

thornado-Hydro (xCFC)

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thornado

<https://github.com/endeve/thornado>



My Website

<https://www.samueljdunham.com>





toolkit for high-order neutrino-radiation hydrodynamics

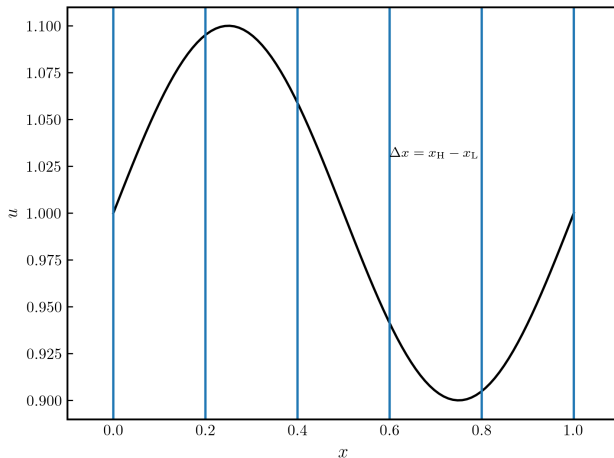
- DG
- SSPRK/IMEX
- GR (xCFC)
- Hydro^a (Valencia)
- Neutrino transport^b (M1)
- Interfaces to tabulated EoS/Opacities (weaklib: <https://github.com/starkiller-astro/weaklib>)
- Fluid self-gravity via Poseidon: <https://github.com/jrober50/Poseidon>
- GPUs via OpenACC or OpenMP pragmas
- MPI parallelism and AMR via AMReX: <https://github.com/AMReX-Codes/amrex>

^aEndeve et al. (2019); Dunham et al. (2020); Pochik et al. (2021)

^bLaiu et al. (2021)

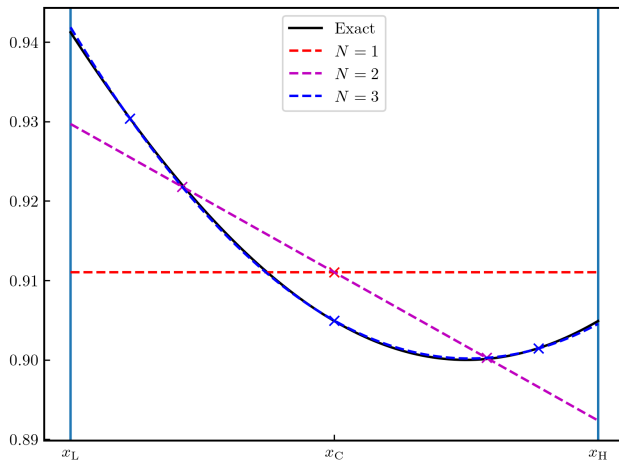
Discontinuous Galerkin (DG) Method

$$u(x) = 1 + 0.1 \sin(2\pi x)$$

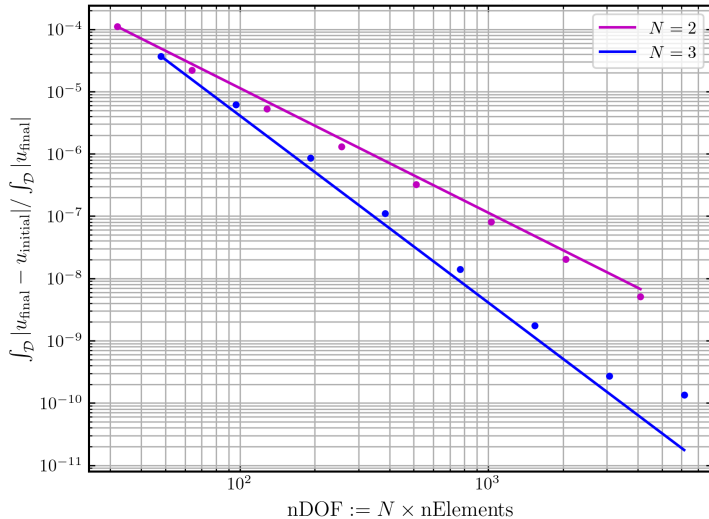


Discontinuous Galerkin (DG) Method

$$u_h(x, t) := \sum_{i=1}^N u_i(t) \ell_i(x)$$

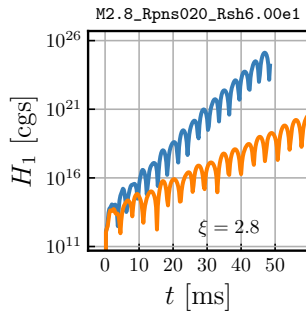
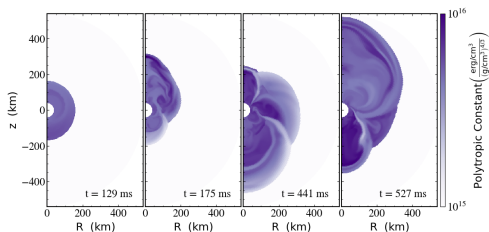


Convergence Rates for Sine Wave Advection (1D)



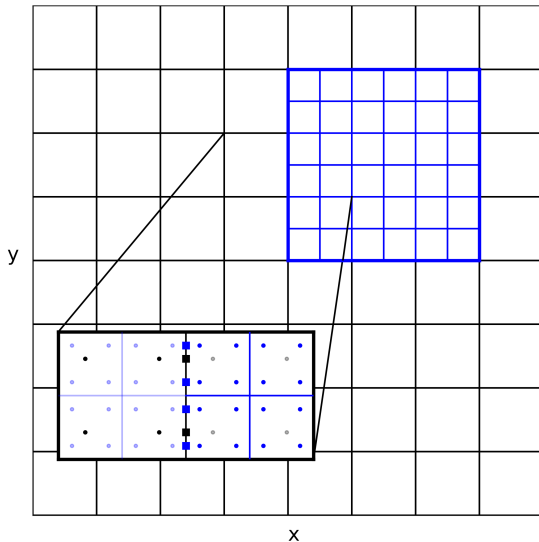
Standing Accretion Shock Instability

Used thornado to investigate the role of GR on the SASI¹

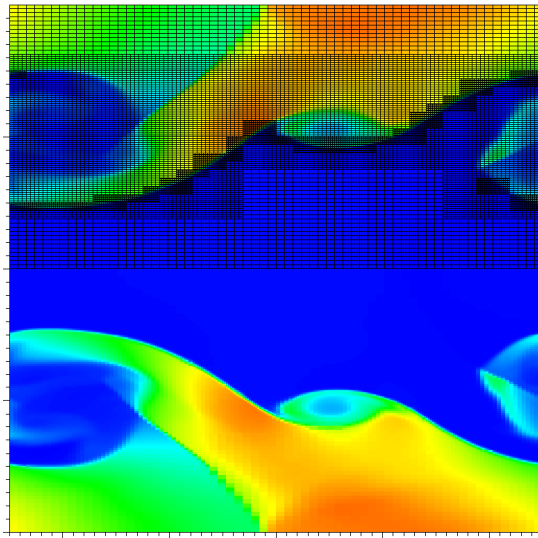


¹Dunham et al. (2020, 2023)

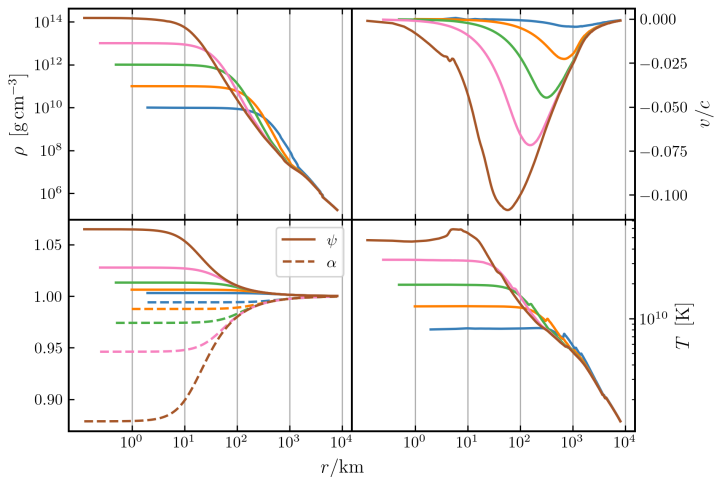
Mesh Refinement



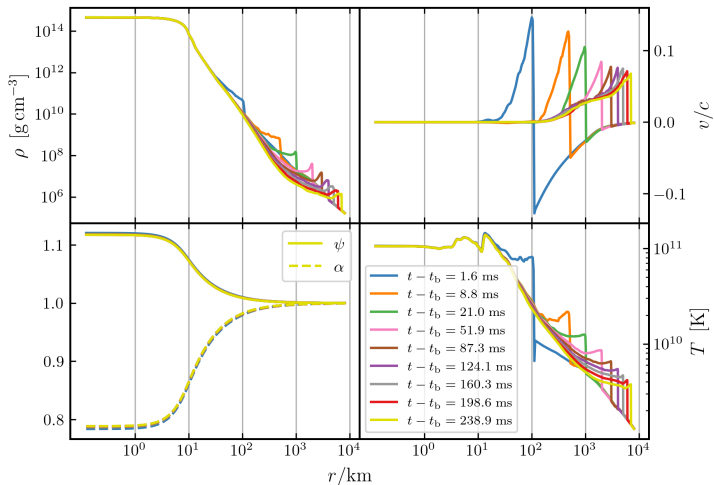
Kelvin–Helmholtz Instability



Adiabatic Collapse (AMR, Collapse Phase)



Adiabatic Collapse (AMR, Post-Bounce Phase)

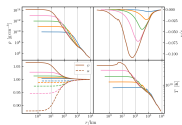
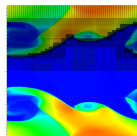


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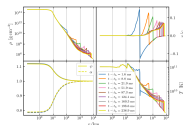
Summary

Can run pure hydro problems in GR with AMR



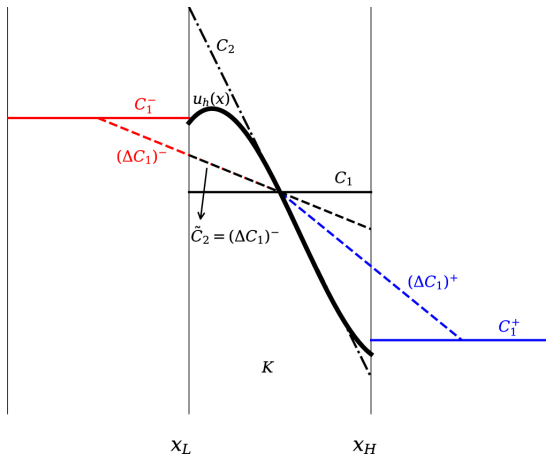
Can run hydro+self-gravity problems in GR with AMR

Working on coupling GR transport to existing hydro+gravity modules

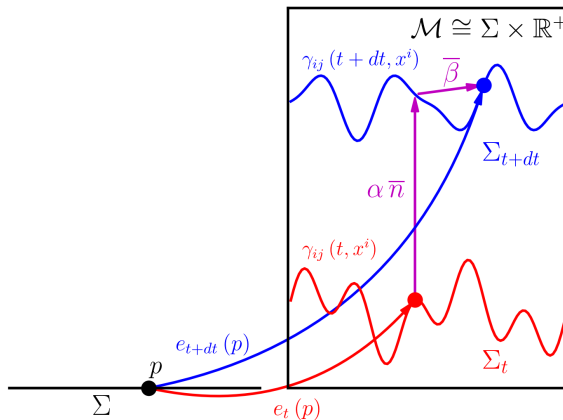


Slope Limiter

$$u_h(x, t) = \sum_{n=1}^N C_n(t) P_n(x) \implies \tilde{u}_h(x, t) = C_1(t) P_1(x) + \tilde{C}_2(t) P_2(x)$$



3+1 Decomposition



$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -\alpha^2 dt^2 + \gamma_{ij} (dx^i + \beta^i dt) (dx^j + \beta^j dt)$$

Conformally-Flat Condition

Developed by Wilson et al. (1996),
extended by Cordero-Carrión et al.
(2009)

Special case: Schwarzschild spacetime
in isotropic coordinates ($G = c = 1$)

$$\gamma_{ij}(x) = \psi^4(x) \bar{\gamma}_{ij}(x^i)$$

$$K = 0, \quad \partial_t K = 0$$

(Always and everywhere)

$$\alpha = \left(1 + \frac{1}{2}\Phi\right) \left(1 - \frac{1}{2}\Phi\right)^{-1}$$

$$\psi = 1 - \frac{1}{2}\Phi$$

$$\beta^i = 0,$$

- Exact in spherical symmetry!
- Hyperbolic \rightarrow Elliptic equations
- Good for long-time simulations

with

$$\Phi(r) := -\frac{M}{r}$$