A Parametric Study of the SASI Comparing General Relativistic and Non-Relativistic Treatments

Samuel J. Dunham

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Paper

A Parametric Study of the SASI Comparing General Relativistic and Non-Relativistic Treatments*

Samuel J. Dunham,[1,2] Eirik Endeve,[3,2] Anthony Mezzacappa,[2] John M. Blondin,[4] Jesse Buffaloe,[2]
and Kelly Holley-Bockelmann[1,5]

[1] *Department of Physics and Astronomy, Vanderbilt University, 6301 Stevenson Center, Nashville, TN 37235*

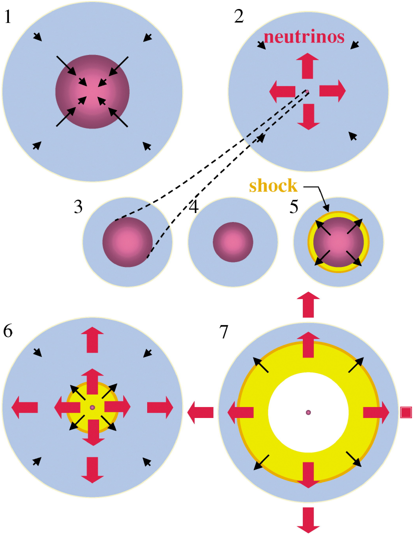
[2] *Department of Physics and Astronomy, University of Tennessee, Knoxville, Nielsen Physics Building, 401, 1408 Circle Drive, Knoxville, TN 37996*

[3] *Computer Science and Mathematics Division, Oak Ridge National Laboratory, Oak Ridge, TN 37831*

[4] *Department of Physics, North Carolina State University, 2401 Stinson Dr., Raleigh, NC 27607*

[5] *Department of Life and Physical Sciences, Fisk University, 1000 17th Ave N, Nasvhille, TN 37208*

Overview



Mezzacappa A. 2005.
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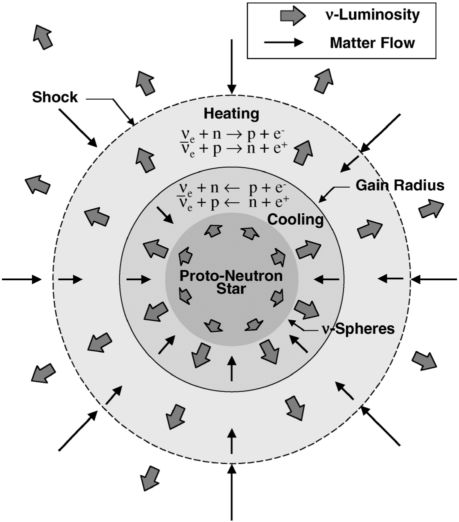

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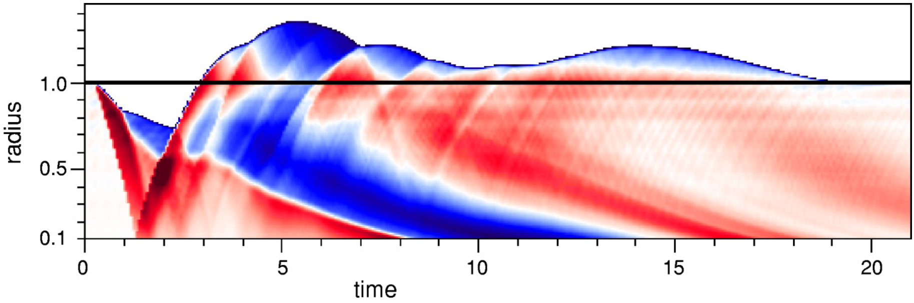


Figure: Equilibrium (white), under- (blue), and over- (red) pressure (**?**).

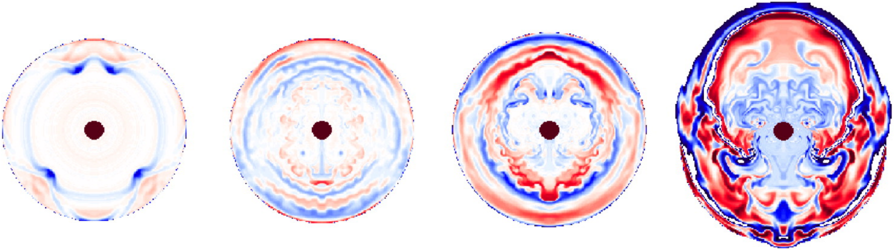
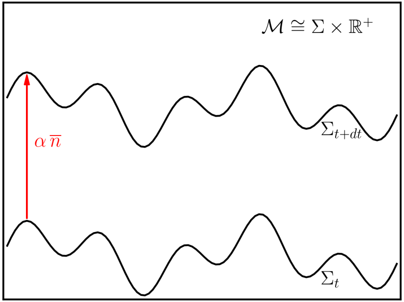


Figure: Equilibrium (white), under- (blue), and over- (red) entropy (**?**).

$d+1$ Decomposition

$$ds^2 = g_{\mu\nu}\, dx^\mu\, dx^\nu$$



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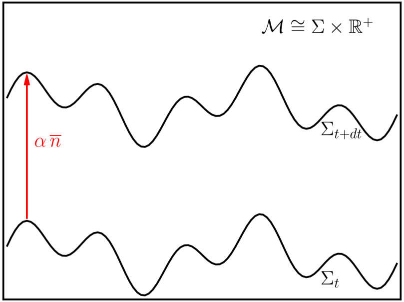

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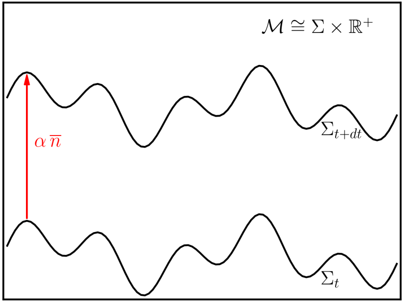

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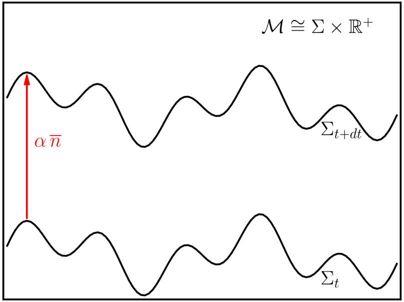

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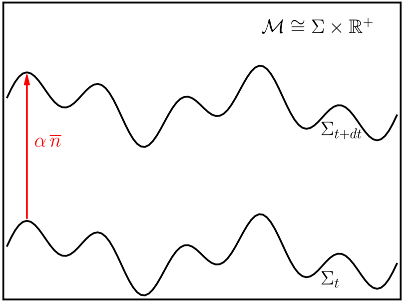
$0 < \alpha \leq 1$: Lapse Function



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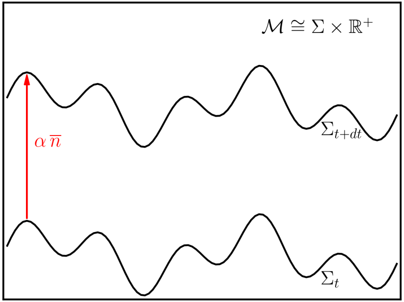
$\overline{n}$: Eulerian four-velocity

$\left(\underline{\underline{g}}\,(\overline{n}, \overline{n}) = \overline{n} \cdot \overline{n} = -1\right)$



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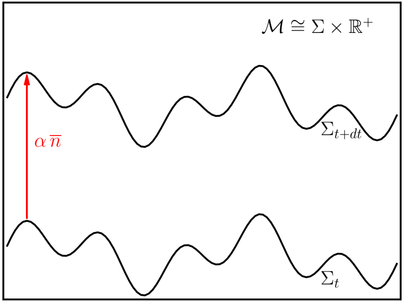
(Also, $\underline{\underline{K}}$: Extrinsic curvature)



Conformally-Flat Condition

??

$$\gamma_{ij} = \psi^4 \, \bar{\gamma}_{ij}$$

$1 \leq \psi < 2$: Conformal factor

$$\bar{\gamma}_{ij} = \mathrm{diag}\left(1, r^2, r^2 \sin^2 \theta\right)$$

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$\bar{\gamma}_{ij} = \mathrm{diag}\left(1, r^2, r^2 \sin^2 \theta\right)$

(Also, $K := \mathrm{Tr}_{\gamma_{ij}}\left(\underline{\underline{K}}\right) = \partial_t \, K = 0$)

Isotropic Coordinates ($c = G = 1$)

?

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$$\beta^i = 0$$

$$K_{ij} = 0$$

Fluid Equations

Units defined such that $c = G = 1$

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$\overline{\overline{T}} := \rho \, h \, \overline{u} \otimes \overline{u} + p \, \overline{\overline{g}} \quad$ (p: comoving thermal pressure,

$h := 1 + (e + p) / \rho$: specific enthalpy, e: comoving internal energy density)

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Five equations with six unknowns ☹

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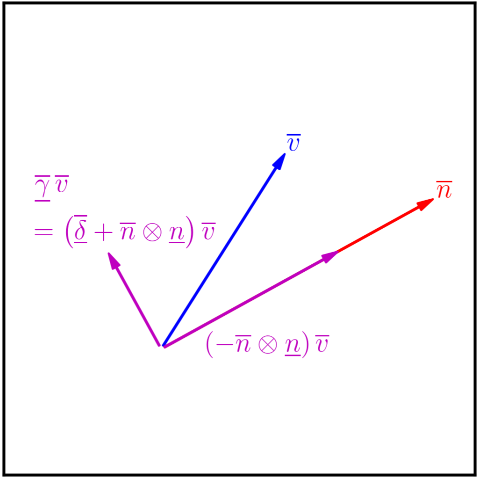
Five equations with six unknowns ☹

Close with an equation of state: $p = p\,(e) := (\Gamma - 1)\,e$, $\Gamma = 4/3$

Valencia Decomposition



Extensible to higher-rank tensors!

Valencia Decomposition

$$E := n_{\mu'} \, n_{\nu'} \, T^{\mu'\nu'}$$

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Math...

Valencia Decomposition

$$\partial_t \boldsymbol{U} + \frac{1}{\sqrt{\gamma}}\,\partial_i\left[\alpha\sqrt{\gamma}\,\boldsymbol{F}^i\left(\boldsymbol{U}\right)\right] = \boldsymbol{S}\left(\boldsymbol{U}\right)$$

GR

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NR

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$\boldsymbol{U} := \left(D_{\mathrm{NR}}, S_j^{\mathrm{NR}}, \tau_{\mathrm{NR}}\right)^\top$

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$$\partial_t \boldsymbol{U} + \frac{1}{\sqrt{\gamma}} \partial_i \left[\alpha \sqrt{\gamma} \boldsymbol{F}^i (\boldsymbol{U}) \right] = \boldsymbol{S} (\boldsymbol{U})$$

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GR

$$\boldsymbol{F}^i\left(\boldsymbol{U}\right) = \begin{pmatrix} \rho\,W\,v^i \\ \rho\,h\,W^2\,v^i\,v_j + p\,\delta^i{}_j \\ \left(\rho\,h\,W^2 - \rho\,W\right)\,v^i \end{pmatrix}$$

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$$\boldsymbol{S} \left(\boldsymbol{U} \right) = \begin{pmatrix} 0 \\ \frac{1}{2} \, \alpha \, P^{ik} \, \partial_j \, \gamma_{ik} - E \, \partial_j \, \alpha \\ -S^j \, \partial_j \, \alpha \end{pmatrix}$$

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$$\Phi \left(r \right) := -M/r$$

Steady-State Solutions (pre-shock)

$$\cancel{\partial_t \boldsymbol{U}}^{\,0} + \frac{1}{\sqrt{\gamma}} \, \partial_r \left[\alpha \sqrt{\gamma} \, \boldsymbol{F}^r \left(\boldsymbol{U} \right) \right] = \boldsymbol{S} \left(\boldsymbol{U} \right)$$

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$\alpha\,\psi^6\,W \times 4\pi\,r^2\,\rho\,v = -\dot{M}$

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$\alpha \, \psi^6 \, W \times 4\pi \, r^2 \, \rho \, v = -\dot{M}$

$\alpha \, h \, W = \mathcal{B}$

$p = K_1 \, \rho^\Gamma$

Steady-State Solutions (pre-shock)

$$\cancel{\partial_t \boldsymbol{U}}^{\,0} + \frac{1}{\sqrt{\gamma}}\,\partial_r\left[\alpha\,\sqrt{\gamma}\,\boldsymbol{F}^r\left(\boldsymbol{U}\right)\right] = \boldsymbol{S}\left(\boldsymbol{U}\right)$$

GR

$$\alpha\,\psi^6\,W \times 4\pi\,r^2\,\rho\,v = -\dot{M}$$

$$\alpha\,h\,W = \mathcal{B}$$

$$p = K_1\,\rho^\Gamma$$

NR

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$$4\pi\,r^2\,\rho\,v = -\dot{M}$$

$$\frac{1}{2}\,v^2 + h_{\rm NR} + \Phi = \mathcal{B}_{\rm NR}$$

Steady-State Solutions (pre-shock)

$$\cancelto{0}{\partial_t \boldsymbol{U}} + \frac{1}{\sqrt{\gamma}} \, \partial_r \left[\alpha \, \sqrt{\gamma} \, \boldsymbol{F}^r \left(\boldsymbol{U} \right) \right] = \boldsymbol{S} \left(\boldsymbol{U} \right)$$

GR

$\alpha \, \psi^6 \, W \times 4\pi \, r^2 \, \rho \, v = -\dot{M}$

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NR

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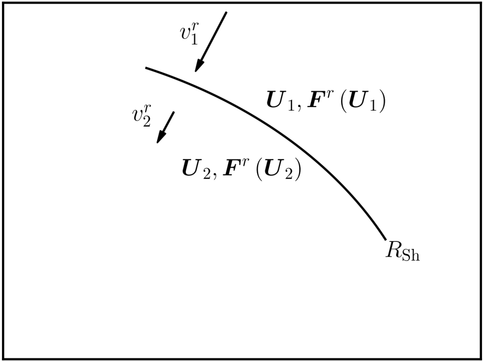
$\frac{1}{2} \, v^2 + h_{\mathrm{NR}} + \Phi = \mathcal{B}_{\mathrm{NR}}$

$p = K_1 \, \rho^\Gamma$

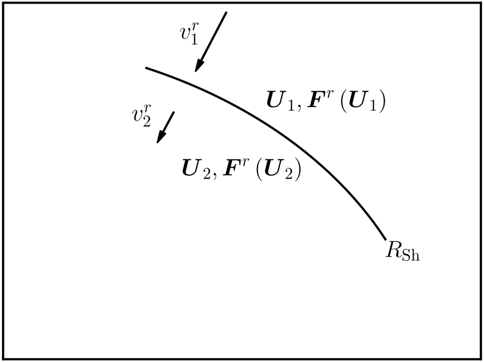
Jump Conditions

$\boldsymbol{U}_1 \neq \boldsymbol{U}_2$



Jump Conditions

$$\boldsymbol{U}_1 \neq \boldsymbol{U}_2$$
$$\boldsymbol{F}^r\left(\boldsymbol{U}_1\right) = \boldsymbol{F}^r\left(\boldsymbol{U}_2\right)$$



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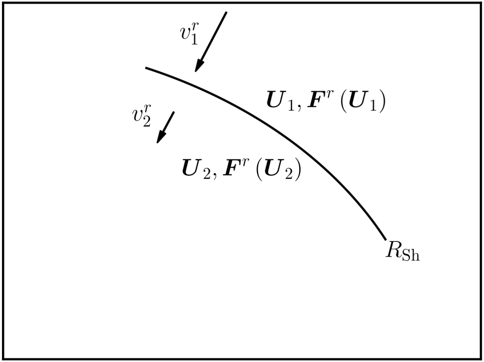
Yields:



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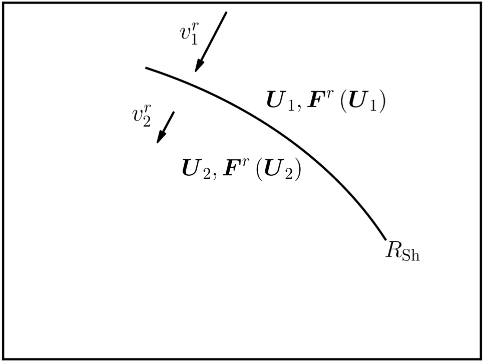
Yields:

$\rho_2 > \rho_1$



Jump Conditions

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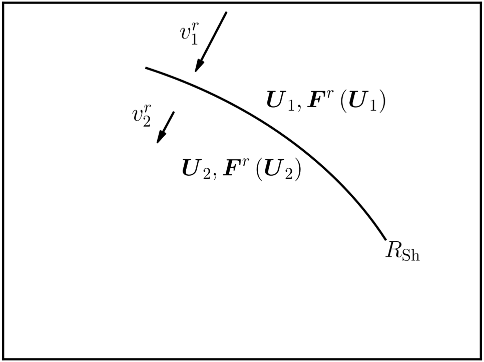
Yields:

$\rho_2 > \rho_1$

$e_2\left(p_2\right) > e_1\left(p_1\right)$



Jump Conditions

$$\boldsymbol{U}_1 \neq \boldsymbol{U}_2$$

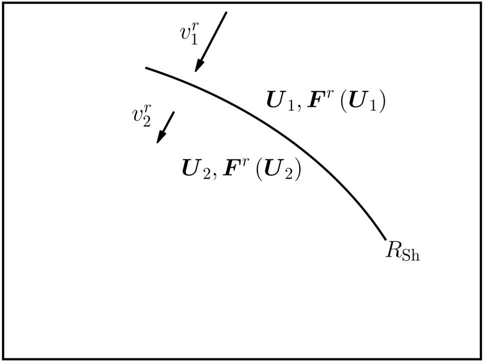
$$\boldsymbol{F}^r(\boldsymbol{U}_1) = \boldsymbol{F}^r(\boldsymbol{U}_2)$$

Yields:

$$\rho_2 > \rho_1$$

$$e_2(p_2) > e_1(p_1)$$

$$K_2 > K_1$$



Jump Conditions

$\boldsymbol{U}_1 \neq \boldsymbol{U}_2$

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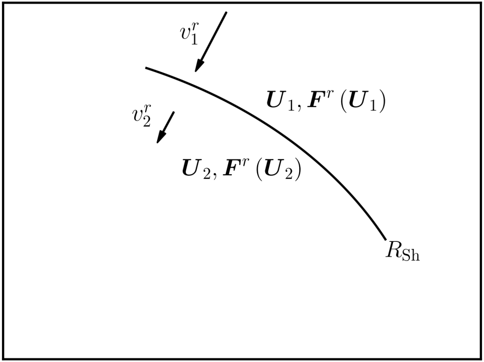
Yields:

$\rho_2 > \rho_1$

$e_2(p_2) > e_1(p_1)$

$K_2 > K_1$

$|v_2^r| < |v_1^r|$



Steady-State Solutions (post-shock)

$$\cancelto{0}{\partial_t \boldsymbol{U}} + \frac{1}{\sqrt{\gamma}}\,\partial_r\left[\alpha\,\sqrt{\gamma}\,\boldsymbol{F}^r\left(\boldsymbol{U}\right)\right] = \boldsymbol{S}\left(\boldsymbol{U}\right)$$

GR

$\alpha\,\psi^6\,W \times 4\pi\,r^2\,\rho\,v = -\dot{M}$

$\alpha\,h\,W = \mathcal{B}$

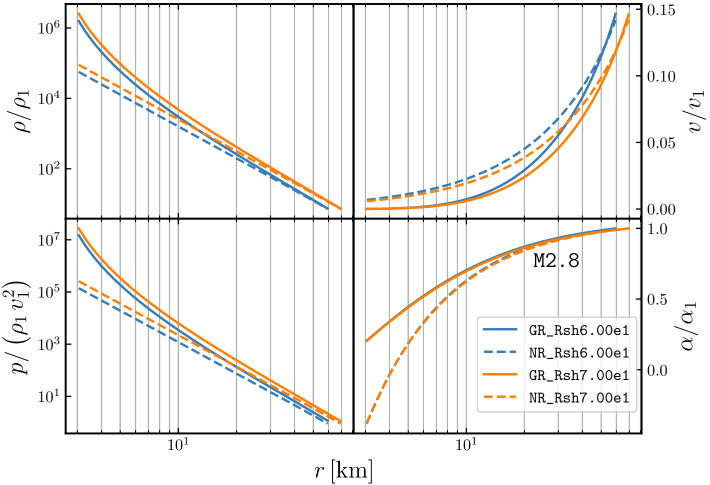
$p = {\color{red}K_2}\,\rho^\Gamma$

NR

$4\pi\,r^2\,\rho\,v = -\dot{M}$

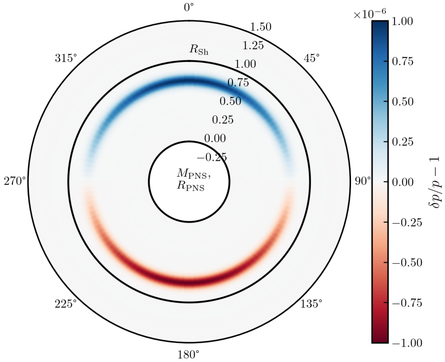
$\frac{1}{2}\,v^2 + h_{\mathrm{NR}} + \Phi = \mathcal{B}_{\mathrm{NR}}$

$p = {\color{red}K_2}\,\rho^\Gamma$



10^6
10^4
10^2
ρ/ρ_1
0.15
0.10
0.05
0.00
v/v_1
10^7
10^5
10^3
10^1
$p/\left(\rho_1\, v_1^2\right)$
1.0
0.5
0.0
α/α_1
M2.8
GR_Rsh6.00e1
NR_Rsh6.00e1
GR_Rsh7.00e1
NR_Rsh7.00e1
10^1
10^1
r [km]

$$\eta(r) := \frac{r - R_{\mathrm{PNS}}}{R_{\mathrm{Sh}} - R_{\mathrm{PNS}}}$$

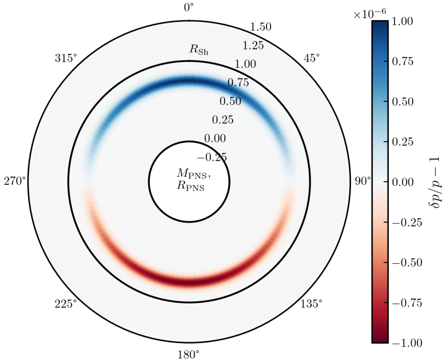


$$\eta\left(r\right) := \frac{r-R_{\mathrm{PNS}}}{R_{\mathrm{Sh}}-R_{\mathrm{PNS}}}$$

$$\frac{\delta p(\eta,\theta)}{p(\eta_{\mathrm{c}})} =$$
$$10^{-6} \times \exp\left[\frac{-(\eta-\eta_{\mathrm{c}})^2}{2\,\sigma^2}\right]\cos\theta$$

$$\eta_{\mathrm{c}} = 0.75$$

$$\sigma = 0.05$$



Parameter Space

Model parameters:
M_{PNS}, R_{PNS},
$R_{\mathrm{Sh}}\left(t=0\right)$, $\dot{M}$, K_1,
$\mathcal{B}_{(\mathrm{NR})}$, Γ

Parameter Space

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M_{PNS}, R_{PNS},
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Parameters we varied:
$\xi = \left(M_{\mathrm{PNS}}/M_{\odot}\right) / \left(R_{\mathrm{PNS}}/20\,\mathrm{km}\right)$ **(?)**, $R_{\mathrm{Sh}}\left(t=0\right)$

Parameter Space

Model parameters:

M_{PNS}, R_{PNS}, $R_{\mathrm{Sh}}\,(t=0)$, $\dot{M}$, K_1, $\mathcal{B}_{(\mathrm{NR})}$, Γ

Parameters we varied:

$\xi = (M_{\mathrm{PNS}}/M_{\odot})\,/\,(R_{\mathrm{PNS}}/20\,\mathrm{km})$ **(?)**, $R_{\mathrm{Sh}}\,(t=0)$

Table 1. Model Parameters

Model	M_{PNS} [$M_{\odot}$]	R_{PNS} [km]	R_{sh} [km]	ξ
M1.4_Rpns040_Rsh1.20e2	1.4	40	120	0.7
M1.4_Rpns040_Rsh1.50e2	1.4	40	150	0.7
M1.4_Rpns040_Rsh1.75e2	1.4	40	175	0.7
M2.8_Rpns020_Rsh6.00e1	2.8	20	60	2.8
M2.8_Rpns020_Rsh7.00e1	2.8	20	70	2.8

Note—Model parameters chosen for the 5 models. All models were run with both GR and NR. The first three rows correspond to the low-compactness models and the last two rows correspond to the high-compactness models.

$A := \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(v^{\theta}\,\sin\theta\right)$ (**?**) `<movie>`

$$A := \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(v^{\theta}\,\sin\theta\right)$$ **(?)** `<movie>`

$$A\left(r,\theta,t\right) = \sum_{\ell'=0}^{\infty} G_{\ell'}\left(r,t\right)P_{\ell'}\left(\cos\theta\right)$$

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$$A\left(r,\theta,t\right) = \sum_{\ell'=0}^{\infty} G_{\ell'}\left(r,t\right) P_{\ell'}\left(\cos\theta\right)$$

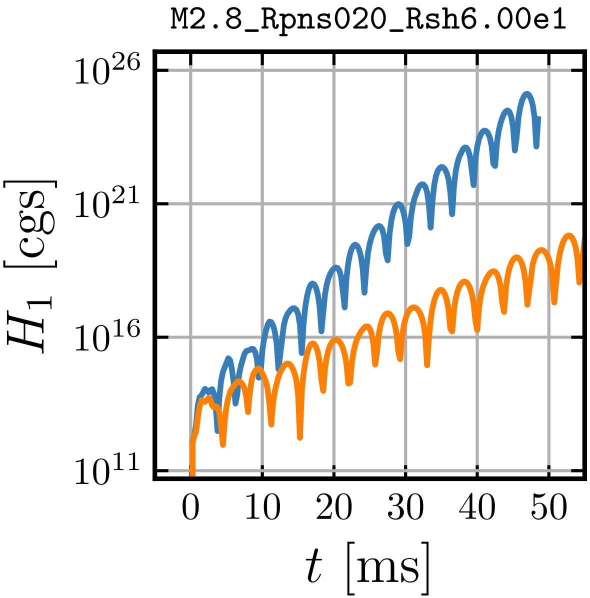
$$\implies G_{\ell}\left(r,t\right) := \frac{1}{N_{\ell}} \int\limits_{0}^{\pi} A\left(r,\theta,t\right)\, P_{\ell}\left(\cos\theta\right)\,\sin\theta\,d\theta$$

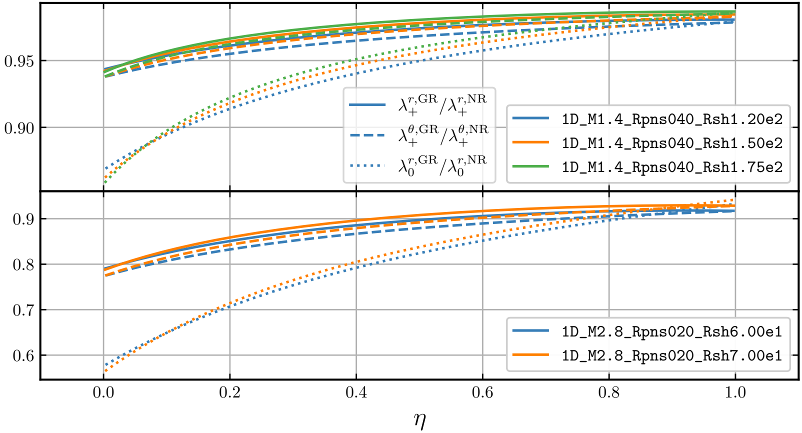
$A := \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(v^{\theta}\,\sin\theta\right)$ (?) <movie>

$$A\left(r,\theta,t\right) = \sum_{\ell'=0}^{\infty} G_{\ell'}\left(r,t\right) P_{\ell'}\left(\cos\theta\right)$$

$$\implies G_{\ell}\left(r,t\right) := \frac{1}{N_{\ell}}\int_{0}^{\pi} A\left(r,\theta,t\right)\,P_{\ell}\left(\cos\theta\right)\,\sin\theta\,d\theta$$

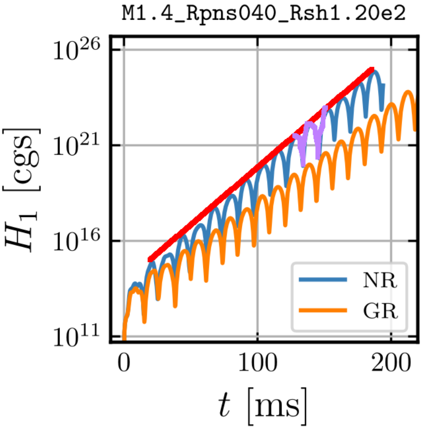
$H_{\ell}\left(t\right) := 4\pi\int_{r_a}^{r_b}\left[G_{\ell}\left(r,t\right)\right]^{2}\left[\psi\left(r\right)\right]^{6} r^{2}\,dr$ <movie>


M2.8_Rpns020_Rsh6.00e1
10^{26}
10^{21}
10^{16}
10^{11}
H_1 [cgs]
0
10
20
30
40
50
t [ms]


0.95
0.90
$\lambda_+^{r,\mathrm{GR}}/\lambda_+^{r,\mathrm{NR}}$
$\lambda_+^{\theta,\mathrm{GR}}/\lambda_+^{\theta,\mathrm{NR}}$
$\lambda_0^{r,\mathrm{GR}}/\lambda_0^{r,\mathrm{NR}}$
1D_M1.4_Rpns040_Rsh1.20e2
1D_M1.4_Rpns040_Rsh1.50e2
1D_M1.4_Rpns040_Rsh1.75e2
0.9
0.8
0.7
0.6
1D_M2.8_Rpns020_Rsh6.00e1
1D_M2.8_Rpns020_Rsh7.00e1
0.0
0.2
0.4
0.6
0.8
1.0
η

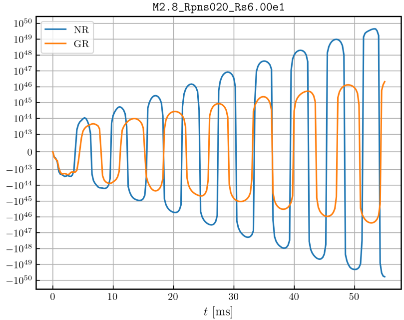
$$F(t) = F(0)\, e^{2\,\omega\, t}\, \sin^2\left(\frac{2\pi\, t}{T} + \delta\right)$$

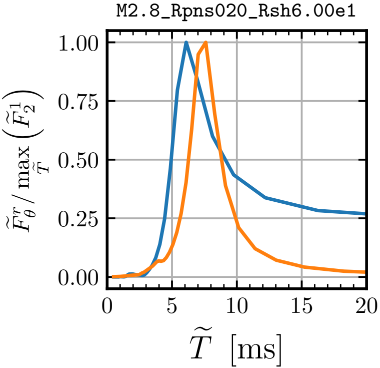
(?)



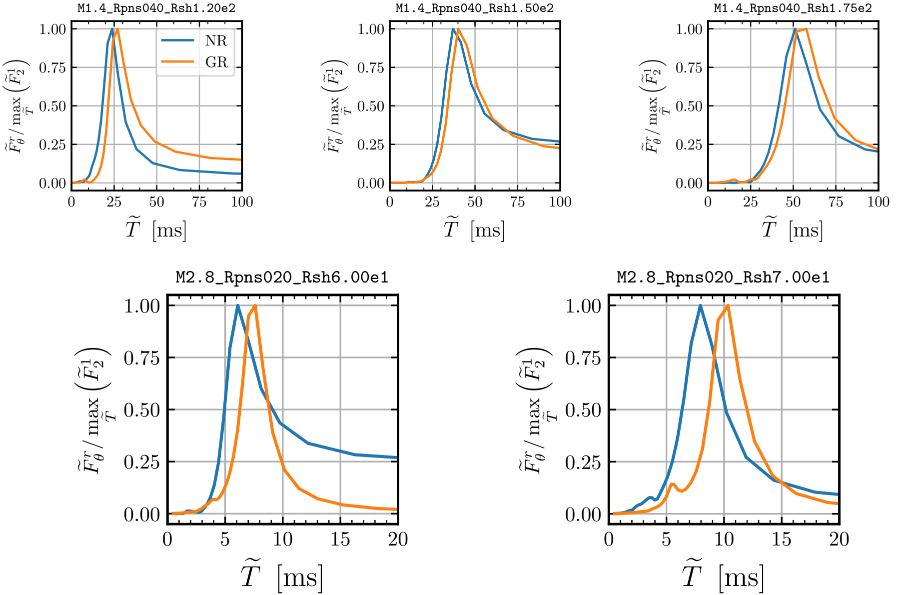
$$F^r_\theta := \alpha\,\psi^6\,h\,W^2 \times \sqrt{\bar{\gamma}}\,\rho\,v^r\,v_\theta$$

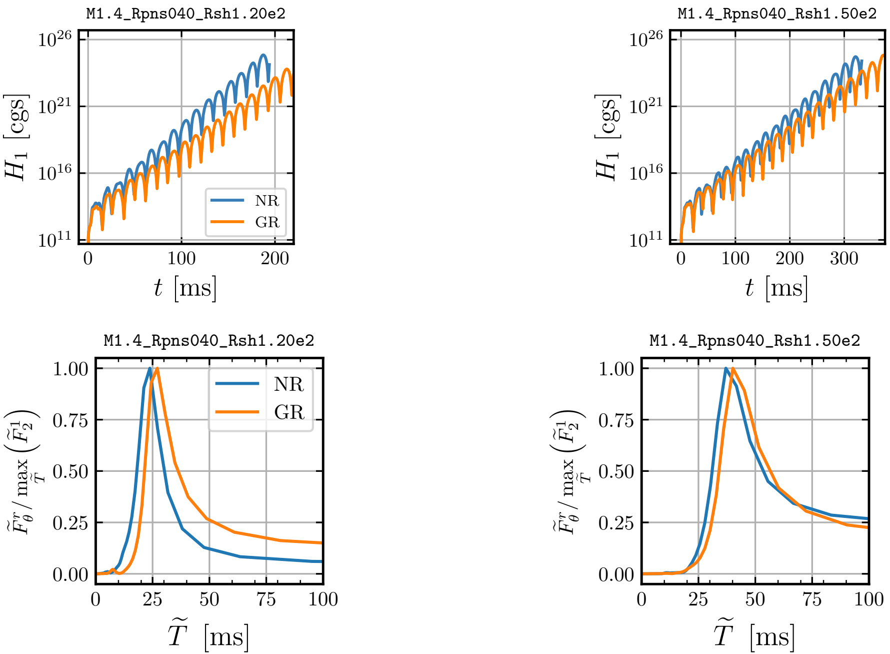
$$\widetilde{F}^r_\theta = \mathrm{FFT}\left\{F^r_\theta\right\}$$

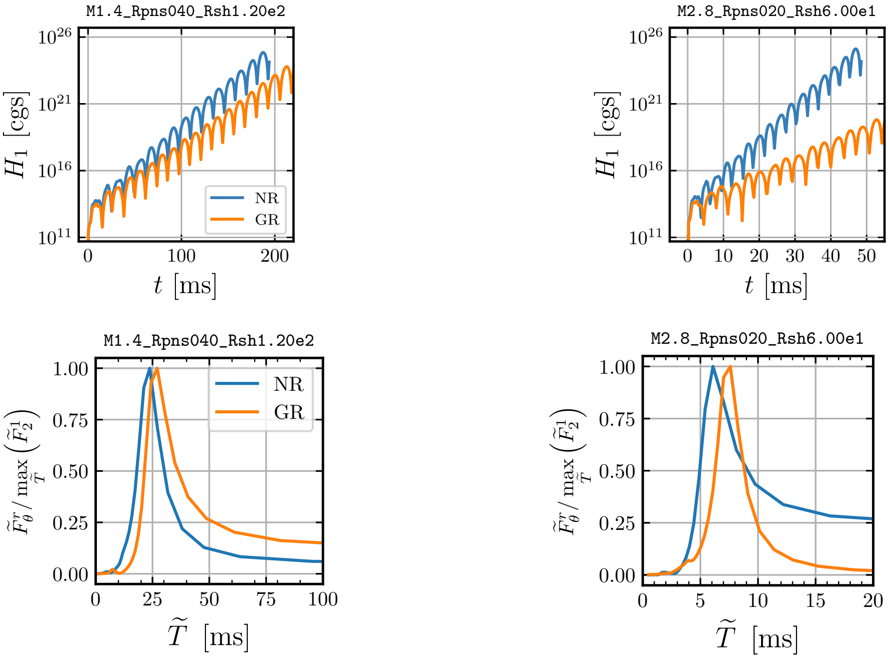


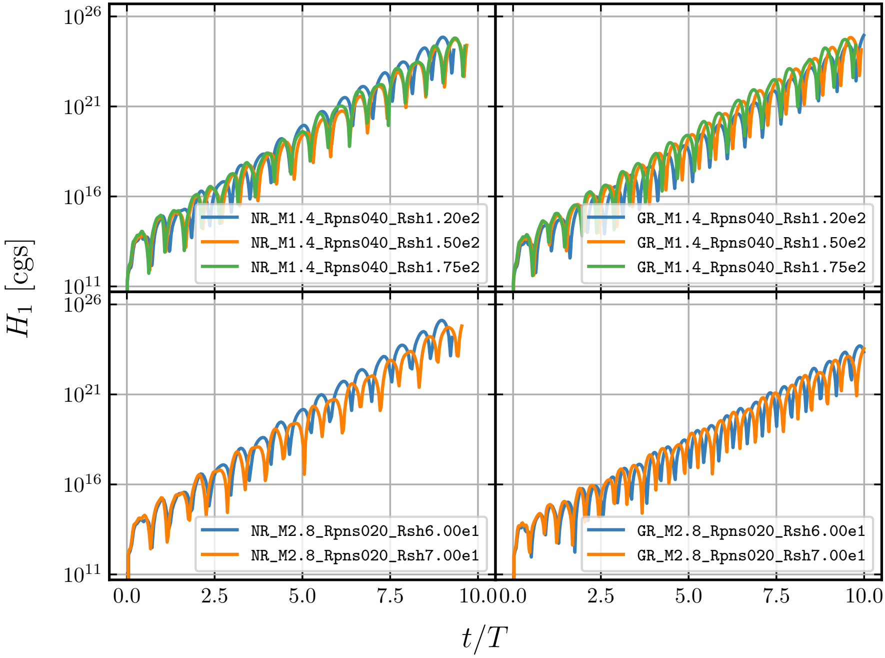


T defined as the unique $\widetilde{T}$ such that $\widetilde{F}^r_\theta\left(\widetilde{T}\right) = 1$

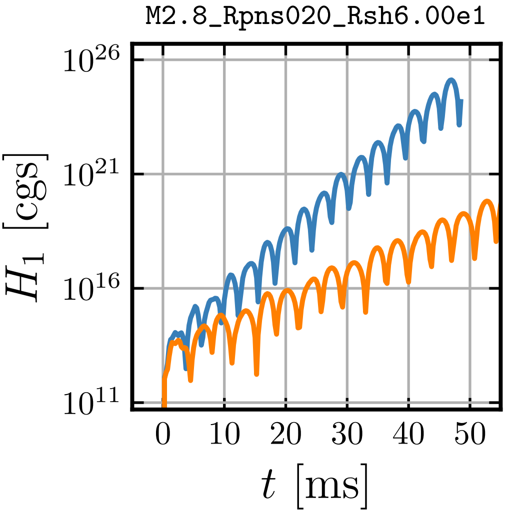

M1.4_Rpns040_Rsh1.20e2
M1.4_Rpns040_Rsh1.50e2
M1.4_Rpns040_Rsh1.75e2
M2.8_Rpns020_Rsh6.00e1
M2.8_Rpns020_Rsh7.00e1
NR
GR
$\widetilde{F}^r_\theta / \max_{\widetilde{T}} \left(\widetilde{F}^1_2\right)$
$\widetilde{T}$ [ms]


M1.4_Rpns040_Rsh1.20e2
M1.4_Rpns040_Rsh1.50e2
H_1 [cgs]
10^{11}
10^{16}
10^{21}
10^{26}
t [ms]
0
100
200
300
NR
GR
$\widetilde{F}^r_\theta \,/\, \max_{\widetilde{T}} \left(\widetilde{F}^1_2\right)$
0.00
0.25
0.50
0.75
1.00
$\widetilde{T}$ [ms]
0
25
50
75
100

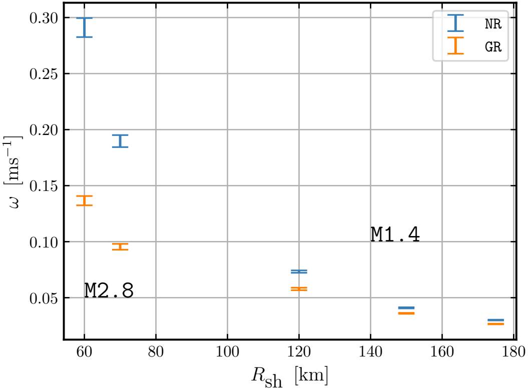

M1.4_Rpns040_Rsh1.20e2
H_1 [cgs]
t [ms]
NR
GR
M2.8_Rpns020_Rsh6.00e1
H_1 [cgs]
t [ms]
M1.4_Rpns040_Rsh1.20e2
$\widetilde{F}^r_\theta \,/\, \max_{\widetilde{T}} \left(\widetilde{F}^1_2\right)$
$\widetilde{T}$ [ms]
NR
GR
M2.8_Rpns020_Rsh6.00e1
$\widetilde{F}^r_\theta \,/\, \max_{\widetilde{T}} \left(\widetilde{F}^1_2\right)$
$\widetilde{T}$ [ms]


10^{26}
10^{21}
10^{16}
10^{11}
H_1 [cgs]
NR_M1.4_Rpns040_Rsh1.20e2
NR_M1.4_Rpns040_Rsh1.50e2
NR_M1.4_Rpns040_Rsh1.75e2
GR_M1.4_Rpns040_Rsh1.20e2
GR_M1.4_Rpns040_Rsh1.50e2
GR_M1.4_Rpns040_Rsh1.75e2
NR_M2.8_Rpns020_Rsh6.00e1
NR_M2.8_Rpns020_Rsh7.00e1
GR_M2.8_Rpns020_Rsh6.00e1
GR_M2.8_Rpns020_Rsh7.00e1
0.0
2.5
5.0
7.5
10.0
t/T

Conclusions



Conclusions



Bibliography

Summary

- Extended study of **?** to include GR
- Showed that GR leads to longer SASI oscillation period than NR
- Showed that GR leads to smaller SASI growth rate than NR
- Found that growth rate is such that ωT is roughly constant for some parameter sets: implications for growth rate mechanism
- Future Work
  - Extend study to 3D
  - Include GR monopole (**?**)