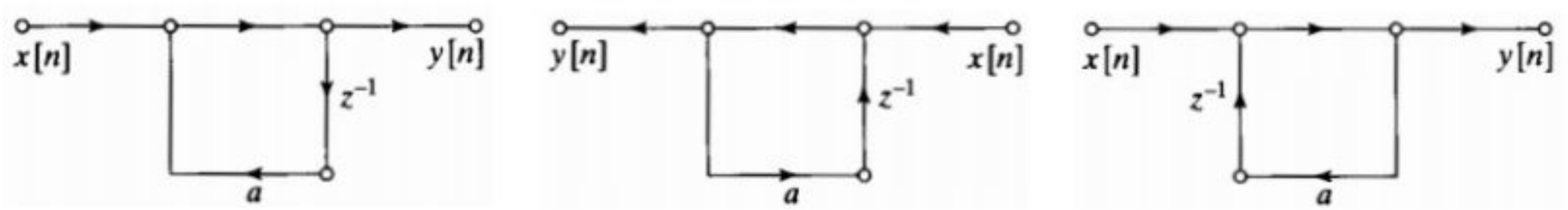


$$w_1[n] = x[n] + aw_2[n] + bw_2[n],$$

$$w_2[n] = cw_1[n],$$

$$y[n] = dx[n] + ew_2[n].$$

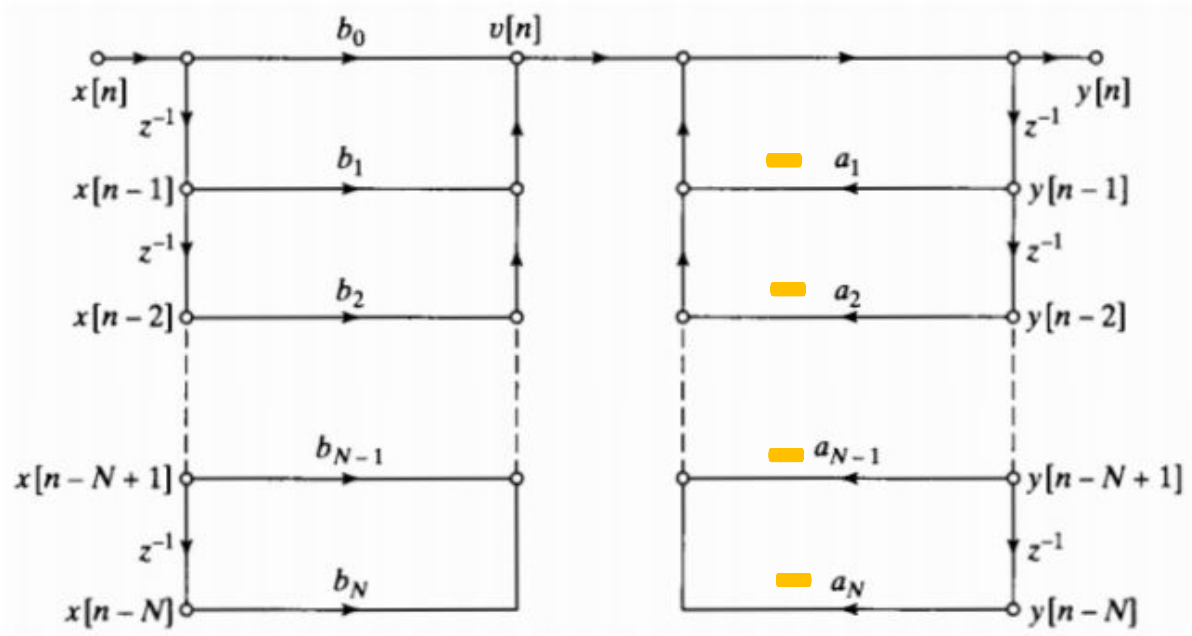
Transposed form

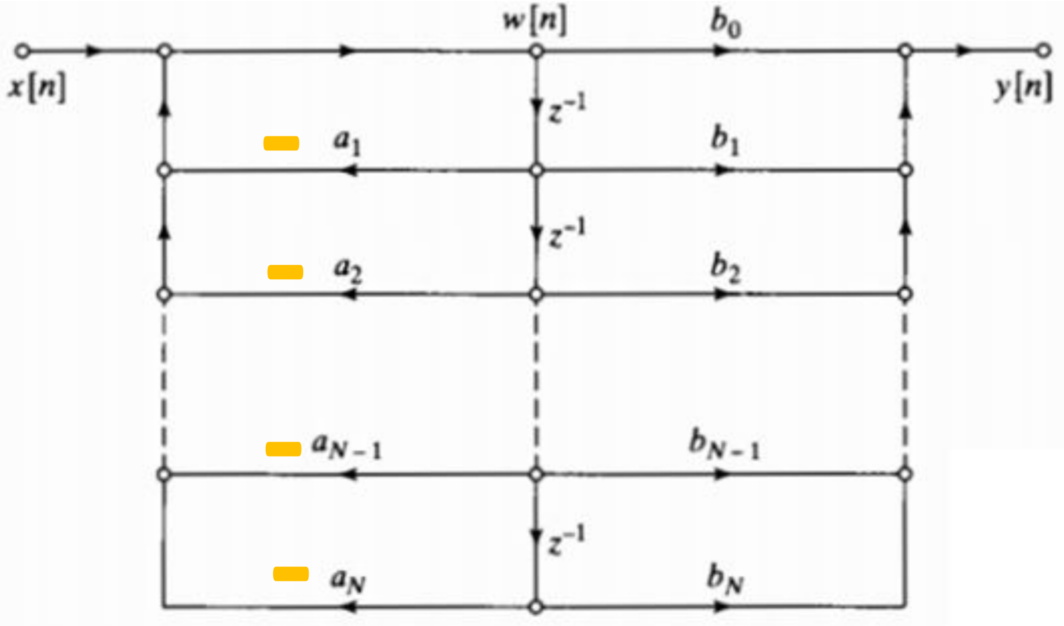


- x[n] <-> y[n]
- arrow: inverse

- Result: the same

Direct form (I) and canonical form (direct II)

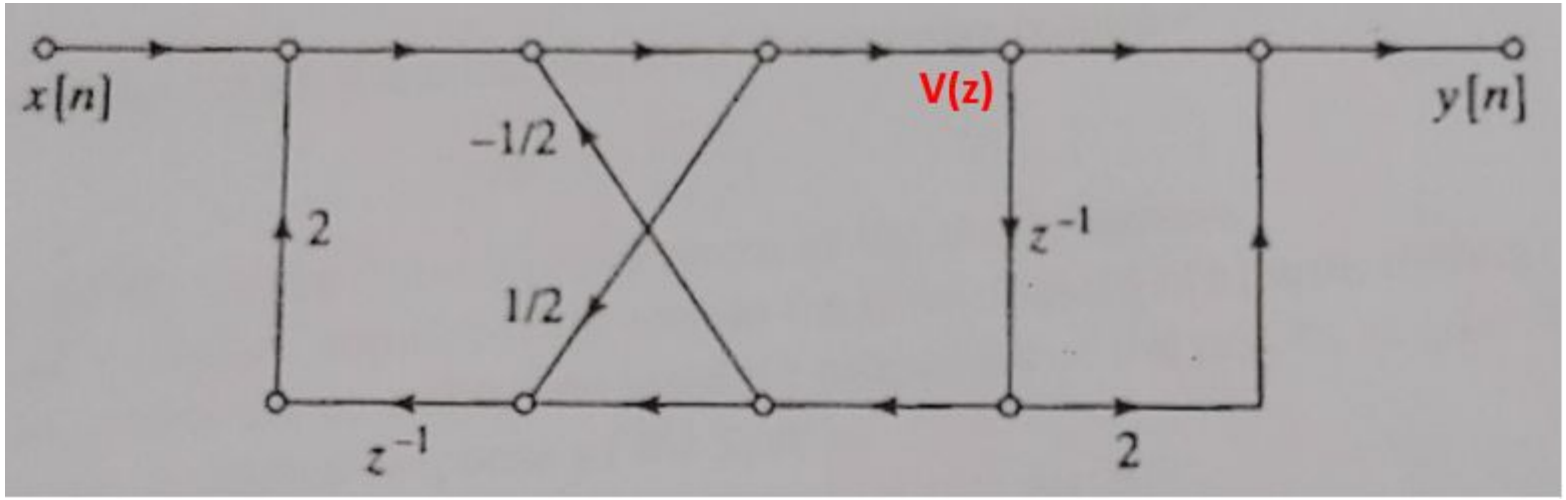




$$H(z)=\frac{N(z)}{D(z)}=\frac{b_0+b_1z^{-1}+b_2z^{-2}+\cdots+b_Lz^{-L}}{a_0+a_1z^{-1}+a_2z^{-2}+\cdots+a_Mz^{-M}}$$

$$y_n=-a_1y_{n-1}-a_2y_{n-2}-\cdots-a_My_{n-M}+b_0x_n+b_1x_{n-1}+b_2x_{n-2}+\cdots+b_Lx_{n-L}$$

Q2 (answer)



b) $H(z) = Y(z)/X(z) = (1 + 2z^{-1})/(1 - 0.5z^{-1} - 2z^{-2})$ □ different equation

e) $h(n) = 0.5h(n - 1) + 2h(n - 2) + \delta(n) + 2\delta(n - 1)$

- causal □ $h(n<0) = 0$, $h(0) = 1$, $h(1) = 2.5$, $h(2) = 3.25$
- $y(2) = x(0).h(2) + x(1).h(1) + x(2).h(0) = 3.25 + 1.25 + 0.25 = 4.75$

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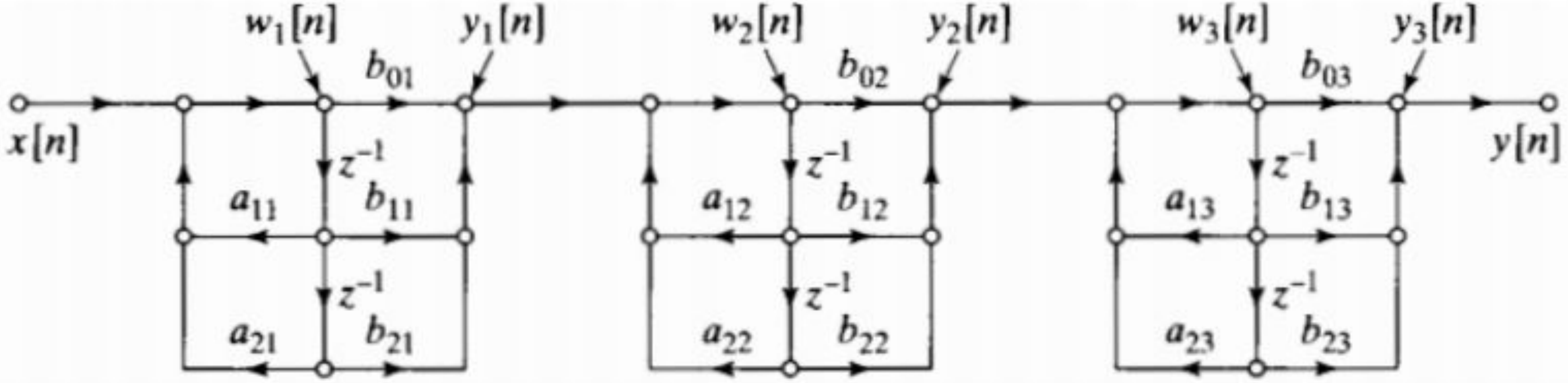
Cascade form (Second Order Section)

$$H(z) = \prod_{k=1}^{N_s} \frac{b_{0k} + b_{1k}z^{-1} + b_{2k}z^{-2}}{1 - a_{1k}z^{-1} - a_{2k}z^{-2}},$$



Parallel form

$$H(z)=\sum_{k=0}^{N_p} C_k z^{-1}+\sum_{k=1}^{N_1}\frac{A_k}{1-c_k z^{-1}}+\sum_{k=1}^{N_2}\frac{B_k(1-e_k z^{-1})}{(1-d_k z^{-1})(1-d_k^* z^{-1})},$$