what is $\frac{\Delta r}{r}$

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1. INTRODUCTION

The contemporary model for supernovae involves a critical phase between the core-collapse of a massive star and the subsequent explosion, which is characterized by a stalled accretion shock. This phase can last on the order of a few hundreds of milliseconds (Mezzacappa 2005). Hydrodynamic simulations have demonstrated the existence of an instability in this stalled accretion shock, which has come to be known as the spherical accretion shock instability, or SASI (Blondin & Mezzacappa 2006).

The SASI has been demonstrated to grow into nonaxisymmetric "spiral" and "sloshing" modes as it evolves, even with no initial angular momentum (Blondin & Mezzacappa 2007). We will investigate the growth of these spiral modes in two-dimensional simulations, by analyzing the development of single arm $m = 1$ spiral modes.

The mechanism of the growth of the SASI is under inquiry, and the two dominant explanations are an acoustic mechanism and an advective-acoustic mechanism. The acoustic mechanism is dominated by transverse propagation, whereas the advective-acoustic cycle involves both transverse and radial propagation (Foglizzo et al. 2007). In a standard circle of inner angle 2π, the radius and circumference of a circle are related by a constant rate, so it is difficult to distinguish radial and transverse propagation by varying the radius of a supernova simulation.

In order to attempt to separate the radial and transverse propagation of the SASI, we will redefine the number of radians in a circle within the simulation model, so that it is possible to change the circumference of a two-dimensional supernova simulation without modifying its radius. In this paper, we seek to simulate supernovae using redefined maximum circular sector angles in order to distinguish between radial and transverse propagation of the SASI.

2. NUMERICAL MODEL

The simulation model presented in this paper is similar to previous two-dimensional supernova simulations (Blondin & Mezzacappa 2006; Blondin et al. 2017). We use the hydrodynamics code VH-1, modified to represent circular sectors with varying maximum angles. The boundaries $\theta = 0$ and $\theta = \theta_{max}$ are set to be periodic, so that matter can freely move across the bounds within the simulation. We use an ideal gas with an adiabatic index of $\gamma = 4/3$ on a disk with an outer radius of 2.0 subject to a Newtonian potential $U = -GM/r$. We use the cooling parameters $\alpha = 3/2$ and $\beta = 5/2$ for a steady state accretion shock given by Houck & Chevalier (1992).

We use cooling rates set so that the steady state shock radius r_{sh} is equal to 1 for all simulations. We then evolve the supernova simulation in one dimension until the SASI begins to develop, and then export the data from the one-dimensional simulation to the two-dimensional simulations in order to reduce computation time.

We use 384 logarithmically placed grid zones in the radial direction for the one and two-dimensional simulations. For the grid count in the direction of θ, we divide θ_{sector} by the grid width in the radial direction, rounded to the nearest multiple of the number of processing cores. This ensures similar grid dimensions across all simulations.

For simulations with no initial specific angular momentum, the sector angle range where the SASI developed as a single $m = 1$ spiral mode was small and centered around 2π. To encourage the growth of spiral modes across multiple circular sector sizes, we initialize the two-dimensional simulations with specific angular momenta of 0.02 and 0.05, following the findings of Blondin et al. (2017) to enhance the growth of the SASI across a broader range of sector angles.

In order to visualize the growth of the SASI in circular sectors with maximum angles larger than 2π, we map the grid zones of the simulations to rectangular blocks. We also mapped the rectangular grid data into regular circular disks of size 2π, in order to compare the growth of the SASI across circular sector sizes.

We ran the simulations for circular sector values in the range of 0.1π to 10.0π, in steps of 0.1π.

We ran the simulations with inner radii r_* of 0.2, 0.4, and 0.55, with an initial angular momentum of 0.02. We also ran a simulation using $r_* = 0.2$ with an initial angular momentum of 0.05.

Given the results of Blondin et al. (2017), we expect that the simulations with a higher initial angular momentum will consistently have a greater angular speed. With the findings of Foglizzo et al. (2007), we expect that the simulation set with an inner radius of 0.55 will have a lower angular speed than the simulation set with an inner radius of 0.2.

3. ANALYSIS

We measure the strength of the SASI by computing the power in the $m = 1$ Fourier components of the deviation of angular momentum from the initial value, in a similar manner to Blondin et al. (2017). Using:

is this the correct quantity?

$$a(r) = \int_0^{2\pi} (h - rv_\phi) cos\phi d\phi \quad (1)$$

$$b(r) = \int_0^{2\pi} (h - rv_\phi) sin\phi d\phi \quad (2)$$