Tutorial 9

- Paper
- Pen
- Calculator

Question 1:

When the input to a causal LTI system is $x(n) = -\frac{1}{3}\left(\frac{1}{2}\right)^n u(n) - \frac{4}{3}2^n u(-n-1)$ and the z-transform of the output is $Y(z) = \frac{1+z^{-1}}{(1-z^{-1})(1+0.5z^{-1})(1-2z^{-1})}$

a. Discuss about the causality property of the input x(n) then find the z-transform of x(n)
b. Find and sketch the pole/zero pattern for the X(z); Y(z)
c. What is the region of convergence of Y(z)?
d. Find the causal impulse response of the system
e. Is the system stable?
f. Realize the system in the canonical form.

Question 2:

The given system for which the z-transform of the impulse response is

$$H(z) = \frac{1-z^3}{1-z^4}$$

a. Find and sketch the Poles/Zeros Pattern of the above system.
b. Find all available impulse responses in the time domain h[n] with their ROCs
c. Discuss about the causality and stability properties of the results in question 2(b)
d. Draft the frequency response of the system.
e. Realize the block diagram of the system in direct form.

Question 3:

A given causal LTI system with the frequency response

$$H(e^{j\omega}) = \left(1+0.4e^{-j(\omega+\pi)}\right)\left(\frac{1+\frac{1}{2}e^{-2j\omega}}{1+e^{-2j\omega}+\frac{1}{4}e^{-4j\omega}}\right) \qquad -\pi < \omega \le \pi$$

a. Find the H(z)?
b. Is the system is FIR or IIR filter? Explain?
c. Find the I/O equation of the system?
d. Sketch the pole/zero pattern of the H(z) then specify the ROC and stability?
e. Based on the pole/zero pattern, draft the amplitude of the given frequency response of the system?
f. Realize the block diagram of the system?

Question 1:

When the input to a causal LTI system is $x(n) = -\frac{1}{3}\left(\frac{1}{2}\right)^n u(n) - \frac{4}{3}2^n u(-n-1)$ and the z-transform of the output is $Y(z) = \frac{1+z^{-1}}{\left(1-z^{-1}\right)\left(1+0.5z^{-1}\right)\left(1-2z^{-1}\right)}$

a. Discuss about the causality property of the input x(n) then find the z-transform of x(n)
b. Find and sketch the pole/zero pattern for the X(z); Y(z)
c. What is the region of convergence of Y(z)?
d. Find the causal impulse response of the system
e. Is the system stable?
f. Realize the system in the canonical form.

Q1 (answer)

- $X(z) = ?$
- $H(z) = Y(z)/X(z) = \ldots = A_1/(1-p_1z^{-1}) + A_2/(1-p_2z^{-1}) + \ldots + B_0 + B_1z^{-1} + \ldots$
- Causal $h(n) = A_1.p_1^nu(n) + A_1.p_1^nu(n) + \ldots + B_0\delta(n) + B_1\delta(n-1) + \ldots$

Question 2:

Consider an LTI system that is stable and for which H(z), the z-transform of the impulse response, is given by

$$H(z)=\frac{2-4z^{-1}+2z^{-2}}{1+3z^{-1}-4z^{-2}}$$

Suppose x(n), the input to the system, is a unit step sequence.

a. Find the output y(n) by evaluating the discrete convolution of x(n) and y(n)
b. Find the output y(n) by computing the inverse z-transform of Y(z)
c. Sketch the Pole/zero pattern of H(z) and the magnitude of the frequency response
d. Realize the above filter with the direct form and Canonical form.

$$H(z) = \frac{2 - 4z^{-1} + 2z^{-2}}{1 + 3z^{-1} - 4z^{-2}}$$

- $H(z) = \ldots = A_1/(1-p_1z^{-1}) + A_2/(1-p_2z^{-1}) + B_0$
- $H(z) = 2(1-z^{-1})^2/\{(1-z^{-1})(1+4z^{-1})\} = (2-2z^{-1})/(1+4z^{-1}) = A_1/(1+4z^{-1}) + B_0$
- Stable □ ROC contains the unit circle □ $ROC_H = \{|z|<|-4|\} = \{|z|<4\}$
- □ $h(n) = B_0\,\delta(n) - A_1(-4)^n u(-n-1)$
- $y(n) = x(n)*h(n) = u(n)*h(n)$

- $Y(z) = H(z).X(z) = \{2(1-z^{-1})/(1+4z^{-1})\}.1/(1-z^{-1}) = 2/(1+4z^{-1})$
- $ROC_Y = ROC_X \cap ROC_H = \{|z|<4\}$
- □ $y(n) = -2(-4)^n u(-n-1)$

Question 3.

a. Determine the unit step response of the causal system for which the z-transform of the impulse response is $H(z) = \dfrac{1-z^3}{1-z^4}$

b. Realize the above filter with the direct form and Canonical form

$x(n) = u(n) \rightarrow X(z) = 1/(1-z^{-1})=z/(z-1)$, $\mathrm{ROC}_X = \{|z|>1\}$

causal system $\rightarrow \mathrm{ROC}_H = \{|z|>1\}$

$\rightarrow \mathrm{ROC}_Y = \mathrm{ROC}_X \cap \mathrm{ROC}_H = \{|z|>1\}$

$\rightarrow Y(z) = H(z).X(z) = -(1+z+z^2)z/\{(1-z).(1+z).(1+z^2)\}$

$= (z^{-3}+z^{-2}+z^{-1})/\{(1-z^{-1}).(1+z^{-1}).(1+z^{-2})\} = \ldots = A_1/(1-p_1z^{-1}) + A_2/(1-p_2z^{-1}) + A_3/(1-p_3z^{-1}) + A_4/(1-p_4z^{-1})$

□ $y(n) = A_1p_1^n u(n) + \ldots$

Question 3.

a. Determine the unit step response of the causal system for which the z-transform of the impulse response is $H(z) = \dfrac{1-z^3}{1-z^4}$

b. Realize the above filter with the direct form and Canonical form

- $y(n) = h(n)*x(n)$
- $H(z) = (1+z+z^2)/\{(1+z).(1+z^2)\} = (z^{-3}+z^{-2}+z^{-1})/\{(1+z^{-1}).(1+z^{-2})\} = \ldots = B_1/(1-p_1z^{-1}) + B_2/(1-p_2z^{-1}) + B_3/(1-p_3z^{-1}) + B_0$
- Causal □ $h(n) = B_1p_1^nu(n) + \ldots + B_0\delta(n)$

- $(1-z^4)H(z) = 1 - z^3$ □ $h(n) - h(n+4) = \delta(n) - \delta(n+3)$ $(m=n+4$□$n=m-4)$
- □ $h(m-4) - h(m) = \delta(m-4) - \delta(m-1)$ $(n=m)$
- □ $h(n-4) - h(n) = \delta(n-4) - \delta(n-1)$ □ $h(n) = h(n-4) - \delta(n-4) + \delta(n-1)$ □
- $h(n<0) = 0$ (causal), $h(0) = 0$, $h(1) = 1$, $h(2) = 0$, $h(3) = 0$, $h(4) = -1$, $h(5)=h(1)=1$, …

Question 2:

The given system for which the z-transform of the impulse response is

$$H(z) = \frac{1 - z^{3}}{1 - z^{4}}$$

a. Find and sketch the Poles/Zeros Pattern of the above system.
b. Find all available impulse responses in the time domain h[n] with their ROCs
c. Discuss about the causality and stability properties of the results in question 2(b)
d. Draft the frequency response of the system.
e. Realize the block diagram of the system in direct form.

Q2 (answer)

- $H(z) = \ldots = A_1/(1-p_1z^{-1}) + A_2/(1-p_2z^{-1}) + A_3/(1-p_3z^{-1})$
- $|p_1|=|p_2|=|p_3|=1$
- □ $ROC_1 = |z|>1$: $h_1(n)= A_1.p_1^n u(n) + A_2.p_2^n u(n) + A_2.p_3^n u(n)$, causal, margin stable
- and $ROC_2 = |z|<1$: $h_2(n)= -A_1.p_1^n u(-n-1) - A_2.p_2^n u(-n-1) - A_2.p_3^n u(-n-1)$, anticausal, margin stable
- Frequency response $H(w) = H(z=e^{jw}) = \ldots$ □ period 2pi
  - $|H(w)|$
  - $\arg H(w)$
- Block diagram: $H(z)=(b_0 + b_1.z^{-1} + b_2.z^{-2} +\ldots)/(1 + a_1.z^{-1} + a_2.z^{-2} +\ldots)$

Question 3:

A given causal LTI system with the frequency response

$$H(e^{j\omega}) = \left(1+0.4e^{-j(\omega+\pi)}\right)\left(\frac{1+\frac{1}{2}e^{-2j\omega}}{1+e^{-2j\omega}+\frac{1}{4}e^{-4j\omega}}\right) \qquad -\pi < \omega \leq \pi$$

a. Find the H(z)?

b. Is the system is FIR or IIR filter? Explain?

c. Find the I/O equation of the system?

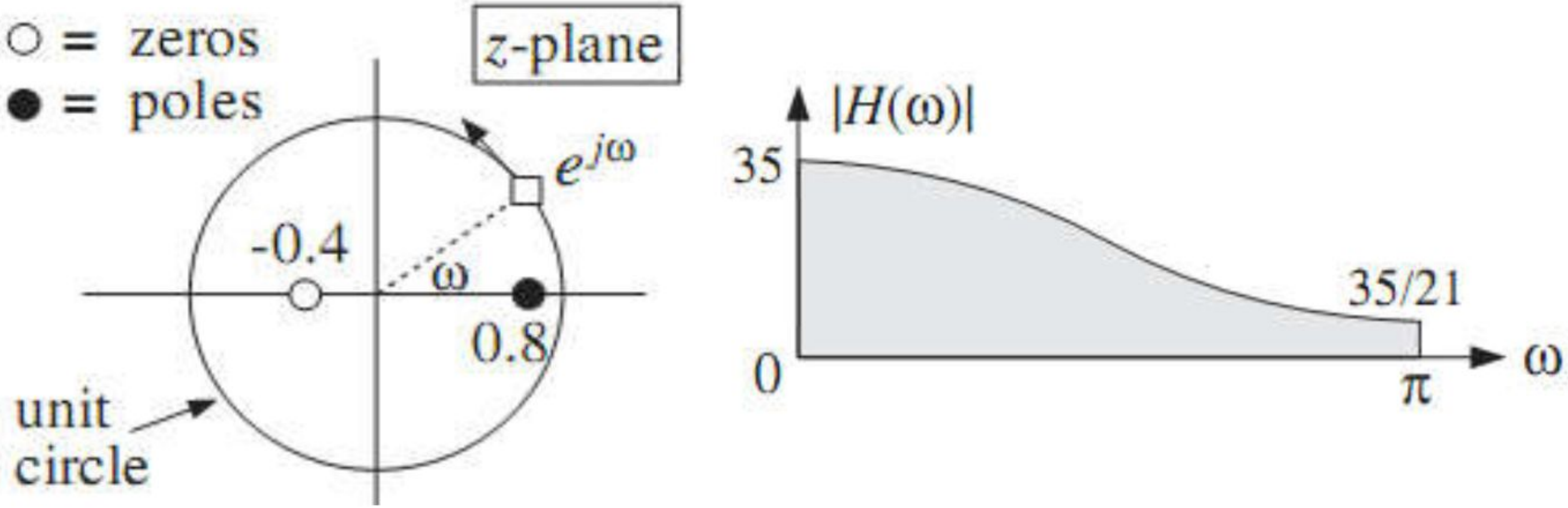
d. Sketch the pole/zero pattern of the H(z) then specify the ROC and stability?

e. Based on the pole/zero pattern, draft the amplitude of the given frequency response of the system?

f. Realize the block diagram of the system?

Q3 (answer)

- $z = e^{jw}$
- $e^{-jw} = z^{-1}$
- $e^{-2jw} = z^{-2}$
- $e^{-j(w+pi)} = e^{-jw}.(e^{-j.pi}) = -z^{-1}$
- $H(z) = \ldots = Y(z)/X(z) = (b_0 + b_1.z^{-1} + b_2.z^{-2} + \ldots)/(1 + a_1.z^{-1} + a_2.z^{-2} + \ldots)$
- I/O equation: $y(n) = b_0x(n) + b_1x(n-1) + b_2x(n-2) + \ldots - a_1y(n-1) - a_2y(n-2) - \ldots$
-



Pole/zero pattern and magnitude response